

# Bottom-up Chart Parsing: the CKY algorithm

## Data Structures and Algorithms for Computational Linguistics III (ISCL-BA-07)

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# Parsing so far

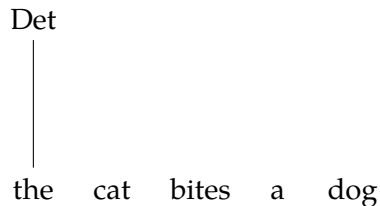
- Parsing is the task of automatic syntactic analysis
- For most practical purposes, context-free grammars are the most useful formalism for parsing
- We can formulate parsing as
  - Top-down: begin with the start symbol, try to *produce* the input string to be parsed
  - Bottom up: begin with the input, and try to *reduce* it to the start symbol
- Both strategies can be cast as search with backtracking
- Backtracking parsers are inefficient: they recompute sub-trees multiple times

# Bottom-up parsing as search

the cat bites a dog

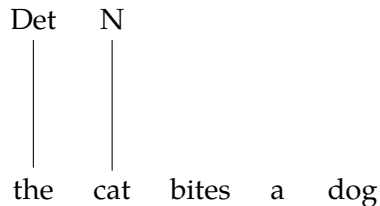
S → NP VP  
NP → Det N  
VP → V NP  
VP → V  
Det → a  
Det → the  
N → cat  
N → dog  
V → bites  
N → bites

# Bottom-up parsing as search



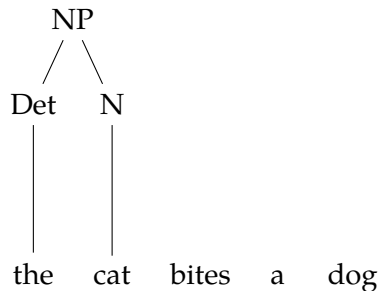
S	→	NP VP
NP	→	Det N
VP	→	V NP
VP	→	V
Det	→	a
Det	→	the
N	→	cat
N	→	dog
V	→	bites
N	→	bites

# Bottom-up parsing as search



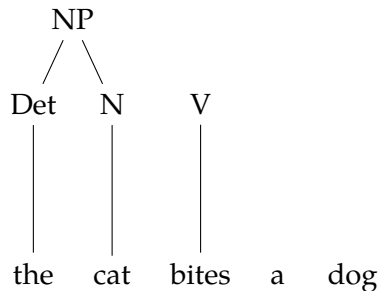
S	→	NP VP
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VP	→	V
Det	→	a
Det	→	the
N	→	cat
N	→	dog
V	→	bites
N	→	bites

# Bottom-up parsing as search



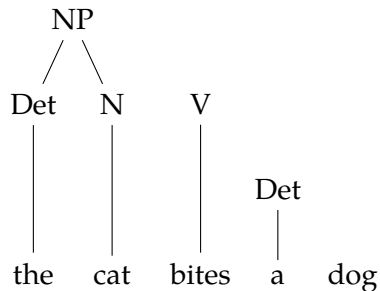
S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Det N  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V  
 Det  $\rightarrow$  a  
 Det  $\rightarrow$  the  
 N  $\rightarrow$  cat  
 N  $\rightarrow$  dog  
 V  $\rightarrow$  bites  
 N  $\rightarrow$  bites

# Bottom-up parsing as search



S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Det N  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V  
 Det  $\rightarrow$  a  
 Det  $\rightarrow$  the  
 N  $\rightarrow$  cat  
 N  $\rightarrow$  dog  
 V  $\rightarrow$  bites  
 N  $\rightarrow$  bites

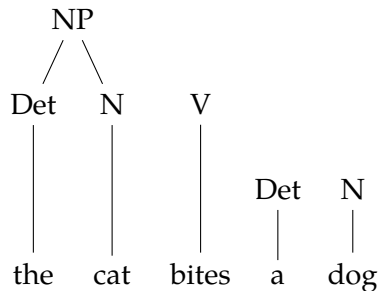
# Bottom-up parsing as search



S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Det N  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V  
 Det  $\rightarrow$  a  
 Det  $\rightarrow$  the  
 N  $\rightarrow$  cat  
 N  $\rightarrow$  dog  
 V  $\rightarrow$  bites  
 N  $\rightarrow$  bites

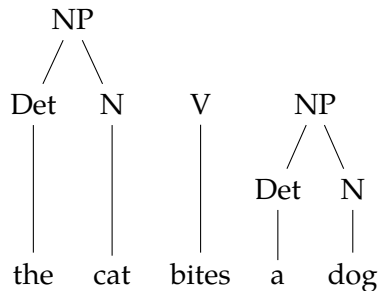


# Bottom-up parsing as search



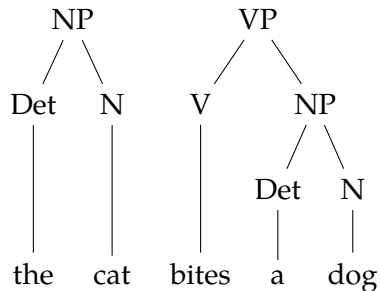
S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Det N  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V  
 Det  $\rightarrow$  a  
 Det  $\rightarrow$  the  
 N  $\rightarrow$  cat  
 N  $\rightarrow$  dog  
 V  $\rightarrow$  bites  
 N  $\rightarrow$  bites

# Bottom-up parsing as search



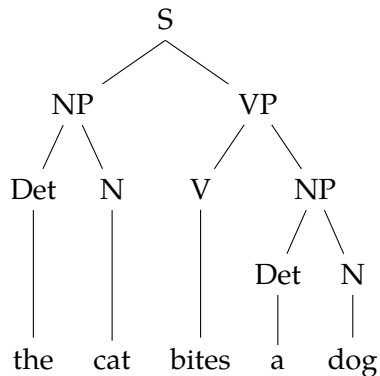
S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Det N  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V  
 Det  $\rightarrow$  a  
 Det  $\rightarrow$  the  
 N  $\rightarrow$  cat  
 N  $\rightarrow$  dog  
 V  $\rightarrow$  bites  
 N  $\rightarrow$  bites

# Bottom-up parsing as search



S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Det N  
 VP  $\rightarrow$  **V NP**  
 VP  $\rightarrow$  V  
 Det  $\rightarrow$  a  
 Det  $\rightarrow$  the  
 N  $\rightarrow$  cat  
 N  $\rightarrow$  dog  
 V  $\rightarrow$  bites  
 N  $\rightarrow$  bites

