# Data Structures and Algorithms III

## Formal languages and automata

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Seminar für Sprachwissenschaft

Winter Semester 2018–2019



## Practical matters

The second part of the course will be somewhat different:

- The focus will shift more towards Computational Linguistics topics / applications
- We will review more specialized data structures and algorithms (e.g., automata, parsing)
- Some overlap with parsing class (but with more emphasis on practical sides)
- Less focus on programming

A quick poll: opinions about switching to Python.





## An overview of the upcoming topics

- Background on formal languages and automata (today)
- Finite state automata and regular languages
- Finite state transducers (FST)
  - FSTs and computational morphology
- Dependency grammars and dependency parsing
- Context-free grammars and constituency parsing





## Assignments

- Assignment policy is similar to the first part of the course
- Two graded assignments:
  - Finite state methods (due early Jan)
  - Parsing (due mid Feb)
- There will be more ungraded assignments – they are part of the course work, they are not 'optional'





## This lecture

### An overview

- Background: some definitions on phrase structure grammars and rewrite rules
- Chomsky hierarchy of (formal) language classes
- Background: computational complexity
- Automata, their relation to formal languages
- Formal languages and automata in natural language processing
- A brief note on learnability of natural languages





## Why study formal languages

- Formal languages are an important area of the theory of computation
- They originate from linguistics, and they have been used in formal/computational linguistics





## Definitions

### Alphabet

- An *alphabet* is a set of symbols
- We generally denote an alphabet using the symbol $\Sigma$
- In our examples, we will use lowercase ASCII letters for the individual symbols, e.g., $\Sigma = \{a, b, c\}$
- Alphabet does not match the every-day use:
  - In some cases one may want to use a binary alphabet, $\Sigma = \{0, 1\}$
  - If we want to define a grammar for arithmetic operations, we may want to have $\Sigma = \{0, 1, 2, 3, \ldots, 9, +, -, \times, /\}$
  - If we are interested in natural language syntax our alphabet is the set of natural language words, $\Sigma = \{\mathrm{the}, \mathrm{on}, \mathrm{cat}, \mathrm{dog}, \mathrm{mat}, \mathrm{sat}, \ldots\}$





## Definitions

### Strings

- A *string* over an alphabet is a finite sequence symbols from the alphabet
  - $a$, $ab$, $acbcaa$ are example strings over $\Sigma = \{a, b, c\}$
- The *empty string* is denoted by $\epsilon$
- The $\Sigma^*$ denotes all strings that can be formed using alphabet $\Sigma$, including the empty string $\epsilon$
- The $\Sigma^+$ is a shorthand for $\Sigma^* - \epsilon$
- Similarly $a^*$ means the symbol $a$ repeated zero or more times, $a+$ means $a$ repeated one or more times
- We use $a^n$ for exactly $n$ repetitions of $a$
- The length of a string $u$ is denoted by $|u|$, e.g., $|abc| = 3$, or if $u = aabbcc$, $|u| = 6$
- Concatenation of two string $u$ and $v$ is denoted by $uv$, e.g., for $u = ab$ and $v = ca$, $uv = abca$

## Definitions
Language

- A (formal) language is a set of string over an alphabet
  - The set of strings of length 2 over $\{0, 1\}$: $\{00, 01, 10, 11\}$
  - The set of strings with even number of 1's over $\{0, 1\}$: $\{\epsilon, 101, 0, 11, 111110, \ldots\}$
  - The set of string that retain alphabetical ordering over $\{a, b, c\}$: $\{a, ab, abc, ac, abcc, \ldots\}$
  - The set of strings of words that form grammatically correct English sentences
- Strings that are member of a language is called *sentences* (or sometimes *words*) of the language





## Definitions
Grammar

- A *grammar* is a finite description of a language
- A common way of specifying a grammar is based on a set of *rewrite rules* (or *phrase structure rules*)
- We represent *non-terminal symbols* with uppercase letters
- We represent *terminal symbols* with lowercase letters
- S is the *start symbol*
- If a string can be generated from S using the rewrite rules, the string is a valid sentence in the language

$S \rightarrow A\,B$
$S \rightarrow S\,A\,B$
$A \rightarrow a$
$B \rightarrow b$

Q: What does this grammar define?





