Algorithmic patterns

Data Structures and Algorithms for Computational Linguistics III
(ISCL-BA-07)

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Overview

- Some common approaches to algorithm design
  - Revisiting recursion
  - Brute force
  - Divide and conquer
  - Greedy algorithms
  - Dynamic programming

Recursion

linear search again

Your task from the first lecture: writing a recursive linear search.

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the complete code

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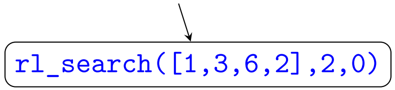
Can we improve this?

How does this recursion work

recursion trace/graph



```
rl_search([1,3,6,2],2,0)
```

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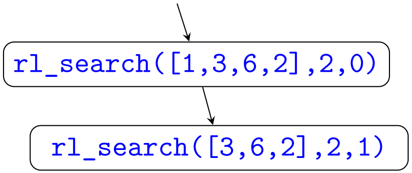

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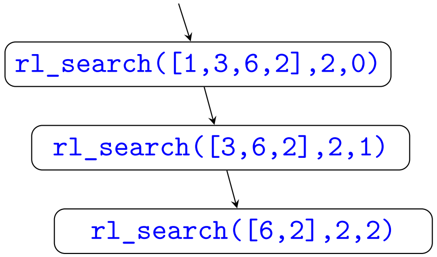

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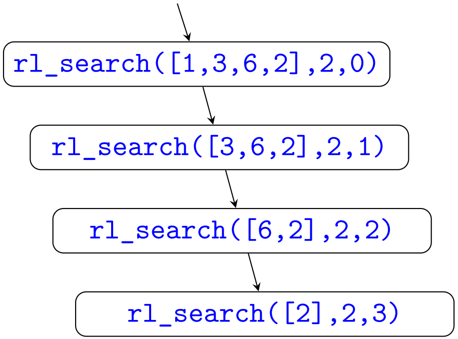

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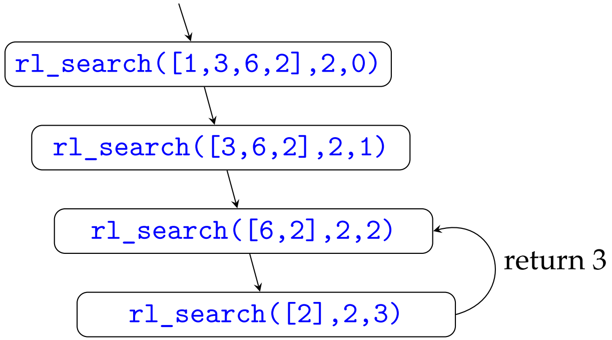

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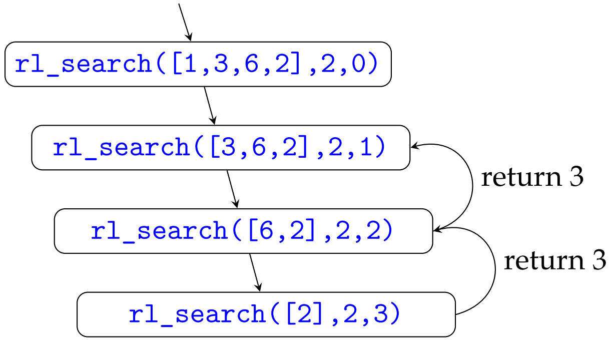

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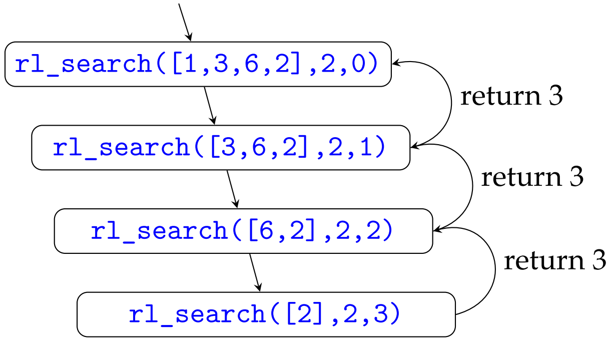

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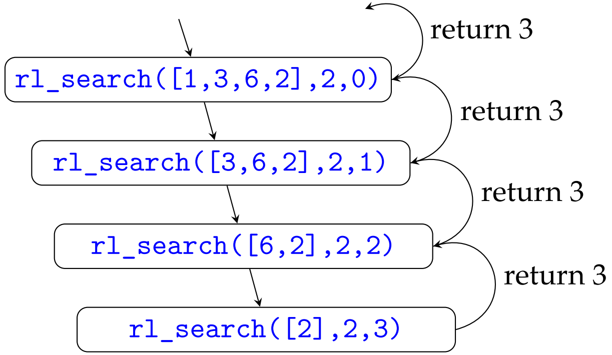

