## CS 61A Fall 2024

# Structure and Interpretation of Computer Programs

FINAL SOLUTIONS

#### INSTRUCTIONS

This is your exam. Complete it either at exam.cs61a.org or, if that doesn't work, by emailing course staff with your solutions before the exam deadline.

This exam is intended for the student with email address <EMAILADDRESS>. If this is not your email address, notify course staff immediately, as each exam is different. Do not distribute this exam PDF even after the exam ends, as some students may be taking the exam in a different time zone.

For questions with **circular bubbles**, you should select exactly *one* choice.

You must choose either this option
Or this one, but not both!

For questions with **square checkboxes**, you may select *multiple* choices.

You could select this choice.

You could select this one too!

You may start your exam now. Your exam is due at <DEADLINE> Pacific Time. Go to the next page to begin.

### Preliminaries

| (a) | What is your full name?         |
|-----|---------------------------------|
|     |                                 |
|     |                                 |
| (b) | What is your student ID number? |

You can complete and submit these questions before the exam starts.

| (c) | What is your @berkeley.edu email address? |
|-----|---|
|     |   |
|     |   |

| ` ' | Sign (or type) your name to confirm that all work on this exam will be your own. The penalty misconduct on an exam is an F in the course. | for academic |
|-----|---|--------------|
|     |   |              |

#### 1. (6.0 points) What Would Python Display?

Assume the following code has been executed.

```
def cut(s):
    this = 1
    for x in s:
        if this:
            this = x
            yield this
        else:
            this = True
def paste(n):
    yield n
    for x in paste(n + 1):
        yield 2 * x
    yield n
def copy(t, k):
    return [next(t) for x in range(k)]
nums = [0, 2, 4, 6, 8]
```

Write the output that would be displayed by printing the result of each expression. If an error occurs, write ERROR.

- (a) (2.0 pt) list(cut(nums))
  - $\bigcirc$  [0, 2, 4, 6, 8]
  - [0, 4, 6, 8]
  - $\bigcirc$  [1, 0, 2, 4, 6, 8]
  - $\bigcirc$  [1, 0, 4, 6, 8]
  - $\bigcirc$  [1, 2, 4, 6, 8]
  - $\bigcirc$  [2, 4, 6, 8]
- (b) (2.0 pt) list(cut(map(lambda x: x 4, cut(nums))))

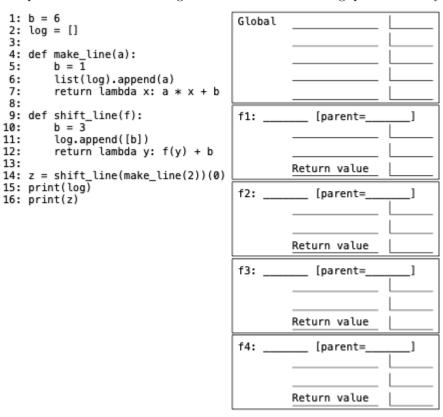
```
[-4, 0, 4]
```

(c) (2.0 pt) copy(paste(0), 3)

```
[0, 2, 8]
```

#### 2. (4.0 points) Line Diagram

Complete the environment diagram to answer the following questions. Only the questions will be scored.



- (a) (2.0 pt) What is displayed by print(log) on line 15?
  - $\bigcirc$  []
  - **(3)**
  - **([3]]**
  - O [[[3]]]
  - [2, 3]
  - O [2, [3]]
  - $\bigcirc$  [2, [[3]]]
  - $\bigcirc$  [[2], [3]]
  - [[2], [[3]]]
- (b) (2.0 pt) What is displayed by print(z) on line 16?
  - O 2
  - **9** 4
  - O 6
  - $\bigcirc$  7
  - O 9
  - O 12

#### 3. (5.0 points) Make Tens

A list of numbers is easier for a person to sum up if it can be split into groups that all sum to 10. Implement tens, which takes a list of positive numbers s. It returns True if, for every positive integer i where 10 \* i <= sum(s), there is a positive k that satisfies sum(s[0:k]) == 10 \* i. Otherwise, it returns False.