## Definitions
Phrase structure grammars: more formally

A phrase structure grammar is a tuple $G = (\Sigma, N, S, R)$ where

- $\Sigma$ is an alphabet of terminal symbols
- $N$ are a set of non-terminal symbols
- $S$ is a special 'start' symbol $\in N$
- $R$ is a set of rules of the form

$$\alpha \rightarrow \beta$$

where $\alpha$ and $\beta$ are strings from $\Sigma \cup N$

A string $u$ is in the language defined by $G$, if it can be derived from $S$.





## Definitions
Grammars and derivations

Grammar

$S \rightarrow A\,B$
$S \rightarrow S\,A\,B$
$A \rightarrow a$
$B \rightarrow b$

Derivation of $abab$

| | |
|---|---|
| $S \Rightarrow SAB$ | $aBAB \Rightarrow abAB$ |
| $SAB \Rightarrow ABAB$ | $abAB \Rightarrow abaB$ |
| $ABAB \Rightarrow aBAB$ | $abaB \Rightarrow abab$ |

- Intermediate strings of terminals and non-terminals are called *sentential forms*
- $S \overset{*}{\Rightarrow} abab$: the string is in the language

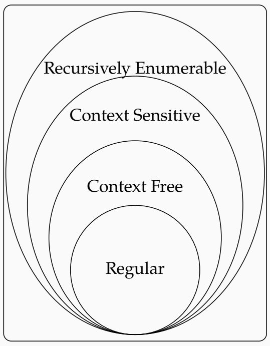
Q: What if string was not in the language?
Q: Is there another derivation sequence?





## Chomsky hierarchy of (formal) languages

- Defined for formalizing natural language syntax
- Definitions are in terms of the restrictions on production rules of the grammar
- Also part of theory of computation
- Each language class corresponds to a class of (abstract) machines
- Other well-studied classes exist








## Regular grammars

Left regular

1. $A \rightarrow a$
2. $A \rightarrow Ba$
3. $A \rightarrow \epsilon$

Right regular

1. $A \rightarrow a$
2. $A \rightarrow aB$
3. $A \rightarrow \epsilon$

- Least expressive, but easy to process
- Used in many NLP applications
- Defines the set of languages expressed by *regular expressions*
- Regular grammars define only regular languages (but reverse is not true)
- We will discuss it in more detail soon





## Regular grammars
an example

Write a right- and a left-regular grammar $ab^*c$

left

$S \rightarrow Ac$
$A \rightarrow Ab$
$A \rightarrow a$

right

$S \rightarrow aA$
$A \rightarrow bA$
$A \rightarrow c$

Can you define a regular grammar for

- $a^n b^n$?
- $a^5 b^5$?

Derive the string $abbbc$ using one of your grammars

left

$S \Rightarrow Ac \Rightarrow Abc \Rightarrow Abbc \Rightarrow Abbbc \Rightarrow abbbc$

right

$S \Rightarrow aA \Rightarrow abA \Rightarrow abbA \Rightarrow abbbA \Rightarrow abbbc$

These grammars are *weakly equivalent*: they generate the same language, but derivations differ





## Context-free grammars (CFG)

CFG rules

$$A \rightarrow \alpha$$

where $A$ is a *single* non-terminal $\alpha$ is a possibly empty sequence of terminals and non-terminals

- More expressive than regular languages
- Syntax of programming languages are based on CFGs
- Many applications for natural languages too (more on this later)

# Context-free grammars

an example

The example grammar:

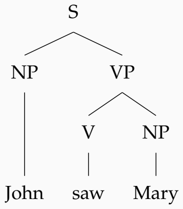
**Example CFG**

| | | | | | |
|---|---|---|---|---|---|
| S | $\rightarrow$ | NP VP | VP | $\rightarrow$ | V NP |
| NP | $\rightarrow$ | John \| Mary | V | $\rightarrow$ | saw |

Exercise: derive 'John saw Mary'

**Derivation**

S $\Rightarrow$NP VP $\Rightarrow$John VP
$\Rightarrow$John V NP $\Rightarrow$John saw NP
$\Rightarrow$John saw Mary
or, S $\overset{*}{\Rightarrow}$John saw Mary








# Contxt-free languages

more exercises / questions

- Define a (non-regular) CFG for language $ab^*c$
- Can you define a CFG for $a^nb^n$?
- Can you define a CFG for $a^nb^nc^n$?
- Can you define a CFG for $a^nb^mc^nd^m$?





