Trees

Data Structures and Algorithms for Computational Linguistics III (ISCL-BA-07)

Çağrı Çöltekin
ccoltekin@sfs.uni-tuebingen.de

University of Tübingen
Seminar für Sprachwissenschaft

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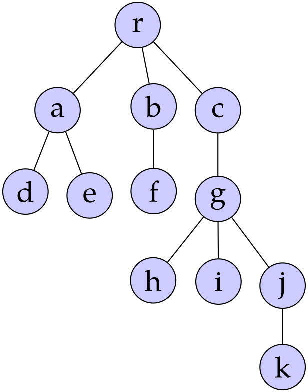
Why study trees

- A tree is a, *hierarchical*, *non-linear* data structure useful in many algorithms
- We have already resorted to descriptions using trees
- A tree is a graph with certain properties, and part of many of the graph algorithms
- It is also very common in (computational) linguistics:
  - Parse trees: we often represent
  - Language trees: trees that trace the relation between languages
  - Decision trees: a well-known algorithm for machine learning, also used for many NLP problems

Definitions

- A tree is a set of nodes organized as hierarchically with the following properties:
  - If a tree is non-empty, it has a special node root
  - Except the root node, every node in the tree has a unique parent (all nodes except the root are children of another node)
- Alternatively, we can define a tree recursively:
  - The empty set of nodes is a tree
  - Otherwise a tree contains a root with sub-trees as its children



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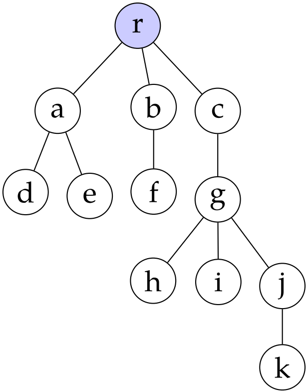

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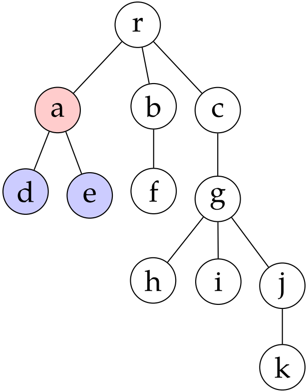
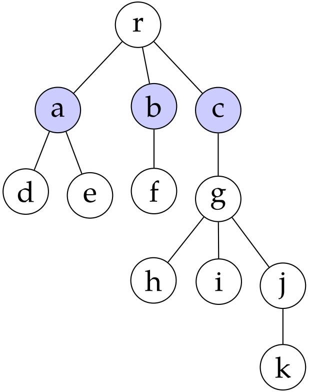


More definitions

- The nodes with the same parent are called siblings
- The nodes with children are called internal nodes
- The nodes without children are the leaf nodes
- A path is a sequence of connected nodes
- Any node in the path from the root to a particular node is its ancestors
- A node is the descendant of its ancestors
- A subtree is a tree rooted by a non-root node
- A depth of a node is the number of edges from root
- A height of a node is the number of edges from the deepest descendant
- The height of a tree is the height of its root



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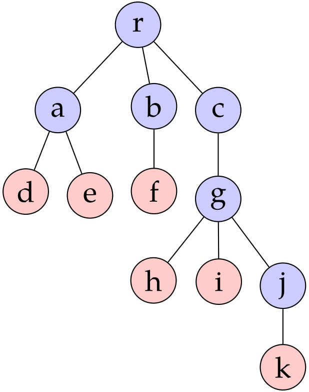

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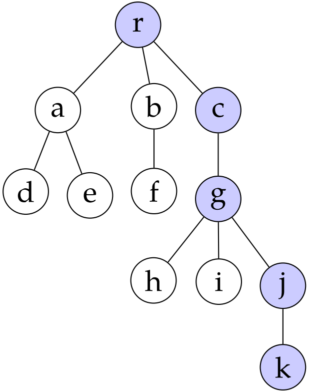

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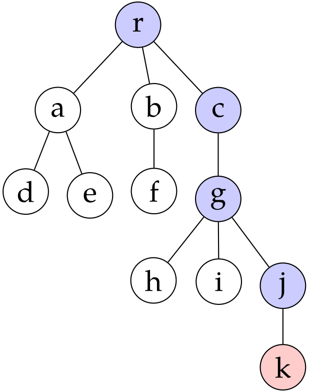

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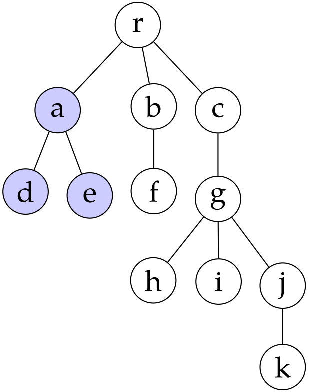

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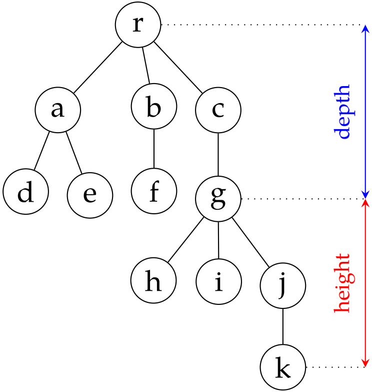


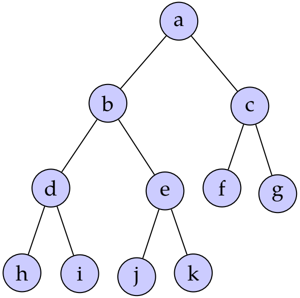
Ordered trees

- A tree is ordered if there is an ordering between siblings. Typical examples include:
  - A tree representing a document (e.g., HTML) structure
  - Parse trees
  - (maybe) a family tree
- In many cases order is not important
  - Class hierarchy in a object-oriented program
  - The tree representing files in a computer

Binary trees

even more definitions

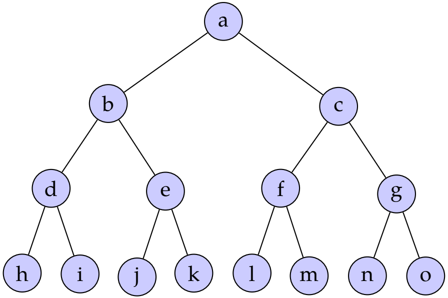
- Binary trees, where nodes can have at most two children, have many applications
- Binary trees have a natural order, each child is either a *left child* or a *right child*
- A binary tree is *proper*, or *full* if every node has either two children or none
- In a *complete* binary tree, every level except possibly the last, is completely filled, and all nodes at the last level is at the left
- A *perfect* binary tree is is a full binary tree whose leaf nodes have the same depth



Some properties of binary trees

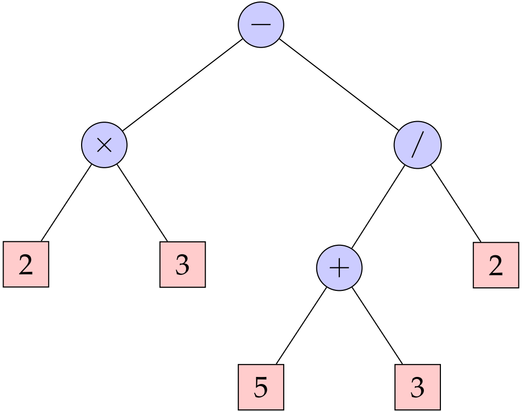
For a binary tree with n_ℓ leaf, n_i internal, n nodes and with height h