def tens(s):

```
"""Return whether every multiple of 10 less than or equal to sum(s) appears as a prefix of s.
>>> tens([3, 2, 2, 3, 6, 2, 2, 4, 1, 5, 2]) # sum(s[:4])==10, sum(s[:7])==20, sum(s[:10])==30
True
>>> tens([3, 2, 2, 3, 6, 2, 6, 1, 5, 2]) \# sum(s[:4])=10, but no slice starting at 0 sums to 20
False
11 11 11
t = 0
for x in s:
    t += x
    if ____:
         (g)
        return _____
                 (h)
    if t == ____:
             (i)
              (j)
```

return True

- (a) (1.0 pt) Fill in blank (g).
  - O t == 10
  - O t != 10
  - t > 10
  - O t < 10
- (b) (2.0 pt) Fill in blank (h). Select all correct answers.
  - False
  - ☐ t % 10 == 0
  - ☐ t % 10 != 0
  - $\square$  sum(s) % 10 == 0
  - $\square$  sum(s) % 10 != 0
- (c) (1.0 pt) Fill in blank (i).
  - $\bigcirc$  0
  - 0 10
  - x
- (d) (1.0 pt) Fill in blank (j).

0

#### 4. (13.0 points) Count Misses

#### (a) (6.0 points)

Implement the Counter class. A Counter has a count of the number of times inc has been invoked on itself or any of its offspring. Its offspring are the Counters created by its spawn method or the spawn method of any of its offspring.

```
class Counter:
```

```
"""Counts how many times inc has been invoked on itself or any of its offspring.
```

```
>>> total = Counter()
>>> odd, even = total.spawn(), total.spawn() # these are offspring of total
>>> one, three = odd.spawn(), odd.spawn()
                                              # these are offspring of odd and total
>>> for c in [one, even, three, even, odd, even]:
        c.inc()
>>> [c.count for c in [one, three, even, odd, total]]
[1, 1, 3, 3, 6]
def __init__(self, parent=None):
    self.parent = parent
      (a)
def inc(self):
    self.count += 1
    ____:
      (b)
          (c)
def spawn(self):
    return _____
             (d)
```

i. (1.0 pt) Fill in blank (a).

```
self.count = 0
```

```
ii. (1.0 pt) Fill in blank (b).
```

- O if parent is not None:
- if self.parent is not None:
- O while parent is not None:
- while self.parent is not None:
- O for p in parent:
- O for p in self.parent:

| iii. (2.0 | ) pt) | Fill | in | blank | (c) | ). |
|-----------|-------|------|----|-------|-----|----|
|-----------|-------|------|----|-------|-----|----|

- O p += 1
- $\bigcirc$  p.count += 1
- O p.inc()
- p.count.inc()
- $\bigcirc$  parent += 1
- O parent.count += 1
- parent.inc()
- parent.count.inc()
- O self.parent += 1
- self.parent.count += 1
- self.parent.inc()
- self.parent.count.inc()

#### iv. (2.0 pt) Fill in blank (d).

- self.parent
- self.parent.spawn()
- self.spawn()
- Ounter()
- Counter().spawn()
- Counter(self)
- Counter(self.count)

#### (b) (7.0 points)

Implement the MissDict class. A MissDict has a dictionary d. Its get method takes an iterable keys, returns a list of all values in d that correspond to those keys, and counts the number of keys that did not appear in d (called *misses*). Printing a MissDict displays a fraction in which:

- The numerator is the number of misses during all calls to get for that particular MissDict instance.
- The denominator is the number of misses during all calls to get for any MissDict instance.

Assume Counter is implemented correctly.

```
class MissDict:
    """Has a dict, gets a list of values for an iterable of keys,
   and counts the number of keys that are not in the dict.
   >>> double = MissDict({1: 2, 2: 4, 3: 6, 5: 10})
   >>> half = MissDict({2: 1.0, 3: 1.5, 4: 2.0})
   >>> double.get([1, 3, 5, 2, 4]) # No value for key 4 (1 miss)
    [2, 6, 10, 4]
   >>> double.get([5, 4, 3, 0, 4]) # No value for keys 4 or 0 or 4 (3 misses)
    [10, 6]
   >>> half.get([1, 3, 5, 2, 4])
                                     # No value for keys 1 or 5 (2 misses)
    [1.5, 1.0, 2.0]
   >>> print(double) # double had 4 misses & half had 2 misses
   4/6 of the misses
    11 11 11
   misses = Counter()
   def __init__(self, d):
        assert isinstance(d, dict)
        self.d = d
        self.misses = _____
                       (e)
   def get(self, keys):
        result = []
        for k in keys:
            if k in self.d:
                  (f)
            else:
                  (g)
        return result
   def __str__(self):
       return f'____ of the misses'
                   (h)
 i. (2.0 pt) Fill in blank (e). Select all correct answers.
   ☐ MissDict.spawn()
   MissDict.misses.spawn()
   ☐ Counter()
   ☐ Counter(MissDict)
    Counter(MissDict.misses)
```

ii. (2.0 pt) Fill in blank (f).

```
result.append(self.d[k])
```

- iii. (1.0 pt) Fill in blank (g).
  - misses.inc()
  - O misses.count += 1
  - self.misses.inc()
  - self.misses.count += 1

