

http://algs4.cs.princeton.edu

## PRIM'S ALGORITHM DEMO

- Prim's algorithm
- lazy implementation
- eager implementation

## PRIM'S ALGORITHM DEMO

Prim's algorithm

lazy implementation

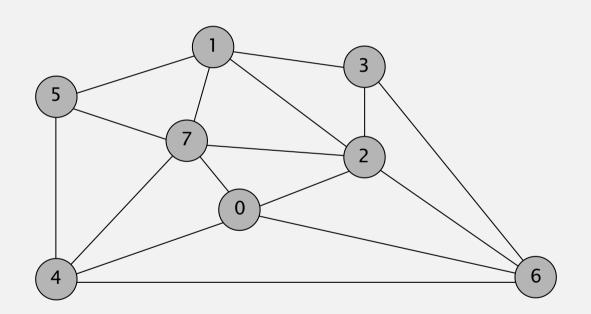
eager implementation



ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



an edge-weighted graph

| 0-7 | 0.10 |
|-----|------|
| 2-3 | 0.17 |
| 1-7 | 0.19 |
| 0-2 | 0.20 |
| 5-7 | 0.28 |
| 1-3 | 0.29 |
| 1-5 | 0.32 |
| 2-7 | 0.34 |
| 4-5 | 0.3  |
| 1-2 | 0.36 |
| 4-7 | 0.37 |
| 0-4 | 0.38 |

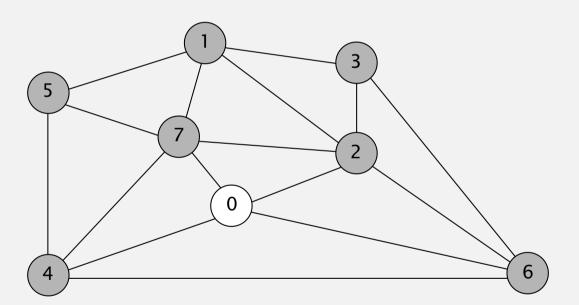
6-2 0.40

3-6 0.52

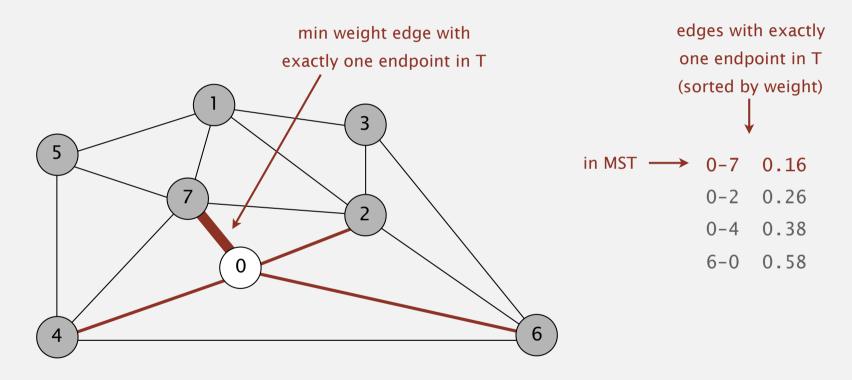
6-0 0.58

6-4 0.93

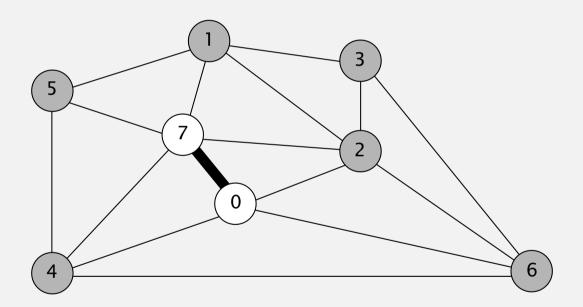
- Start with vertex 0 and greedily grow tree T.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V 1 edges.



- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

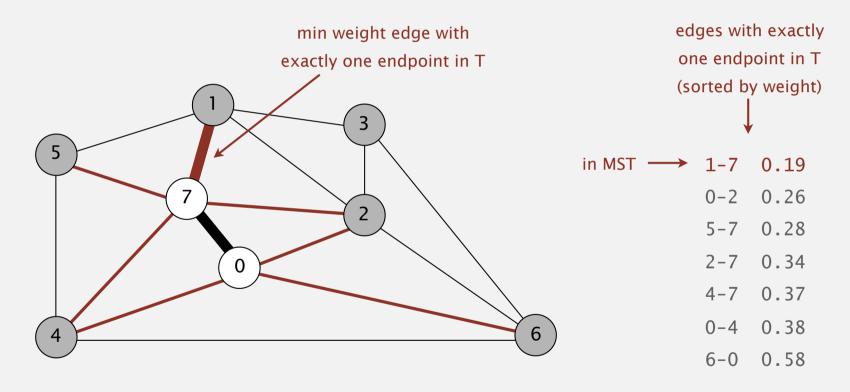


- Start with vertex 0 and greedily grow tree T.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



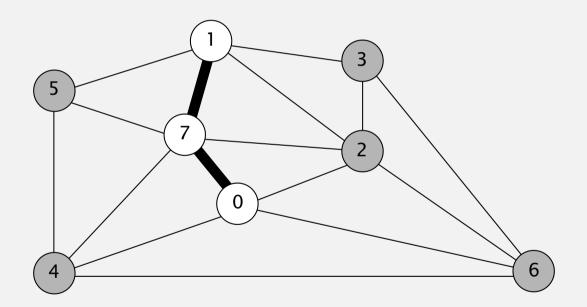
**MST edges** 

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



**MST** edges

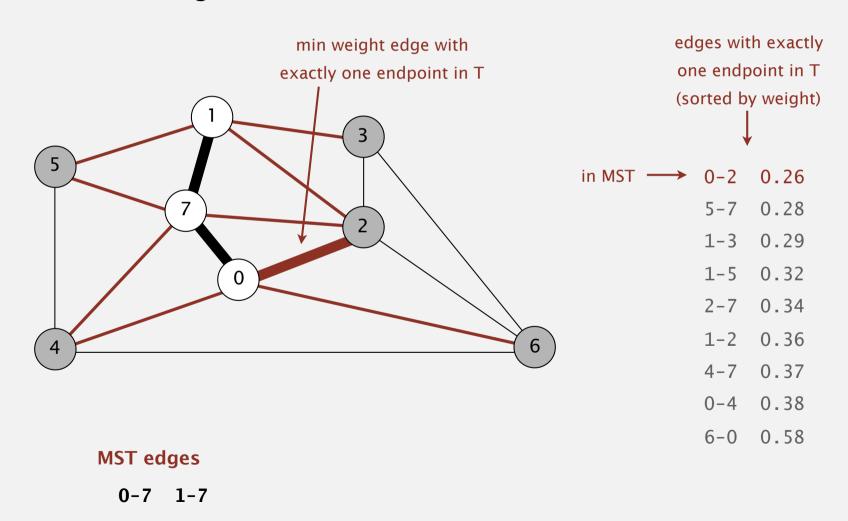
- Start with vertex 0 and greedily grow tree T.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



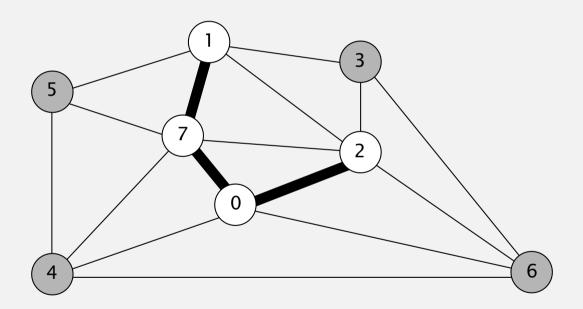
**MST** edges

0-7 1-7

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



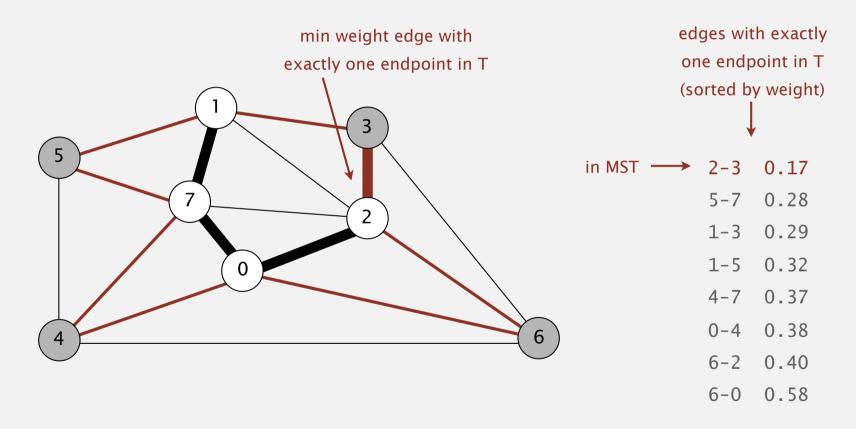
- Start with vertex 0 and greedily grow tree T.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



