

# 3 Phase Inverter MATLAB Simulation

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## Introduction

In this MATLAB Simulation, we simulate a three-phase inverter using the mathematical model discussed during class. In this report, we show that the three-phase inverter can be mathematically modelled wherein the line-to-line voltages can be represented by the switching function discussed in class. We also discuss the effects of overmodulation and how the third harmonic injection can be used to keep the inverter well within linear mode. We also observe the effects of variable voltage, variable frequency, as well as a quick look on the Fourier Transform to check for harmonics.

## Code

We start by asking the user for simulation options: inject 3<sup>rd</sup> harmonic component, do a frequency sweep, and/or do amplitude sweep for the input voltage. Setting the 3<sup>rd</sup> injection off, causes the 3<sup>rd</sup> harmonic to be multiplied to 0 (line 88) else will be multiplied to 1. Setting frequency sweep causes the pole voltages to have increasing frequency as the simulation runs. Similar setting applies to the amplitude sweep.

```
5      %% Ask User for Simulation Options
6
7      prompt_3rd = 'Inject 3rd Harmonic Component? [Yes: 1, No: 0]: ';
8      cond_3rd = input(prompt_3rd,"s");
9      if cond_3rd == "1"
10         third_harmonic_injection = 1;
11     else
12         third_harmonic_injection = 0;
13     end
14
15     prompt_f = 'Frequency Sweep? [Yes: 1, No: 0]: ';
16     cond_f = input(prompt_f, "s");
17     if cond_f == "1"
18         frequency_sweep = 1;
19     else
20         frequency_sweep = 0; |
21     end
22
23     prompt_a = 'Amplitude Sweep? [Yes: 1, No: 0]: ';
24     cond_a = input(prompt_a,"s");
25     if cond_a == "1"
26         amplitude_sweep = 1;
27         amp_init = 220; % starting amplitude of sweep (RMS)
28         amp_fin = 720; % final amplitude of sweep (RMS)
29     else
30         amplitude_sweep = 0;
31     end
32
```

<Figure 1: Simulation Options>

The following section finishes the setup process by declaring the DC bus voltage, frequencies, carrier, and reference waveforms.

```

33 %% Simulation Setup
34 % DC comes from a three-phase DBR connected to 3 ph Voltage Source;
35 % 220 Vrms
36 Vdc = sqrt(2)*sqrt(3)*220;
37 Vdc_2= Vdc/2;
38
39 % variable voltage and variable frequency...
40 % Frequency command
41 f=60;
42 % Carrier frequency
43 fs = 3e3;
44 % This is for plotting.
45 Fs = 1e6;
46 dt = 1/Fs;
47
48 % Number of cycles and time duration
49 if frequency_sweep== 0
50     no=4;
51 else
52     no = 6;
53 end
54
55 T = no*1/f;
56 t = 0:dt:T-dt;
57
58
59
60
61 Vref = 380;
62
63
64 if amplitude_sweep == 0
65     Vm = Vref/sqrt(3)*sqrt(2);
66 else
67     sweep_start_amp = amp_init/sqrt(3)*sqrt(2);
68     sweep_end_amp = amp_fin/sqrt(3)*sqrt(2);
69     Vm = linspace(sweep_start_amp,sweep_end_amp,length(t));
70 end
71
72 % To control your voltage output
73 ma = Vm/Vdc_2;
74
75 % Phase voltage magnitude(Pole Voltage)
76 if frequency_sweep == 0
77     Van= Vm.*sin(2*pi*f*t);
78     Vbn= Vm.*sin(2*pi*f*t-120/360*2*pi);
79     Vcn= Vm.*sin(2*pi*f*t+120/360*2*pi);
80 else
81     Van= Vm.*chirp(t,60,0.03,120,"linear",-90);
82     Vbn= Vm.*chirp(t,60,0.03,120,"linear",-90-120);
83     Vcn= Vm.*chirp(t,60,0.03,120,"linear",-90+120);
84 end
85
86 % To inject a 3rd harmonic component
87 V_3rd = -Vm/6.*sin(2*pi*f*3*t);
88 V_3rd = third_harmonic_injection*V_3rd;
89
90 %line to line voltage
91 Vab = Van-Vbn;
92 Vbc = Vbn-Vcn;
93 Vca = Vcn-Van;
94
95 % triangular waveform (carrier waveform)
96 Vtri = sawtooth(2*pi*fs*t,1/2)*Vdc_2;
97
98 % Reference Voltages used in the modulation
99 Va_cmd = Van-V_3rd;
100 Vb_cmd = Vbn-V_3rd;
101 Vc_cmd = Vcn-V_3rd;

```

<Figure 2: Simulation Setup>

For the modulation, the switching action is similar with the single-phase inverter: when the reference signal's magnitude is higher than the carrier, the switch on that pole is on ( $S = 1$ ) else, is off ( $S = 0$ ).

```

133 for i = 1:length(t)
134     if Va_cmd(i) >= Vtri(i)
135         Sa = 1; % switch is on, same goes for other phases
136     else
137         Sa = 0; % switch is off, same goes for other phases
138     end
139
140     if Vb_cmd(i) >= Vtri(i)
141         Sb = 1;
142     else
143         Sb = 0;
144     end
145
146     if Vc_cmd(i) >= Vtri(i)
147         Sc = 1;
148     else
149         Sc = 0;
150     end
151
152
153     Van_pwm(i) = Vdc/2*(2*Sa-1);
154     Vbn_pwm(i) = Vdc/2*(2*Sb-1);
155     Vcn_pwm(i) = Vdc/2*(2*Sc-1);
156
157     % phase voltages can be represented in terms of switching functions
158     % and pole voltage
159     Vas(i) = Vdc/3*(2*Sa-Sb-Sc);
160     Vbs(i) = Vdc/3*(2*Sb-Sc-Sa);
161     Vcs(i) = Vdc/3*(2*Sc-Sa-Sb);
162
163     % Line to line voltage (Vab = Vas- Vbs = Van - Vbn)
164     Vab_pwm(i) = Vas(i) - Vbs(i);
165     Vbc_pwm(i) = Vbs(i) - Vcs(i);
166     Vca_pwm(i) = Vcs(i) - Vas(i);
167
168