  - MissDict.misses.count += 1
- iv. (2.0 pt) Fill in blank (h).

  - ( \{\text{misses.count}\} / \{\text{self.MissDict.misses.count}\}

  - ( \{\self.misses.count\} / \{\self.misses.count\}

  - {self.misses.count} / {MissDict.misses.count}

#### 5. (16.0 points) Promotion to CS 61B

**Definition.** Given a sequence s and positions (indices) i and j in s with i < j, promoting an element to i from j means reordering s so that the element originally at position j is now at position i, all elements originally positioned between i and j increase their position (index) by one, and all other elements stay where they are. The beginning of s is position 0. For example, in the list [30, 60, 90, 120, 150, 180], promoting to 2 from 4 would place the number 150 (original position 4) just before 90 (originally position 2) and increases the positions of both 90 and 120 by one, resulting in [30, 60, 150, 90, 120, 180].

#### (a) (4.0 points)

Implement promote, which takes a list of numbers s and two non-negative integers i and j. It returns a new list promoting to i from j in s. Do not mutate s.

```
Hint: For any list s, s[len(s):] evaluates to [].
def promote(s, i, j):
    """Return a list in which s[j] is at index i without mutating s.
    >>> promote([3, 6, 9, 12, 15, 18], 2, 4)
    [3, 6, 15, 9, 12, 18]
    >>> promote([3, 6, 9, 12, 15, 18], 0, 4)
    [15, 3, 6, 9, 12, 18]
    assert i \ge 0 and i < j and j < len(s)
    return s[:i] + ____ + ___ + s[___:]
(a) (b) (c)
 i. (2.0 pt) Fill in blank (a). Select all correct answers.
   □ s[j]
   [s[i]]
   ☐ list(s[j])
   □ s[j:j]
   s[j:j+1]
ii. (1.0 pt) Fill in blank (b).
   s[i:j]
   ○ s[i+1:j]
   ○ s[i:j+1]
   ○ s[i+1:j+1]
iii. (1.0 pt) Fill in blank (c).
   \bigcirc i
   ○ i+1
   O j
    🛑 j+1
```

 $\bigcirc$  s[j] == t[1]

#### (b) (4.0 points)

Implement promotions, which takes two lists of numbers s and t that have the same elements, possibly in different orders. It returns the minimum number of promotions that must be applied to s so that it has the same order as t. Assume promote is implemented correctly.

```
def promotions(s, t):
    """Return the minimum times promote must be called to start from s and return t.
   >>> promotions([2, 4, 6, 8, 10, 12],
                   [2, 6, 8, 4, 12, 10]) # promote (1, 2) then (2, 3) then (4, 5)
   3
   >>> promotions([6, 1, 6, 1, 6, 1],
                   [1, 1, 6, 6, 1, 6])
                                          # promote (0, 1) then (1, 5)
   >>> promotions([1, 2, 3], [1, 2, 3])
                                                # no promotions needed
   >>> promotions([1, 2, 1, 2], [2, 1, 2, 1]) # promote (0, 3)
    assert sorted(s) == sorted(t) # Check that s & t have the same elements (disregarding order)
    if len(s) == 0:
        return 0
    elif ____:
           (d)
        return promotions(s[1:], t[1:])
    else:
        return 1 + min([promotions(_____, t[1:]) for j in range(1, len(s)) if _____])
                                     (e)
                                                                                   (f)
 i. (1.0 pt) Fill in blank (d).
   \bigcirc s == t
   ○ s != t
   \bigcirc s[0] != t[0]
     sorted(s) == t
    s == sorted(t)
ii. (2.0 pt) Fill in blank (e).
     promote(s, 0, j)[1:] OR s[:j] + s[j+1:]
iii. (1.0 pt) Fill in blank (f).
   \bigcirc s[0] == t[j]
   \bigcirc s[1] == t[j]
```