**MST edges** 

0-7 1-7 0-2

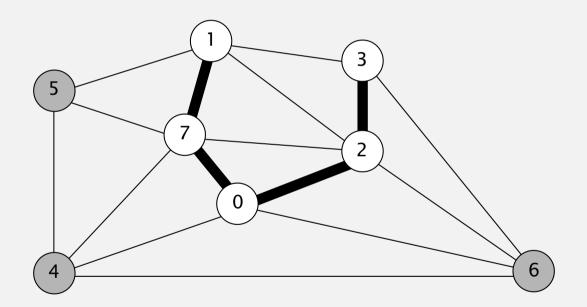
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST** edges

0-7 1-7 0-2

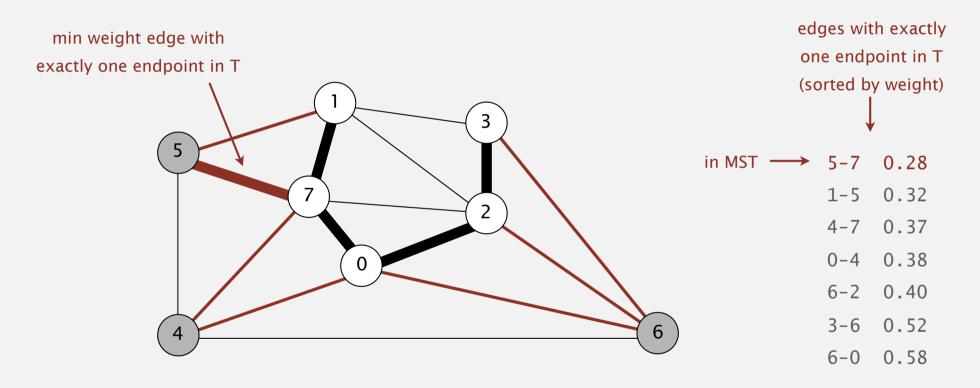
- Start with vertex 0 and greedily grow tree T.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



#### **MST edges**

0-7 1-7 0-2 2-3

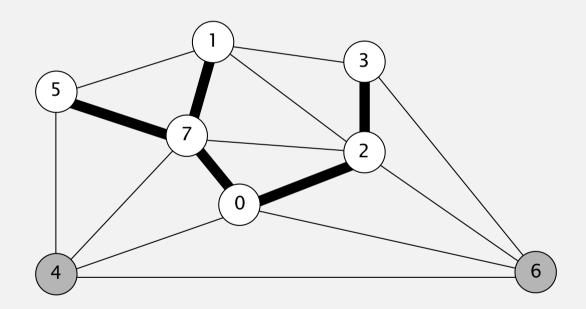
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



**MST** edges

0-7 1-7 0-2 2-3

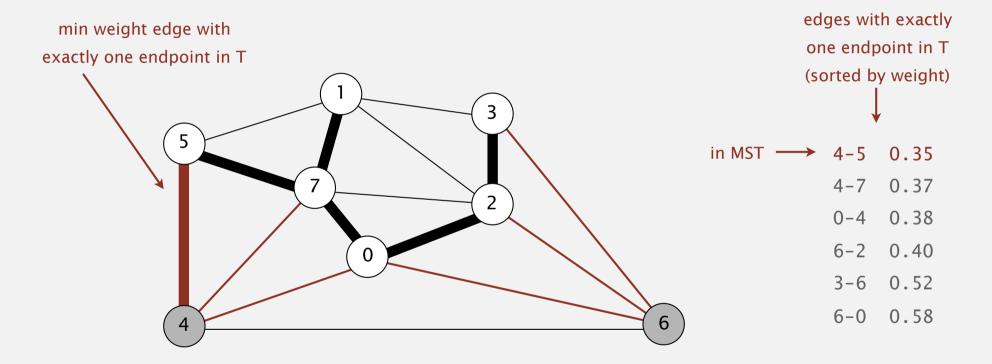
- Start with vertex 0 and greedily grow tree T.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



#### **MST edges**

0-7 1-7 0-2 2-3 5-7

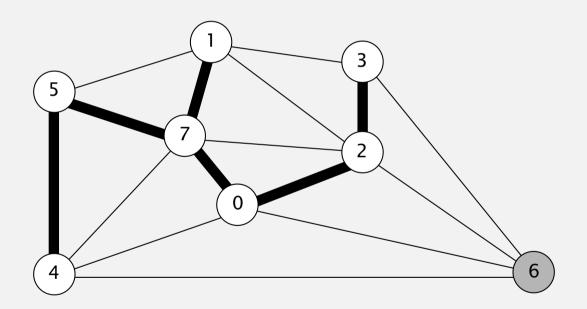
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



#### **MST** edges

0-7 1-7 0-2 2-3 5-7

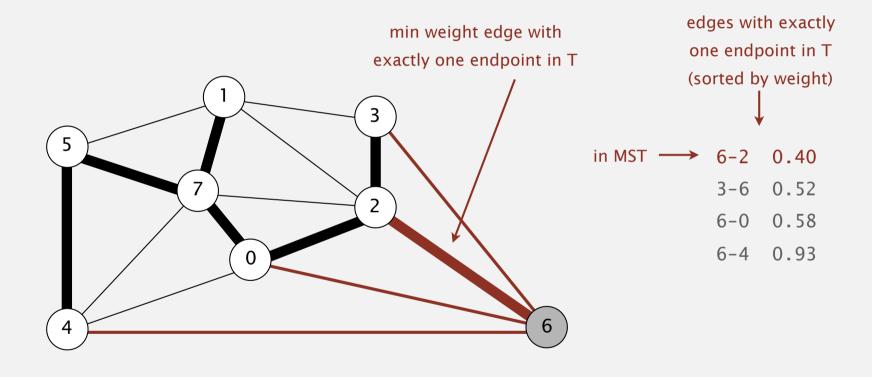
- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



#### **MST edges**

0-7 1-7 0-2 2-3 5-7 4-5

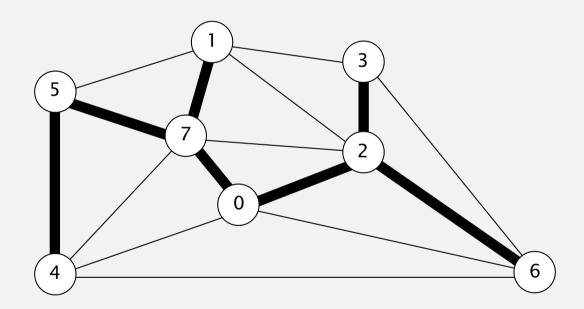
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



#### **MST** edges

0-7 1-7 0-2 2-3 5-7 4-5

- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



#### **MST edges**

0-7 1-7 0-2 2-3 5-7 4-5 6-2

## PRIM'S ALGORITHM DEMO

Prim's algorithm

lazy implementation

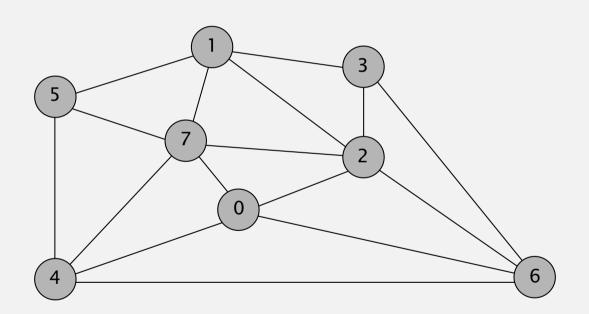
eager implementation

Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

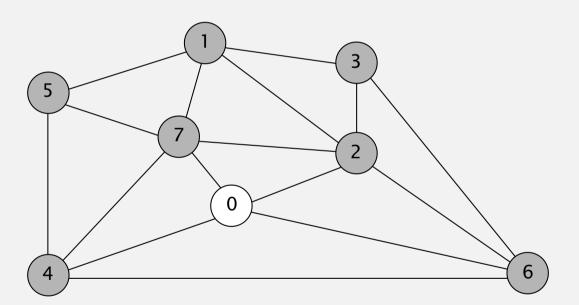
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



an edge-weighted graph

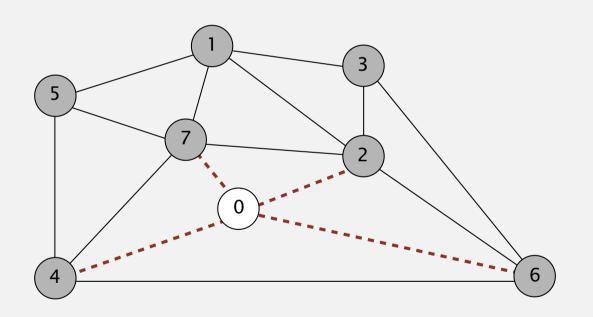
- 0-7 0.16
- 2-3 0.17
- 1-7 0.19
- 0-2 0.26
- 5-7 0.28
- 1-3 0.29
- 1-5 0.32
- 2-7 0.34
- 4-5 0.35
- 1-2 0.36
- 4-7 0.37
- 0-4 0.38
- 6-2 0.40
- 3-6 0.52
- 6-0 0.58
- $6-4 \quad 0.93$

- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

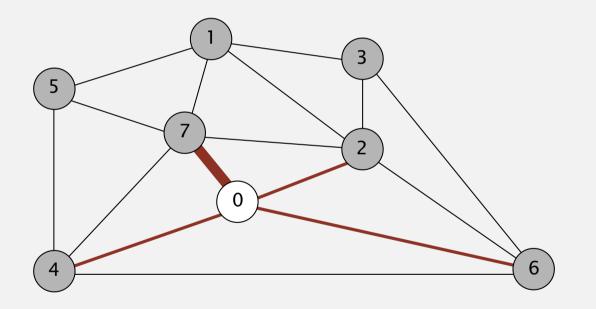
#### add to PQ all edges incident to 0



- \* 0-7 0.16
- \* 0-2 0.26
- \* 0-4 0.38
- **\*** 6-0 0.58

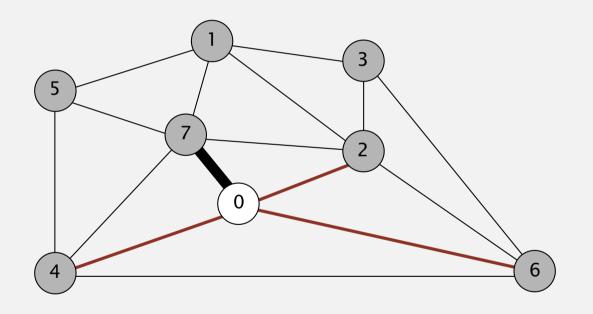
- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.