# Bottom-up parsing as search



S → NP VP

NP → Det N

VP → V NP

VP → V

Det → a

Det → the

N → cat

N → dog

V → bites

N → bites

# Dealing with ambiguity

I saw her duck

S  $\rightarrow$  NP VP

NP  $\rightarrow$  Prn N

NP  $\rightarrow$  Prn

VP  $\rightarrow$  V NP

VP  $\rightarrow$  V

VP  $\rightarrow$  V S

N  $\rightarrow$  duck

V  $\rightarrow$  duck

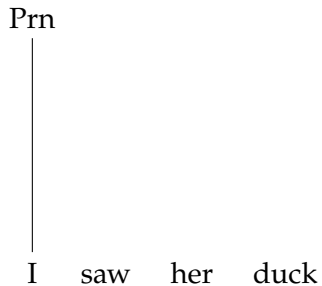
V  $\rightarrow$  saw

Prn  $\rightarrow$  I  $\leftarrow$

Prn  $\rightarrow$  she

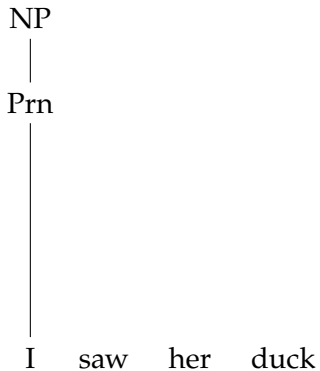
Prn  $\rightarrow$  her

# Dealing with ambiguity



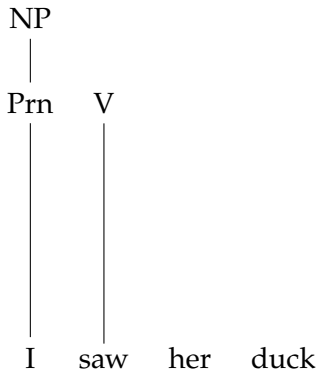
S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Prn N  
 NP  $\rightarrow$  Prn  $\leftarrow$   
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V  
 VP  $\rightarrow$  V S  
 N  $\rightarrow$  duck  
 V  $\rightarrow$  duck  
 V  $\rightarrow$  saw  
 Prn  $\rightarrow$  I  
 Prn  $\rightarrow$  she  
 Prn  $\rightarrow$  her

# Dealing with ambiguity



S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Prn N  
 NP  $\rightarrow$  Prn  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V  
 VP  $\rightarrow$  V S  
 N  $\rightarrow$  duck  
 V  $\rightarrow$  duck  
 V  $\rightarrow$  saw  $\leftarrow$   
 Prn  $\rightarrow$  I  
 Prn  $\rightarrow$  she  
 Prn  $\rightarrow$  her

# Dealing with ambiguity



S  $\rightarrow$  NP VP

NP  $\rightarrow$  Prn N

NP  $\rightarrow$  Prn

VP  $\rightarrow$  V NP

VP  $\rightarrow$  V

VP  $\rightarrow$  V S

N  $\rightarrow$  duck

V  $\rightarrow$  duck

V  $\rightarrow$  saw

Prn  $\rightarrow$  I

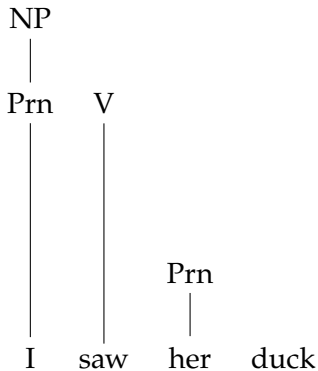
Prn  $\rightarrow$  she

Prn  $\rightarrow$  her

$\leftarrow$



# Dealing with ambiguity



S → NP VP

NP → Prn N

NP → **Prn** ←

VP → V NP

VP → V

VP → V S

N → **duck**

V → **duck**

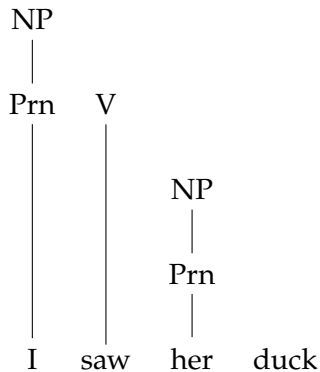
V → saw

Prn → I

Prn → she

Prn → her

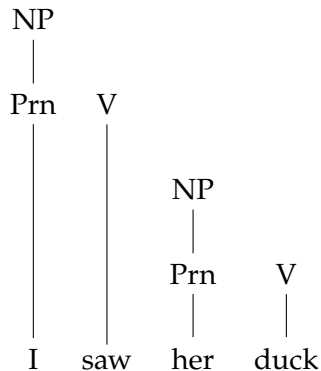
# Dealing with ambiguity


 $S \rightarrow NP VP$ 
 $NP \rightarrow Prn N$ 
 $NP \rightarrow Prn$ 
 $VP \rightarrow V NP$ 
 $VP \rightarrow V$ 
 $VP \rightarrow V S$ 
 $N \rightarrow \text{duck}$ 
 $V \rightarrow \text{duck}$ 

←

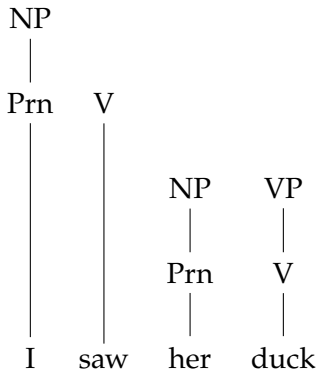
 $V \rightarrow \text{saw}$ 
 $Prn \rightarrow I$ 
 $Prn \rightarrow \text{she}$ 
 $Prn \rightarrow \text{her}$

# Dealing with ambiguity



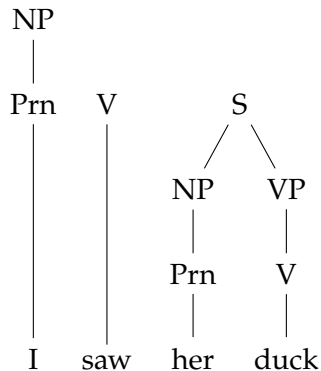
$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $NP \rightarrow Prn$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V \quad \leftarrow$   
 $VP \rightarrow V S$   
 $N \rightarrow duck$   
 $V \rightarrow duck$   
 $V \rightarrow saw$   
 $Prn \rightarrow I$   
 $Prn \rightarrow she$   
 $Prn \rightarrow her$