#### (c) (8.0 points)

Implement promote\_link, which takes a Link instance s (a non-empty linked list) and two non-negative integers i and j with i<j and j less than the length of s. It mutates s by promoting to i from j and then returns s.

```
The Link class appears on Page 2 (left side) of the Midterm 2 Study Guide.
```

```
def promote_link(s, i, j):
    """Mutate linked list s so that the item at index j is at index i.
   >>> a = Link(3, Link(6, Link(9, Link(12, Link(15, Link(18)))))
   >>> print(promote_link(a, 2, 4))
    <3 6 15 9 12 18>
   >>> print(promote_link(a, 0, 4))
   <12 3 6 15 9 18>
   >>> promote_link(a, 1, 3) is a
   True
   assert i \ge 0 and i < j
   if i > 0:
          (g)
    else:
        insert, tail = s.first, s.rest
        while j > 0:
            _____ # Hint: use multiple assignment: ___ , ___ = ___ , ___
              (h)
            tail, j = tail.rest, j-1
        _____ = insert
          (i)
   return _____
             (j)
 i. (2.0 pt) Fill in blank (g).
   promote_link(s.rest, i-1, j)
       return promote_link(s.rest, i-1, j)
   promote_link(s.rest, i, j-1)
       return promote_link(s.rest, i, j-1)
   promote_link(s.rest, i-1, j-1)
   O return promote_link(s.rest, i-1, j-1)
ii. (2.0 \text{ pt}) Fill in blank (h). Hint: Use multiple assignment: ___ , ___ = ___ , ___
      insert, tail.first = tail.first, insert
```

O <9 3 5 7>

```
iii. (1.0 pt) Fill in blank (i).
   s.first
   ○ s.rest.first
   O tail.first
   O tail.rest.first
iv. (1.0 pt) Fill in blank (j).
   s
   \bigcirc tail
   Link(insert, tail.rest)
   Link(s.first, tail.rest)
v. (2.0 pt) What is displayed by the call to print in this code?
   odds = Link(3, Link(5, Link(7, Link(9))))
   for i in range(3):
       promote_link(odds, 0, 3)
   print(odds)
   O <3 5 7 9>
   O <3 9 7 5>
   <5 7 9 3>
   <5 9 7 3>
   0 <9 7 5 3>
```

#### 6. (12.0 points) Fresh Produce

#### (a) (5.0 points)

Implement products, which takes a Tree instance t with positive integer labels and a positive integer n. It returns True if every path from the root of t to a leaf has labels that equal n when multiplied together.

The Tree class appears on Page 2 (left side) of the Midterm 2 Study Guide.

```
def products(t, n):
    """Return whether the product of labels along every root-to-leaf path is n.
    >>> products(Tree(1, [Tree(2, [Tree(3)]), Tree(6)]), 6)
    >>> products(Tree(1, [Tree(2, [Tree(3)]), Tree(6)]), 12)
    False
    >>> products(Tree(1, [Tree(2, [Tree(3)]), Tree(5)]), 6)
    False
    >>> products(Tree(1, [Tree(5, [Tree(2)]), Tree(12)]), 12)
    False
    assert type(n) == int
    if t.is_leaf():
        return _____
                 (a)
    if ____:
         (b)
        return False
    return _____
             (c)
 i. (1.0 pt) Fill in blank (a).
   ○ True
   ○ False
    n == t.label

    n % t.label == 0

    n % t.label > 0
ii. (1.0 pt) Fill in blank (b).
   On != t.label
   \bigcirc n < t.label
   \bigcirc n > t.label
    n % t.label > 0
```

iii. (3.0 pt) Fill in blank (c). Important: The second argument to a call to products must be an integer (int).

```
all([products(b, n // t.label) for b in t.branches])
```

t.label + 1

#### (b) (7.0 points)

**Definition.** An *increasing sequence* is a sequence of integers in which each element after the first is larger than the previous element.