#### delete 0-7 and add to MST



| edges on PQ        |      |  |
|--------------------|------|--|
| (sorted by weight) |      |  |
| 0-7                | 0.16 |  |
| 0-2                | 0.26 |  |
| 0-4                | 0.38 |  |
| 6-0                | 0.58 |  |

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

0-2 0.26

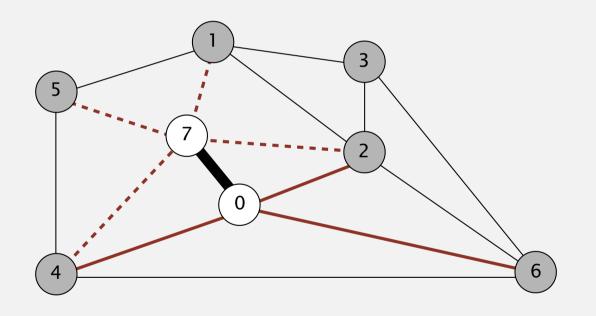
0-4 0.38

6-0 0.58

**MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### add to PQ all edges incident to 7



edges on PQ (sorted by weight)

\* 1-7 0.19

0-2 0.26

**\*** 5-7 0.28

**\*** 2-7 0.34

**\*** 4-7 0.37

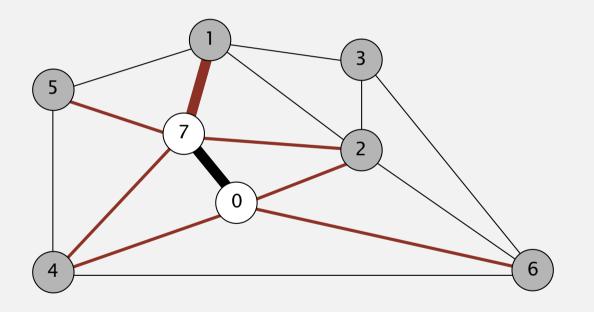
0-4 0.38

6-0 0.58

**MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 1-7 and add to MST



edges on PQ (sorted by weight)

1-7 0.19

0-2 0.26

5-7 0.28

2-7 0.34

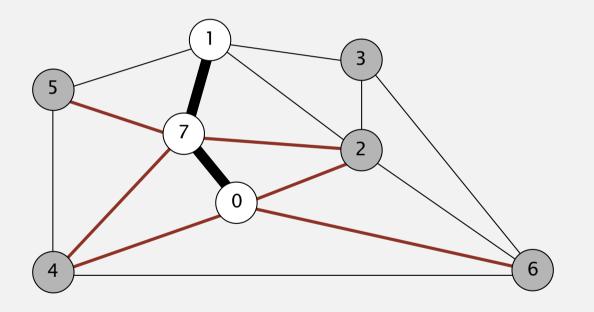
4-7 0.37

0-4 0.38

6-0 0.58

**MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

0-2 0.26

5-7 0.28

2-7 0.34

4-7 0.37

0-4 0.38

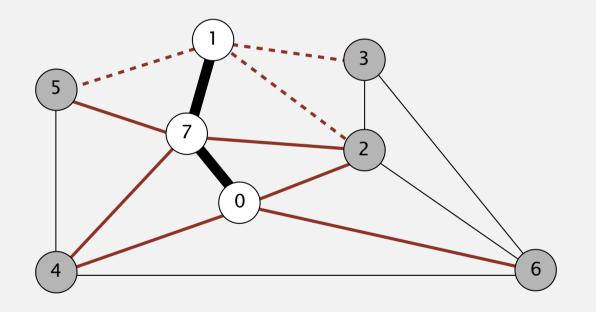
6-0 0.58

**MST** edges

0-7 1-7

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### add to PQ all edges incident to 1



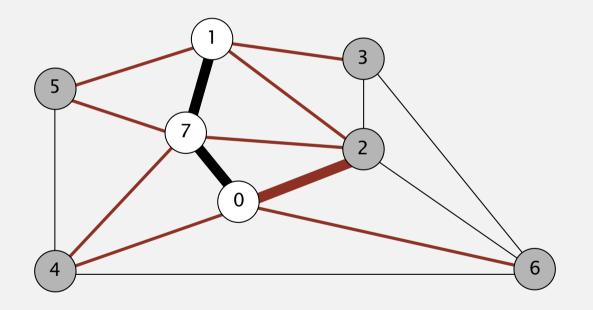
**MST** edges

0-7 1-7

| 0-2  | Λ | . 26         |
|------|---|--------------|
| ()-/ |   | . <i>/</i> n |

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### delete edge 0-2 and add to MST

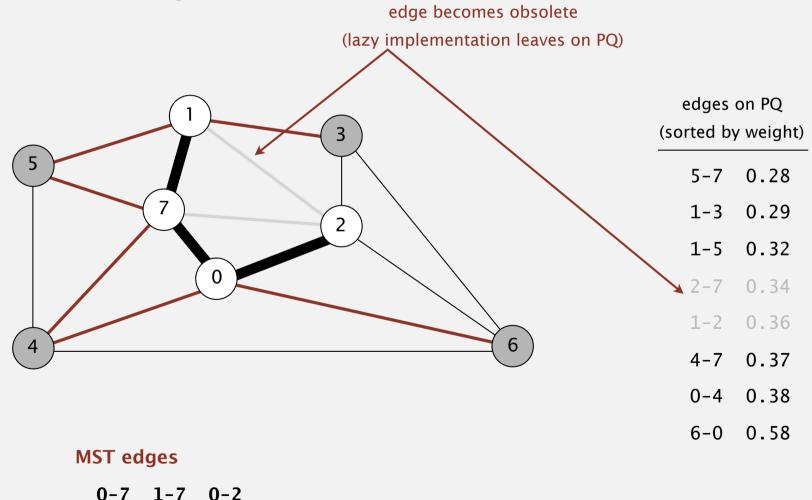


**MST** edges

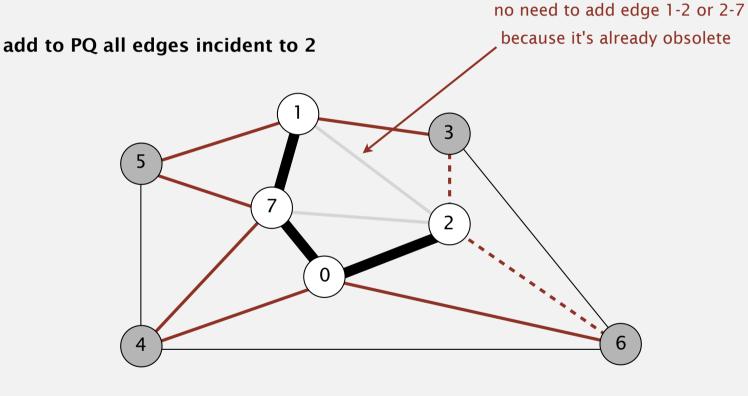
0-7 1-7

## edges on PQ (sorted by weight) 0-2 0.26 5-7 0.28 1-3 0.29 1-5 0.32 2-7 0.34 1-2 0.36 4-7 0.37 0-4 0.386-0 0.58

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



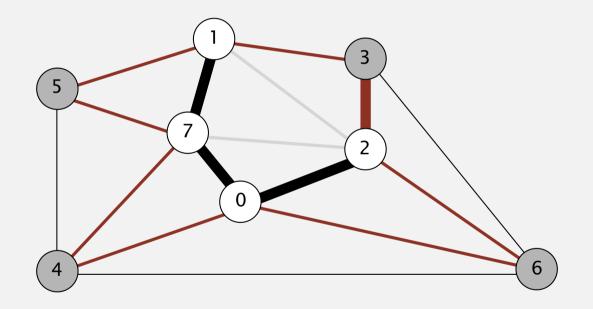
#### **MST** edges

0-7 1-7 0-2

| * | 2-3 | 0 | 17 |
|---|-----|---|----|

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 2-3 and add to MST

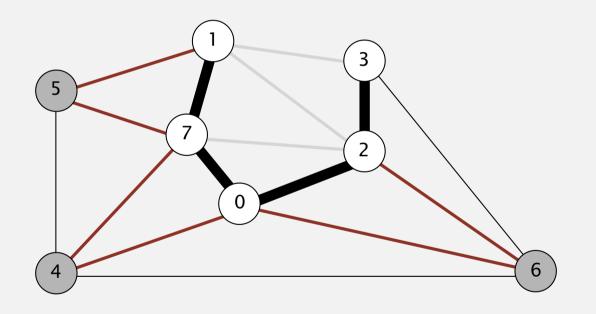


#### **MST** edges

0-7 1-7 0-2

- **\*** 2-3 0.17
  - 5-7 0.28
  - 1-3 0.29
  - 1-5 0.32
  - 2-7 0.34
  - 1-2 0.36
  - 4-7 0.37
  - 0-4 0.38
- \* 6-2 0.40
  - 6-0 0.58