# Dealing with ambiguity



$S \rightarrow \text{NP VP} \leftarrow$   
 $\text{NP} \rightarrow \text{Prn N}$   
 $\text{NP} \rightarrow \text{Prn}$   
 $\text{VP} \rightarrow \text{V NP}$   
 $\text{VP} \rightarrow \text{V}$   
 $\text{VP} \rightarrow \text{V S}$   
 $\text{N} \rightarrow \text{duck}$   
 $\text{V} \rightarrow \text{duck}$   
 $\text{V} \rightarrow \text{saw}$   
 $\text{Prn} \rightarrow \text{I}$   
 $\text{Prn} \rightarrow \text{she}$   
 $\text{Prn} \rightarrow \text{her}$

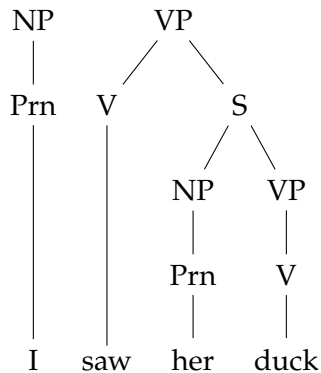
# Dealing with ambiguity


 $S \rightarrow NP VP$ 
 $NP \rightarrow Prn N$ 
 $NP \rightarrow Prn$ 
 $VP \rightarrow V NP$ 
 $VP \rightarrow V$ 
 $VP \rightarrow V S$ 

←

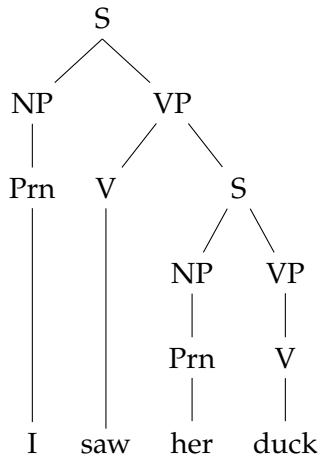
 $N \rightarrow \text{duck}$ 
 $V \rightarrow \text{duck}$ 
 $V \rightarrow \text{saw}$ 
 $Prn \rightarrow I$ 
 $Prn \rightarrow \text{she}$ 
 $Prn \rightarrow \text{her}$

# Dealing with ambiguity



$S \rightarrow \text{NP VP} \quad \leftarrow$   
 $\text{NP} \rightarrow \text{Prn N}$   
 $\text{NP} \rightarrow \text{Prn}$   
 $\text{VP} \rightarrow \text{V NP}$   
 $\text{VP} \rightarrow \text{V}$   
 $\text{VP} \rightarrow \text{V S}$   
 $\text{N} \rightarrow \text{duck}$   
 $\text{V} \rightarrow \text{duck}$   
 $\text{V} \rightarrow \text{saw}$   
 $\text{Prn} \rightarrow \text{I}$   
 $\text{Prn} \rightarrow \text{she}$   
 $\text{Prn} \rightarrow \text{her}$

