Data Structures and Algorithms III

Formal languages and automata

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University of Tübingen
Seminar für Sprachwissenschaft

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Practical matters

The second part of the course will be somewhat different:

- The focus will shift more towards Computational Linguistics topics / applications
- We will review more specialized data structures and algorithms (e.g., automata, parsing)
- Some overlap with parsing class (but with more emphasis on practical sides)
- Less focus on programming

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A quick poll: opinions about switching to Python.

An overview of the upcoming topics

- Background on formal languages and automata (today)
- Finite state automata and regular languages
- Finite state transducers (FST)
  - FSTs and computational morphology
- Dependency grammars and dependency parsing
- Context-free grammars and constituency parsing

Assignments

- Assignment policy is similar to the first part of the course
- Two graded assignments:
  - Finite state methods (due early Jan)
  - Parsing (due mid Feb)
- There will be more ungraded assignments – they are part of the course work, they are not 'optional'

This lecture

An overview

- Background: some definitions on phrase structure grammars and rewrite rules
- Chomsky hierarchy of (formal) language classes
- Background: computational complexity
- Automata, their relation to formal languages
- Formal languages and automata in natural language processing
- A brief note on learnability of natural languages

Why study formal languages

- Formal languages are an important area of the theory of computation
- They originate from linguistics, and they have been used in formal/computational linguistics

Definitions
Alphabet

- An *alphabet* is a set of symbols
- We generally denote an alphabet using the symbol Σ
- In our examples, we will use lowercase ASCII letters for the individual symbols, e.g., $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$
- Alphabet does not match the every-day use:
  - In some cases one may want to use a binary alphabet, $\Sigma = \{0, 1\}$
  - If we want to define a grammar for arithmetic operations, we may want to have $\Sigma = \{0, 1, 2, 3, \ldots, 9, +, -, \times, /\}$
  - If we are interested in natural language syntax our alphabet is the set of natural language words, $\Sigma = \{\mathtt{the}, \mathtt{on}, \mathtt{cat}, \mathtt{dog}, \mathtt{mat}, \mathtt{sat}, \ldots\}$

Definitions

Strings

- A *string* over an alphabet is a finite sequence symbols from the alphabet
  - a, ab, $acbcaa$ are example strings over $\Sigma = \{a, b, c\}$
- The *empty string* is denoted by ϵ
- The Σ^* denotes all strings that can be formed using alphabet Σ, including the empty string ϵ
- The Σ^+ is a shorthand for $\Sigma^* - \epsilon$
- Similarly a^* means the symbol a repeated zero or more times, $a+$ means a repeated one or more times
- We use a^n for exactly n repetitions of a
- The length of a string u is denoted by $|u|$, e.g., $|abc| = 3$, or if $u = aabbcc$, $|u| = 6$
- Concatenation of two string u and v is denoted by uv, e.g., for $u = ab$ and $v = ca$, $uv = abca$

Definitions

Language

- A (formal) language is a set of string over an alphabet
  - The set of strings of length 2 over $\{0, 1\}$:
    $\{00, 01, 10, 11\}$
  - The set of strings with even number of 1′s over $\{0, 1\}$:
    $\{\epsilon, 101, 0, 11, 111110, \ldots\}$
  - The set of string that retain alphabetical ordering over $\{a, b, c\}$:
    $\{a, ab, abc, ac, abcc, \ldots\}$
  - The set of strings of words that form grammatically correct English sentences
- Strings that are member of a language is called *sentences* (or sometimes *words*) of the language

Definitions

Grammar

- A *grammar* is a finite description of a language
- A common way of specifying a grammar is based on a set of *rewrite rules* (or *phrase structure rules*)
- We represent *non-terminal symbols* with uppercase letters
- We represent *terminal symbols* with lowercase letters
- S is the *start symbol*
- If a string can be generated from S using the rewrite rules, the string is a valid sentence in the language

$$
\begin{aligned}
\mathrm{S} &\rightarrow \mathrm{A\ B} \\
\mathrm{S} &\rightarrow \mathrm{S\ A\ B} \\
\mathrm{A} &\rightarrow \mathrm{a} \\
\mathrm{B} &\rightarrow \mathrm{b}
\end{aligned}
$$

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\begin{aligned}
S &\rightarrow A\,B \\
S &\rightarrow S\,A\,B \\
A &\rightarrow a \\
B &\rightarrow b
\end{aligned}
$$

Q: What does this grammar define?