- $h + 1 \leqslant n \leqslant 2^{h+1} - 1$
- $1 \leqslant n_\ell \leqslant 2^h$
- $h \leqslant n_i \leqslant 2^h - 1$
- $\log(n+1) - 1 \leqslant h \leqslant n - 1$
- For any proper binary tree, $n_\ell = n_i + 1$



Binary tree example: expression trees

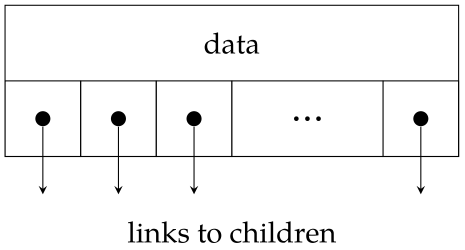
$2 \times 3 + (5 + 3)/2$



Implementation of trees

general case: linked data structures

N-way



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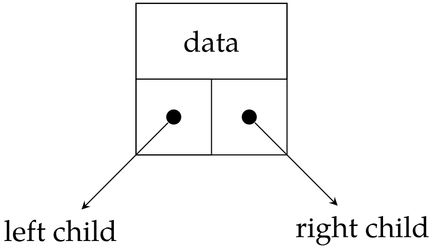
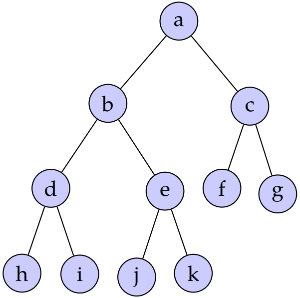
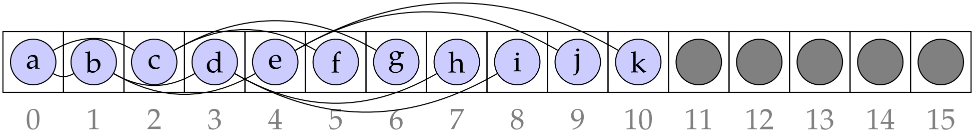


Implementation of trees

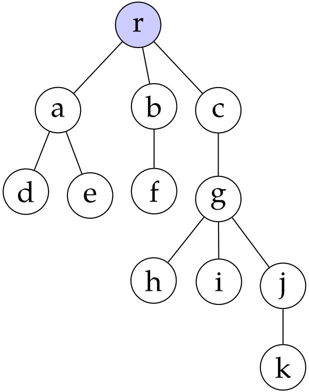
array implementation of binary trees

- Binary trees can also be implemented with arrays:
  - the root node is stored at index 0
  - the left child of the node at index i is stored at $2i + 1$
  - the right child of the node at index i is stored at $2i + 2$
  - the parent of the node at index i is at index $\lfloor i/2 \rfloor$
- If the binary tree is complete, this representation does not waste (much) space





Breadth first traversal

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def breadth_first(root):
  queue = []
  queue.append(root)
  while queue:
    node = queue.pop(0)
    # process the node
    print(node.data)
    for child in node.children:
      queue.append(child)
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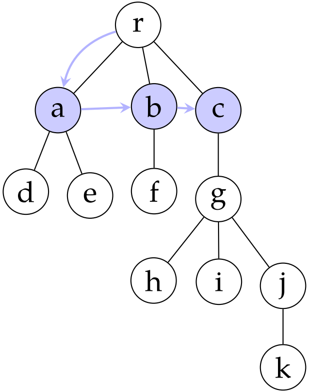


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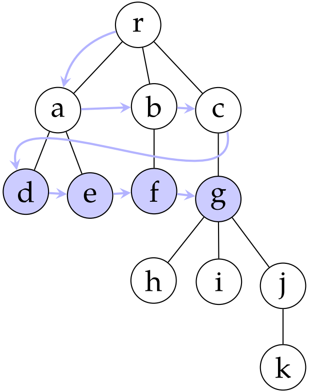


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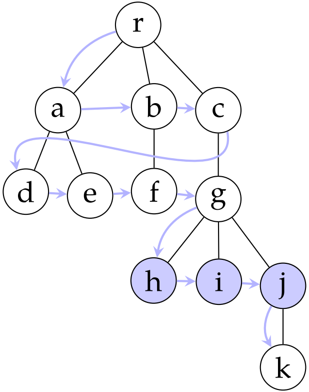


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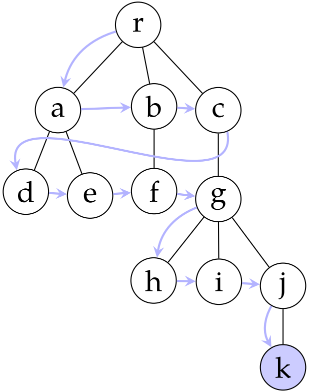

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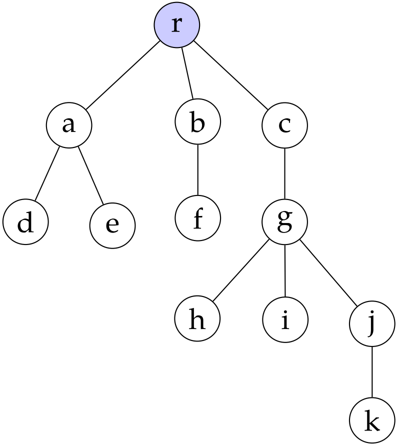


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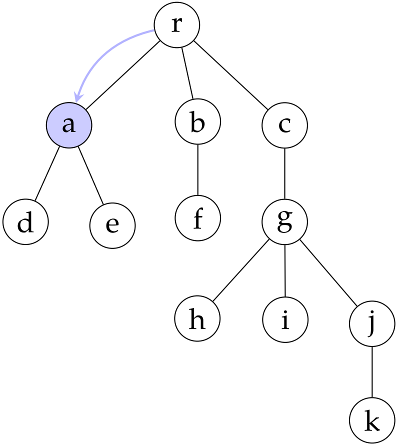


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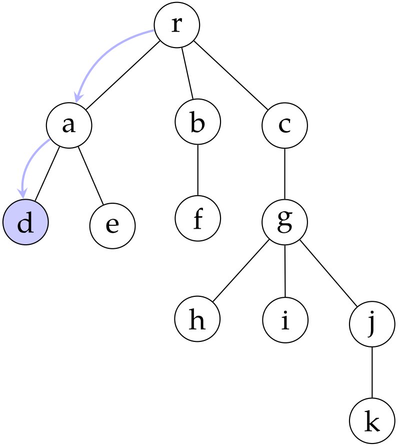


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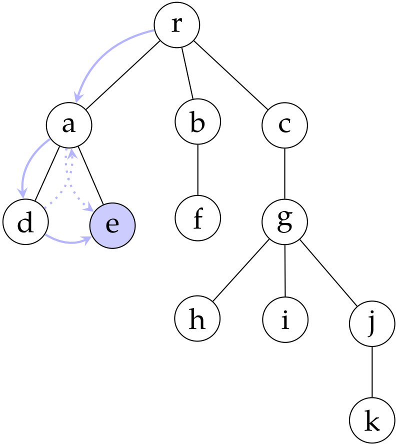


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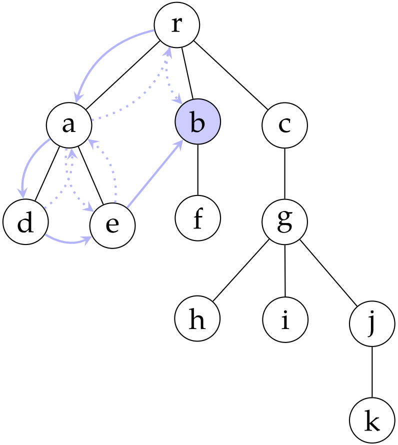


