

String matching

Data Structures and Algorithms for Computational Linguistics III
(ISCL-BA-07)

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Finding patterns in a string

- Finding a pattern in a larger text is a common problem in many applications
- Typical example is searching in a text editor or word processor
- There are many more:
 - DNA sequencing / bioinformatics
 - Plagiarism detection
 - Search engines / information retrieval
 - Spell checking
 - ...

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Introduction, motivation Brute force Rabin-Karp FPM KMP Edit-Distance

Types of problems

- The efficiency and usability of algorithms depend on some properties of the problem
- Typical applications are based on finding multiple occurrences of a single pattern in a text, where the pattern is much shorter than the pattern
- The efficiency of the algorithms may depend on the
 - relative size of the pattern
 - expected number of repetitions
 - size of the alphabet
 - whether the pattern is used once or many times
- Another related problem is searching for multiple patterns at once
- In some cases, fuzzy / approximate search may be required
- In some applications, preprocessing (indexing) the text to be searched may be beneficial

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Problem definition

and some terminology

text: A A T A G A C G G C T A G C A A

pattern: A G C A C

- We want to find all occurrences of pattern p (length m) in text t (length n)
- The characters in both t and p are from an alphabet Σ , in the example $\Sigma = \{A, C, G, T\}$
- The size of the alphabet (q) is often an important factor
- p occurs in t with shift s if $p[0 : m] == t[s : s + m]$, we have a match at $s = 3$ in the example
- A string x is a prefix of string y , if $y = xw$ for a possibly empty string w
- A string x is a suffix of string y , if $y = wx$ for a possibly empty string w

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Brute-force string search

i 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
t: A A T A G A C G G C T A G C A A
p: A G C A C
j 0 1 2 3

- Start from the beginning, of $i = 0$ and $j = 0$
 - if $j == m$, announce success with $s = i$
 - if $t[i] == p[j]$: shift p ($i += 1, j = 0$)
 - otherwise: compare the next character ($i += 1, j += 1$, repeat)

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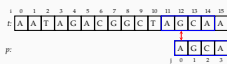
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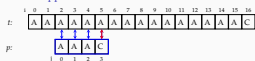
Brute-force approach: worst case



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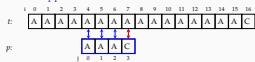
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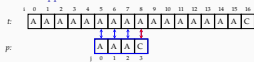
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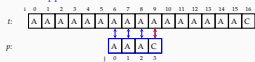
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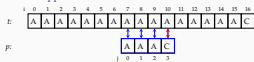
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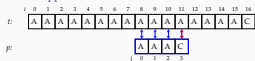
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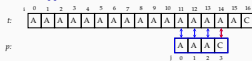
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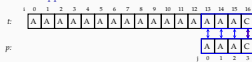
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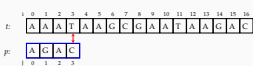
Brute-force approach: worst case



- Worst-case complexity of the method is $O(nm)$
- Crucially, most of the comparisons are redundant
 - for $i > 0$ and any comparison with $j = 0, 1, 2$, we already inspected corresponding i values
- The main idea for more advanced algorithms is to avoid this unnecessary comparisons

Boyer-Moore algorithm

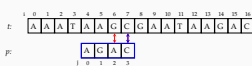
slightly simplified version



- The main idea is to start comparing from the end of p
- If $t[i]$ does not occur in p , shift m steps
- Otherwise, align the last occurrence of $t[i]$ in p with $t[i]$

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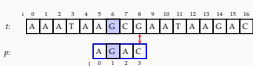
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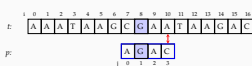
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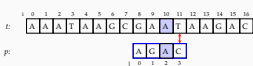
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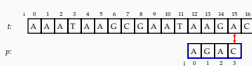
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Boyer-Moore algorithm

implementation and analysis

- On average the algorithm performs better than brute-force
- In worst case the complexity of the algorithm is $O(nm)$, example: $t = aaaa \dots a, p = baaa \dots a$
- Faster versions exist ($O(n + m + q)$)