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



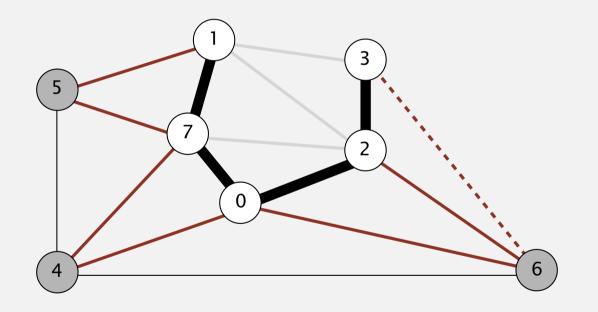
#### **MST** edges

0-7 1-7 0-2 2-3

| ГЭ             | $\wedge$ | . 28 |
|----------------|----------|------|
| <b>\ \ -</b> / | ()       | / X  |
| <i></i>        |          |      |

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### add to PQ all edges incident to 3



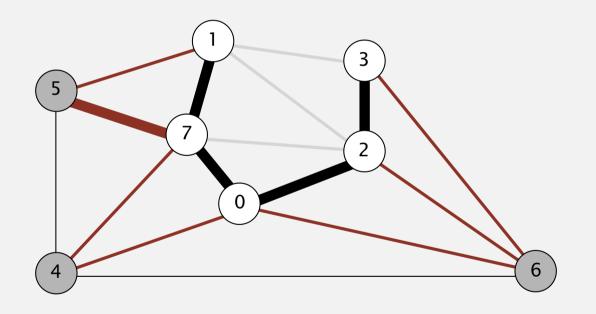
#### **MST** edges

0-7 1-7 0-2 2-3

| ГЭ         | $\sim$ | 28 |
|------------|--------|----|
| 5-7        | ()     | /X |
| <i>J</i> 1 | •      |    |

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### delete 5-7 and add to MST



#### **MST** edges

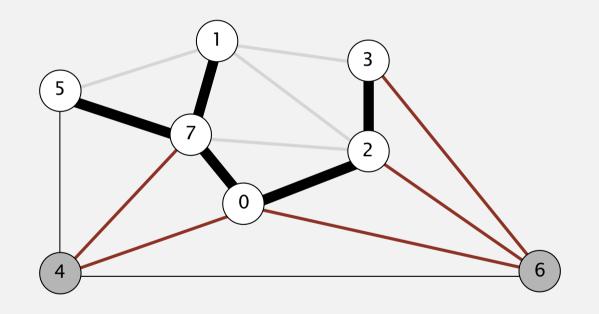
0-7 1-7 0-2 2-3

|     | _  | ~ ~ |
|-----|----|-----|
| 5-/ | 0. | 28  |

$$1-3 \quad 0.29$$

$$6-2$$
 0.40

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



#### **MST** edges

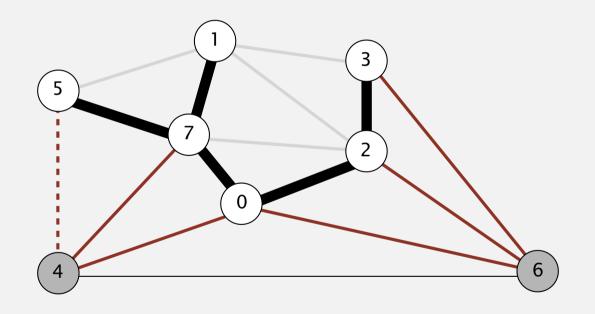
0-7 1-7 0-2 2-3 5-7

| - 1 | -3  |     | 20 |
|-----|-----|-----|----|
|     | - < | ( ) | 79 |

$$6-2$$
 0.40

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### add to PQ all edges incident to 5



#### **MST** edges

0-7 1-7 0-2 2-3 5-7

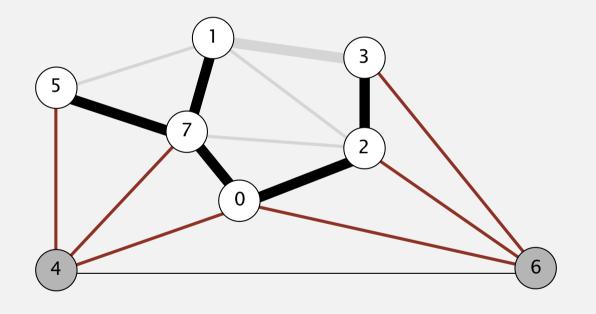
# edges on PQ (sorted by weight)

| 1 | -3 | $\cap$ | 29 |
|---|----|--------|----|

$$1-5$$
 0.32

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### delete 1-3 and discard obsolete edge



#### **MST** edges

0-7 1-7 0-2 2-3 5-7

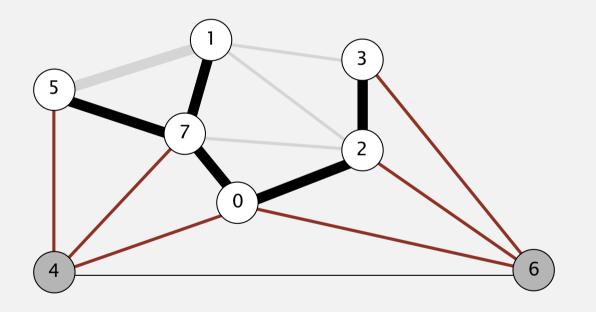
# edges on PQ (sorted by weight)

| - 1 | _ 3 | $\cap$ | 20 |
|-----|-----|--------|----|

$$6-2$$
 0.40

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### delete 1-5 and discard obsolete edge



#### **MST** edges

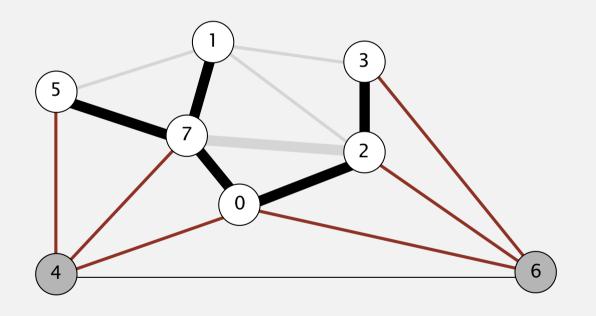
0-7 1-7 0-2 2-3 5-7

# edges on PQ (sorted by weight)

| 1 | _ 5 | $\cap$ | 37 |
|---|-----|--------|----|

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 2-7 and discard obsolete edge



# edges on PQ (sorted by weight)

2-7 0.34

4-5 0.35

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

3-6 0.52

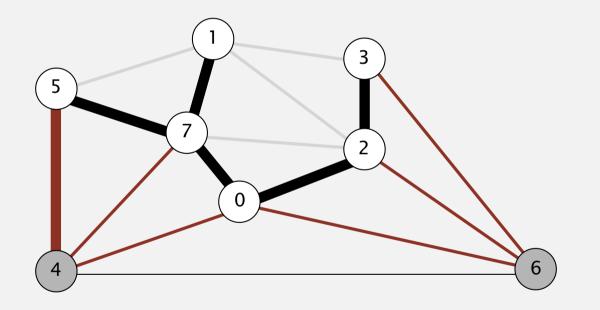
6-0 0.58

#### **MST** edges

0-7 1-7 0-2 2-3 5-7

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### delete 4-5 and add to MST



edges on PQ (sorted by weight)