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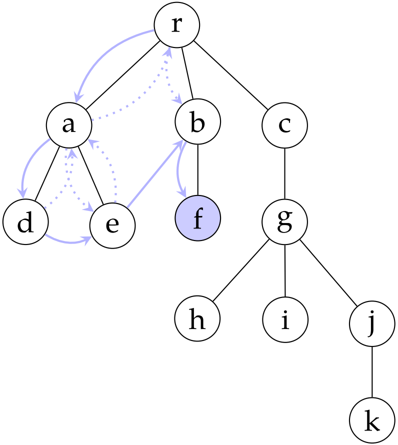


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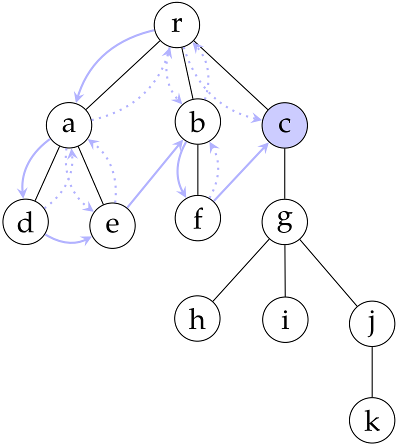


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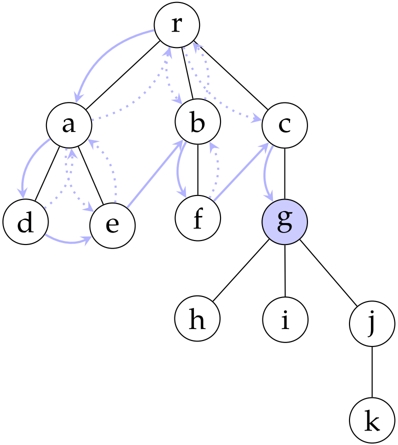


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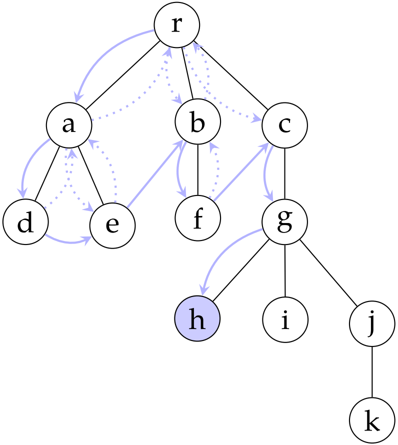


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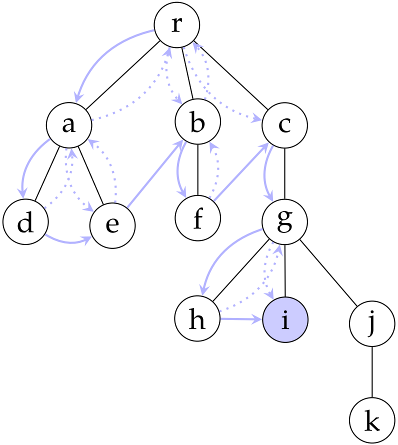


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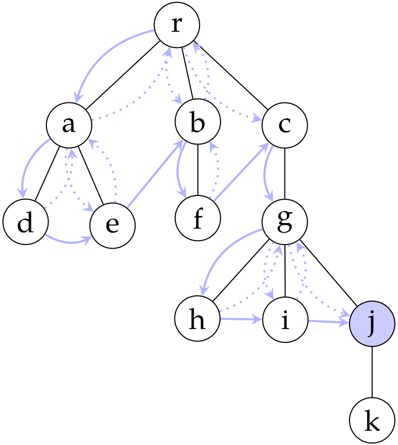


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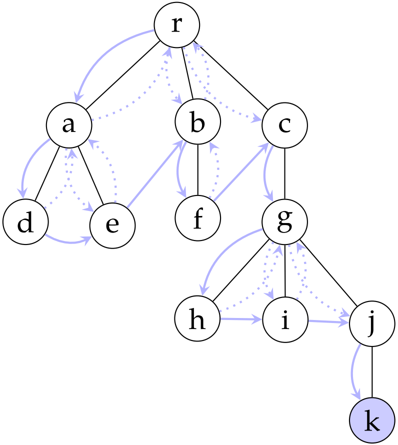


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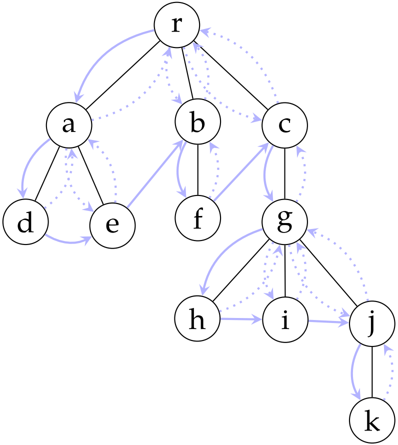

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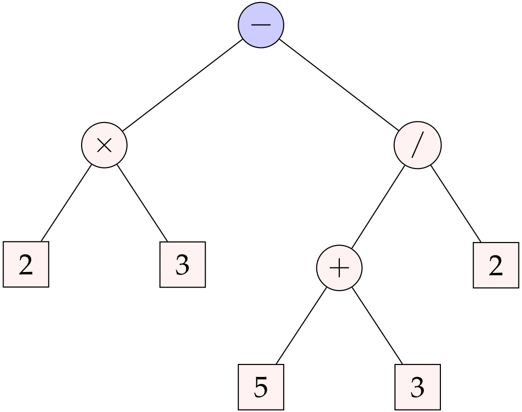
Example: pre-order in an expression tree



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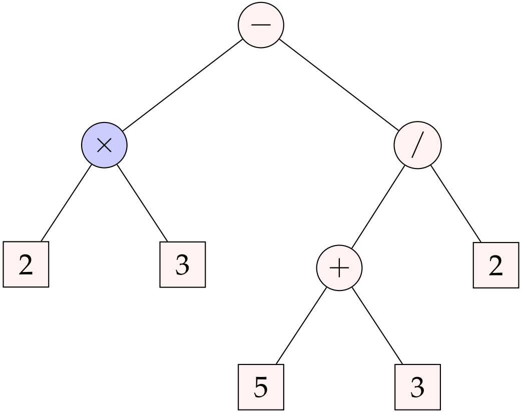

$- \quad \times$

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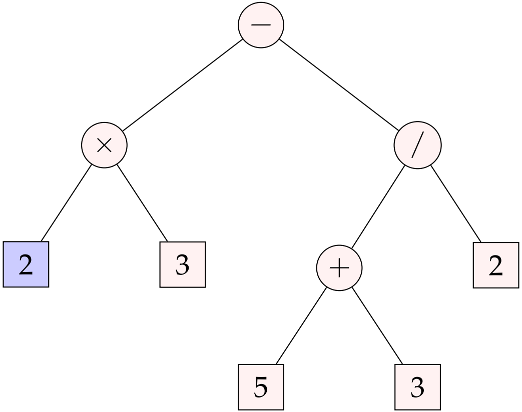

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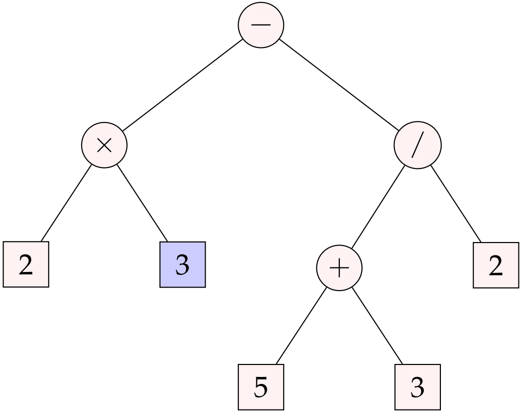