# Context-sensitive grammars

**Context-sensitive rules**

$$\alpha A \beta \rightarrow \alpha \gamma \beta$$

where $A$ is a non-terminal symbol, $\alpha$ and $\beta$ are possibly empty strings of terminals and non-terminals, and $\gamma$ is a non-empty string of terminal and non-terminal symbols.

- There is also an alternative definition through non-contracting grammars
- A rule of the form $S \rightarrow \epsilon$ is allowed





# Context-sensitive grammars

an example

- Can you define a context-sensitive grammar for $a^nb^nc^n$?
- Can you define a context-sensitive grammar for $a^nb^mc^nd^m$?





# Unrestricted grammars

- The most expressive class of languages in the Chomsky hierarchy is *recursively enumerable* (RE) languages
- RE languages are those for which there is an algorithm to enumerate all sentences
- RE languages are generated by *unrestricted grammars*
- Unrestricted grammars do not limit the rewrite rules in any way (except LHS cannot be empty)
- Mostly theoretical interest, not much practical use





# A(nother) review of computational complexity

Big-O notation

*Big-O notation* is used for describing *worst-case order of complexity* of algorithms

| | |
|---|---|
| $O(1)$ | constant |
| $O(\log n)$ | logarithmic |
| $O(n)$ | linear |
| $O(n \log n)$ | log linear |
| $O(n^2)$ | quadratic |
| $O(n^3)$ | cubic |
| $O(2^n)$ | exponential |
| $O(n!)$ | factorial |

Given $T(n)$, what is $O(n)$?

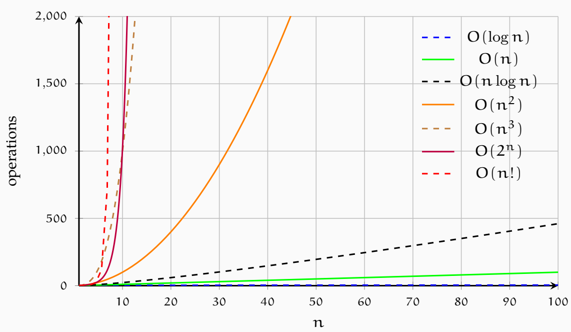
- $T(n) = \log(5n)$
- $T(n) = 5n$
- $T(n) = n + \log n$
- $T(n) = n^2 + 10$
- $T(n) = n^5 + n^4$
- $T(n) = n^5 + 4^n$
- $T(n) = n! + 2^n$





# Big-O notation and order of complexity

the picture








# A(nother) review of computational complexity

P, NP, NP-complete and all that

- A major division of complexity classes according to Big-O notation is between
  - P polynomial time algorithms
  - NP non-deterministic polynomial time algorithms
- A big question in computing is whether P = NP
- All problems in NP can be reduced in polynomial time to a problem in a subclass of NP, (*NP-complete*)
  - Solving an NP complete problem in P would mean proving P = NP

Video from `https://www.youtube.com/watch?v=YX40hbAHx3s`

## Grammars and automata

| Language | Grammar | Automata |
|---|---|---|
| Regular | Regular | Finite-state |
| Context-free | Context-free | Push-down |
| Context-sensitive | Context-sensitive | Linear-bounded |
| Recursively-enumerable | Unrestricted | Turing machines |





## RE languages and Turing machines

- Recursively enumerable languages can be generated by *Turing machines*
- Turing machine is an simple model of computation that can compute any computable function
  - A Turing machine manipulates symbols on an infinite tape, using a finite table of rules
- A Turing machine can enumerate all string defined by an unrestricted phrase structure grammar
- The membership problem of RE languages is not decidable





## Context-sensitive languages and LBA

- Context-sensitive languages can be generated using a restricted form of Turing machine, called *linear-bounded automata*
- Although decidable, recognition of a string with a context-sensitive grammar is computationally intractable (PSPACE-complete)