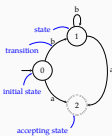
```

last = {}
for j in range(m):
    last[P[j]] = j
i, j = m-1, m-1
while i < n:
    if T[i] == P[j]:
        return i
    i -= 1
    j -= 1
else:
    k = last.get(T[i], -1)
    i += m - min(j, k+1)
    j = m - 1
return None

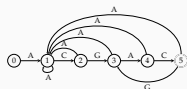
```

A quick introduction to FSA

- Another efficient way to search a string is building a finite state automaton for the pattern
- An FSA is a directed graph where edges have labels
- One of the states is the *initial state*
- Some states are accepting states
- We will study FSA more in-depth soon



An FSA for the pattern ACGAC

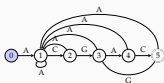


- Start at state 0, switch states based on the input
- All unspecified transitions go to state 0
- When at the accepting state, announce success

FSA pattern matching

demonstration

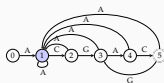
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	A	A	C	G	A	C	G	A	C	A	T	A	C	G	A	C



FSA pattern matching

demonstration

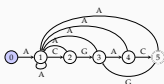
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FSA pattern matching

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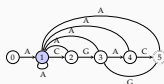
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FSA pattern matching

demonstration

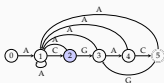
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FSA pattern matching

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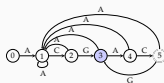
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FSA pattern matching

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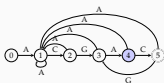
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FSA pattern matching

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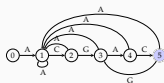
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FSA pattern matching

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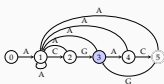
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FSA pattern matching

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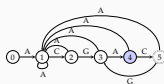
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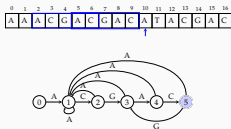
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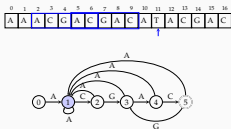
FSA pattern matching

demonstration



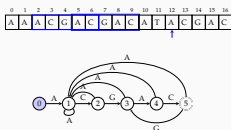
FSA pattern matching

demonstration



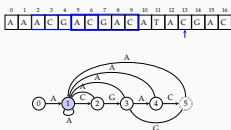
FSA pattern matching

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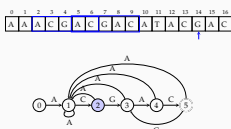
FSA pattern matching

demonstration



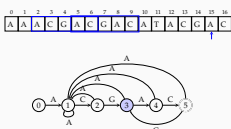
FSA pattern matching

demonstration



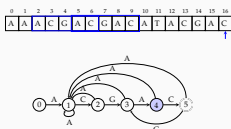
FSA pattern matching

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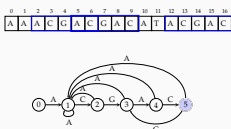
FSA pattern matching

demonstration



FSA pattern matching

demonstration



FSA for string matching

how to build the automaton

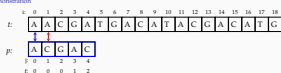
- An FSA results in $O(n)$ time matching, however, we need to first build the automaton
- At any state of the automaton, we want to know which state to go for the failing matches
- Given substring s recognized by a state and a non-matching input symbol a , we want to find the longest prefix of s such that it is also a suffix of sa
- A naive attempt results in $O(qm^2)$ time for building the automaton (where q is the size of the alphabet m is the length of the pattern)
- If stored in a matrix, the space requirement is $O(m^2)$
- Better (faster) algorithms exist for construction these automaton (we will cover some later in this course)

Knuth-Morris-Pratt (KMP) algorithm

- The KMP algorithm is probably the most popular algorithm for string matching
- The idea is similar to the FSA approach: on failure, continue comparing from the longest matched prefix so far
- However, we rely on a simpler data structure (a function/table that tells us where to back up)
- Construction of the table is also faster