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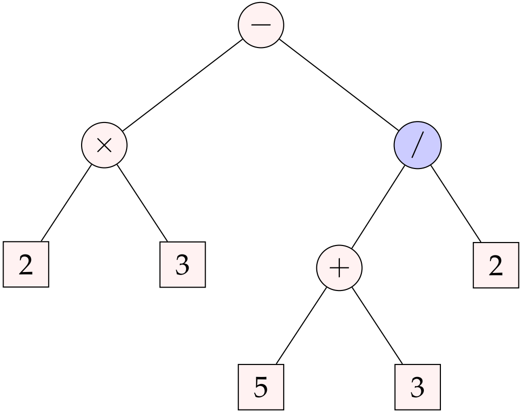

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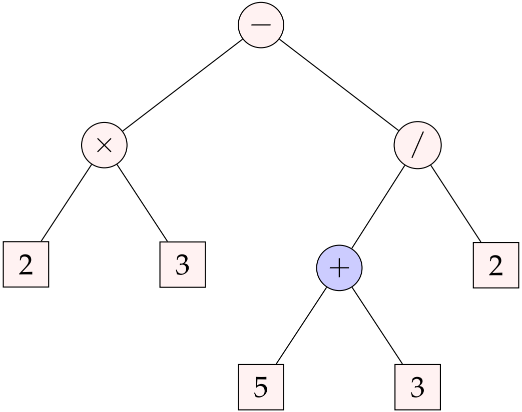

$$- \quad \times \quad 2 \;\; 3 \quad / \quad +$$

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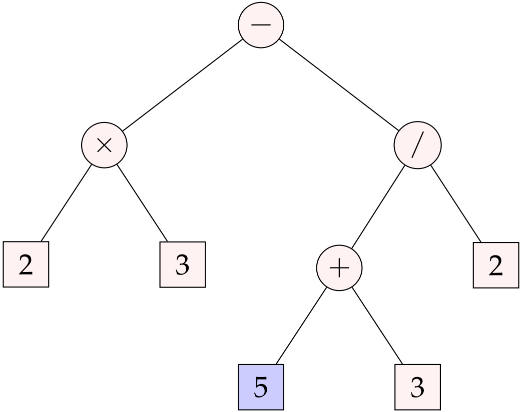

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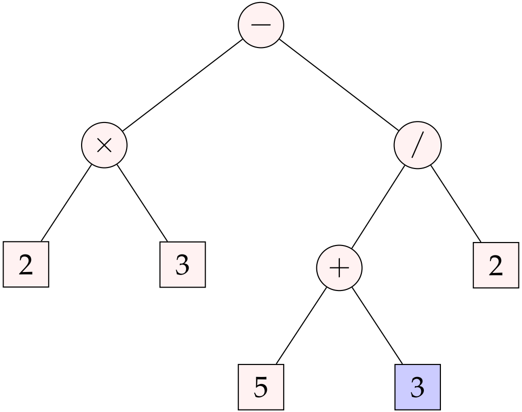

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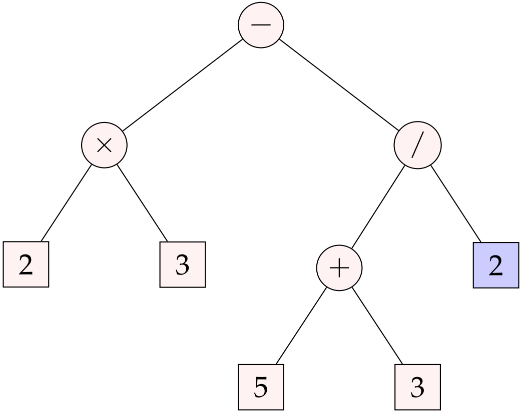

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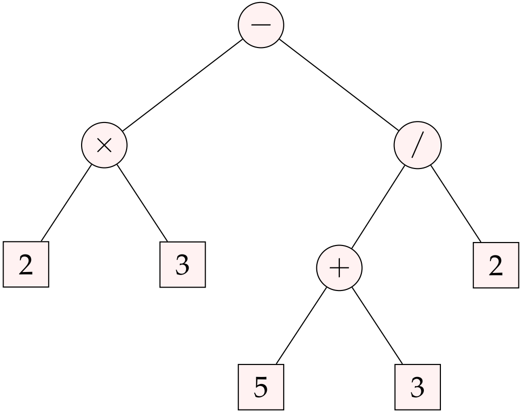
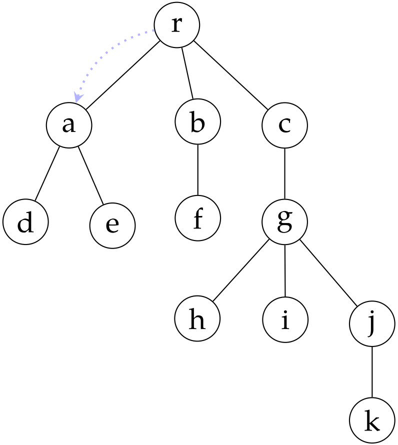