## Context-free languages and pushdown automata

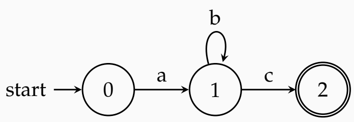
- Context-free languages are recognized by *pushdown automata*
- Pushdown automata consist of a finite-state control mechanism and a stack
- Computationally feasible solutions exists for many problems related to context-free grammars
- There are polynomial time algorithms for recognizing strings of context-free languages (we will return to these in lectures on parsing)





## Regular languages and FSA

- Regular languages can be recognized using *finite-state automata* (FSA)
- A FSA consist of a finite set of states with directed edges between them
- Edges are labeled with the terminal symbols, and tell the automation to which state to move on a given input symbol








## Chomsky hierarchy and natural language syntax

### Where do natural languages fit?

- The class of grammars adequate for formally describing natural languages has been an important question for (computational) linguistics
- For the most part, context-free grammars are enough, but there are some examples, e.g., from Swiss German (Shieber 1985)
  Jan säit das…

| …mer | em Hans | es huss | hälfed | aastriiche |
|---|---|---|---|---|
| …we | Hans (DAT) | the house (ACC) | helped | paint |

Note that this resembles $a^n b^m c^n d^m$.

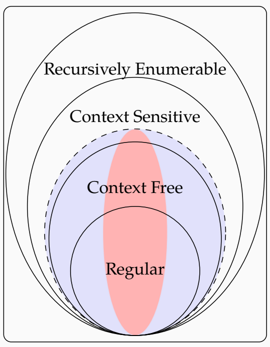




## Where do natural languages fit?

### the picture

- Often a superset of CF languages, *mildly context-sensitive languages* are considered adequate
- Note, though, we do not even need full RE expressivity
- Modern/computational theories of grammars range from mildly CS (TAG, CCG) to Turing complete (HPSG, LFG?)








## Learnability natural languages

### language acquisition & nature vs. nurture

- A central question in linguistics have been about ‘learnability’ of the languages
- Some linguists claim that natural languages are not learnable, hence, humans born with a innate *language acquisition device*
- A poplar theory of the *language acquisition device* is called *principles and parameters*
- This has created a long-lasting debate, which is also related to even longer-lasting debate on nature vs. nurture

## Formal languages and learnability

- Some of the arguments in the learnability debate has been based on results on formal languages
- It is shown (Gold 1967) that none of the languages in the Chomsky hierarchy are learnable from positive input
- The applicability of such results to human language acquisition is questionable
- Computational modeling/experiments may help here (another job for computational linguists)





## Wrapping up

- Formal languages has a central role in the theory of computation, as well as in formal/computational linguistics
- Practically-useful classes of languages in Chomsky hierarchy is regular and context-free languages (we will return to these in more detail)
- Natural language syntax can be described mostly by CFGs

Next:

- Finite state automata





## References / additional reading material

- The classic reference for theory of computation is Hopcroft and Ullman (1979) (and its successive editions)
- Sipser (2006) is another good textbook on the topic
- A popular nativist account of language acquisition debate is Pinker (1994)
- A popular non-nativist (somewhat empiricist) book on language acquisition is Clark and Lappin (2011), which also covers discussion of (Gold 1967) and later work





## References / additional reading material (cont.)


- Clark, Alexander and Shalom Lappin (2011). *Linguistic Nativism and the Poverty of the Stimulus*. Oxford: Wiley-Blackwell. ISBN: 978-1-4051-8785-5.
- Gold, E. Mark (1967). "Language identification in the limit". In: *Information and Control* 10.5, pp. 447–474.
- Hopcroft, John E. and Jeffrey D. Ullman (1979). *Introduction to Automata Theory, Languages, and Computation*. Addison-Wesley Series in Computer Science and Information Processing. Addison-Wesley. ISBN: 9780201029888.
- Pinker, Steven (1994). *The language instinct: the new science of language and mind*. Penguin Books.
- Shieber, Stuart M. (1985). "Evidence against the context-freeness of natural language". In: *Linguistics and Philosophy* 8.3, pp. 333–343. DOI: 10.1007/BF00630917.
- Sipser, Michael (2006). *Introduction to the Theory of Computation*. second. Thomson Course Technology. ISBN: 0-534-95097-3.