Definitions

Phrase structure grammars: more formally

A phrase structure grammar is a tuple $G = (\Sigma, N, S, R)$ where

- Σ is an alphabet of terminal symbols
- N are a set of non-terminal symbols
- S is a special 'start' symbol $\in N$
- R is a set of rules of the form

$$\alpha \rightarrow \beta$$

where α and β are strings from $\Sigma \cup N$

A string u is in the language defined by G,
if it can be derived from S.

Definitions

Grammars and derivations

Grammar

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\begin{aligned}
S &\rightarrow A\,B \\
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Definitions

Grammars and derivations

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Derivation of $abab$

$$S \Rightarrow SAB$$

Definitions

Grammars and derivations

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$SAB \Rightarrow ABAB$

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$SAB \Rightarrow ABAB$

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$aBAB \Rightarrow abAB$

Definitions

Grammars and derivations

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$ABAB \Rightarrow aBAB$

$aBAB \Rightarrow abAB$

$abAB \Rightarrow abaB$

Definitions

Grammars and derivations

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S &\rightarrow A\,B \\
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Derivation of $abab$

$S \Rightarrow SAB$	$aBAB \Rightarrow abAB$
$SAB \Rightarrow ABAB$	$abAB \Rightarrow abaB$
$ABAB \Rightarrow aBAB$	$abaB \Rightarrow abab$

Definitions

Grammars and derivations

Grammar

$$\begin{aligned} S &\rightarrow A\,B \\ S &\rightarrow S\,A\,B \\ A &\rightarrow a \\ B &\rightarrow b \end{aligned}$$

Derivation of $abab$

$S \Rightarrow SAB$	$aBAB \Rightarrow abAB$
$SAB \Rightarrow ABAB$	$abAB \Rightarrow abaB$
$ABAB \Rightarrow aBAB$	$abaB \Rightarrow abab$

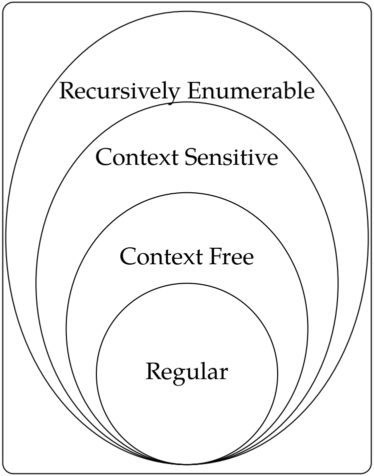
- Intermediate strings of terminals and non-terminals are called *sentential forms*
- $S \overset{*}{\Rightarrow} abab$: the string is in the language

Q: What if string was not in the language?

Q: Is there another derivation sequence?

Chomsky hierarchy of (formal) languages

- Defined for formalizing natural language syntax
- Definitions are in terms of the restrictions on production rules of the grammar
- Also part of theory of computation
- Each language class corresponds to a class of (abstract) machines
- Other well-studied classes exist



Regular grammars

Left regular	Right regular
1. $A \rightarrow a$	1. $A \rightarrow a$
2. $A \rightarrow Ba$	2. $A \rightarrow aB$
3. $A \rightarrow \epsilon$	3. $A \rightarrow \epsilon$

- Least expressive, but easy to process
- Used in many NLP applications
- Defines the set of languages expressed by *regular expressions*
- Regular grammars define only regular languages (but reverse is not true)
- We will discuss it in more detail soon

Regular grammars

an example

Write a right- and a left-regular grammar ab^*c

Regular grammars

an example

Write a right- and a left-regular grammar ab^*c

left	right
$S \rightarrow Ac$	$S \rightarrow aA$
$A \rightarrow Ab$	$A \rightarrow bA$
$A \rightarrow a$	$A \rightarrow c$

Regular grammars

an example

Write a right- and a left-regular grammar ab^*c

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$S \rightarrow Ac$	$S \rightarrow aA$
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Can you define a regular grammar for

- a^nb^n?
- a^5b^5?