Recursion: practical issues

recursion depth and tail recursion

- Each function call requires some bookkeeping
- Compilers/interpreters allocate space on a stack for the bookkeeping for each function call
- Most environments limit the number of recursive calls: long chains of recursion are likely to cause errors
- *Tail recursion* (e.g., our recursive search example) is easy to convert to iteration
- It is also easy to optimize, and optimized by many compilers (not by the Python interpreter)

Another recursive example

every algorithm course is required to introduce Fibonacci numbers

Fibonacci numbers are defined as:

$$
\begin{aligned}
F_0 &= 0 \\
F_1 &= 1 \\
F_n &= F_{n-1} + F_{n-2} \quad \text{for} \quad n > 1
\end{aligned}
$$

```python
def fib(n):
  if n <= 1:
    return n
  return fib(n-2) + fib(n-1)
```

- Recursion is common in math, and maps well to the recursive algorithms

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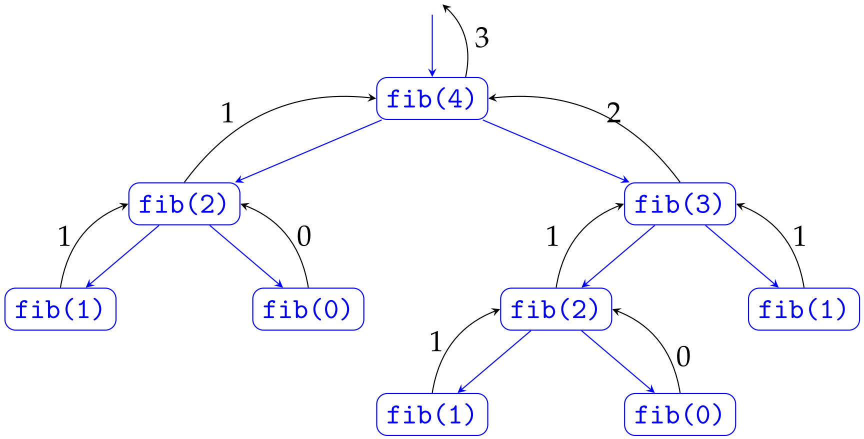
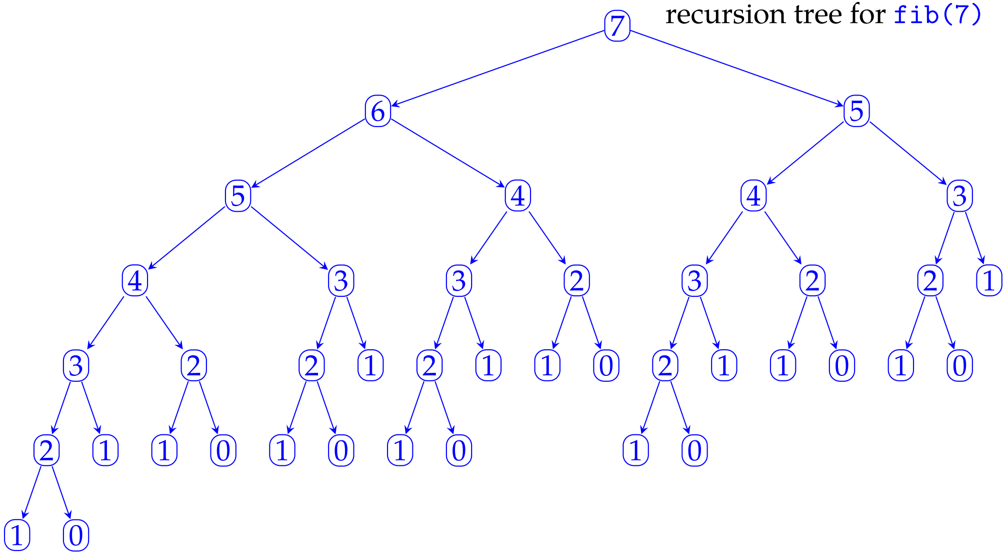
- Note that we now have binary recursion, each function call creates two calls to self
- We follow the math exactly, but is this code efficient?

Visualizing binary recursion



Complexity of (naive) Fibonacci algorithm

recursion tree for `fib(7)`



Brute force

- In some cases, we may need to enumerate all possible cases (e.g., to find the best solution)
- Common in combinatorial problems
- Often intractable, practical only for small input sizes
- It is also typically the beginning of finding a more efficient approach

Brute force

example: finding all possible ways to segment a string

- Segmentation is prevalent in CL
  - Examples include finding words: tokenization (particularly for writing systems that do not use white space)
  - Finding sub-word units (e.g., morphemes, or more specialized application: compound splitting)
  - Psycholinguistics: how do people extract words from continuous speech?
- We consider the following problem:
  - Given a metric or score to determine the ”best” segmentation
  - We enumerate all possible ways to segment, pick the one with the best score

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- We consider the following problem:
  - Given a metric or score to determine the ”best” segmentation
  - We enumerate all possible ways to segment, pick the one with the best score
- How can we enumerate all possible segmentations of a string?