4-5 0.35

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

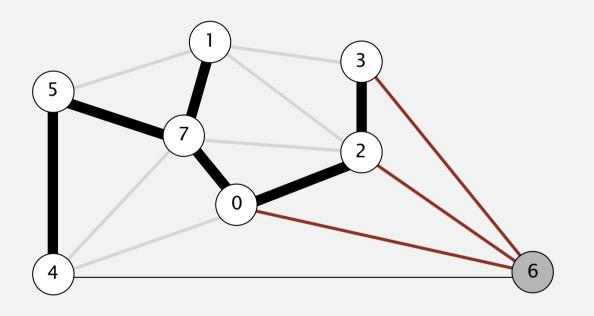
3-6 0.52

6-0 0.58

#### **MST** edges

0-7 1-7 0-2 2-3 5-7

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



# edges on PQ (sorted by weight)

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

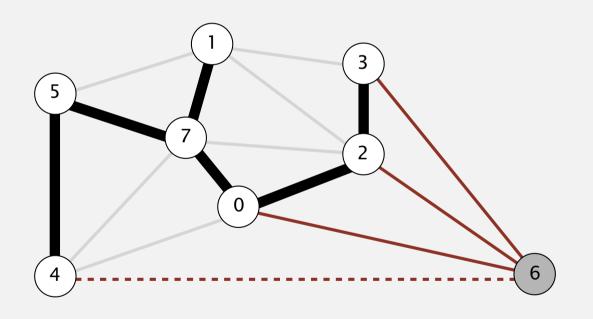
3-6 0.52

6-0 0.58

#### **MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### add to PQ all edges incident to 4



# edges on PQ (sorted by weight)

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

3-6 0.52

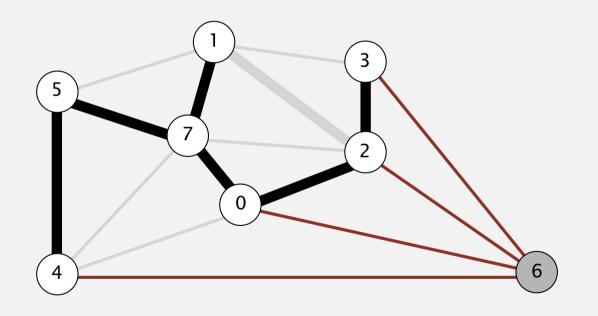
6-0 0.58

**\*** 6-4 0.93

#### **MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 1-2 and discard obsolete edge



## edges on PQ (sorted by weight)

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

3-6 0.52

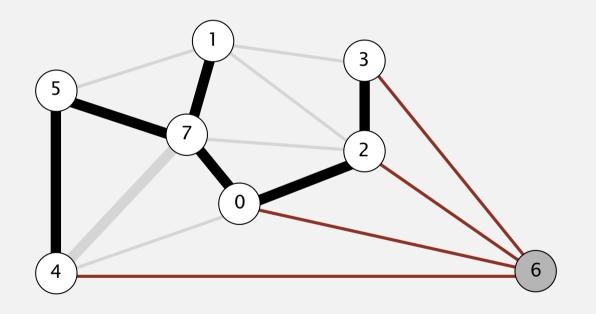
6-0 0.58

6-4 0.93

#### **MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### delete 4-7 and discard obsolete edge



## edges on PQ (sorted by weight)

4-7 0.37

0-4 0.38

6-2 0.40

3-6 0.52

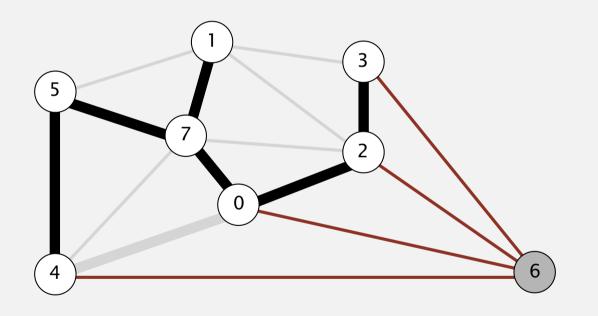
6-0 0.58

6-4 0.93

#### **MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 0-4 and discard obsolete edge



## edges on PQ (sorted by weight)

0-4 0.38

6-2 0.40

3-6 0.52

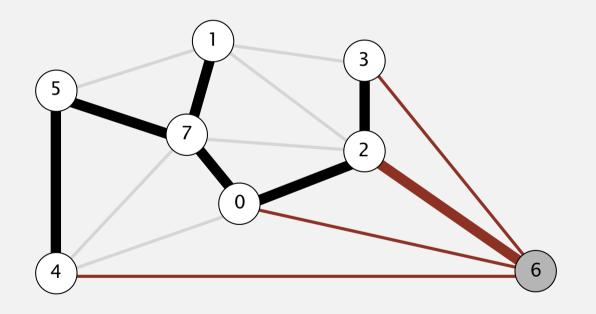
6-0 0.58

6-4 0.93

#### **MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 6-2 and add to MST



edges on PQ (sorted by weight)

6-2 0.40

3-6 0.52

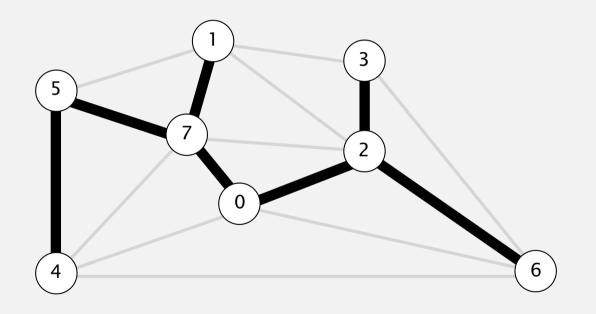
6-0 0.58

6-4 0.93

**MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

#### delete 6-2 and add to MST



edges on PQ (sorted by weight)

3-6 0.52

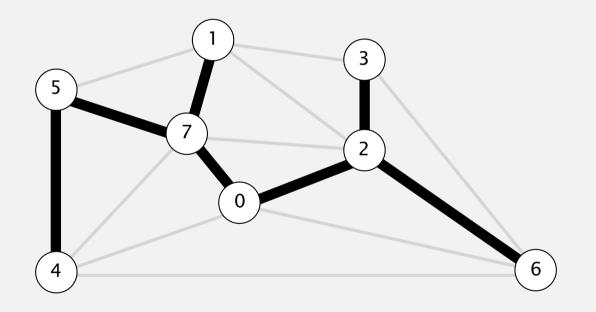
6-0 0.58

 $6-4 \quad 0.93$ 

#### **MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### stop since V-1 edges



# edges on PQ (sorted by weight)

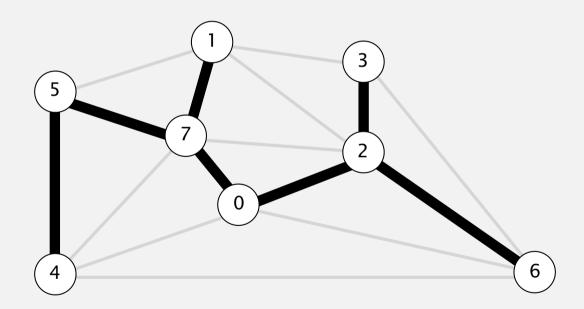
3-6 0.52

6-0 0.58

 $6-4 \quad 0.93$ 

#### **MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



#### **MST edges**

## PRIM'S ALGORITHM DEMO

Prim's algorithm

Jazy implementation

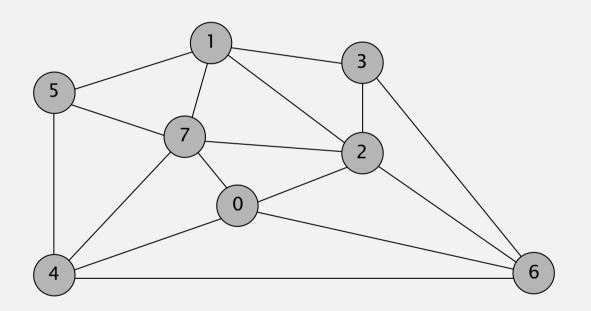
eager implementation



ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

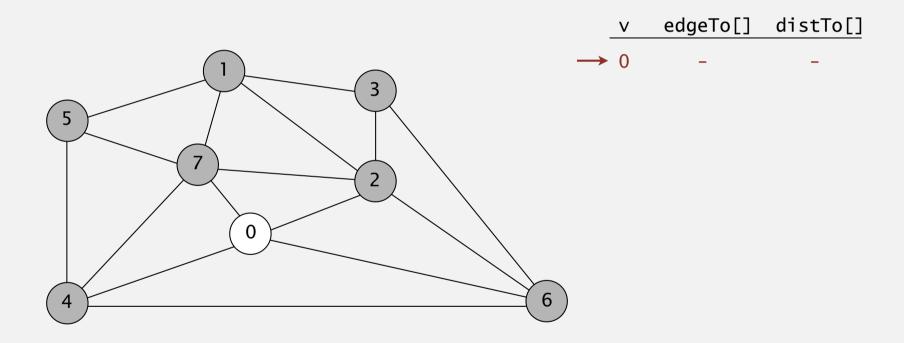
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



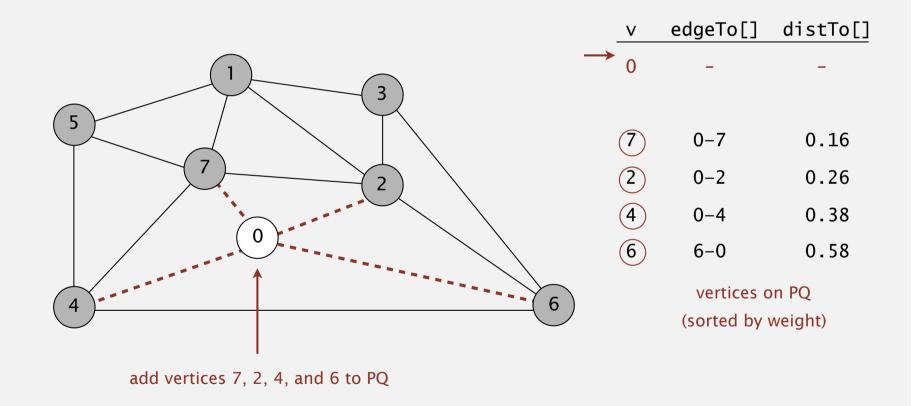
an edge-weighted graph

- 0-7 0.16
- 2-3 0.17
- 1-7 0.19
- 0-2 0.26
- 5-7 0.28
- 1-3 0.29
- 1-5 0.32
- 2-7 0.34
- 4-5 0.35
- 1-2 0.36
- 4-7 0.37
- 0-4 0.38
- 6-2 0.40
- 3-6 0.52
- 6-0 0.58
- 6-4 0.93