# Dealing with ambiguity



$S \rightarrow NP VP$

$NP \rightarrow Prn N$

$NP \rightarrow Prn$

$VP \rightarrow V NP$

$VP \rightarrow V$

$VP \rightarrow V S$

$N \rightarrow \text{duck}$

$V \rightarrow \text{duck}$

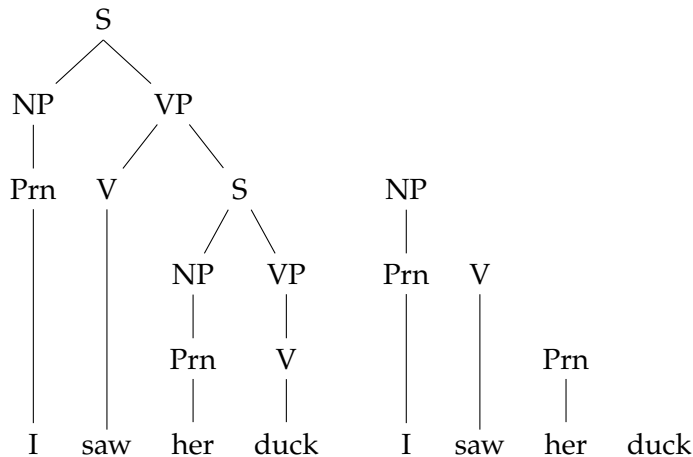
$V \rightarrow \text{saw}$

$Prn \rightarrow I$

$Prn \rightarrow \text{she}$

$Prn \rightarrow \text{her}$

# Dealing with ambiguity



$$S \rightarrow NP VP$$

$$NP \rightarrow Prn N$$

$$NP \rightarrow \text{Prn}$$

$$VP \rightarrow V NP$$

$$VP \rightarrow V$$

$$VP \rightarrow V S$$

$$N \rightarrow \text{duck} \quad \leftarrow$$

$$V \rightarrow \text{duck}$$

$$V \rightarrow \text{saw}$$

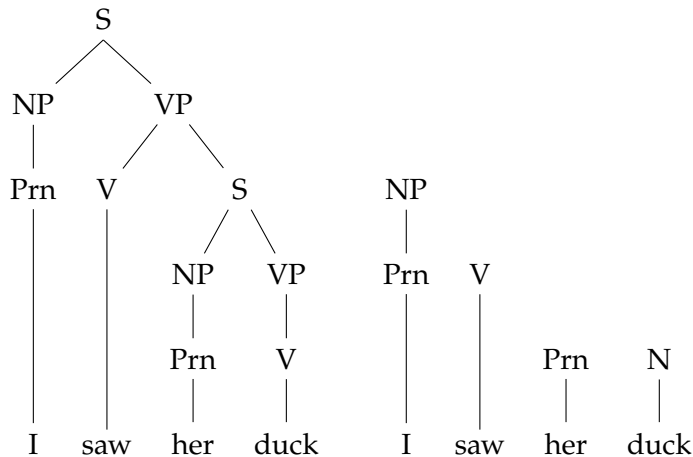
$$Prn \rightarrow I$$

$$Prn \rightarrow \text{she}$$

$$Prn \rightarrow \text{her}$$

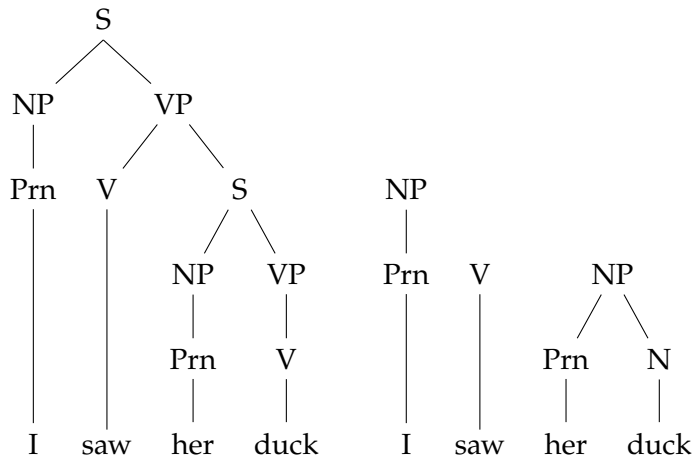


# Dealing with ambiguity



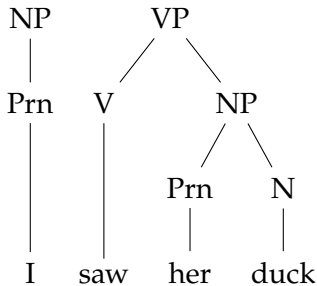
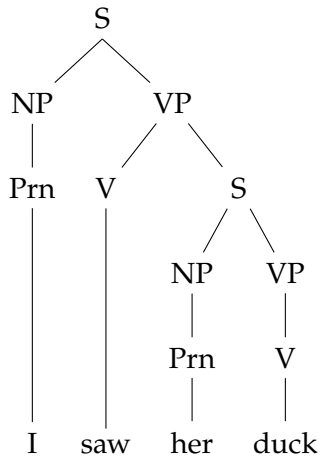
$S \rightarrow NP VP$   
 $NP \rightarrow \text{Prn } N \quad \leftarrow$   
 $NP \rightarrow \text{Prn}$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V$   
 $VP \rightarrow V S$   
 $N \rightarrow \text{duck}$   
 $V \rightarrow \text{duck}$   
 $V \rightarrow \text{saw}$   
 $\text{Prn} \rightarrow I$   
 $\text{Prn} \rightarrow \text{she}$   
 $\text{Prn} \rightarrow \text{her}$

# Dealing with ambiguity



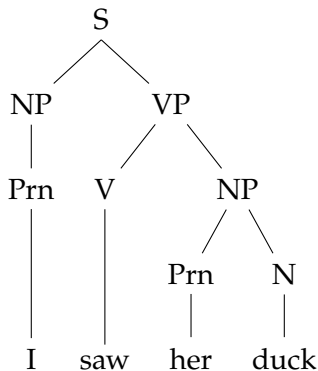
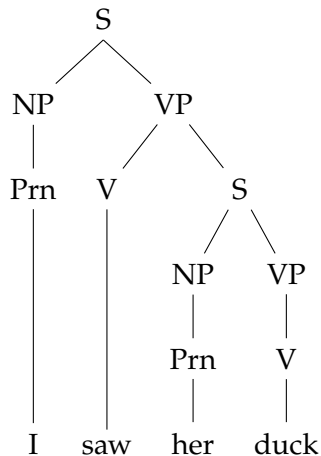
$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $NP \rightarrow Prn$   
 $VP \rightarrow V NP \quad \leftarrow$   
 $VP \rightarrow V$   
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 $N \rightarrow duck$   
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 $V \rightarrow saw$   
 $Prn \rightarrow I$   
 $Prn \rightarrow she$   
 $Prn \rightarrow her$

# Dealing with ambiguity



$S \rightarrow \text{NP VP} \quad \leftarrow$   
 $\text{NP} \rightarrow \text{Prn N}$   
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 $\text{V} \rightarrow \text{saw}$   
 $\text{Prn} \rightarrow \text{I}$   
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# Dealing with ambiguity



$S \rightarrow NP VP$

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$N \rightarrow \text{duck}$

$V \rightarrow \text{duck}$

$V \rightarrow \text{saw}$

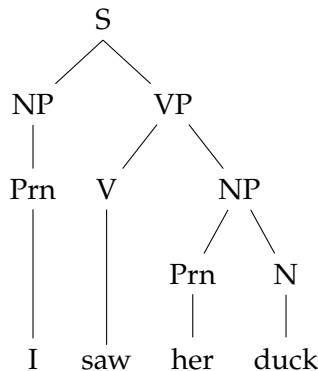
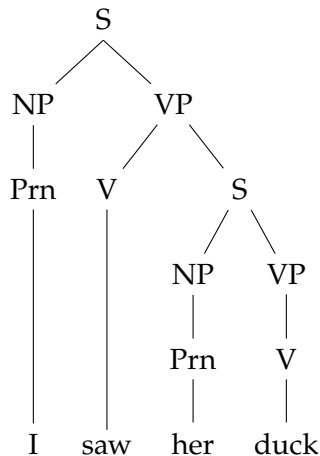
$Prn \rightarrow I$

$Prn \rightarrow \text{she}$

$Prn \rightarrow \text{her}$

# How to represent multiple parses

parse forest grammar



$$S_{0:4} \rightarrow NP_{0:1} VP_{1:4}$$

$$NP_{0:1} \rightarrow Prn_{0:1}$$

$$Prn_{0:1} \rightarrow I_{0:1}$$

$$VP_{1:4} \rightarrow V_{1:2} S_{2:4}$$

$$V_{1:2} \rightarrow \text{saw}_{1:2}$$

$$S_{2:4} \rightarrow Prn_{2:3} V_{3:4}$$

$$V_{3:4} \rightarrow \text{duck}_{3:4}$$


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$$VP_{1:4} \rightarrow V_{1:2} NP_{2:4}$$

$$NP_{2:4} \rightarrow Prn_{2:3} N_{3:4}$$

# CKY algorithm

- The CKY (Cocke–Kasami–Younger) parsing algorithm is a dynamic programming algorithm
- It processes the input *bottom up*, and saves the intermediate results on a *chart*
- Time complexity for *recognition* is  $O(n^3)$
- Space complexity is  $O(n^2)$
- It requires the CFG to be in *Chomsky normal form* (CNF) (can somewhat be relaxed, but not common)

# Chomsky normal form (CNF)

- A CFG is in CNF, if the rewrite rules are in one of the following forms
  - $A \rightarrow BC$
  - $A \rightarrow a$

where  $A, B, C$  are non-terminals and  $a$  is a terminal

- Any CFG can be converted to CNF
- Resulting grammar is *weakly equivalent* to the original grammar:
  - it generates/accepts the same language
  - but the derivations are different