```

<Figure 3: Modulation>

Since the output requires a low pass filter to extract the fundamental, we also add the filter in this for loop to avoid adding another loop at the end.

```

168
169
170     % With the output, we take extract the fundamental using a low
171     % pass filter. We use the indicated filter from above.
172
173     % low pass filter
174     if i < 3
175         if i == 1
176             Vabf(i) = Vab_pwm(i);
177             Vbcf(i) = Vbc_pwm(i);
178             Vcaf(i) = Vca_pwm(i);
179         else
180             Vabf(2) = Vab_pwm(1);
181             Vbcf(2) = Vbc_pwm(1);
182             Vcaf(2) = Vca_pwm(1);
183         end
184     else
185         % IIR Filter
186         Vabf(i) = 0.999*Vabf(i-1)+0.001*Vab_pwm(i);
187         Vbcf(i) = 0.999*Vbcf(i-1)+0.001*Vbc_pwm(i);
188         Vcaf(i) = 0.999*Vcaf(i-1)+0.001*Vca_pwm(i);
189     end
190 end

```

<Figure 4: Low Pass Filter for the Line-to-Line Voltages>

Once the modulation and filter for loop is done, we plot the results.

```

191 %% Plot Modulation Results
192
193 figure; sgtitle("Brief Summary of Modulation Results");
194 subplot(3,1,1); plot(t, Van_pwm); grid on;
195 xlabel("time (s)"); ylabel("Van");
196 subplot(3,1,2); plot(t, Vas); grid on;
197 xlabel('time (s)'); ylabel("Vas");
198 subplot(3,1,3); plot(t, Vab_pwm); grid on;
199 xlabel("time (s)"); ylabel("Vab");
200
201 % Pole Voltages
202 figure; sgtitle("Full Modulation Results");
203 subplot(3,3,1); plot(t, Van_pwm); grid on;
204 xlabel("time (s)"); ylabel("Van");
205 subplot(3,3,2); plot(t, Vbn_pwm); grid on;
206 xlabel("time (s)"); ylabel("Vbn");
207 subplot(3,3,3); plot(t, Vcn_pwm); grid on;
208 xlabel("time (s)"); ylabel("Vcn");
209
210
211 % Line to Line PWM
212
213 subplot(3,3,7); plot(t, Vab_pwm); grid on;
214 xlabel("time (s)"); ylabel("Vab");
215 subplot(3,3,8); plot(t, Vbc_pwm); grid on;
216 xlabel("time (s)"); ylabel("Vbc");
217 subplot(3,3,9); plot(t, Vca_pwm); grid on;
218 xlabel("time (s)"); ylabel("Vca");
219
220 % Phase Voltages
221
222 subplot(3,3,4); plot(t, Vas); grid on;
223 xlabel("time (s)"); ylabel("Vas");
224 subplot(3,3,5); plot(t, Vbs); grid on;
225 xlabel("time (s)"); ylabel("Vbs");
226 subplot(3,3,6); plot(t, Vcs); grid on;
227 xlabel("time (s)"); ylabel("Vcs");
228
229

```

<Figure 5: Modulation Results>

```

230 %% Plot PWM Result
231 figure; sgtitle(" Brief PWM Result");
232 plot(t, Vab_pwm, t, Vabf); grid on;
233 xlabel("time(s)");
234 ylabel("Vab and Vab\_LPF");
235
236 figure; sgtitle("PWM Results");
237 subplot(3,1,1); plot(t, Vab_pwm, t, Vabf); grid on;
238 xlabel("time(s)"); ylabel("Vab and Vab\_LPF");
239 subplot(3,1,2); plot(t, Vbc_pwm, t, Vbcf); grid on;
240 xlabel("time(s)"); ylabel("Vbc and Vbc\_LPF");
241 subplot(3,1,3); plot(t, Vca_pwm, t, Vcaf); grid on;
242 xlabel("time(s)"); ylabel("Vca and Vca\_LPF");
243
244 figure; sgtitle("Output of 3\phi-phase Inverter");
245 plot(t, Vabf, t, Vbcf, t, Vcaf); grid on;
246 xlabel("time(s)"); ylabel("Magnitude");
247

```

<Figure 6: PWM Result>

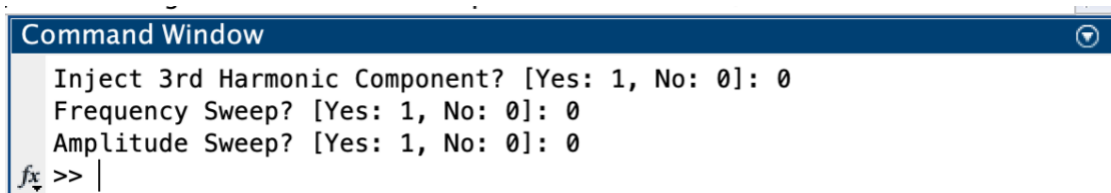
Finally, we setup the code for the harmonics using `fft()`.

```
248 %% Check Harmonics
249 x = Vab_pwm; % check harmonics on Vab line to line voltage
250 % x = Van_pwm;
251
252 f = 60;
253 Fs = 1e6;
254
255 L = length(t);
256 T = 1/Fs;
257 t = (0:L-1)*T;
258
259 Y = fft(x);
260 P2 = abs(Y/L);
261 P1 = P2(1:L/2+1);
262 P1(2:end-1) = 2*P1(2:end-1);
263
264 figure;
265 f = Fs*(0:(L/2))/L;
266 plot(f,P1); grid minor;
267 title("Single-Sided Amplitude Spectrum of Vab\_pwm");
```

