

Data Structures and Algorithms III

Formal languages and automata

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Seminar für Sprachwissenschaft

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Practical matters

The second part of the course will be somewhat different:

- The focus will shift more towards Computational Linguistics topics / applications
- We will review more specialized data structures and algorithms (e.g., automata, parsing)
- Some overlap with parsing class (but with more emphasis on practical sides)
- Less focus on programming

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A quick poll: opinions about switching to Python.

An overview of the upcoming topics

- Background on formal languages and automata (today)
- Finite state automata and regular languages
- Finite state transducers (FST)
 - FSTs and computational morphology
- Dependency grammars and dependency parsing
- Context-free grammars and constituency parsing

Assignments

- Assignment policy is similar to the first part of the course
- Two graded assignments:
 - Finite state methods (due early Jan)
 - Parsing (due mid Feb)
- There will be more ungraded assignments – they are part of the course work, they are not ‘optional’

This lecture

An overview

- Background: some definitions on phrase structure grammars and rewrite rules
- Chomsky hierarchy of (formal) language classes
- Background: computational complexity
- Automata, their relation to formal languages
- Formal languages and automata in natural language processing
- A brief note on learnability of natural languages

Why study formal languages

- Formal languages are an important area of the theory of computation
- They originate from linguistics, and they have been used in formal/computational linguistics

Definitions

Alphabet

- An *alphabet* is a set of symbols
- We generally denote an alphabet using the symbol Σ
- In our examples, we will use lowercase ASCII letters for the individual symbols, e.g., $\Sigma = \{a, b, c\}$
- Alphabet does not match the every-day use:
 - In some cases one may want to use a binary alphabet, $\Sigma = \{0, 1\}$
 - If we want to define a grammar for arithmetic operations, we may want to have $\Sigma = \{0, 1, 2, 3, \dots, 9, +, -, \times, /\}$
 - If we are interested in natural language syntax our alphabet is the set of natural language words, $\Sigma = \{\text{the, on, cat, dog, mat, sat, } \dots\}$

Definitions

Strings

- A *string* over an alphabet is a finite sequence symbols from the alphabet
 - $a, ab, acbcaa$ are example strings over $\Sigma = \{a, b, c\}$
- The *empty string* is denoted by ϵ
- The Σ^* denotes all strings that can be formed using alphabet Σ , including the empty string ϵ
- The Σ^+ is a shorthand for $\Sigma^* - \epsilon$
- Similarly a^* means the symbol a repeated zero or more times, a^+ means a repeated one or more times
- We use a^n for exactly n repetitions of a
- The length of a string u is denoted by $|u|$, e.g., $|abc| = 3$, or if $u = aabbcc$, $|u| = 6$
- Concatenation of two string u and v is denoted by uv , e.g., for $u = ab$ and $v = ca$, $uv = abca$

Definitions

Language

- A (formal) language is a set of string over an alphabet
 - The set of strings of length 2 over $\{0, 1\}$:
 $\{00, 01, 10, 11\}$
 - The set of strings with even number of 1's over $\{0, 1\}$:
 $\{\epsilon, 101, 0, 11, 111110, \dots\}$
 - The set of string that retain alphabetical ordering over $\{a, b, c\}$:
 $\{a, ab, abc, ac, abcc, \dots\}$
 - The set of strings of words that form grammatically correct English sentences
- Strings that are member of a language is called *sentences* (or sometimes *words*) of the language

Definitions

Grammar

- A *grammar* is a finite description of a language
- A common way of specifying a grammar is based on a set of *rewrite rules* (or *phrase structure rules*)
- We represent *non-terminal symbols* with uppercase letters
- We represent *terminal symbols* with lowercase letters
- S is the *start symbol*
- If a string can be generated from S using the rewrite rules, the string is a valid sentence in the language

$$S \rightarrow A B$$
$$S \rightarrow S A B$$
$$A \rightarrow a$$
$$B \rightarrow b$$

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$$S \rightarrow AB$$

$$S \rightarrow SAB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

Q: What does this grammar define?