$$-(\ \times(\ 2\ \ 3)\ \ /(\ +(\ 5\ \ 3)\ \ 2)\ \ )\ \ )$$

Post-order traversal




```python
def post_order(node):
  for child in node.children:
    post_order(child)
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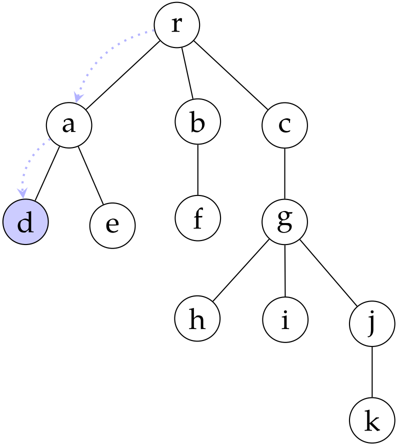


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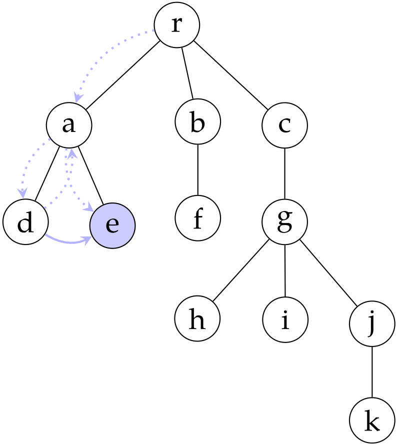


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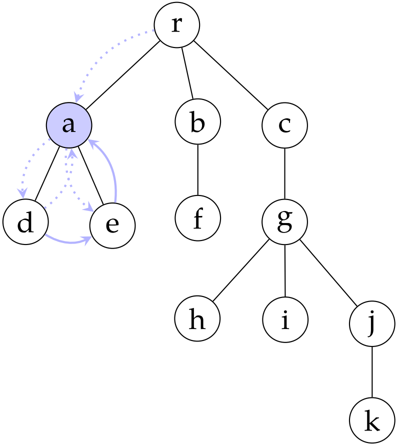


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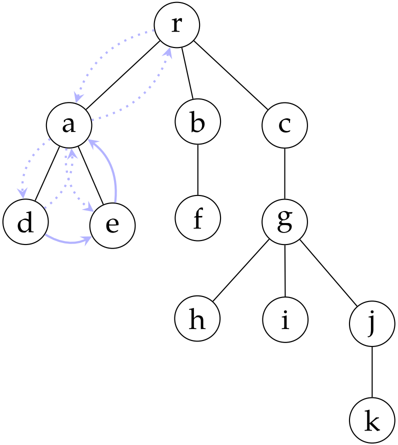


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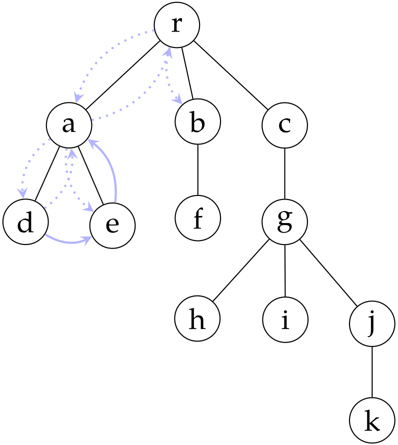


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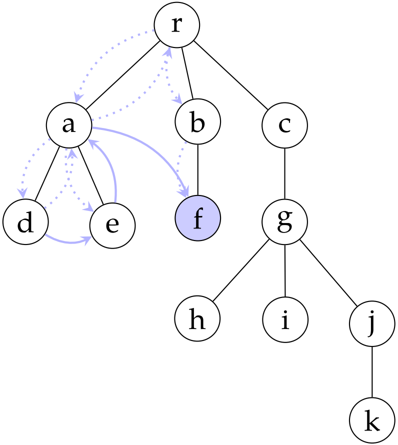


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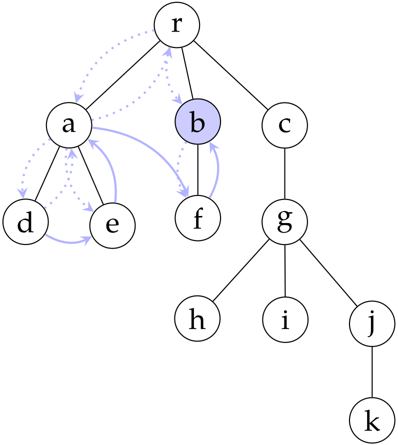


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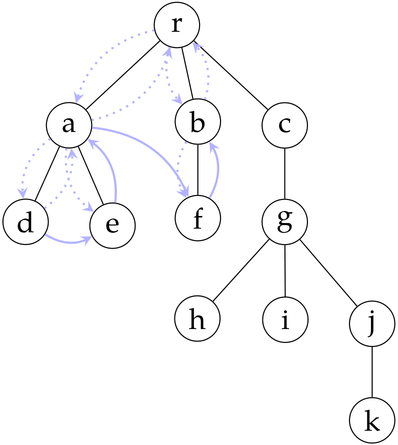


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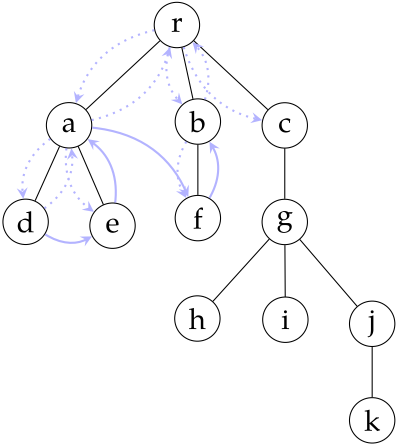


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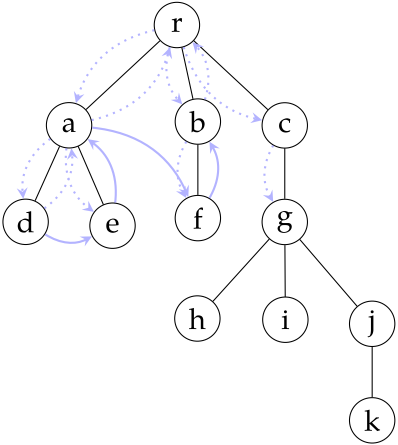


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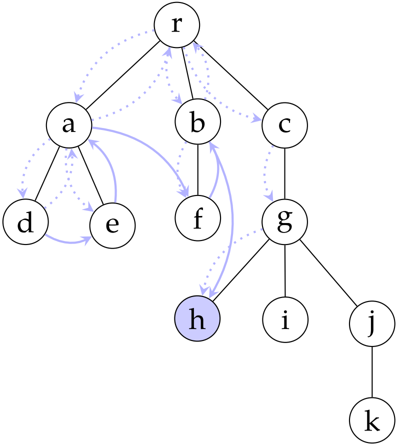


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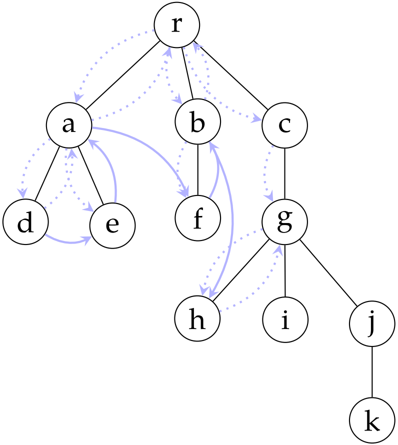


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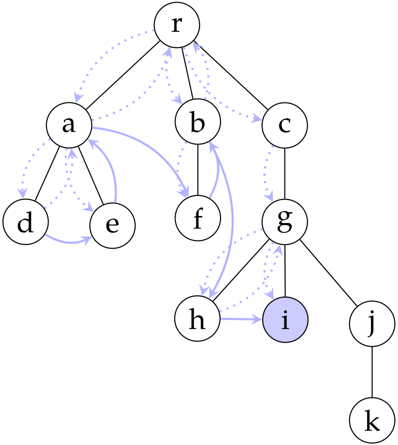


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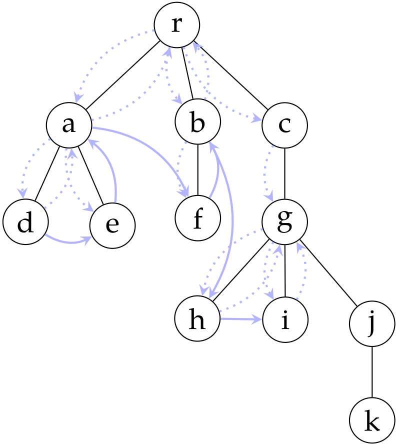


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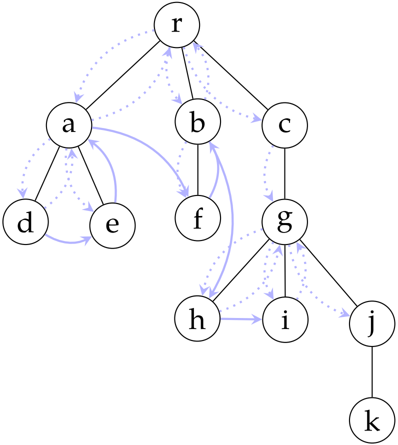


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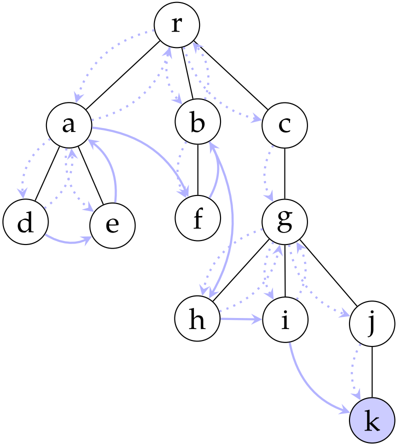


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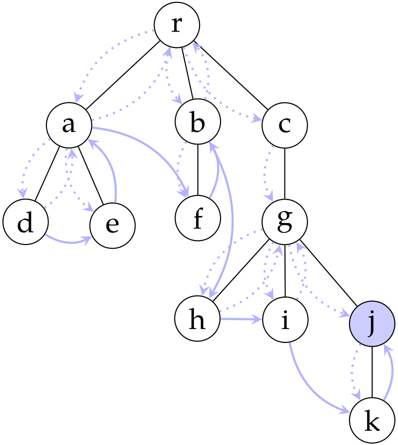


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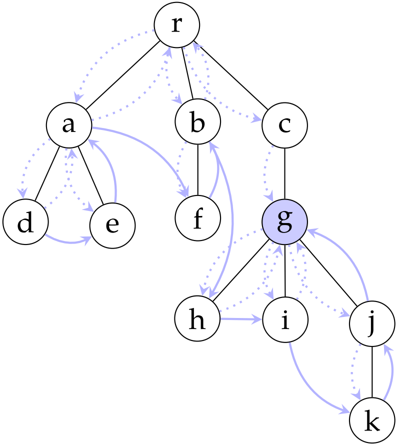


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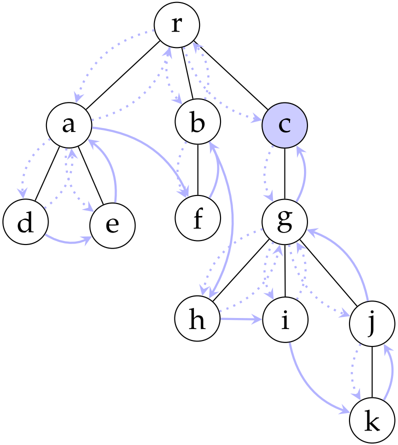


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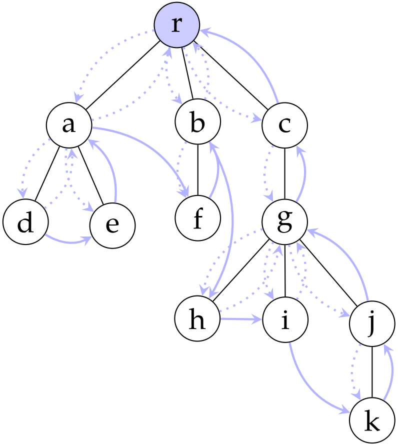
Post-order traversal



d e a f b h i k j g c r