<Figure 7: Harmonics>

## Results

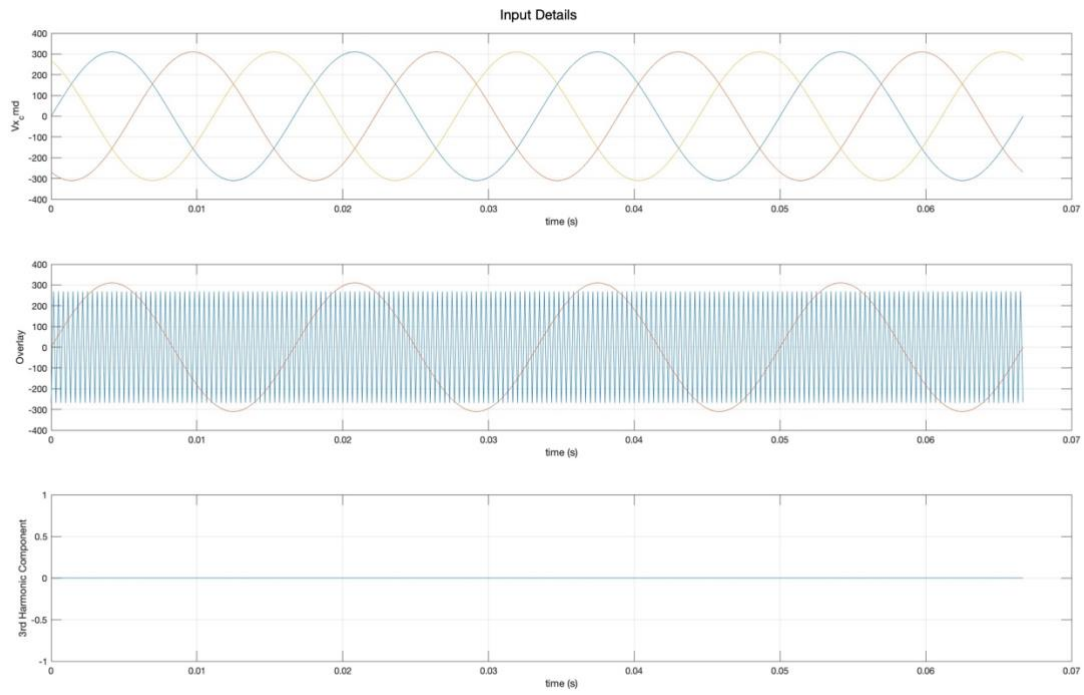
We start with the basic case: no harmonic injection, no sweeps, with desired output of 380 V RMS (line-to-line).



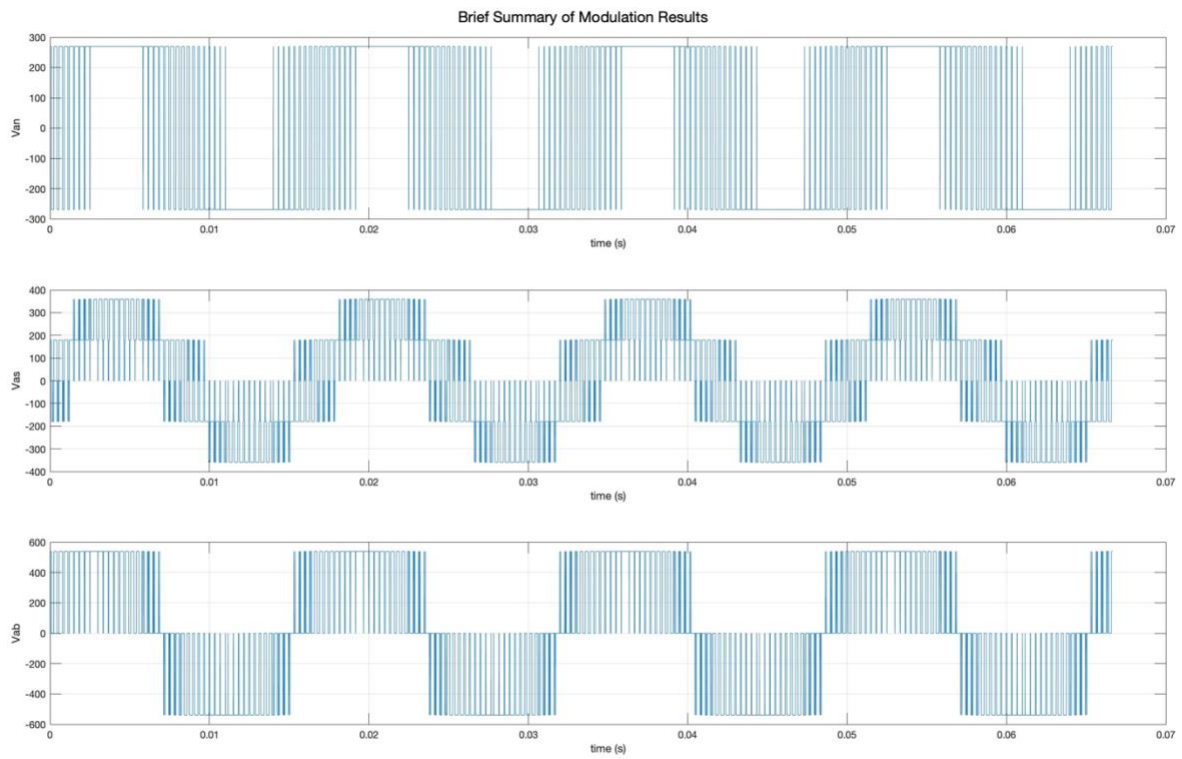
<Figure 8: Command Window for base case>

As shown in figure 9 (next page), we see that the voltage that we want at the output exceeds the magnitude of the carrier waveform, which is the DC bus limit for the inverter. We also see that there is no 3<sup>rd</sup> harmonic component in the waveform.