Regular grammars

an example

Write a right- and a left-regular grammar ab^*c

left
$S \rightarrow Ac$
$A \rightarrow Ab$
$A \rightarrow a$

right
$S \rightarrow aA$
$A \rightarrow bA$
$A \rightarrow c$

Derive the string $abbbc$ using one of your grammars

Can you define a regular grammar for

- $a^n b^n$?
- $a^5 b^5$?

Regular grammars

an example

Write a right- and a left-regular grammar ab^*c

left

$$S \to Ac$$
$$A \to Ab$$
$$A \to a$$

right

$$S \to aA$$
$$A \to bA$$
$$A \to c$$

Can you define a regular grammar for

- $a^n b^n$?
- $a^5 b^5$?

Derive the string $abbbc$ using one of your grammars

left

$S \Rightarrow Ac \Rightarrow Abc \Rightarrow Abbc \Rightarrow Abbbc \Rightarrow abbbc$

right

$S \Rightarrow aA \Rightarrow abA \Rightarrow abbA \Rightarrow abbbA \Rightarrow abbbc$

Regular grammars

an example

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Derive the string $abbbc$ using one of your grammars

left

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right

$S \Rightarrow aA \Rightarrow abA \Rightarrow abbA \Rightarrow abbbA \Rightarrow abbbc$

These grammars are *weakly equivalent*: they generate the same language, but derivations differ

Context-free grammars (CFG)

CFG rules

$$A \rightarrow \alpha$$

where A is a *single* non-terminal α is a possibly empty sequence of terminals and non-terminals

- More expressive than regular languages
- Syntax of programming languages are based on CFGs
- Many applications for natural languages too (more on this later)

Context-free grammars

an example

The example grammar:

Example CFG

S	$\rightarrow$	NP VP	VP	$\rightarrow$	V NP
NP	$\rightarrow$	John | Mary	V	$\rightarrow$	saw

Exercise: derive 'John saw Mary'

Context-free grammars

an example

The example grammar:

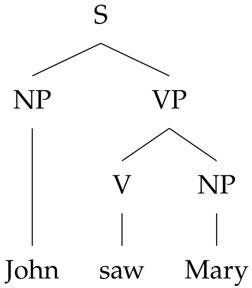
Example CFG

S	$\rightarrow$	NP VP	VP	$\rightarrow$	V NP
NP	$\rightarrow$	John | Mary	V	$\rightarrow$	saw

Exercise: derive 'John saw Mary'

Derivation

$$\mathrm{S} \Rightarrow \mathrm{NP\ VP} \Rightarrow \mathrm{John\ VP}$$
$$\Rightarrow \mathrm{John\ V\ NP} \Rightarrow \mathrm{John\ saw\ NP}$$
$$\Rightarrow \mathrm{John\ saw\ Mary}$$
or, $\mathrm{S} \overset{*}{\Rightarrow} \mathrm{John\ saw\ Mary}$



Contxt-free languages

more exercises / questions

- Define a (non-regular) CFG for language ab^*c

Contxt-free languages

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- Define a (non-regular) CFG for language ab^*c
- Can you define a CFG for $a^n b^n$?

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- Can you define a CFG for $a^n b^n$?
- Can you define a CFG for $a^n b^n c^n$?
- Can you define a CFG for $a^n b^m c^n d^m$?

Context-sensitive grammars

Context-sensitive rules

$$\alpha A \beta \to \alpha \gamma \beta$$

where A is a non-terminal symbol, α and β are possibly empty strings of terminals and non-terminals, and γ is a non-empty string of terminal and non-terminal symbols.

- There is also an alternative definition through non-contracting grammars
- A rule of the form $S \to \epsilon$ is allowed

Context-sensitive grammars

an example

- Can you define a context-sensitive grammar for $a^n b^n c^n$?
- Can you define a context-sensitive grammar for $a^n b^m c^n d^m$?

Unrestricted grammars

- The most expressive class of languages in the Chomsky hierarchy is *recursively enumerable* (RE) languages
- RE languages are those for which there is an algorithm to enumerate all sentences
- RE languages are generated by *unrestricted grammars*
- Unrestricted grammars do not limit the rewrite rules in any way (except LHS cannot be empty)
- Mostly theoretical interest, not much practical use

A(nother) review of computational complexity

Big-O notation

Big-O notation is used for describing *worst-case order of complexity* of algorithms

$O(1)$	constant
$O(\log n)$	logarithmic
$O(n)$	linear
$O(n \log n)$	log linear
$O(n^2)$	quadratic
$O(n^3)$	cubic
$O(2^n)$	exponential
$O(n!)$	factorial

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Given $T(n)$, what is $O(n)$?

