

# Finite state transducers

Data Structures and Algorithms for Computational Linguistics III  
(ISCL-BA-07)

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# Finite state transducers

A quick introduction

- A *finite state transducer* (FST) is a finite state machine where transitions are conditioned on pairs of symbols
- The machine moves between the states based on an *input* symbol, while it outputs the corresponding *output* symbol
- An FST encodes a *relation*, a mapping from a set to another
- The relation defined by an FST is called a *regular* (or *rational*) relation



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## Formal definition

A finite state transducer is a tuple  $(\Sigma_i, \Sigma_o, Q, q_0, F, \Delta)$

$\Sigma_i$  is the *input* alphabet

$\Sigma_o$  is the *output* alphabet

$Q$  a finite set of states

$q_0$  is the start state,  $q_0 \in Q$

$F$  is the set of accepting states,  $F \subseteq Q$

$\Delta$  is a relation  $(\Delta : Q \times \Sigma_i \rightarrow Q \times \Sigma_o)$

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## Where do we use FSTs?

example 1: morphological analysis



In this lecture, we treat an FSA as a simple PST that outputs its input: the edge label 'a' is a shorthand for 'aa'.

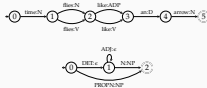
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## Where do we use FSTs?

example 2: POS tagging / shallow parsing



Note: (1) It is important to express the ambiguity. (2) This gets interesting if we can 'compose' these automata.

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## Closure properties of FSTs

Like FSA, FSTs are closed under some operations.

- Concatenation
- Kleene star
- Complement**
- Reversal
- Union
- Intersection**
- Inversion**
- Composition**

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## FST inversion

- Since an FST encodes a relation, it can be reversed
- Inverse of an FST swaps the input symbols with output symbols
- We indicate inverse of an FST  $M$  with  $M^{-1}$



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## FST composition

sequential application



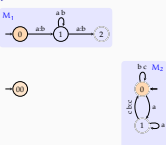
- Can we compose two FSTs without running them sequentially?

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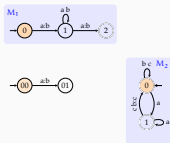
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## FST composition



## FST composition

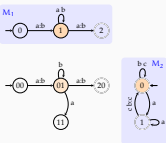


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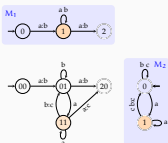
## FST composition



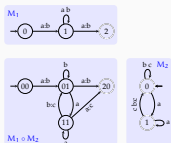
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## FST composition



## FST composition



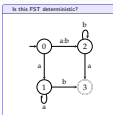
## Projection

- Projection turns an FST into a FSA, accepting either the input language or the output language



## FST determinization

- A deterministic FST has unambiguous transitions from every state on any input symbol
- We can extend the subset construction to FSTs
- Determinization of FSTs means converting to a *subsequential* FST
- However, not all FSTs can be determinized



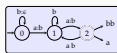
## Sequential FSTs

- A sequential FST has a single transition from each state on every input symbol
- Output symbols can be strings, as well as  $\epsilon$
- The recognition is linear in the length of input
- However, sequential FSTs do not allow ambiguity



## Subsequential FSTs

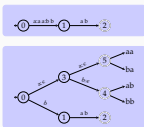
- A *k*-subsequential FST is a sequential FST which can output up to  $k$  strings at an accepting state
- Subsequential transducers allow limited ambiguity
- Recognition time is still linear



- The 2-subsequential FST above maps every string it accepts to two strings, e.g.:
  - $baa \rightarrow bba$
  - $baa \rightarrow bbbb$

## An exercise

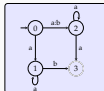
Convert the following FST to a subsequential FST



## Determinizing FSTs

Another example

Can you convert the following FST to a subsequential FST?



Note that we cannot 'determine' the output on first input, until reaching the final input.

## FSA vs FST

- FSA are *acceptors*, FSTs are *transducers*
- FSA accept or reject their input, FSTs produce output(s) for the inputs they accept
- FSA define sets, FSTs define relations between sets
- FSTs share many properties of FSAs. However,
  - FSTs are not closed under intersection and complement
  - We can compose (and invert) the FSTs
  - Determinizing FSTs is not always possible
- Both FSA and FSTs can be **weighted** (not covered in this course)

## References / additional reading material

- Jurafsky and Martin (2009, Ch. 3)
- Additional references include:
  - Roche and Schabes (1996) and Roche and Schabes (1997): FSTs and their use in NLP
  - Mohri (2009): weighted FSTs

## References / additional reading material (cont.)

- Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*, second edition. Pearson Prentice Hall. [sasc: 978-0-13-504196-3](#).
- Mohri, Mehryar (2009). "Weighted automata algorithms". In: *Handbook of Weighted Automata*. Monographs in Theoretical Computer Science. Springer, pp. 213–254.
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