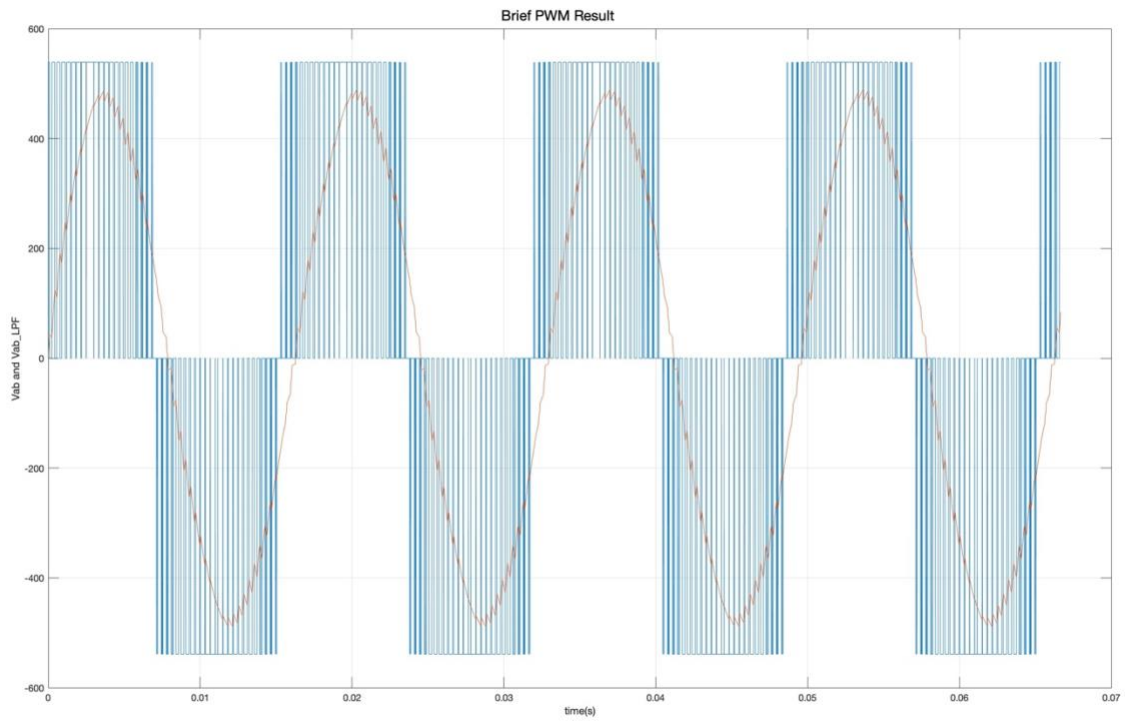
Figure 10 (next page) shows the results of the modulation. As seen in the first graph, the pole voltage with respect to the fictitious node has a segment of time where the switch is kept on (because of overmodulation). The second graph show that the phase voltage indeed has 5 levels as discussed, and the final graph shows the unipolar SPWM resulting waveform.