Definitions

Phrase structure grammars: more formally

A phrase structure grammar is a tuple $G = (\Sigma, N, S, R)$ where

Σ is an alphabet of terminal symbols

N are a set of non-terminal symbols

S is a special 'start' symbol $\in N$

R is a set of rules of the form

$$\alpha \rightarrow \beta$$

where α and β are strings from $\Sigma \cup N$

A string u is in the language defined by G ,
if it can be derived from S .

Definitions

Grammars and derivations

Grammar

$$S \rightarrow AB$$
$$S \rightarrow SAB$$
$$A \rightarrow a$$
$$B \rightarrow b$$

Definitions

Grammars and derivations

Grammar

$$S \rightarrow AB$$
$$S \rightarrow SAB$$
$$A \rightarrow a$$
$$B \rightarrow b$$

Derivation of abab

$$S \Rightarrow SAB$$

Definitions

Grammars and derivations

Grammar

$$S \rightarrow AB$$
$$S \rightarrow SAB$$
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$$S \Rightarrow SAB$$
$$SAB \Rightarrow ABAB$$

Definitions

Grammars and derivations

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Grammars and derivations

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$$S \rightarrow SAB$$
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Derivation of abab

$$S \Rightarrow SAB$$
$$aBAB \Rightarrow abAB$$
$$SAB \Rightarrow ABAB$$
$$ABAB \Rightarrow aBAB$$

Definitions

Grammars and derivations

Grammar

$$S \rightarrow AB$$

$$S \rightarrow SAB$$

$$A \rightarrow a$$

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Derivation of abab

$$S \Rightarrow SAB$$

$$aBAB \Rightarrow abAB$$

$$SAB \Rightarrow ABAB$$

$$abAB \Rightarrow abaB$$

$$ABAB \Rightarrow aBAB$$

Definitions

Grammars and derivations

Grammar

$$S \rightarrow AB$$

$$S \rightarrow SAB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

Derivation of abab

$$S \Rightarrow SAB \qquad aBAB \Rightarrow abAB$$

$$SAB \Rightarrow ABAB \qquad abAB \Rightarrow abaB$$

$$ABAB \Rightarrow aBAB \qquad abaB \Rightarrow abab$$

Definitions

Grammars and derivations

Grammar

$$\begin{aligned} S &\rightarrow AB \\ S &\rightarrow SAB \\ A &\rightarrow a \\ B &\rightarrow b \end{aligned}$$

Derivation of abab

$$\begin{aligned} S &\Rightarrow SAB & aBAB &\Rightarrow abAB \\ SAB &\Rightarrow ABAB & abAB &\Rightarrow abaB \\ ABAB &\Rightarrow aBAB & abaB &\Rightarrow abab \end{aligned}$$

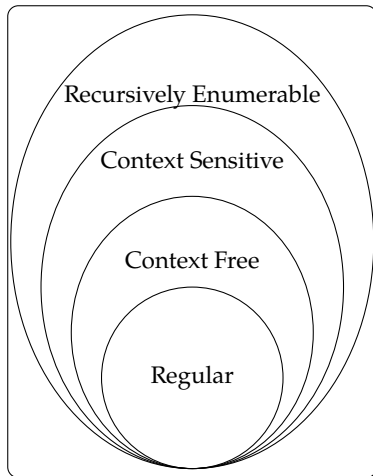
- Intermediate strings of terminals and non-terminals are called *sentential forms*
- $S \xRightarrow{*} abab$: the string is in the language

Q: What if string was not in the language?

Q: Is there another derivation sequence?