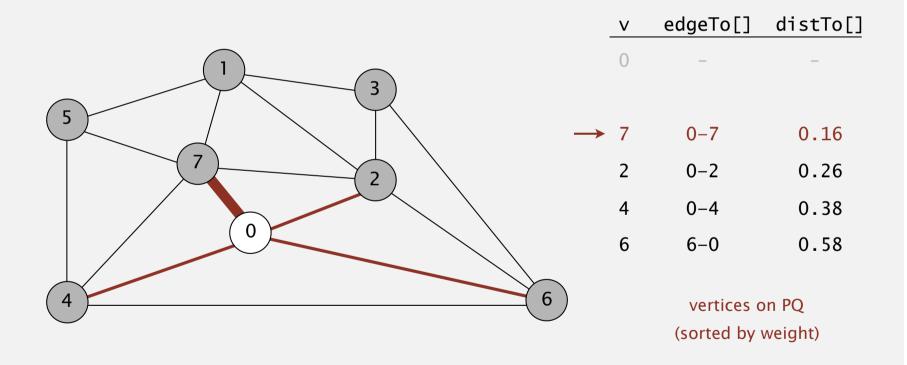
- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



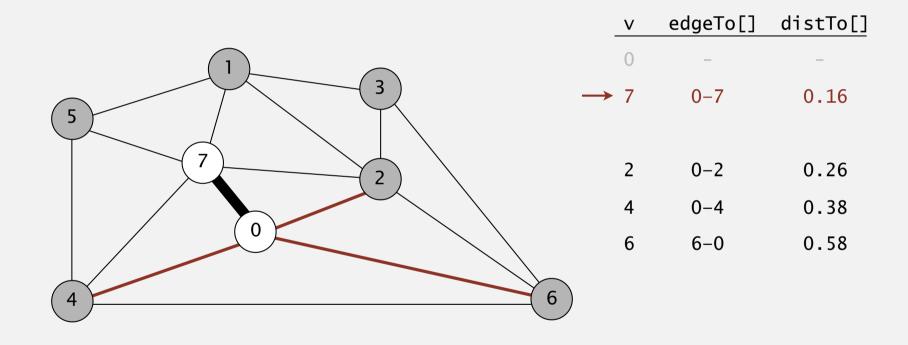
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V 1 edges.



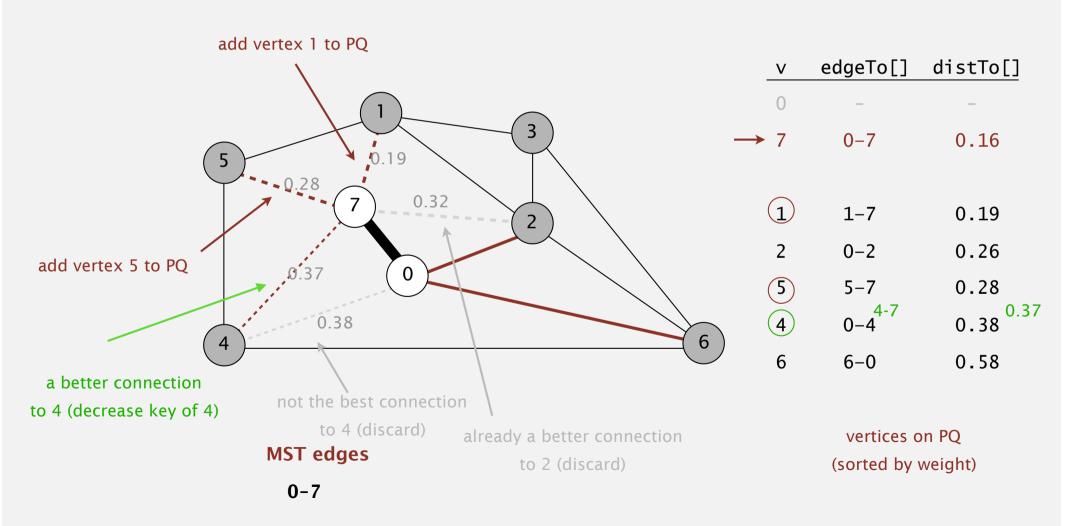
- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



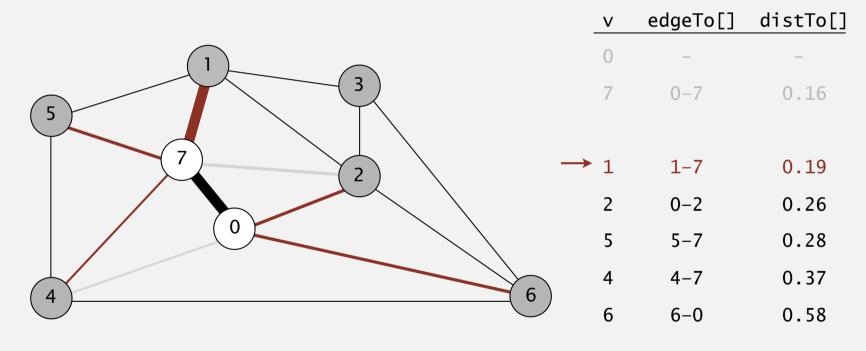
**MST** edges

0-7

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

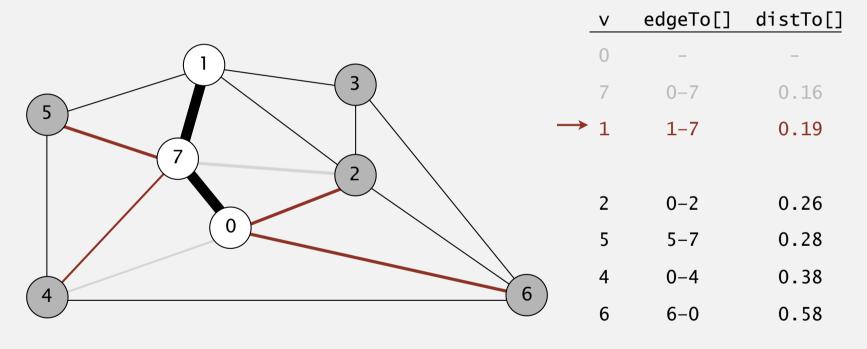


MST edges

0-7 1-7

vertices on PQ (sorted by weight)

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.

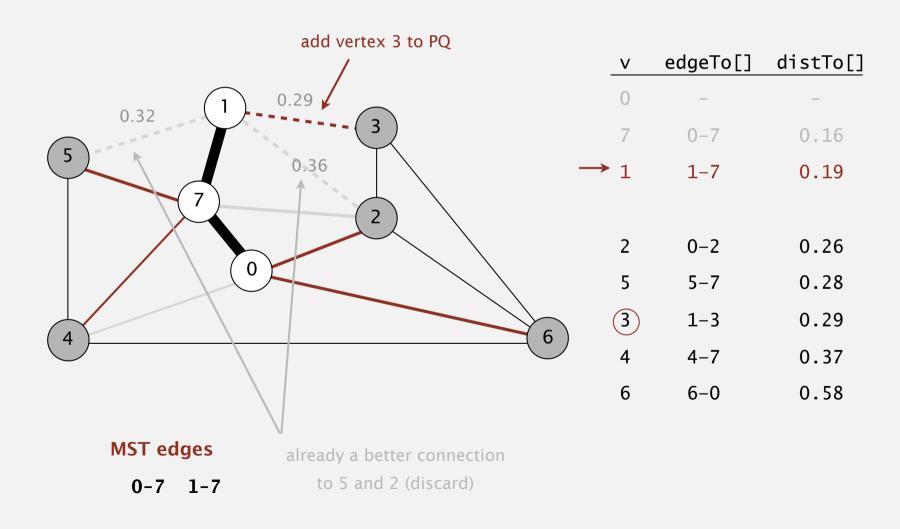


MST edges

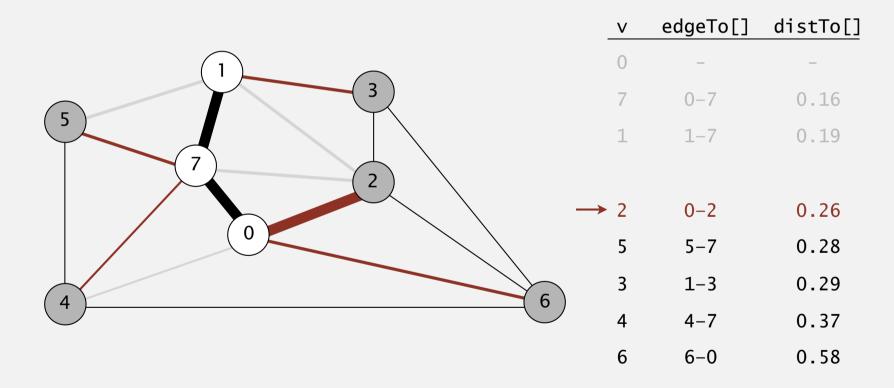
0-7 1-7

vertices on PQ (sorted by weight)