- $T(n) = \log(5n)$

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- $T(n) = n + \log n$

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- $T(n) = n^2 + 10$

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- $T(n) = n^2 + 10$
- $T(n) = n^5 + n^4$

A(nother) review of computational complexity

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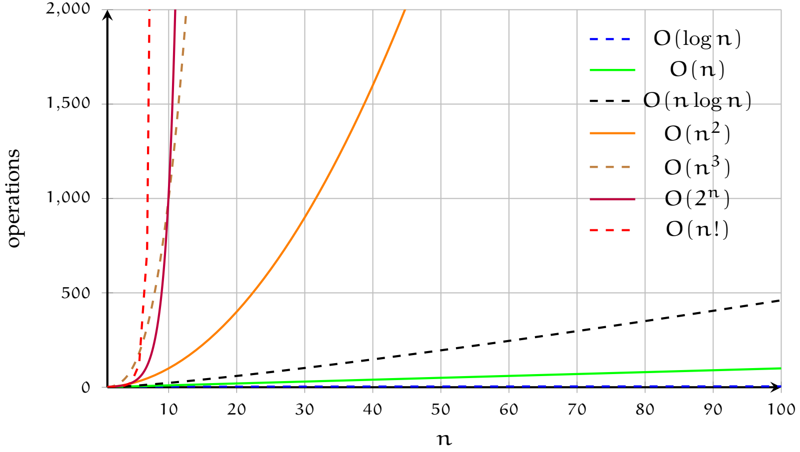
- $T(n) = n^2 + 10$
- $T(n) = n^5 + n^4$
- $T(n) = n^5 + 4^n$
- $T(n) = n! + 2^n$

Big-O notation and order of complexity

the picture



A(nother) review of computational complexity

P, NP, NP-complete and all that

- A major division of complexity classes according to Big-O notation is between

  P polynomial time algorithms

  NP non-deterministic polynomial time algorithms

- A big question in computing is whether P = NP
- All problems in NP can be reduced in polynomial time to a problem in a subclass of NP, (*NP-complete*)
  - Solving an NP complete problem in P would mean proving P = NP

Video from https://www.youtube.com/watch?v=YX40hbAHx3s

Grammars and automata

Language	Grammar	Automata
Regular	Regular	Finite-state
Context-free	Context-free	Push-down
Context-sensitive	Context-sensitive	Linear-bounded
Recursively-enumerable	Unrestricted	Turing machines

RE languages and Turing machines

- Recursively enumerable languages can be generated by *Turing machines*
- Turing machine is an simple model of computation that can compute any computable function
  - A Turing machine manipulates symbols on an infinite tape, using a finite table of rules
- A Turing machine can enumerate all string defined by an unrestricted phrase structure grammar
- The membership problem of RE languages is not decidable

Context-sensitive languages and LBA

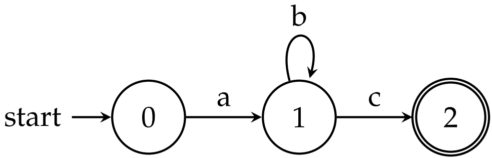
- Context-sensitive languages can be generated using a restricted form of Turing machine, called *linear-bounded automata*
- Although decidable, recognition of a string with a context-sensitive grammar is computationally intractable (PSPACE-complete)

Context-free languages and pushdown automata

- Context-free languages are recognized by *pushdown automata*
- Pushdown automata consist of a finite-state control mechanism and a stack
- Computationally feasible solutions exists for many problems related to context-free grammars
- There are polynomial time algorithms for recognizing strings of context-free languages (we will return to these in lectures on parsing)

Regular languages and FSA

- Regular languages can be recognized using *finite-state automata* (FSA)
- A FSA consist of a finite set of states with directed edges between them
- Edges are labeled with the terminal symbols, and tell the automation to which state to move on a given input symbol



Chomsky hierarchy and natural language syntax

Where do natural languages fit?