Chomsky hierarchy of (formal) languages

- Defined for formalizing natural language syntax
- Definitions are in terms of the restrictions on production rules of the grammar
- Also part of theory of computation
- Each language class corresponds to a class of (abstract) machines
- Other well-studied classes exist



Regular grammars

Left regular

1. $A \rightarrow a$
2. $A \rightarrow Ba$
3. $A \rightarrow \epsilon$

Right regular

1. $A \rightarrow a$
2. $A \rightarrow aB$
3. $A \rightarrow \epsilon$

- Least expressive, but easy to process
- Used in many NLP applications
- Defines the set of languages expressed by *regular expressions*
- Regular grammars define only regular languages (but reverse is not true)
- We will discuss it in more detail soon

Regular grammars

an example

Write a right- and a left-regular
grammar ab^*c

Regular grammars

an example

Write a right- and a left-regular grammar ab^*c

left

$$S \rightarrow Ac$$

$$A \rightarrow Ab$$

$$A \rightarrow a$$

right

$$S \rightarrow aA$$

$$A \rightarrow bA$$

$$A \rightarrow c$$

Regular grammars

an example

Write a right- and a left-regular grammar ab^*c

left	right
$S \rightarrow Ac$	$S \rightarrow aA$
$A \rightarrow Ab$	$A \rightarrow bA$
$A \rightarrow a$	$A \rightarrow c$

Can you define a regular grammar for

- $a^n b^n$?
- $a^5 b^5$?

Regular grammars

an example

Write a right- and a left-regular grammar ab^*c

Derive the string $abbbc$ using one of your grammars

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Regular grammars

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Derive the string $abbbc$ using one of your grammars

left

$$S \Rightarrow Ac \Rightarrow Abc \Rightarrow Abbc \Rightarrow Abbbc \Rightarrow abbbc$$

right

$$S \Rightarrow aA \Rightarrow abA \Rightarrow abbA \Rightarrow abbbA \Rightarrow abbbc$$

Regular grammars

an example

Write a right- and a left-regular grammar ab^*c

left

$$\begin{aligned} S &\rightarrow Ac \\ A &\rightarrow Ab \\ A &\rightarrow a \end{aligned}$$

right

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow bA \\ A &\rightarrow c \end{aligned}$$

Can you define a regular grammar for

- $a^n b^n$?
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Derive the string $abbbc$ using one of your grammars

left

$$S \Rightarrow Ac \Rightarrow Abc \Rightarrow Abbc \Rightarrow Abbbc \Rightarrow abbbc$$

right

$$S \Rightarrow aA \Rightarrow abA \Rightarrow abbA \Rightarrow abbbA \Rightarrow abbbc$$

These grammars are *weakly equivalent*: they generate the same language, but derivations differ

Context-free grammars (CFG)

CFG rules

$$A \rightarrow \alpha$$

where A is a *single* non-terminal α is a possibly empty sequence of terminals and non-terminals

- More expressive than regular languages
- Syntax of programming languages are based on CFGs
- Many applications for natural languages too (more on this later)

Context-free grammars

an example

The example grammar:

Example CFG

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$NP \rightarrow \text{John} \mid \text{Mary}$

$V \rightarrow \text{saw}$

Exercise: derive 'John saw Mary'

Context-free grammars

an example

The example grammar:

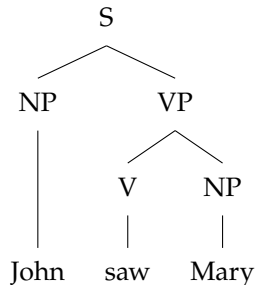
Example CFG

$S \rightarrow NP VP$	$VP \rightarrow V NP$
$NP \rightarrow John \mid Mary$	$V \rightarrow saw$

Exercise: derive 'John saw Mary'

Derivation

$S \Rightarrow NP VP \Rightarrow John VP$
 $\Rightarrow John V NP \Rightarrow John saw NP$
 $\Rightarrow John saw Mary$
 or, $S \xRightarrow{*} John saw Mary$



Context-free languages

more exercises / questions

- Define a (non-regular) CFG for language ab^*c

Context-free languages

more exercises / questions

- Define a (non-regular) CFG for language ab^*c
- Can you define a CFG for $a^n b^n$?