KMP algorithm

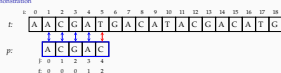
demonstration



- In case of a match, increment both i and j
- On failure, or at the end of the pattern, decide which new $p[j]$ compare with $t[i]$ based on a function f
- $f[j - 1]$ tells which j value to resume the comparisons from

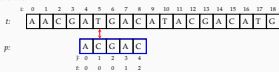
KMP algorithm

demonstration



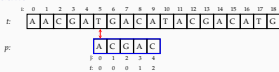
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KMP algorithm demonstration



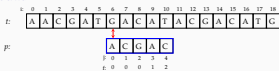
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- $f[j - 1]$ tells which j value to resume the comparisons from

KMP algorithm demonstration



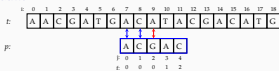
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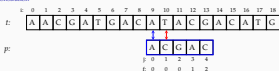
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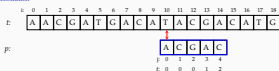
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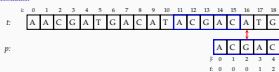
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Complexity of the KMP algorithm

- In the while loop, we either increase i , or shift the comparison
- As a result, the loop runs at most 2n times, complexity is $O(n)$

```

i, j = 0, 0
while i < n:
    if T[i] == P[j]:
        if j == m - 1:
            return j - m + 1
        else:
            i += 1
            j += 1
    elif j > 0:
        j = fail[k - 1]
    else:
        j = 1
    j += 1
return None

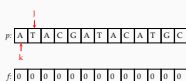
```

Building the failure table

```

f = [0] * m
j, k = 1, 0
while j < m:
    if P[j] == P[k]:
        f[j] = k + 1
        j += 1
        k += 1
    elif k > 0:
        k = fail[k - 1]
    else:
        j += 1

```

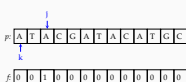


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f = [0] * m
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```

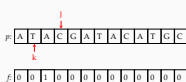


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        f[j] = k + 1
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    elif k > 0:
        k = fail[k - 1]
    else:
        j += 1

```



Rabin-Karp algorithm

- Rabin-Karp string matching algorithm is another interesting algorithm
- The idea is instead of matching the string itself, matching the hash of it (based on a hash function)
- If a match found, we need to verify – the match may be because of a hash collision
- Otherwise, the algorithm makes a single comparison for each position in the text
- However, a hash should be computed for each position (with size m)
- Rolling hash functions avoid this complication

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9
h = 39

p: 4 3 8 5 7 9 4 3 h(p) = 43

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9
h = 37

p: 4 3 8 5 7 9 4 3 h(p) = 43

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9
h = 43

p: 4 3 8 5 7 9 4 3 h(p) = 43

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9
h = 49

p: 4 3 8 5 7 9 4 3 h(p) = 43

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9
h = 47

p: 4 3 8 5 7 9 4 3 h(p) = 43

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9
h = 43

p: 4 3 8 5 7 9 4 3 h(p) = 43

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9
h = 48

p: 4 3 8 5 7 9 4 3 h(p) = 43

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Summary

- String matching is an important problem with wide range of applications
 - The choice of algorithm largely depends on the problem
 - We will revisit the problem on regular expressions and finite-state automata
 - Reading: Goodrich, Tamassia, and Goldwasser (2013, chapter 13)
- Next:
- Algorithms on strings: edit distance / alignment, tries
 - Reading: Goodrich, Tamassia, and Goldwasser (2013, chapter 13), Jurafsky and Martin (2009, section 3.11, or 2.5 in online draft)

Acknowledgments, credits, references

- Goodrich, Michael T., Roberto Tamassia, and Michael H. Goldwasser (2013). *Data Structures and Algorithms in Python*. John Wiley & Sons, Incorporated. [9781118476734](https://doi.org/10.11184/76734).
- Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*, second edition. Pearson Prentice Hall. [978-0-13-504196-3](https://doi.org/10.11184/76734).