```
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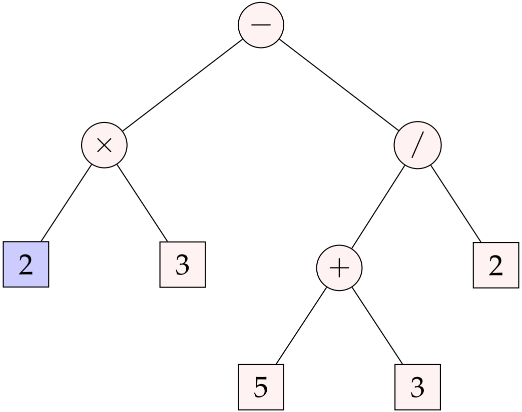
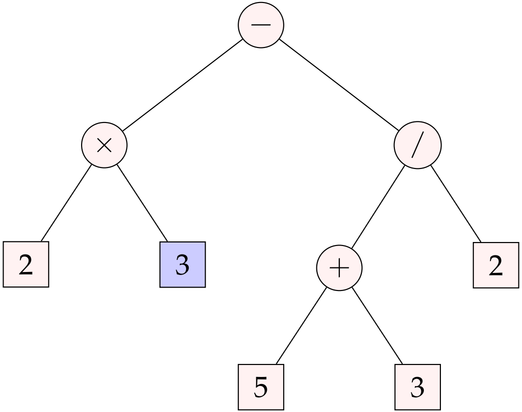
Example: post-order in an expression tree