Context-free languages

more exercises / questions

- Define a (non-regular) CFG for language ab^*c
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- Can you define a CFG for $a^n b^n c^n$?

Context-free languages

more exercises / questions

- Define a (non-regular) CFG for language ab^*c
- Can you define a CFG for $a^n b^n$?
- Can you define a CFG for $a^n b^n c^n$?
- Can you define a CFG for $a^n b^m c^n d^m$?

Context-sensitive grammars

Context-sensitive rules

$$\alpha A \beta \rightarrow \alpha \gamma \beta$$

where A is a non-terminal symbol, α and β are possibly empty strings of terminals and non-terminals, and γ is a non-empty string of terminal and non-terminal symbols.

- There is also an alternative definition through non-contracting grammars
- A rule of the form $S \rightarrow \epsilon$ is allowed

Context-sensitive grammars

an example

- Can you define a context-sensitive grammar for $a^n b^n c^n$?
- Can you define a context-sensitive grammar for $a^n b^m c^n d^m$?

Unrestricted grammars

- The most expressive class of languages in the Chomsky hierarchy is *recursively enumerable* (RE) languages
- RE languages are those for which there is an algorithm to enumerate all sentences
- RE languages are generated by *unrestricted grammars*
- Unrestricted grammars do not limit the rewrite rules in any way (except LHS cannot be empty)
- Mostly theoretical interest, not much practical use

A(nother) review of computational complexity

Big-O notation

Big-O notation is used for describing *worst-case order of complexity* of algorithms

$O(1)$ constant

$O(\log n)$ logarithmic

$O(n)$ linear

$O(n \log n)$ log linear

$O(n^2)$ quadratic

$O(n^3)$ cubic

$O(2^n)$ exponential

$O(n!)$ factorial

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Given $T(n)$, what is $O(n)$?

- $T(n) = \log(5n)$

A(nother) review of computational complexity

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Given $T(n)$, what is $O(n)$?

- $T(n) = \log(5n)$
- $T(n) = 5n$
- $T(n) = n + \log n$

A(nother) review of computational complexity

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- $T(n) = n + \log n$
- $T(n) = n^2 + 10$

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- $T(n) = n^5 + n^4$

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- $T(n) = n^5 + 4^n$

A(nother) review of computational complexity

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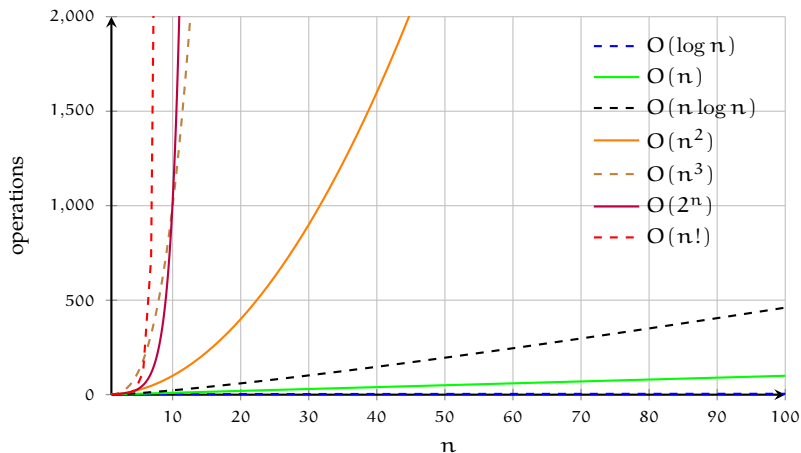
$O(n!)$ factorial

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- $T(n) = n + \log n$
- $T(n) = n^2 + 10$
- $T(n) = n^5 + n^4$
- $T(n) = n^5 + 4^n$
- $T(n) = n! + 2^n$