# Converting to CNF: example

S → NP VP  
S → Aux NP VP  
NP → the N  
VP → V NP  
VP → V  
N → cat  
N → dog  
V → bites  
N → bites



# Converting to CNF: example

S  $\rightarrow$  NP VP

S  $\rightarrow$  Aux NP VP

NP  $\rightarrow$  the N

VP  $\rightarrow$  V NP

VP  $\rightarrow$  V

N  $\rightarrow$  cat

N  $\rightarrow$  dog

V  $\rightarrow$  bites

N  $\rightarrow$  bites

- S  $\rightarrow$  Aux NP VP  
 S  $\rightarrow$  Aux NP VP  $\Rightarrow$  S  $\rightarrow$  Aux X  
 X  $\rightarrow$  NP VP

# Converting to CNF: example

$S \rightarrow NP VP$

$S \rightarrow Aux NP VP$

$NP \rightarrow the N$

$VP \rightarrow V NP$

$VP \rightarrow V$

$N \rightarrow cat$

$N \rightarrow dog$

$V \rightarrow bites$

$N \rightarrow bites$

- $S \rightarrow Aux NP VP$   
 $S \rightarrow Aux NP VP \Rightarrow S \rightarrow Aux X$   
 $X \rightarrow NP VP$
- $NP \rightarrow the N$   
 $NP \rightarrow the N \Rightarrow NP \rightarrow X N$   
 $X \rightarrow the$

# Converting to CNF: example

$S \rightarrow NP VP$

$S \rightarrow \text{Aux } NP VP$

$NP \rightarrow \text{the } N$

$VP \rightarrow V NP$

$VP \rightarrow V$

$N \rightarrow \text{cat}$

$N \rightarrow \text{dog}$

$V \rightarrow \text{bites}$

$N \rightarrow \text{bites}$

- $S \rightarrow \text{Aux } NP VP$   
 $S \rightarrow \text{Aux } NP VP \Rightarrow S \rightarrow \text{Aux } X$   
 $X \rightarrow NP VP$
- $NP \rightarrow \text{the } N$   
 $NP \rightarrow \text{the } N \Rightarrow NP \rightarrow X N$   
 $X \rightarrow \text{the}$
- $VP \rightarrow V$   
 $VP \rightarrow V \Rightarrow VP \rightarrow \text{bites}$

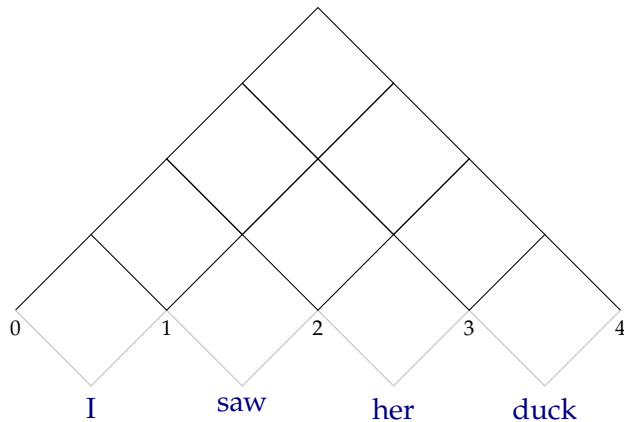
# Converting to CNF

1. Eliminate the  $\epsilon$  rules: if  $A \rightarrow \epsilon$  is in the grammar
  - replace any rule  $B \rightarrow \alpha A \beta$  with two rules
 
$$B \rightarrow \alpha \beta$$

$$B \rightarrow \alpha A' \beta$$
  - add  $A' \rightarrow \alpha$  for all  $\alpha$  (except  $\epsilon$ ) whose LHS is  $A$
  - repeat the process for newly created  $\epsilon$  rules
  - remove the rules with  $\epsilon$  on the RHS (except  $S \rightarrow \epsilon$ )
2. Eliminate unit rules: for a rule  $A \rightarrow B$ 
  - Replace the rule with  $A \rightarrow \alpha_1 \mid \dots \mid \alpha_n$ , where  $\alpha_1, \dots, \alpha_n$  are all RHS or rule  $B$
  - Remove the rule  $A \rightarrow B$
  - Repeat the process until no unit rules remain
3. Binarize all the non-binary rules with non-terminal on the RHS: for a rule  $A \rightarrow X_1 X_2 \dots X_n$ :
  - Replace the rule with  $A \rightarrow A_1 X_2 \dots X_n$ , and add  $A_1 \rightarrow X_1 X_2$
  - Repeat the process until all new rules are binary

# CKY demonstration

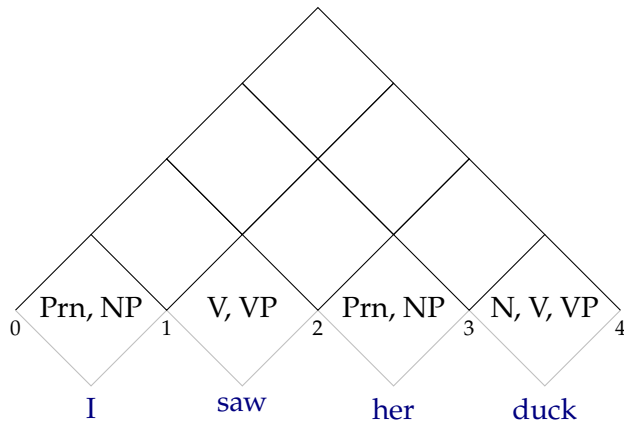
an ambiguous example



$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V S$   
 $N \rightarrow \text{duck}$   
 $VP \rightarrow \text{duck} \mid \text{saw}$   
 $V \rightarrow \text{duck} \mid \text{saw}$   
 $Prn \rightarrow I \mid \text{she} \mid \text{her}$   
 $NP \rightarrow I \mid \text{she} \mid \text{her}$

# CKY demonstration

an ambiguous example

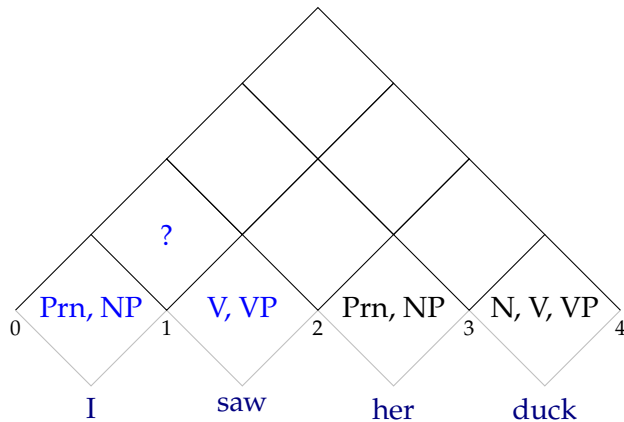


$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V S$   
 $N \rightarrow duck$   
 $VP \rightarrow duck \mid saw$   
 $V \rightarrow duck \mid saw$   
 $Prn \rightarrow I \mid she \mid her$   
 $NP \rightarrow I \mid she \mid her$