<Figure 9: Input Details>

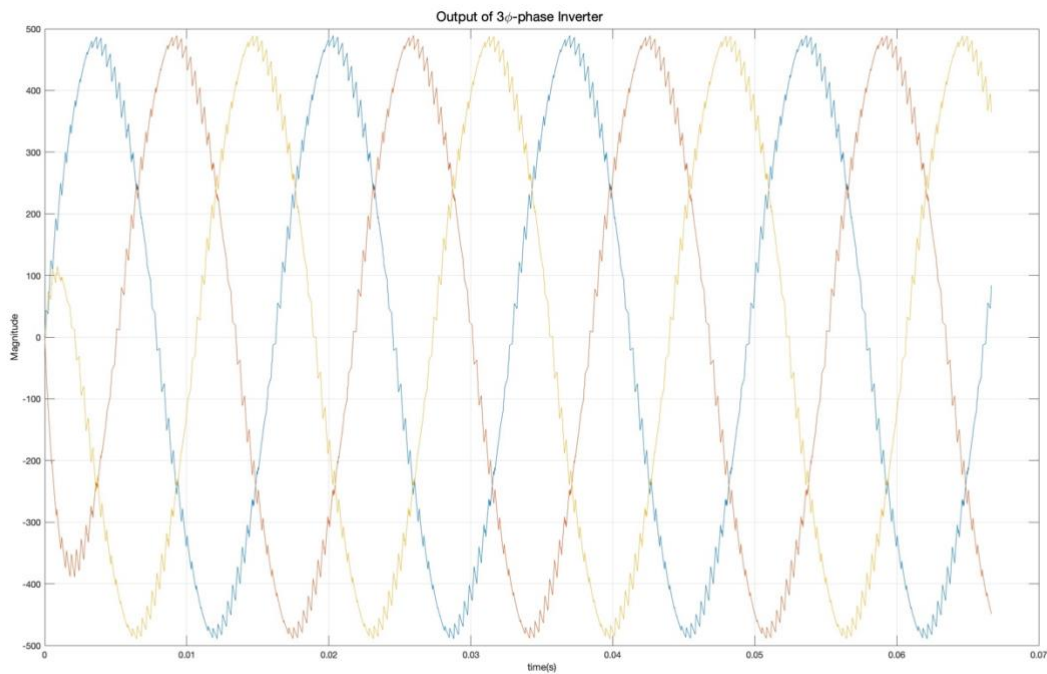


<Figure 10: Modulation Results>



<Figure 11: PWM resulting waveform + filtered result>

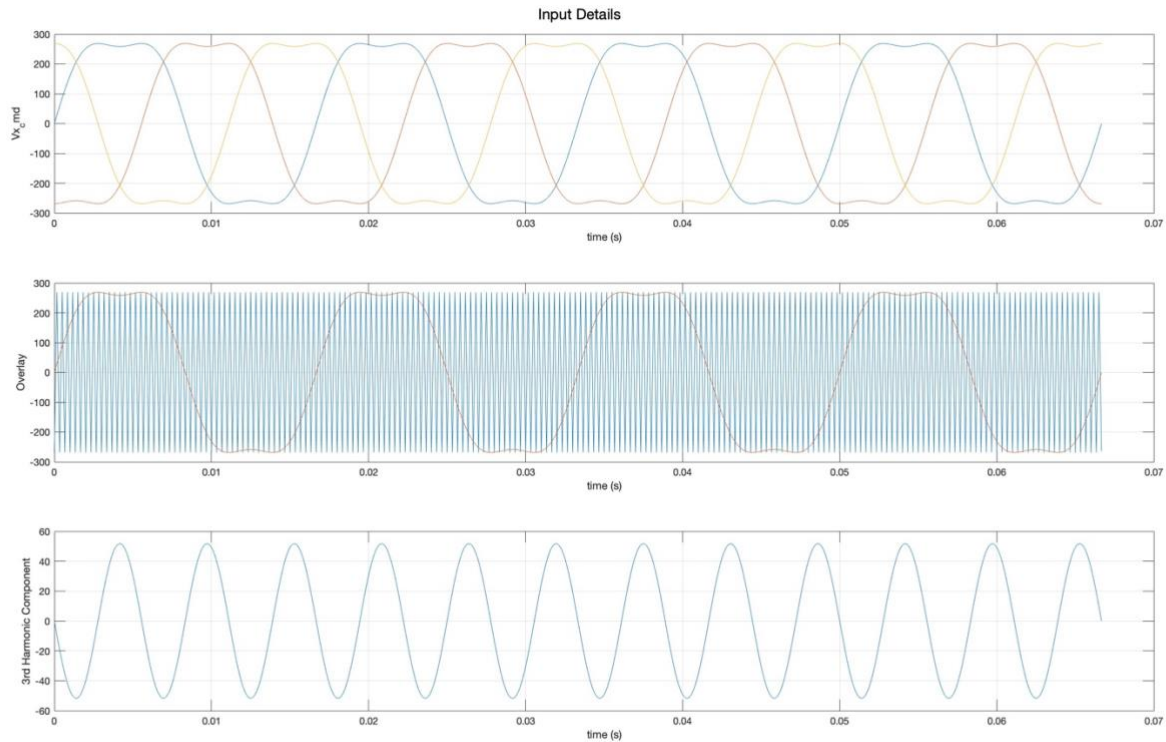
Shown in figure 11 is the filtered PWM waveform to extract the fundamental. We can also see that there is delay due to the filter. Figure 12 below shows the overall output of the three-phase inverter.



<Figure 12: Output of the 3-phase SPWM Inverter>



Before we check the harmonics of the line-to-line voltage, we first observe the results of the base case with the 3<sup>rd</sup> harmonic injection option selected. Figure 13 shows the input waveform. As seen, in the second graph, the reference voltage has the 'tip' cut-off to ensure that the inverter avoids overmodulation. The third graph shows the third harmonic component injected.