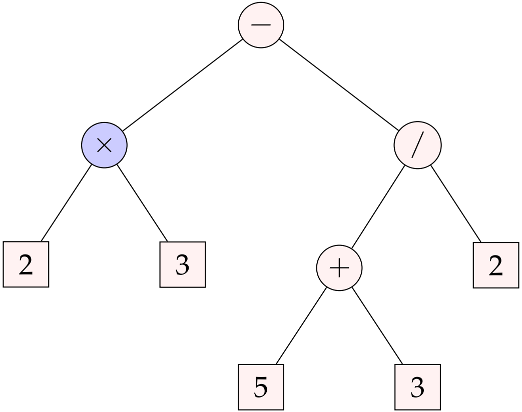
2

Example: post-order in an expression tree



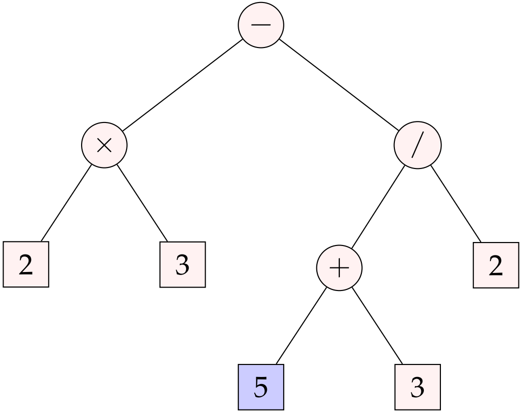
2 3

Example: post-order in an expression tree



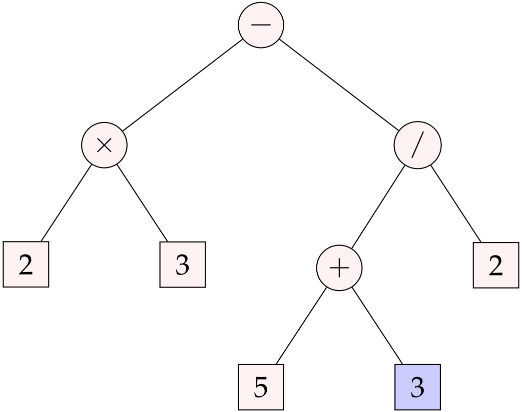
$2\ 3\ \times$

Example: post-order in an expression tree



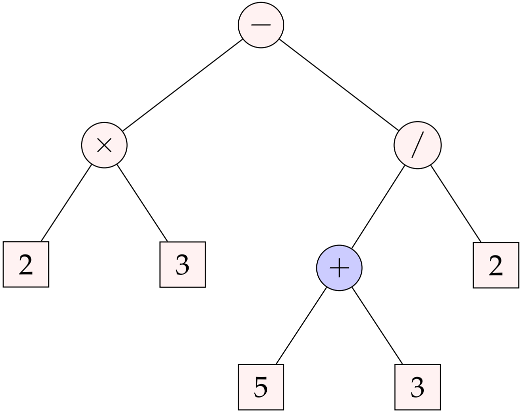
$2\ 3 \times 5$

Example: post-order in an expression tree



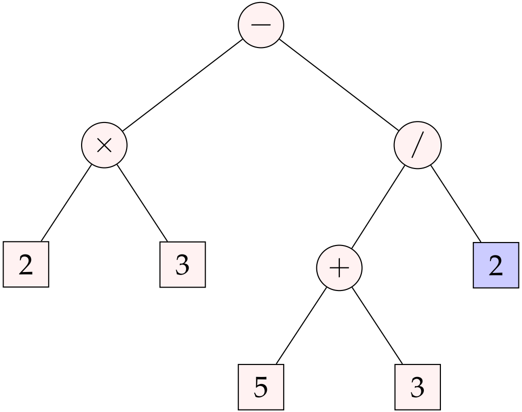
$$2\;3\;\times\;5\;3$$

Example: post-order in an expression tree



$$2\ 3 \times 5\ 3 +$$

Example: post-order in an expression tree



$$2\;3\times 5\;3+2$$

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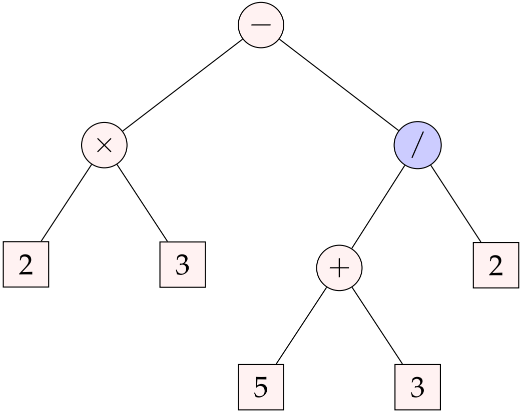
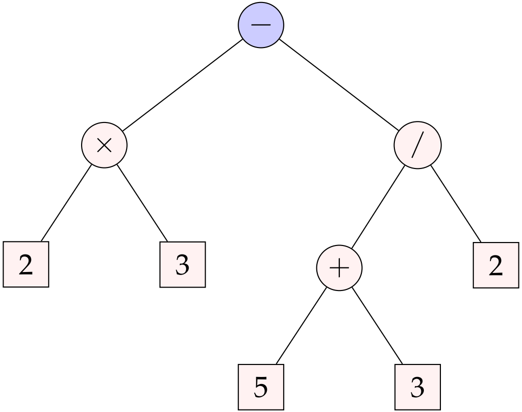


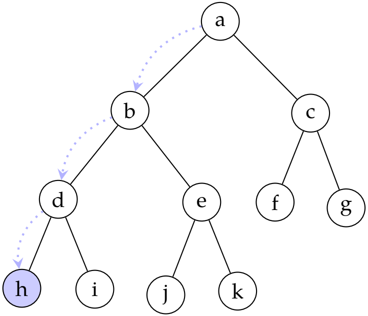
$$2\ 3\ \times\ 5\ 3\ +\ 2\ /$$

Example: post-order in an expression tree



$$2\;3\;\times\;5\;3\;+\;2\;/\;-$$

In-order traversal

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def in_order(node):
  in_order(node.left)
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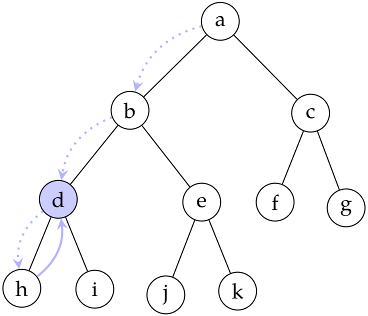
In-order traversal

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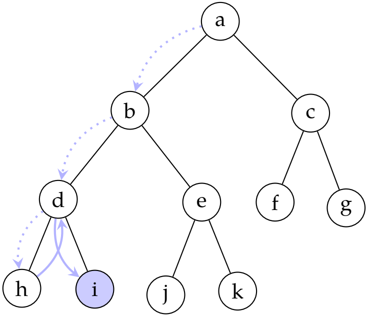


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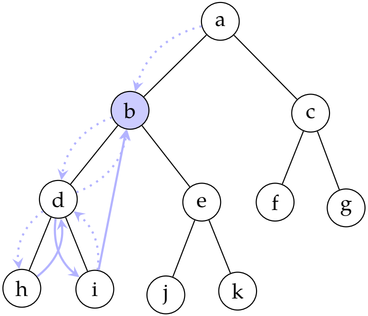


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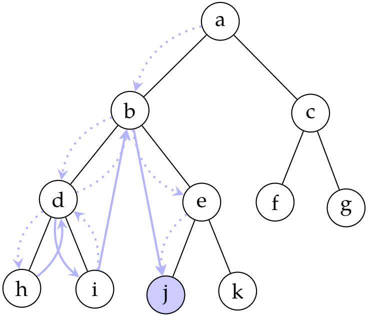


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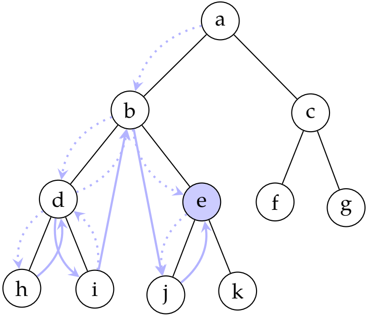


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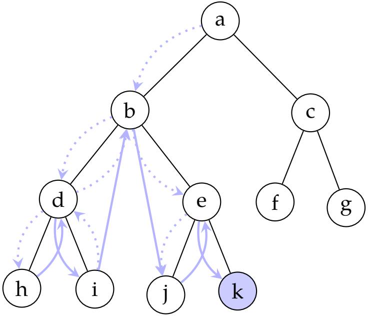


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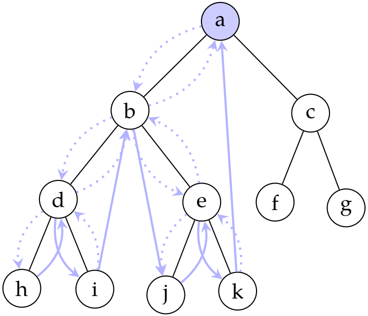


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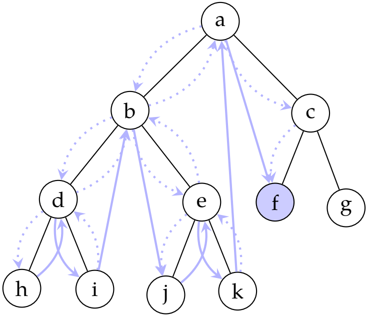


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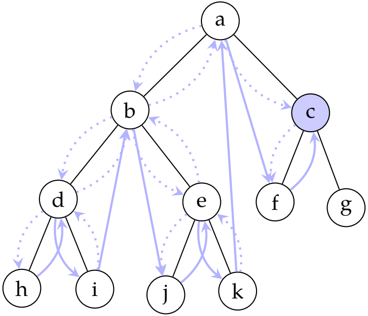


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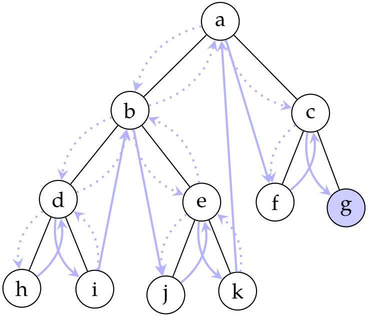


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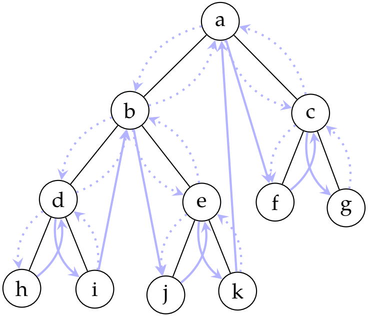

h d i b j e k a f c g

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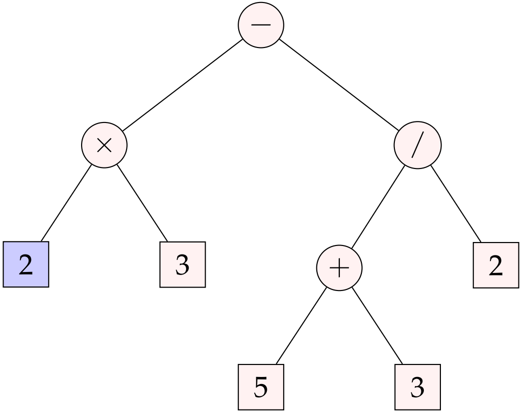