Implement produce, which takes a positive integer n. It returns a Tree of positive integers in which:

- The product of the labels along every root-to-leaf path is n,
- Every increasing sequence of integers starting with 1 that has product n is a root-to-leaf path, and
- Every sequence of siblings (nodes with a common parent) is an increasing sequence.

```
def produce(n):
    """Return the largest tree in which the labels for every root-to-label path
    are increasing and have product n. Put all siblings in increasing order.
    >>> produce(12)
    Tree(1, [Tree(2, [Tree(6)]), Tree(3, [Tree(4)]), Tree(12)])
    >>> print(produce(24))  # Paths are 1-2-3-4, 1-2-12, 1-3-8, 1-4-6, and 1-24
    1
      2
        3
        12
      3
        8
      4
        6
      24
    11 11 11
    def grow(t, x):
        for k in range(____, x + 1):
(d)
            if _____ % k == 0:
                 branch = _____
                 if ____:
                     t.branches.append(branch)
        return t
    return grow(Tree(1), n)
 i. (1.0 pt) Fill in blank (d).
   \bigcirc 0
   \bigcirc 1
   \bigcirc x
   O t.label
```

#### 7. (14.0 points) A Pair of Schemes

(a) (4.0 points)

```
Implement the Scheme procedure all-pairs, which takes a procedure f and a list s. It returns #t if (f x y) is #t for every pair of adjacent elements x and y in s. Assume f always returns either #t or #f.
```

```
(define (inc x y) (= (+ x 1) y)); Whether x+1 equals y
;;; Return #t if (f x y) is #t for every pair of adjacent values (x, y) in list s.
;;; scm> (all-pairs inc '(3 4 5 6 7 8))
;;; scm> (all-pairs inc '(3 4 5 8 7 8))
;;; scm> (all-pairs inc '(3))
;;; #t
(define (all-pairs f s)
  (or (null? s) (null? (cdr s))
      (and _____ (all-pairs f ____ ))))
(a) (b)
 i. (3.0 pt) Fill in blank (a). Select all correct answers.
   \square (car s) (cdr s)
   ☐ (car s) (car (cdr s))
   \Box f (car s) (cdr s)
   ☐ f (car s) (car (cdr s))
   \Box f((car s) (cdr s))
   \square f((car s) (car (cdr s)))
   \square (f (car s) (cdr s))
   (f (car s) (car (cdr s)))
   ☐ (apply f (car s) (cdr s))
   ☐ (apply f (car s) (car (cdr s)))
ii. (1.0 pt) Fill in blank (b).
   () s
   (cdr s)
   (cdr (cdr s))
   (cons (car s) (cdr s))
   (cons (car s) (cdr (cdr s)))
```

#### (b) (6.0 points)

| Implement the Scheme    | procedure   | <pre>show-pairs,</pre> | which | takes a | a list | s and | $\operatorname{returns}$ | a list | of | ${\rm every}$ | pair | of |
|-------------------------|-------------|------------------------|-------|---------|--------|-------|--------------------------|--------|----|---------------|------|----|
| adjacent elements in s. | A pair is a | two-element            | list. |         |        |       |                          |        |    |               |      |    |

ii. (3.0 pt) Fill in blank (b).

```
(list (car s) (car (cdr s)))
```

iii. (1.0 pt) Fill in blank (c).

- $\bigcirc$  s
- (cdr s)

append

- (cdr (cdr s))
- (cons (car s) (cdr s))
- (cons (car s) (cdr (cdr s)))
- iv. (1.0 pt) What order of growth describes the time it takes to execute (unpair s) in terms of the length of the input list s, assuming that car, cdr, cons, and null? are all constant-time operations?

- O constant
- O logarithmic
- linear
- O quadratic
- exponential

#### (c) (4.0 points)

Implement the Scheme procedure all-pairs-exp, which takes a procedure name proc-name (a symbol) and a list s. It returns an and expression that calls the procedure named by proc-name on every adjacent pair of elements in s. Assume show-pairs is implemented correctly.

ii. (3.0 pt) Fill in blank (b).

```
(lambda (pair) (cons proc-name pair)) or (lambda (pair) (list proc-name (car pair) (car (cdr pair))))
```

#### 8. (5.0 points) Arcane Jobs

The Council of Piltover wants a list of all the people living in regions it governs that have unique jobs. Create a table with columns labeled name and job that contains one row for each person living in a region governed by the "council" who is the only person with their job among everybody living in regions ruled by the council.

The who table has one row per person and columns for their name (string; each row has a unique value), the region (string) they live in, and their job (string). The gov table has one row per region and columns for its name called place (string; each row has a unique value) and the group that rules over it called ruler (string).