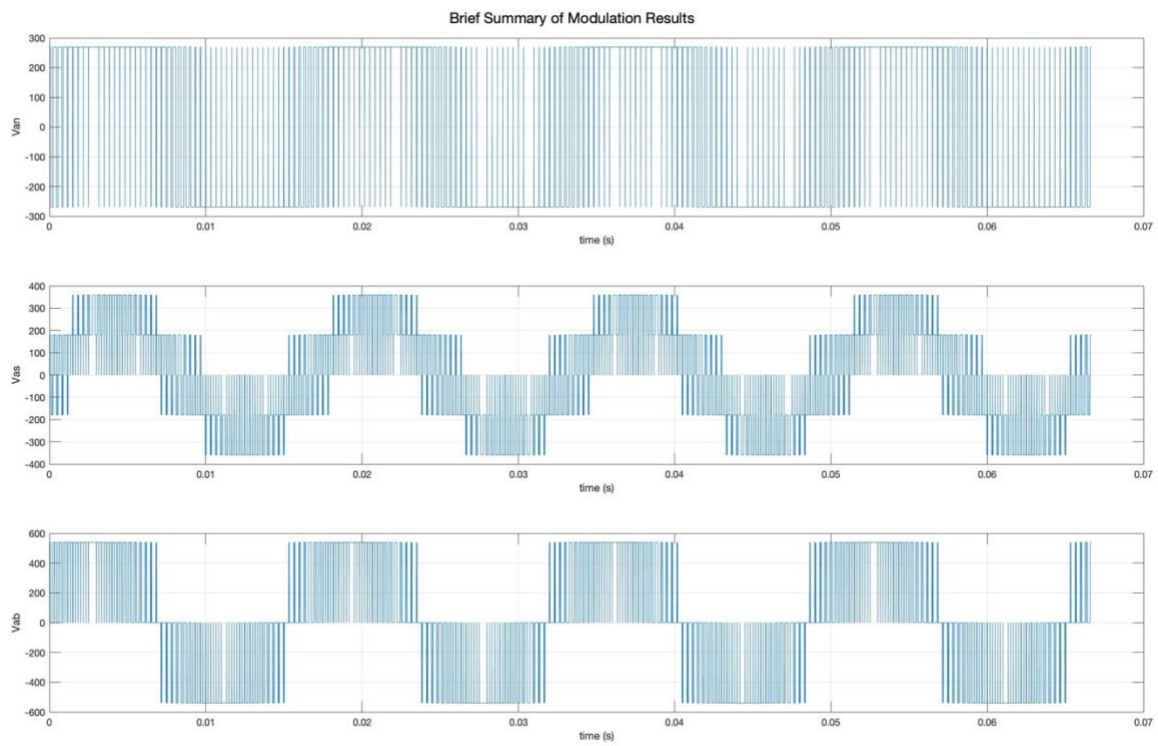


<Figure 13: 3<sup>rd</sup> Harmonic Injection Input>

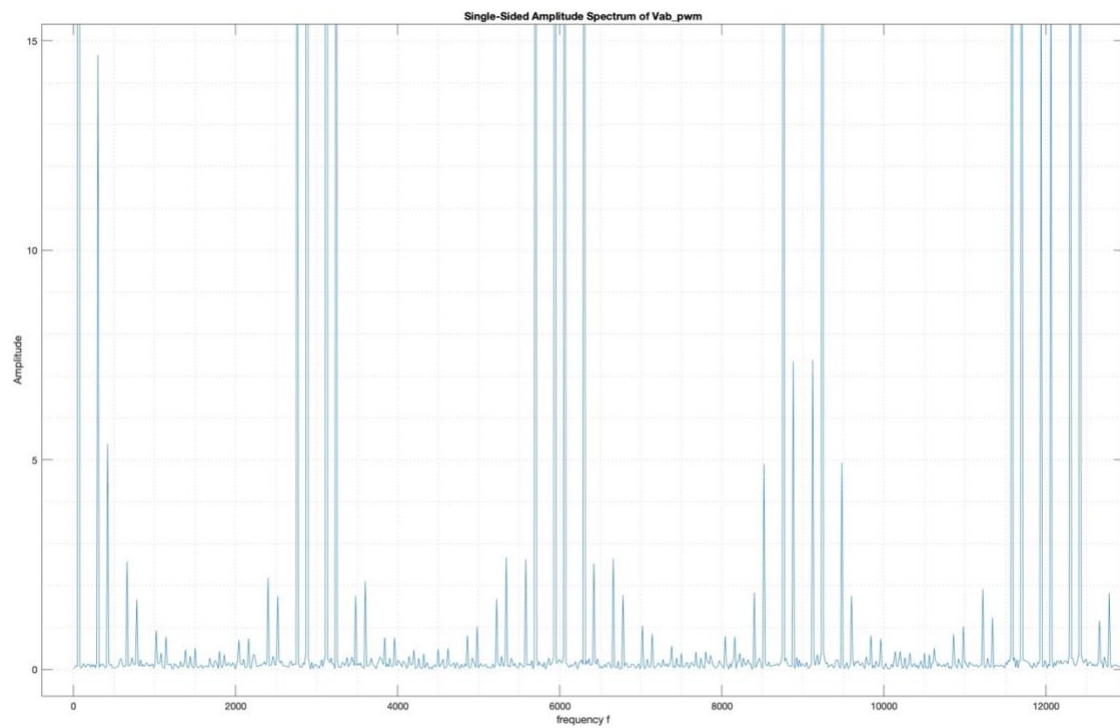
The use and benefit of injecting 3<sup>rd</sup> harmonic component waveform to the input of the inverter is shown in figure 14 (next page). Clearly from the first graph, the white space has been removed if not reduced which is an indication of improvement in the input and modulation of the inverter. Figure 15 (next page) shows that the PWM output follows the same form - the extracted fundamental is still well within the DC bus limit.

We compare the harmonics of both cases. Figure 16 and 17 shows the harmonics for the base case and 3<sup>rd</sup> harmonic injection case respectively. As observed, the overmodulated base case suffers from harmonics around the fundamental while injecting the 3<sup>rd</sup> harmonic component removes the harmonics around the fundamental. This shows that the 3<sup>rd</sup> harmonic injection technique allows the inverter not to be overmodulated.