# CKY demonstration

an ambiguous example

$S \rightarrow NP VP$

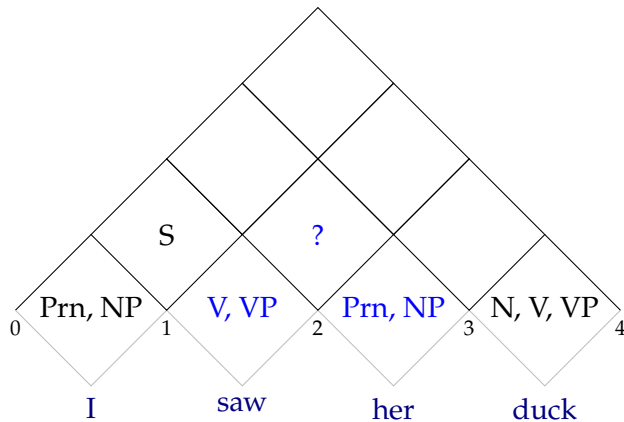


$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V S$   
 $N \rightarrow duck$   
 $VP \rightarrow duck \mid saw$   
 $V \rightarrow duck \mid saw$   
 $Prn \rightarrow I \mid she \mid her$   
 $NP \rightarrow I \mid she \mid her$

# CKY demonstration

an ambiguous example

VP  $\rightarrow$  V NP



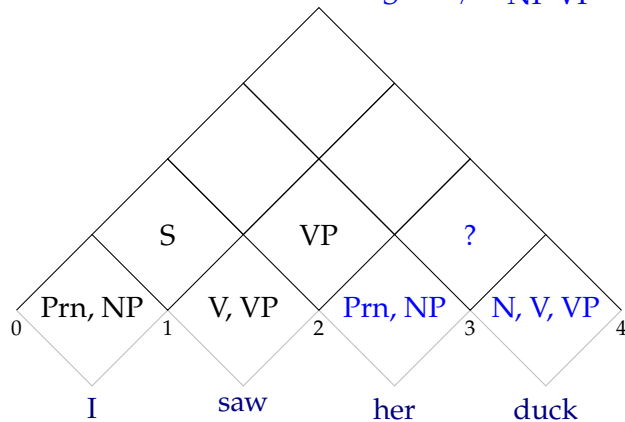
S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Prn N  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V S  
 N  $\rightarrow$  duck  
 VP  $\rightarrow$  duck | saw  
 V  $\rightarrow$  duck | saw  
 Prn  $\rightarrow$  I | she | her  
 NP  $\rightarrow$  I | she | her



# CKY demonstration

an ambiguous example

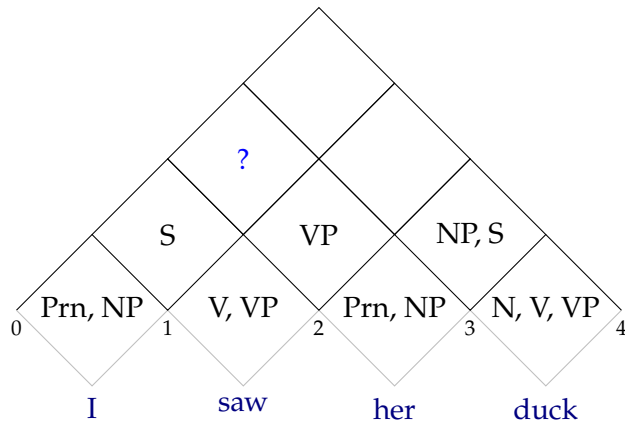
NP  $\rightarrow$  Prn N  
S  $\rightarrow$  NP VP



S  $\rightarrow$  NP VP  
NP  $\rightarrow$  Prn N  
VP  $\rightarrow$  V NP  
VP  $\rightarrow$  V S  
N  $\rightarrow$  duck  
VP  $\rightarrow$  duck | saw  
V  $\rightarrow$  duck | saw  
Prn  $\rightarrow$  I | she | her  
NP  $\rightarrow$  I | she | her

# CKY demonstration

an ambiguous example

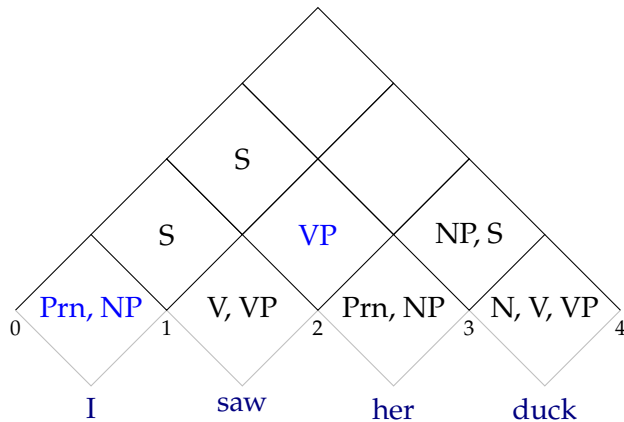


$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V S$   
 $N \rightarrow duck$   
 $VP \rightarrow duck \mid saw$   
 $V \rightarrow duck \mid saw$   
 $Prn \rightarrow I \mid she \mid her$   
 $NP \rightarrow I \mid she \mid her$

# CKY demonstration

an ambiguous example

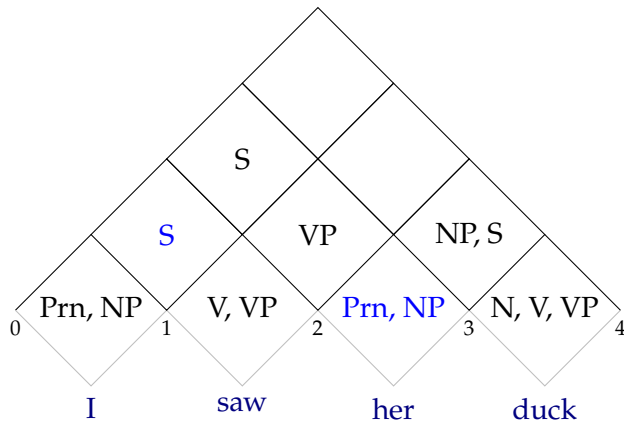
$S \rightarrow NP VP$



$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V S$   
 $N \rightarrow duck$   
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## CKY demonstration