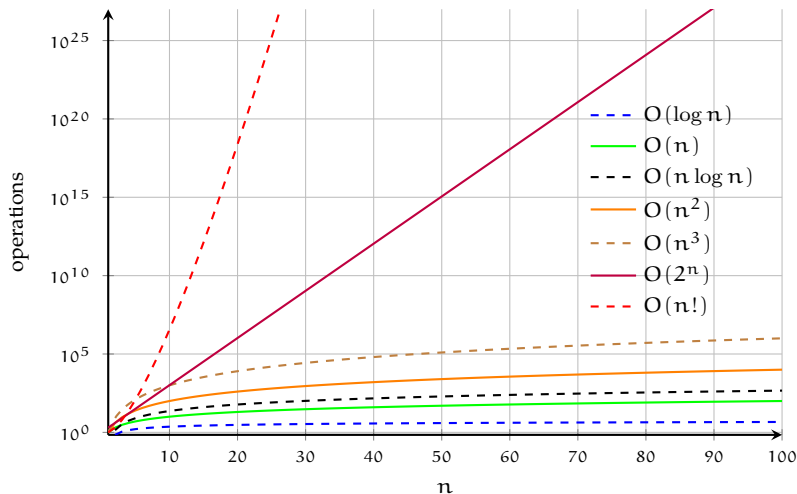
Big-O notation and order of complexity

the picture



Big-O notation and order of complexity

the picture (with log y-axis)



A(nother) review of computational complexity

P, NP, NP-complete and all that

- A major division of complexity classes according to Big-O notation is between
 - P polynomial time algorithms
 - NP non-deterministic polynomial time algorithms
- A big question in computing is whether $P = NP$
- All problems in NP can be reduced in polynomial time to a problem in a subclass of NP, (*NP-complete*)
 - Solving an NP complete problem in P would mean proving $P = NP$

Video from <https://www.youtube.com/watch?v=YX40hbAHx3s>

Grammars and automata

Language	Grammar	Automata
Regular	Regular	Finite-state
Context-free	Context-free	Push-down
Context-sensitive	Context-sensitive	Linear-bounded
Recursively-enumerable	Unrestricted	Turing machines

RE languages and Turing machines

- Recursively enumerable languages can be generated by *Turing machines*
- Turing machine is a simple model of computation that can compute any computable function
 - A Turing machine manipulates symbols on an infinite tape, using a finite table of rules
- A Turing machine can enumerate all strings defined by an unrestricted phrase structure grammar
- The membership problem of RE languages is not decidable

Context-sensitive languages and LBA

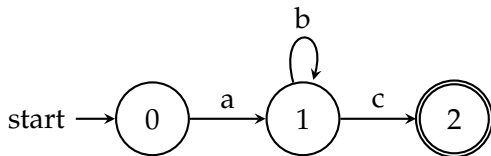
- Context-sensitive languages can be generated using a restricted form of Turing machine, called *linear-bounded automata*
- Although decidable, recognition of a string with a context-sensitive grammar is computationally intractable (PSPACE-complete)

Context-free languages and pushdown automata

- Context-free languages are recognized by *pushdown automata*
- Pushdown automata consist of a finite-state control mechanism and a stack
- Computationally feasible solutions exist for many problems related to context-free grammars
- There are polynomial time algorithms for recognizing strings of context-free languages (we will return to these in lectures on parsing)

Regular languages and FSA

- Regular languages can be recognized using *finite-state automata* (FSA)
- A FSA consist of a finite set of states with directed edges between them
- Edges are labeled with the terminal symbols, and tell the automation to which state to move on a given input symbol

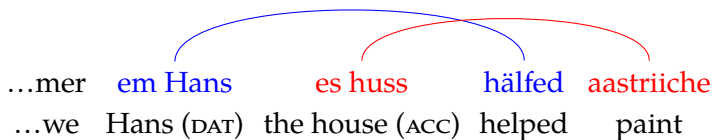


Chomsky hierarchy and natural language syntax

Where do natural languages fit?