- The class of grammars adequate for formally describing natural languages has been an important question for (computational) linguistics
- For the most part, context-free grammars are enough, but there are some examples, e.g., from Swiss German (Shieber 1985)

  Jan säit das…

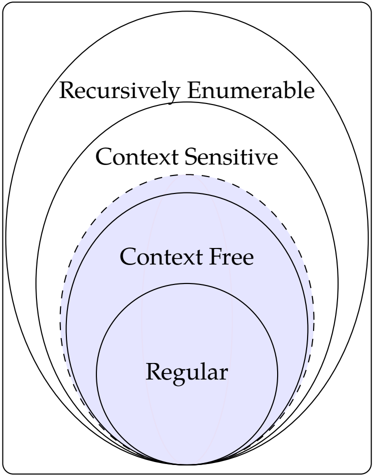
…mer	em Hans	es huss	hälfed	aastriiche
…we	Hans (DAT)	the house (ACC)	helped	paint

Note that this resembles $a^n b^m c^n d^m$.

Where do natural languages fit?

the picture

- Often a superset of CF languages, *mildly context-sensitive languages* are considered adequate



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the picture

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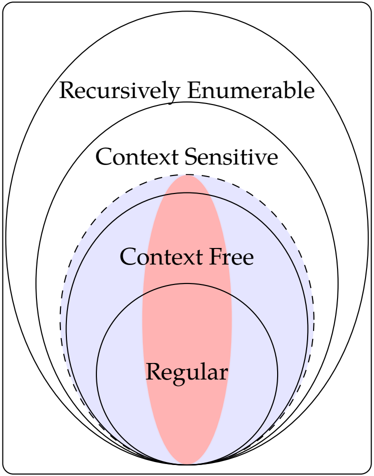
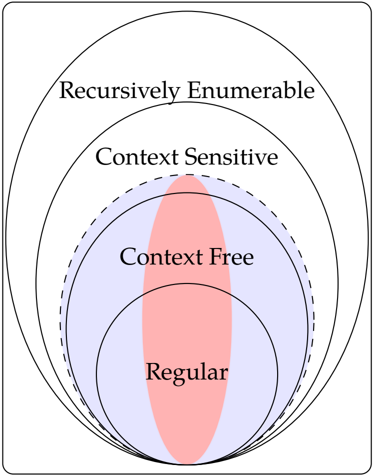
- Note, though, we do not even need full RE expressivity



Where do natural languages fit?

the picture

- Often a superset of CF languages, *mildly context-sensitive languages* are considered adequate
- Note, though, we do not even need full RE expressivity
- Modern/computational theories of grammars range from mildly CS (TAG, CCG) to Turing complete (HPSG, LFG?)



Learnability natural languages

language acquisition & nature vs. nurture

- A central question in linguistics have been about 'learnability' of the languages
- Some linguists claim that natural languages are not learnable, hence, humans born with a innate *language acquisition device*
- A poplar theory of the *language acquisition device* is called *principles and parameters*
- This has created a long-lasting debate, which is also related to even longer-lasting debate on nature vs. nurture

Formal languages and learnability

- Some of the arguments in the learnability debate has been based on results on formal languages
- It is shown (Gold 1967) that none of the languages in the Chomsky hierarchy are learnable from positive input
- The applicability of such results to human language acquisition is questionable
- Computational modeling/experiments may help here (another job for computational linguists)

Wrapping up

- Formal languages has a central role in the theory of computation, as well as in formal/computational linguistics
- Practically-useful classes of languages in Chomsky hierarchy is regular and context-free languages (we will return to these in more detail)
- Natural language syntax can be described mostly by CFGs

Wrapping up

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- Natural language syntax can be described mostly by CFGs

Next:

- Finite state automata

References / additional reading material

- The classic reference for theory of computation is Hopcroft and Ullman (1979) (and its successive editions)
- Sipser (2006) is another good textbook on the topic
- A popular nativist account of language acquisition debate is Pinker (1994)
- A popular non-nativist (somewhat empiricist) book on language acquisition is Clark and Lappin (2011), which also covers discussion of (Gold 1967) and later work

References / additional reading material (cont.)

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