Here's an example of the contents of the who and gov tables and the expected result for a query based on these.

| 14 | h |   | • |
|----|---|---|---|
| w  | ш | ľ | , |
|    |   |   |   |

| name         | region   | job       |
|--------------|----------|-----------|
| caitlyn      | piltover | enforcer  |
| vi           | zaun     | enforcer  |
| jayce        | piltover | scientist |
| victor       | piltover | scientist |
| ekko         | zaun     | firelight |
| heimerdinger | piltover | scientist |
| jinx         | zaun     | rebel     |
| mel          | noxus    | councilor |
| ambessa      | noxus    | warlord   |
| vander       | zaun     | barkeep   |
| silco        | zaun     | chembaron |

| ov: | place      | ruler       |  |  |
|-----|------------|-------------|--|--|
|     | noxus      | triumvirate |  |  |
|     | zaun       | council     |  |  |
|     | piltover   | council     |  |  |
|     | bilgewater | gangs       |  |  |

expected result:

| name   | job       |
|--------|-----------|
| vander | barkeep   |
| silco  | chembaron |
| ekko   | firelight |
| jinx   | rebel     |

SELECT name, job FROM \_\_\_\_\_ WHERE \_\_\_\_ AND ruler = "council" \_\_\_\_\_;

(a) (b) (c)

- (a) (1.0 pt) Fill in blank (a).
  - O who
  - O who AS a, who AS b
  - who, gov
  - who AS a, who AS b, gov
- (b) (2.0 pt) Fill in blank (b).
  - $\bigcirc$  COUNT(\*) = 1
  - place = "council"
  - $\bigcirc$  a.job = b.job
  - O a.job != b.job
  - region = place
  - a.region = place
  - a.region = b.region AND a.region = place
  - a.job = b.job AND a.region = place

(c) (2.0 pt) Fill in blank (c). You may write AND to continue the WHERE clause (but you don't have to). You may also include other clauses such as GROUP BY, ORDER BY, HAVING, and LIMIT (but you don't have to).

GROUP BY job HAVING COUNT(\*) = 1

#### 9. (0.0 points) Ape Pull Us

These two A+ questions are not worth any points. They can only affect your course grade if you have a high A and might receive an A+. Finish the rest of the exam first!

(a) Implement all-next, a macro that takes an expression x-expr and a list of numbers. It returns #t if every element in s besides the first is equal to the value of x-expr when x is bound to the previous element in s. It returns #f otherwise. Assume x-expr does not contain the symbol y.

You may call all-pairs from Q6. Important: The template has a quasiquote before the blank.

```
;;; Return whether every value in s that follows another value x is equal
;;; to the x-expr evaluated when x is the previous value.
;;;
;;; scm> (all-next (+ x 2) '(3 5 7 9 11 13))
;;; #t
;;; scm> (all-next (* x 2) '(2 4 8 16 32 64))
;;; #t
;;; scm> (all-next (+ x 2) '(3 5 7 8 11 13))
;;; #f
;;; scm> (all-next (+ x 4) '(3 5 7 9 11 13))
;;; #f
(define-macro (all-next x-expr s) ` _____)
```

```
(all-pairs (lambda (x y) (= ,x-expr y)) ,s)
```

(b) Implement make\_tens by filling in the blank in repromote. The make\_tens function takes a list of numbers s. It returns a list t of the same elements reordered so that tens(t) returns True. Return an order that requires the fewest promotions to reorder s into t. You may use tens from Q2 and promote from Q4.

```
def make_tens(s):
    """Return a list t for which tens(t) is true and promotions(s, t) is as small as possible.
   If there is no reordering t of s for which tens(t) is true, return None.
   >>> make_tens([4, 2, 2, 2, 4, 6, 1, 3, 3, 2, 5]) # promote (6, 7), (7, 9), and (8, 10)
    [4, 2, 2, 2, 4, 6, 3, 2, 5, 1, 3]
   >>> make_tens([4, 2, 2, 2, 4, 5, 1, 4, 3, 4]) # promote (2, 4), (4, 7), (5, 9), and (8, 9)
    [4, 2, 4, 2, 4, 4, 2, 5, 3, 1]
    11 11 11
   for t in promute(s):
        if tens(t):
            return t
def promute(s):
    for k in range(len(s)):
        yield from repromote(s, k)
def repromote(s, k):
   if k == 0:
       yield s
    elif len(s) > 1:
        for j in range(len(s)):
            for rest in ____:
                yield [s[j]] + rest
  repromote(s[:j] + s[j+1:], k - min(1, j))
```

No more questions.