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



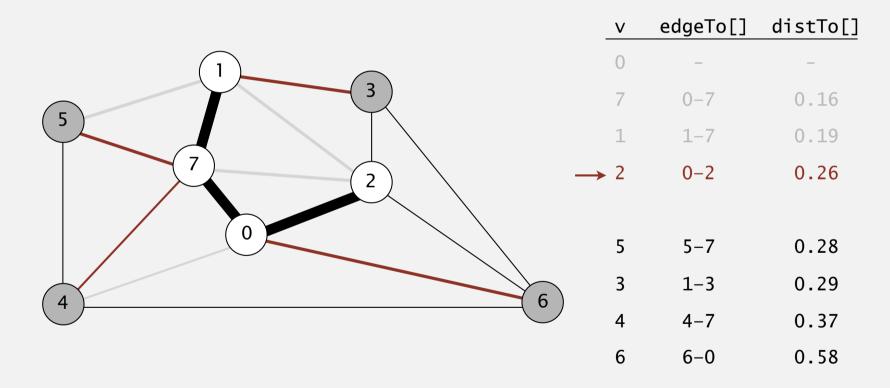
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



**MST** edges

0-7 1-7

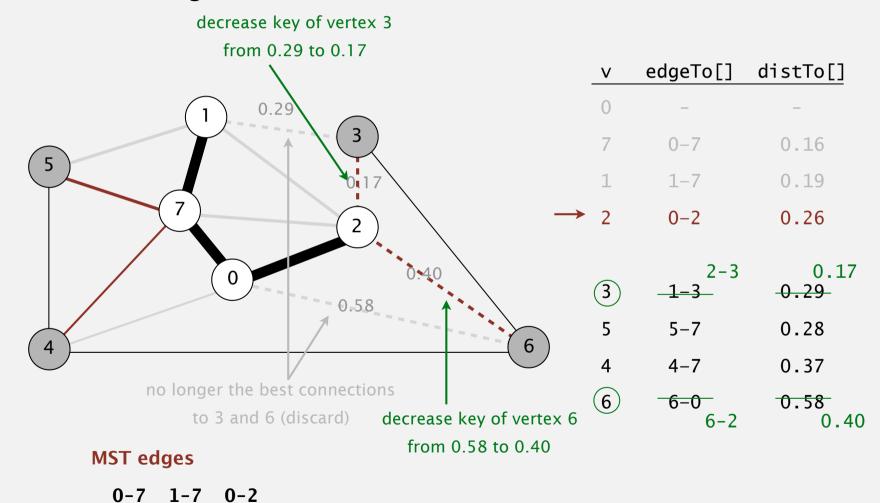
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



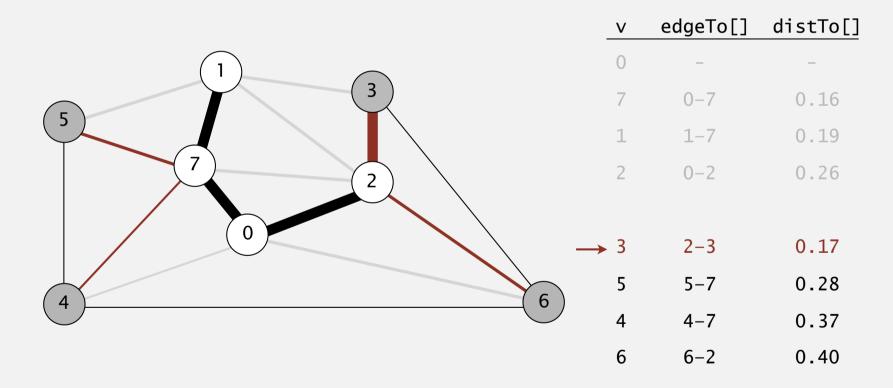
**MST edges** 

0-7 1-7 0-2

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



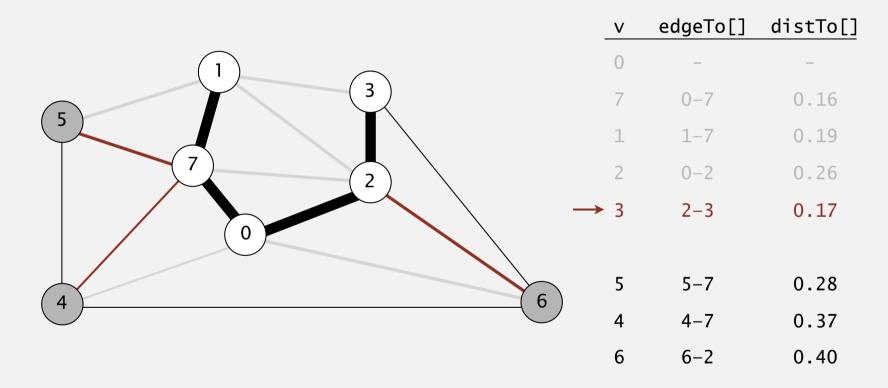
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



**MST edges** 

0-7 1-7 0-2 2-3

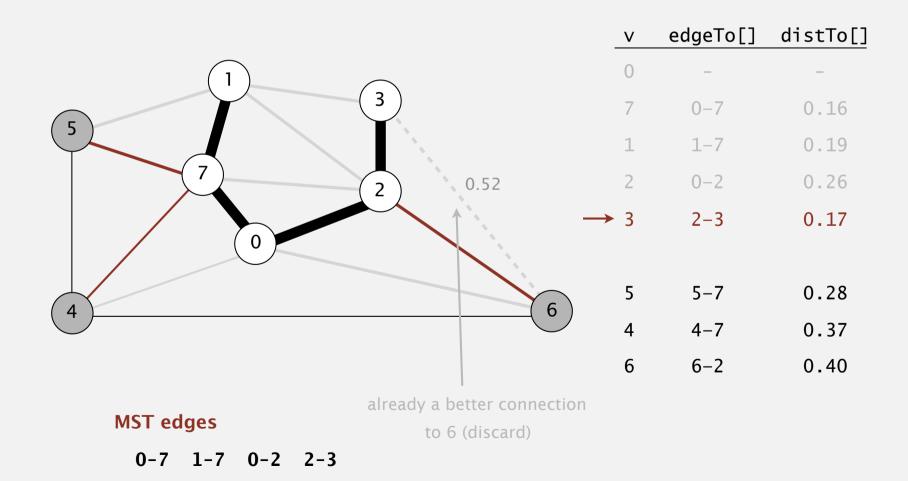
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



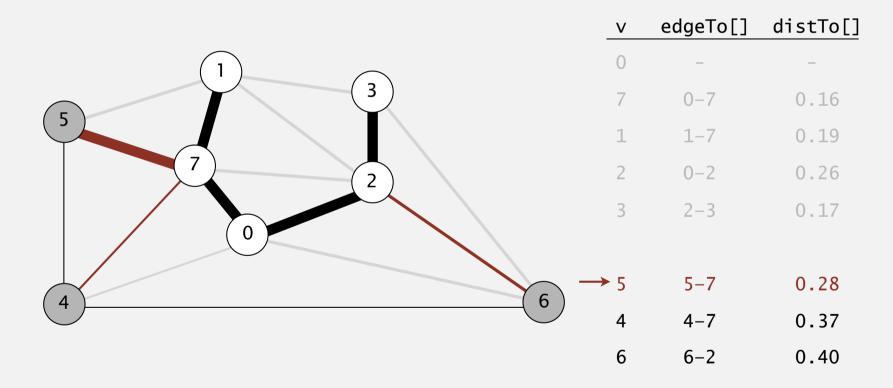
**MST edges** 

0-7 1-7 0-2 2-3

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



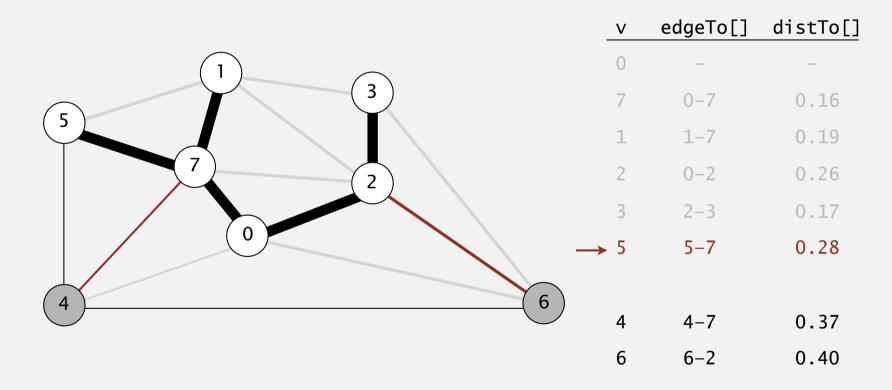
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST edges** 