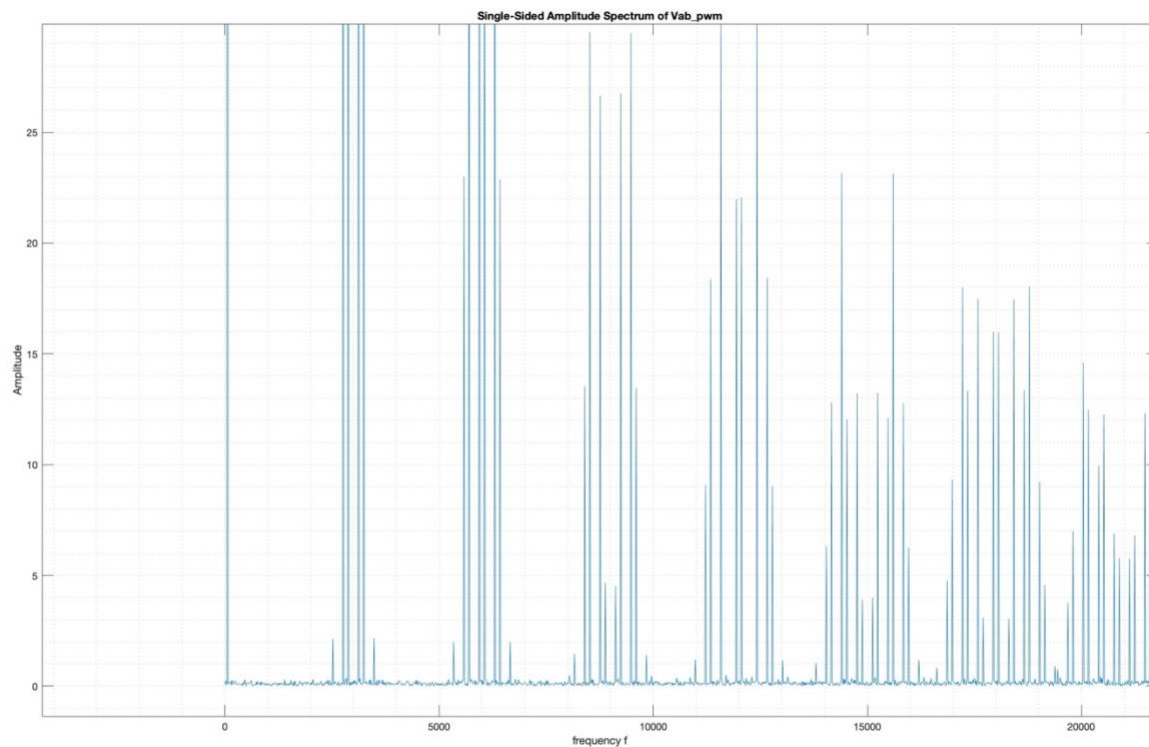




<Figure 14: Modulation Results with 3<sup>rd</sup> Harmonic Injection>



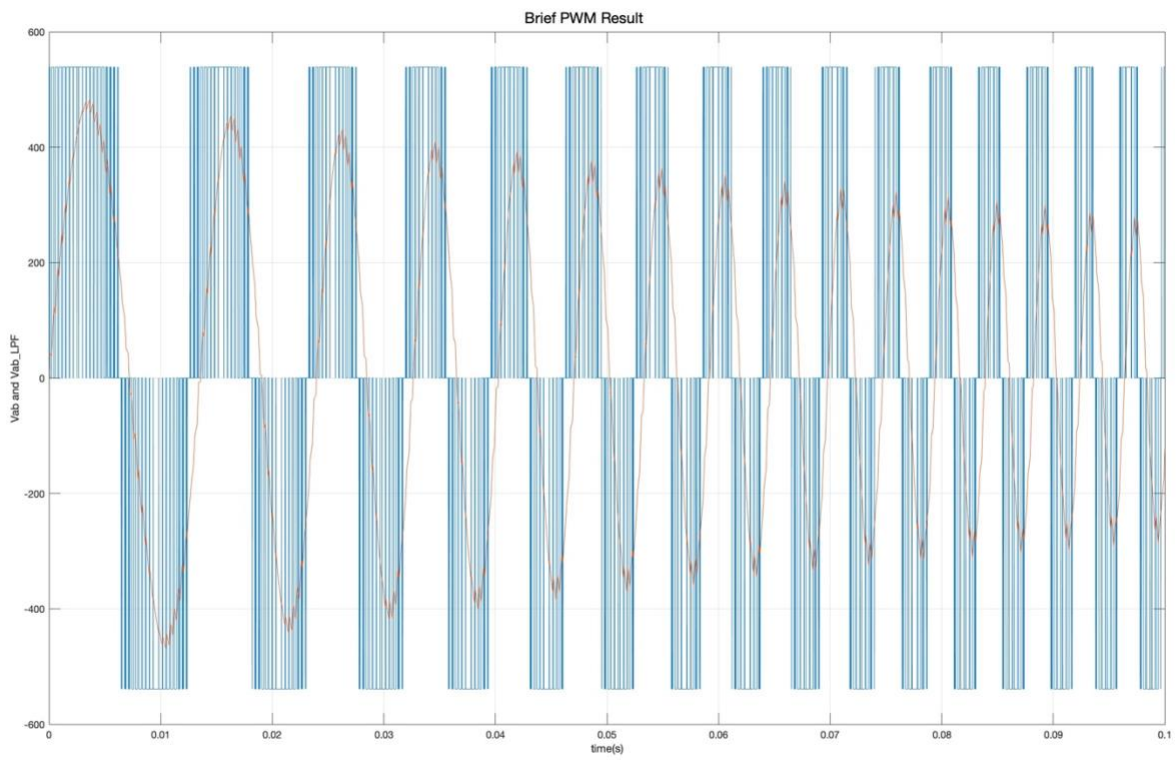
<Figure 15: Harmonics of the Base Case>



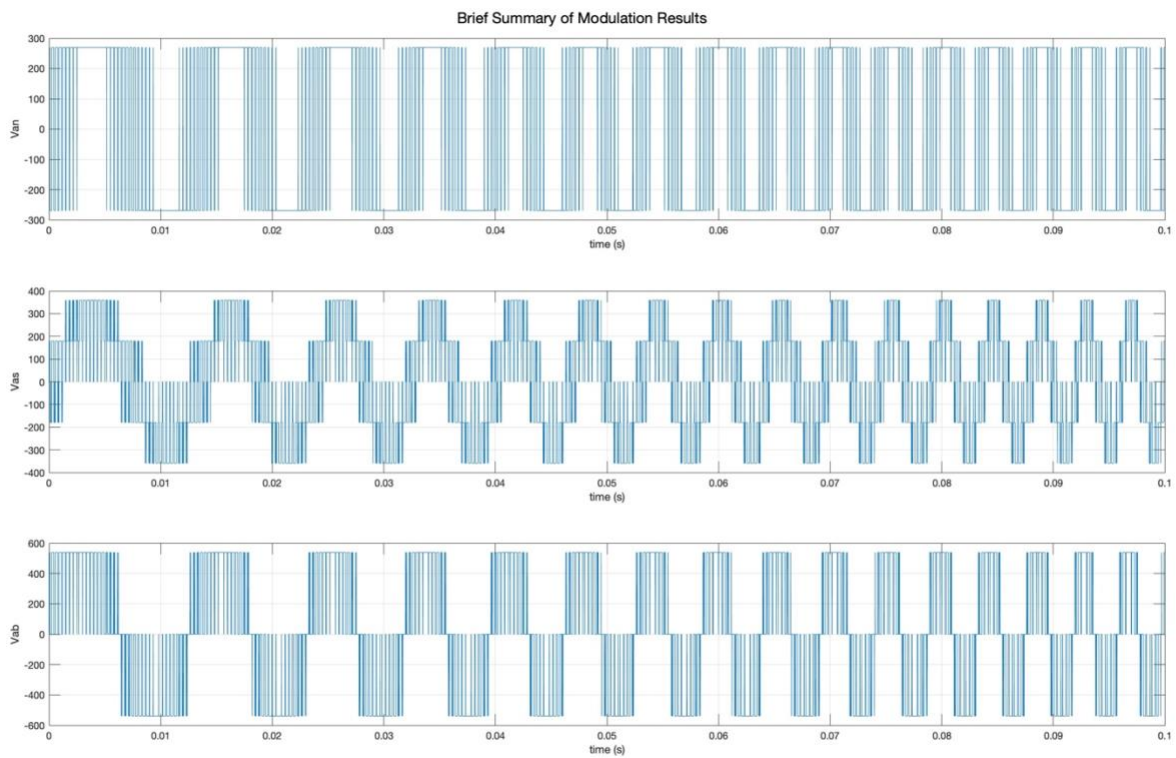
<Figure 16: Harmonics with 3<sup>rd</sup> Harmonic Injection>