- The class of grammars adequate for formally describing natural languages has been an important question for (computational) linguistics
- For the most part, context-free grammars are enough, but there are some examples, e.g., from Swiss German (Shieber 1985)

Jan säit das...

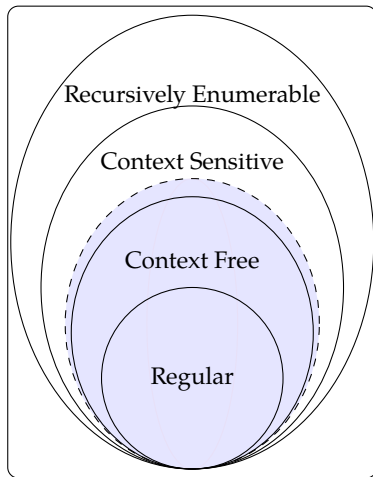


Note that this resembles $a^n b^m c^n d^m$.

Where do natural languages fit?

the picture

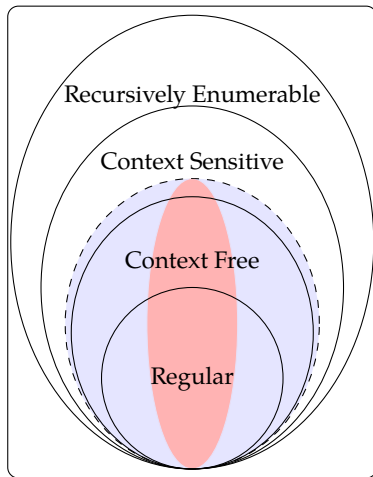
- Often a superset of CF languages, *mildly context-sensitive languages* are considered adequate



Where do natural languages fit?

the picture

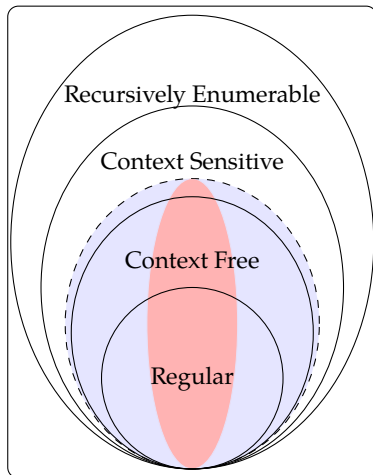
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- Note, though, we do not even need full RE expressivity



Where do natural languages fit?

the picture

- Often a superset of CF languages, *mildly context-sensitive languages* are considered adequate
- Note, though, we do not even need full RE expressivity
- Modern/computational theories of grammars range from mildly CS (TAG, CCG) to Turing complete (HPSG, LFG?)



Learnability natural languages

language acquisition & nature vs. nurture

- A central question in linguistics have been about ‘learnability’ of the languages
- Some linguists claim that natural languages are not learnable, hence, humans born with a innate *language acquisition device*
- A poplar theory of the *language acquisition device* is called *principles and parameters*
- This has created a long-lasting debate, which is also related to even longer-lasting debate on nature vs. nurture

Formal languages and learnability

- Some of the arguments in the learnability debate has been based on results on formal languages
- It is shown (Gold 1967) that none of the languages in the Chomsky hierarchy are learnable from positive input
- The applicability of such results to human language acquisition is questionable
- Computational modeling/experiments may help here (another job for computational linguists)

Wrapping up

- Formal languages has a central role in the theory of computation, as well as in formal/computational linguistics
- Practically-useful classes of languages in Chomsky hierarchy is regular and context-free languages (we will return to these in more detail)
- Natural language syntax can be described mostly by CFGs

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Next:

- Finite state automata

References / additional reading material

- The classic reference for theory of computation is Hopcroft and Ullman (1979) (and its successive editions)
- Sipser (2006) is another good textbook on the topic
- A popular nativist account of language acquisition debate is Pinker (1994)
- A popular non-nativist (somewhat empiricist) book on language acquisition is Clark and Lappin (2011), which also covers discussion of (Gold 1967) and later work

References / additional reading material (cont.)

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