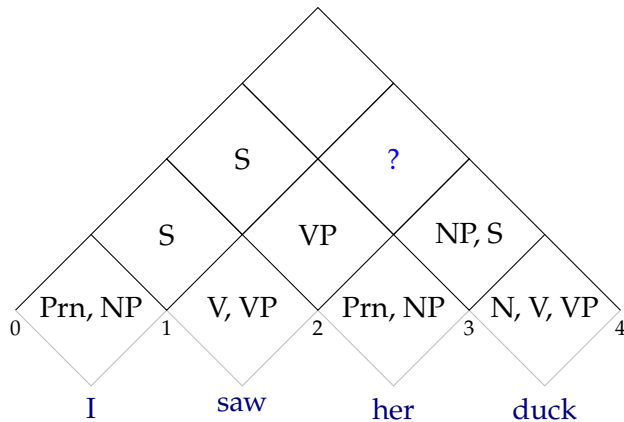
an ambiguous example



S  $\rightarrow$  NP VP  
NP  $\rightarrow$  Prn N  
VP  $\rightarrow$  V NP  
VP  $\rightarrow$  V S  
N  $\rightarrow$  duck  
VP  $\rightarrow$  duck | saw  
V  $\rightarrow$  duck | saw  
Prn  $\rightarrow$  I | she | her  
NP  $\rightarrow$  I | she | her

# CKY demonstration

an ambiguous example

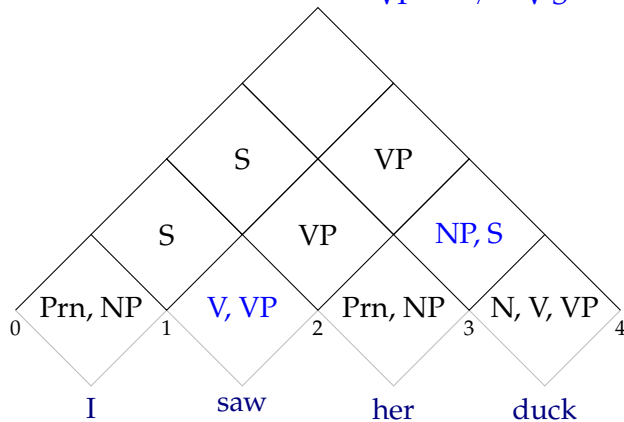


$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V S$   
 $N \rightarrow duck$   
 $VP \rightarrow duck \mid saw$   
 $V \rightarrow duck \mid saw$   
 $Prn \rightarrow I \mid she \mid her$   
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# CKY demonstration

an ambiguous example

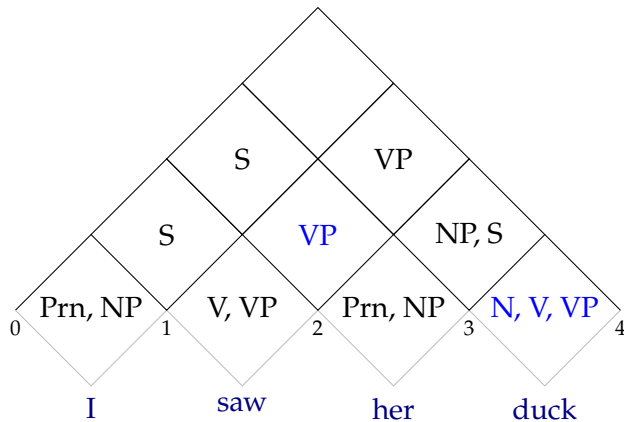
VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V S



S  $\rightarrow$  NP VP  
 NP  $\rightarrow$  Prn N  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V S  
 N  $\rightarrow$  duck  
 VP  $\rightarrow$  duck | saw  
 V  $\rightarrow$  duck | saw  
 Prn  $\rightarrow$  I | she | her  
 NP  $\rightarrow$  I | she | her

# CKY demonstration

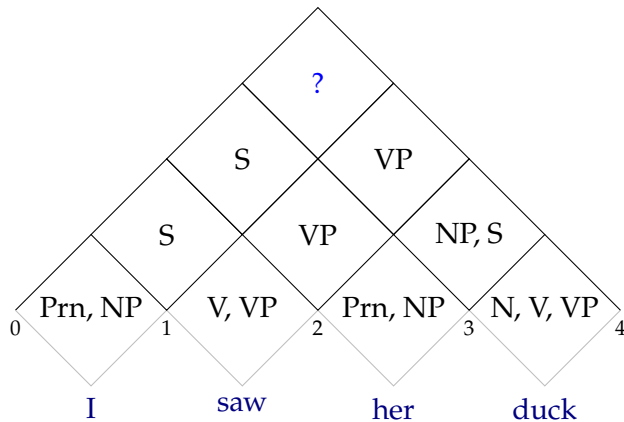
an ambiguous example



$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V S$   
 $N \rightarrow duck$   
 $VP \rightarrow duck \mid saw$   
 $V \rightarrow duck \mid saw$   
 $Prn \rightarrow I \mid she \mid her$   
 $NP \rightarrow I \mid she \mid her$

# CKY demonstration

an ambiguous example



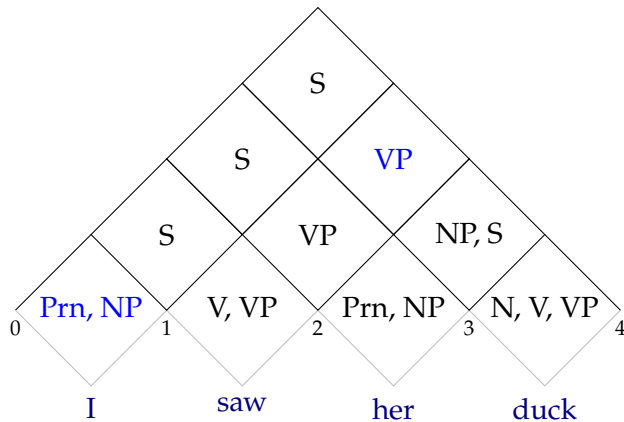
$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V S$   
 $N \rightarrow duck$   
 $VP \rightarrow duck \mid saw$   
 $V \rightarrow duck \mid saw$   
 $Prn \rightarrow I \mid she \mid her$   
 $NP \rightarrow I \mid she \mid her$