2

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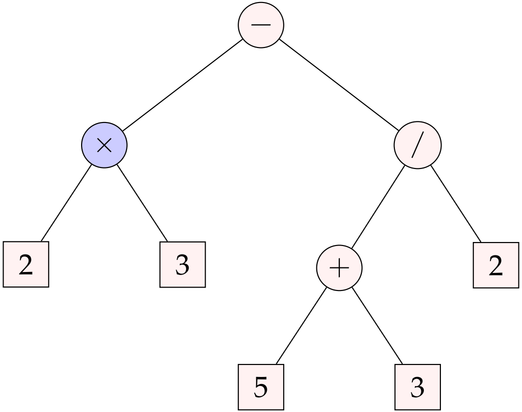

$2 \times$

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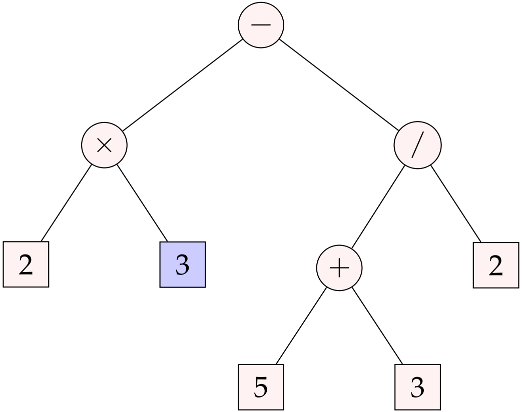

$$2 \times 3$$

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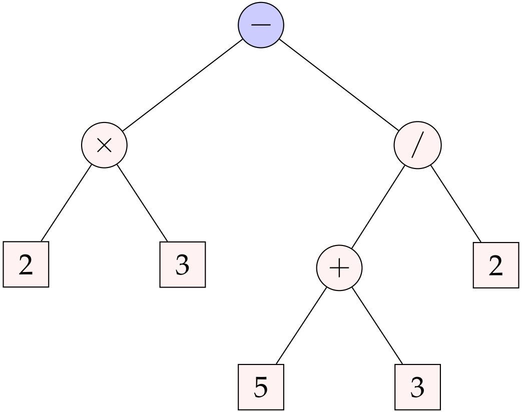

$$2 \times 3 \quad -$$

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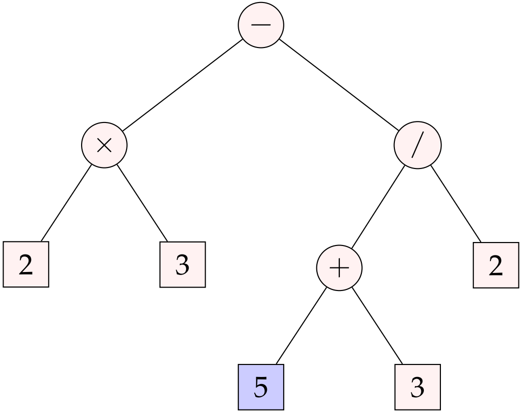

$$2 \times 3 \quad - \quad 5$$

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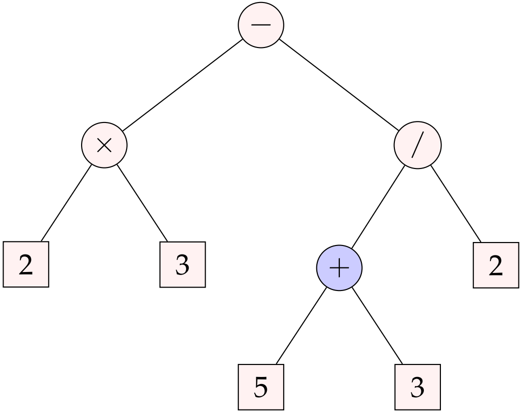

$$2 \times 3 \quad - \quad 5 +$$

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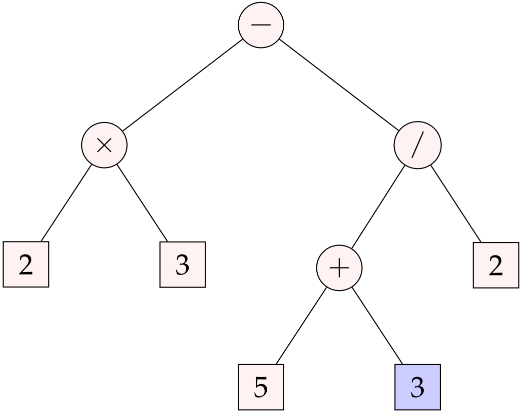

$$2 \times 3 \quad - \quad 5 + 3$$

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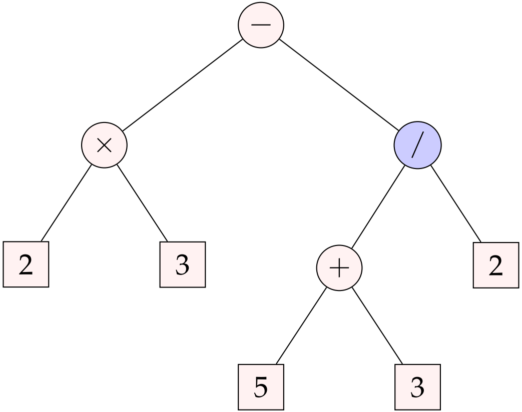

$$2 \times 3 \quad - \quad 5 + 3 \quad /$$

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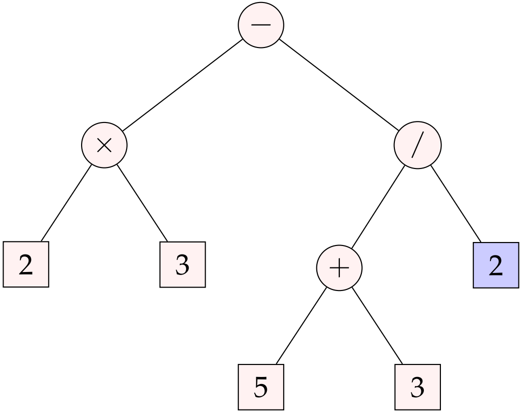
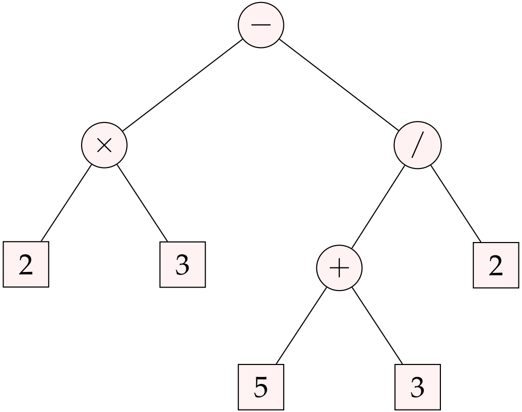


$$2 \times 3 \quad - \quad 5 + 3 \quad / \; 2$$

Example: in-order in an expression tree



$$((2 \times 3) - ((5 + 3) / 2))$$

Summary

- Trees are hierarchical data structures useful in many applications
- We will often return to trees and properties of trees in the rest of the course
- Reading on trees: Goodrich, Tamassia, and Goldwasser (2013, chapter 8), and optionally the chapter on *search trees* (Goodrich, Tamassia, and Goldwasser 2013, ch. 11)

Next:

- Heaps and priority queues
- Reading: Reading: Goodrich, Tamassia, and Goldwasser (2013, chapter 9)

Acknowledgments, credits, references

Goodrich, Michael T., Roberto Tamassia, and Michael H. Goldwasser (2013). *Data Structures and Algorithms in Python*. John Wiley & Sons, Incorporated. ISBN: 9781118476734.

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