0-7 1-7 0-2 2-3

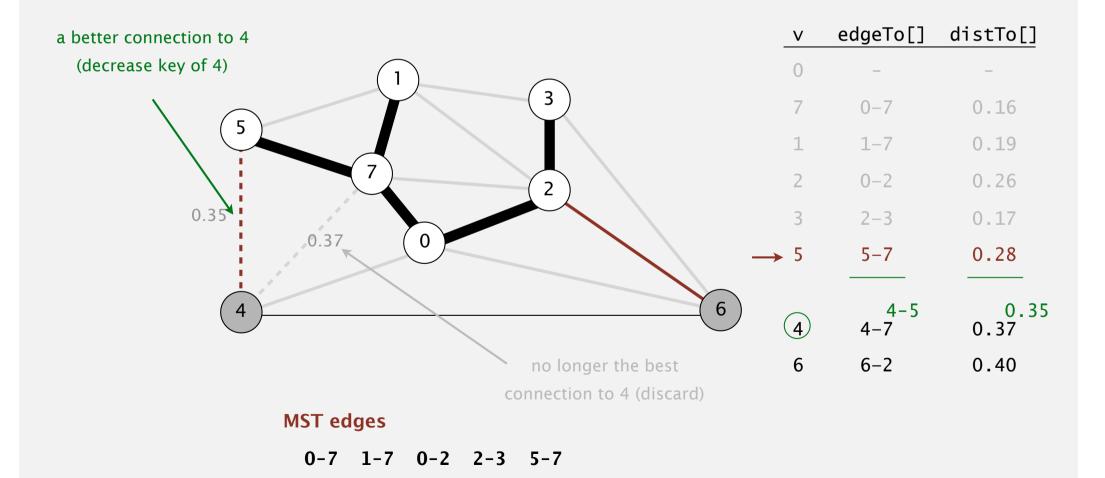
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



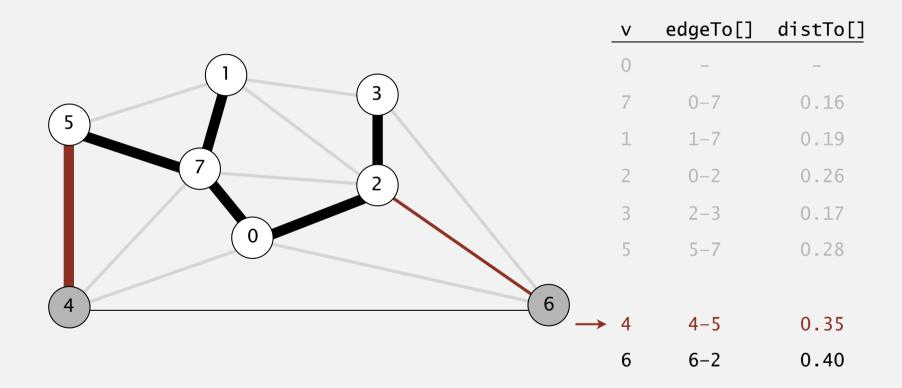
#### **MST** edges

0-7 1-7 0-2 2-3 5-7

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V 1 edges.



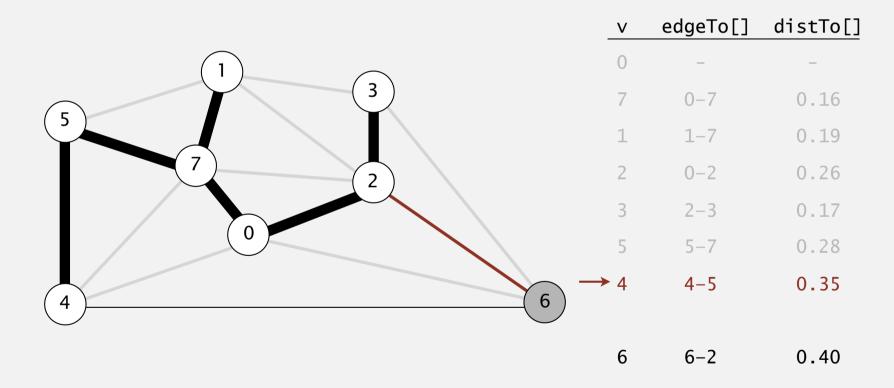
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST** edges

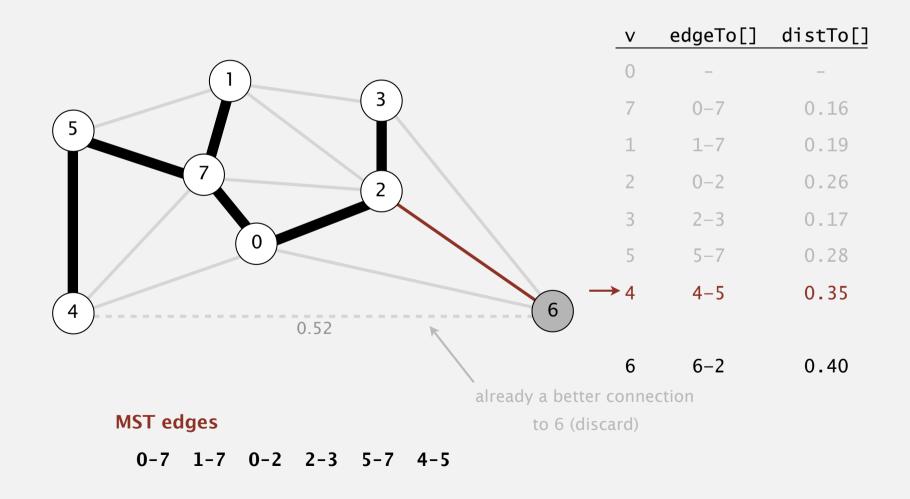
0-7 1-7 0-2 2-3 5-7

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

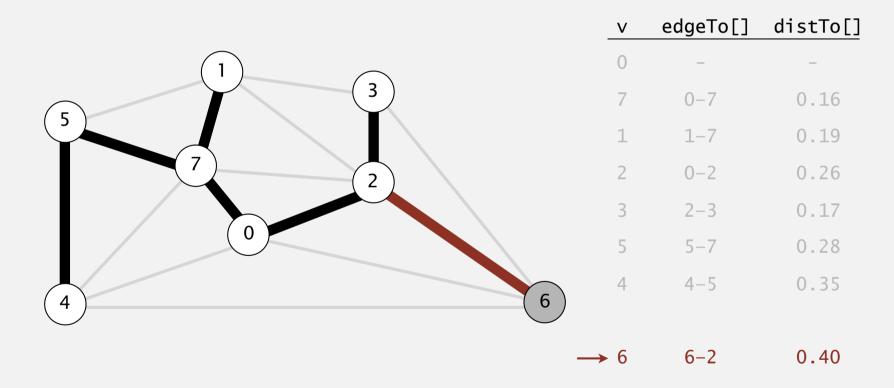


**MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

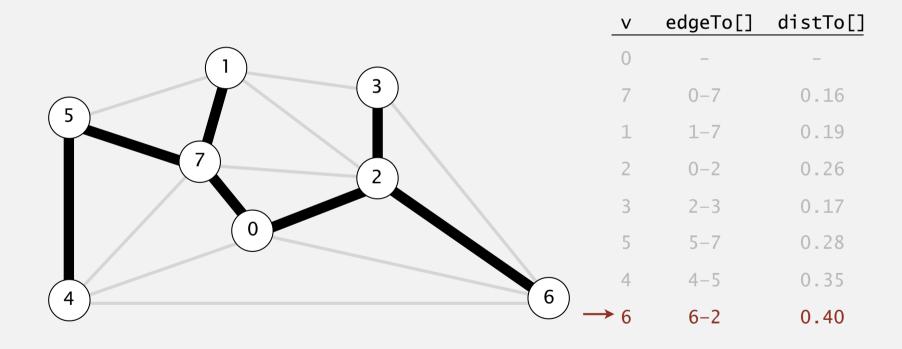


- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



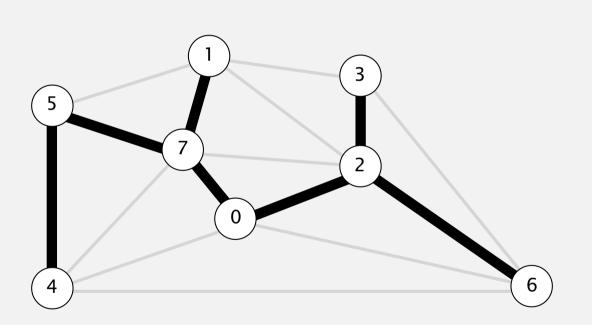
**MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



#### **MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to *T* the min weight edge with exactly one endpoint in *T*.
- Repeat until V-1 edges.



| V | edgeTo[] | distTo[] |
|---|----------|----------|
| 0 | -        | -        |
| 7 | 0-7      | 0.16     |
| 1 | 1-7      | 0.19     |
| 2 | 0-2      | 0.26     |
| 3 | 2–3      | 0.17     |
| 5 | 5-7      | 0.28     |
| 4 | 4–5      | 0.35     |
| 6 | 6–2      | 0.40     |

**MST edges**