# CKY demonstration

an ambiguous example

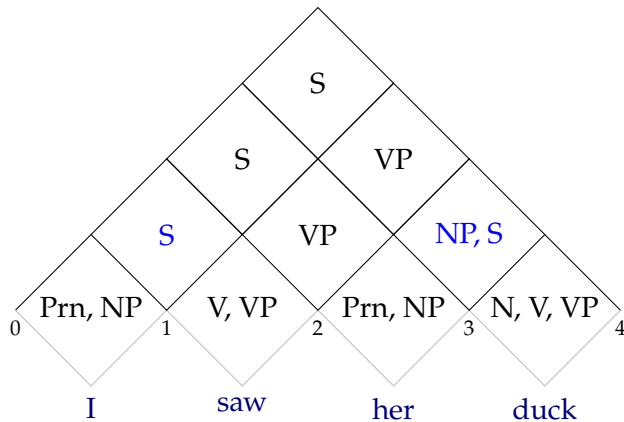
$S \rightarrow NP VP$



$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V S$   
 $N \rightarrow duck$   
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 $V \rightarrow duck \mid saw$   
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# CKY demonstration

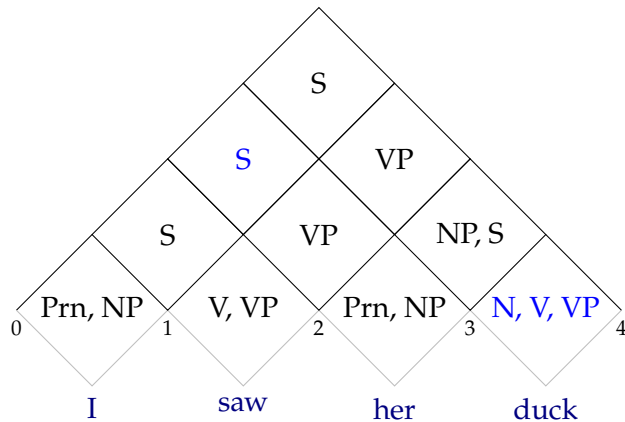
an ambiguous example



$S \rightarrow NP VP$   
 $NP \rightarrow Prn N$   
 $VP \rightarrow V NP$   
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 $N \rightarrow duck$   
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# CKY demonstration

an ambiguous example



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# CKY demonstration: the chart

our chart is a 2D array

NP, Prn	S	S	S					
	V, VP	VP	VP					
		Prn	NP, S					
			V, N, NP					
0	she	1	saw	2	her	3	duck	4

Space complexity is  $O(n^2)$ .

# CKY demonstration: the chart

our chart is a 2D array – this is more convenient for programming

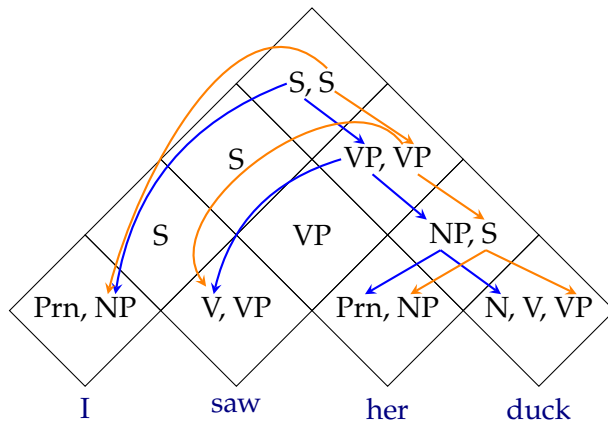
S				
S	VP			
S	VP	NP, S		
NP, Prn	V, VP	Prn, NP	V, N, NP	
0	1	2	3	4
she	saw	her	duck	

Space complexity is  $O(n^2)$ .

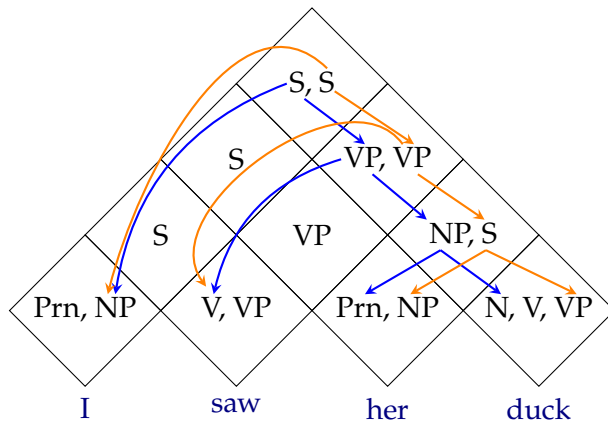
# Parsing vs. recognition

- We went through a recognition example
- Note that the algorithm is not directional: it takes the complete input
- Recognition accepts or rejects a sentence based on a grammar
- For parsing, we want to know the derivations that yielded a correct parse
- To recover parse trees, we
  - we follow the same procedure as recognition
  - add back links to keep track of the derivations

# Chart parsing example (CKY parsing)



# Chart parsing example (CKY parsing)



The chart stores a *parse forest* efficiently.



## Summary

- + CKY avoids re-computing the analyses by storing the earlier analyses (of sub-spans) in a table
- It still computes lower level constituents that are not allowed by the grammar
- CKY requires the grammar to be in CNF
  - CKY has  $O(n^3)$  recognition complexity
  - For parsing we need to keep track of backlinks
  - CKY can efficiently store all possible parses in a chart
  - Enumerating all possible parses have exponential complexity (worst case)
  - Suggested reading: Jurafsky and Martin (2009, draft 3rd ed, section 13.2)

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Next:

- Top-down chart parsing: Earley algorithm
- Suggested reading:
  - Jurafsky and Martin (2009, section 13.2.4)
  - Grune and Jacobs (2007, section 7.2)

# Acknowledgments, references, additional reading material



Grune, Dick and Ceriel J.H. Jacobs (2007). *Parsing Techniques: A Practical Guide*. second. Monographs in Computer Science. The first edition is available at [http://dickgrune.com/Books/PTAPG\\_1st\\_Edition/BookBody.pdf](http://dickgrune.com/Books/PTAPG_1st_Edition/BookBody.pdf). Springer New York. ISBN: 9780387689548.



Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second edition. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.