We now observe the effects of varying frequency using the frequency sweep option. Our focus would be on the filtered output and its respective PWM waveform shown in figures 17 and 18. Note that the frequency sweep is set starting from 60 Hz to 120 Hz.

Figure 17 shows that increasing frequency causes the filtered fundamental to have its magnitude lower due to the low pass filter. By figure 18, we can verify that overmodulation does not depend on the frequency as we can still see that white space in the middle of each section of each set of pulses.

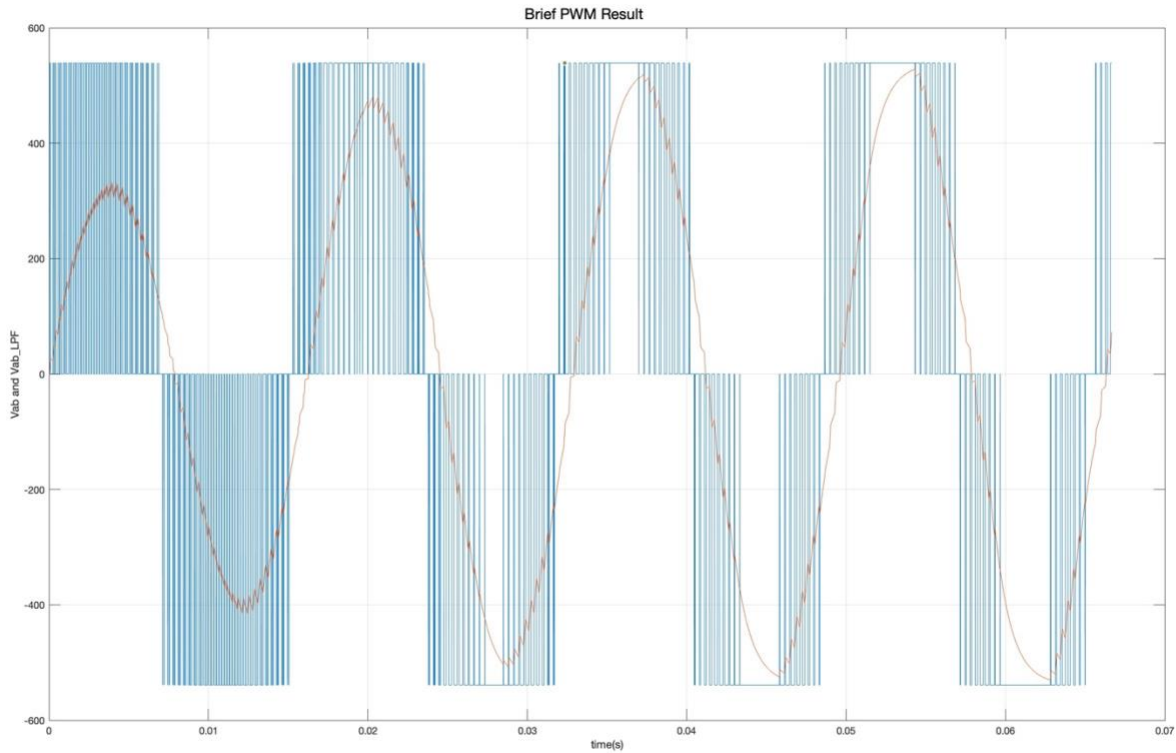


<Figure 17: PWM Result for Frequency Sweep>

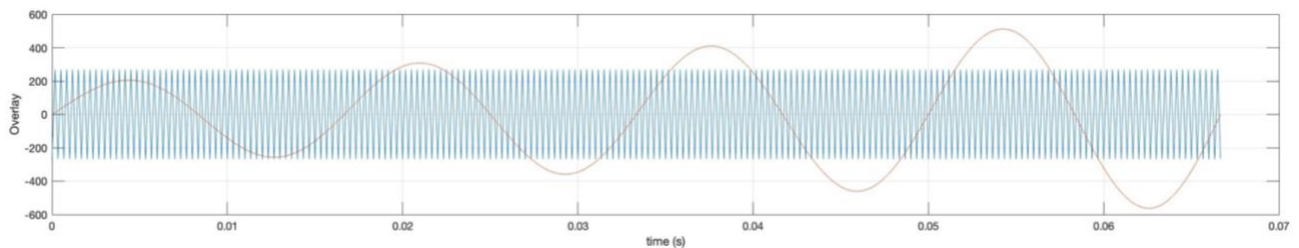


<Figure 18: Frequency Sweep Modulation Results>

Finally, we observe the effects of varying the magnitude using the amplitude sweep option. In this case, the voltage sweeps from 220 V RMS to 720 V RMS. We focus on figure 19 which shows the PWM result with the filtered sine wave on top of the graph. As shown below, increasing the voltage amplitude of the reference causes the inverter to be overmodulated as indicated by the increasing white space in between each set of pulses. To add to this observation, figure 20 shows that the overmodulation is caused by the fact that the increasing reference voltage would cause the reference voltage to be bigger than the voltage at the DC bus, hence the white spaces.



<Figure 19: PWM Output + Fundamental>



<Figure 20: Increasing Voltage causes Overmodulation>