Segmentation

a recursive solution

```python
def segment_r(seq):
    if len(seq) == 1:
        yield [seq]
    else:
        for seg in segment_r(seq[1:]):
            yield [seq[0]] + seg
            yield [seq[0] + seg[0]] + seg[1:]
```

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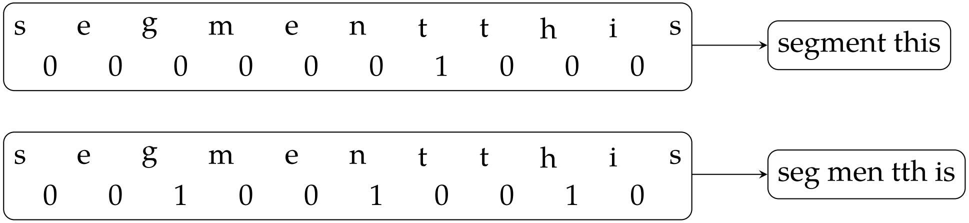
```

- Can you think of a non-recursive solution?

Enumerating segmentations

sketch of a non-recursive solution



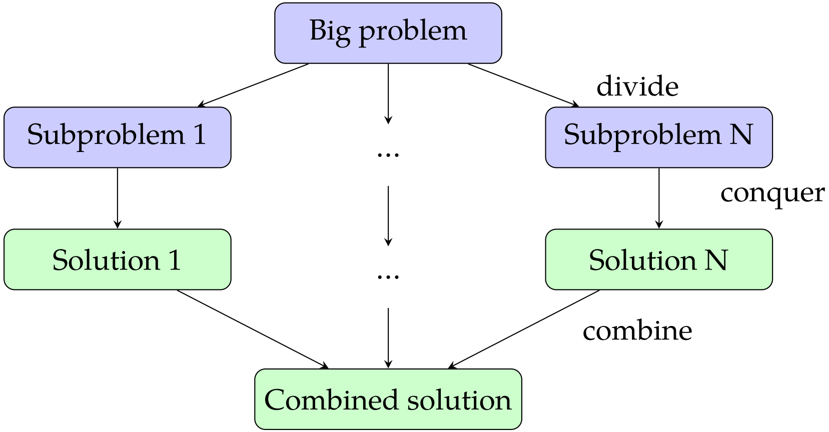
- ‘1’ means there is a boundary at this position
- Problem is now enumerating all possible binary strings of length $n - 1$ (this is binary counting)

Divide and conquer

- The general idea is dividing the problem into smaller parts until it becomes trivial to solve
- Once small parts are solved, the results are combined
- Goes well with recursion
- We have already seen a particular flavor: binary search
- The algorithms like binary search are sometimes called *decrease and conquer*

Divide and conquer

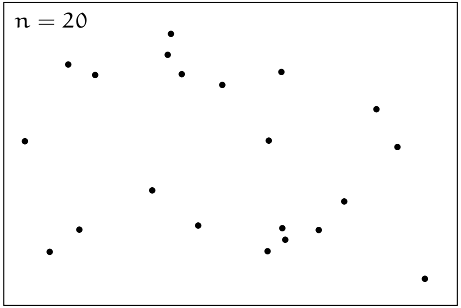
General idea



Divide and conquer

an example: nearest neighbors (only a sketch)

- Task: find the closest two points



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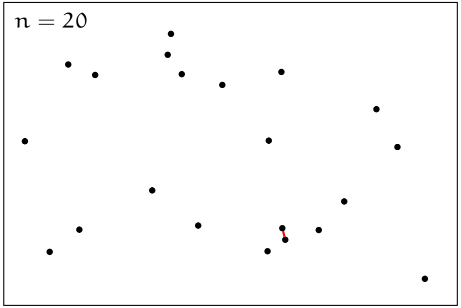
- Task: find the closest two points
- Direct solution:
  $20 \times 20 = 400$ comparisons[1]



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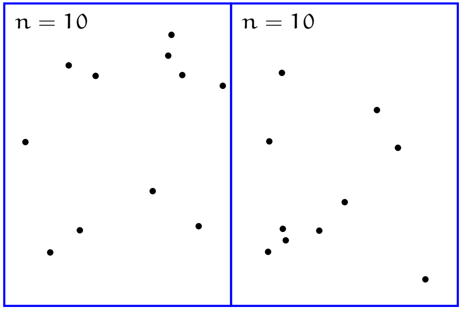

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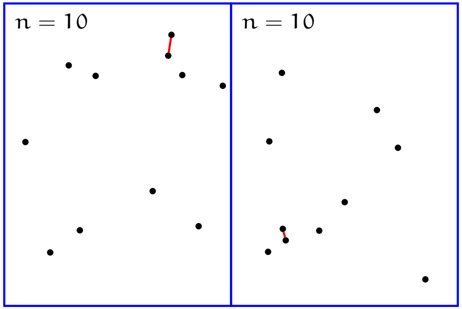
- Solve separately (conquer): $10 \times 10 + 10 \times 10 = 200$ comparisons



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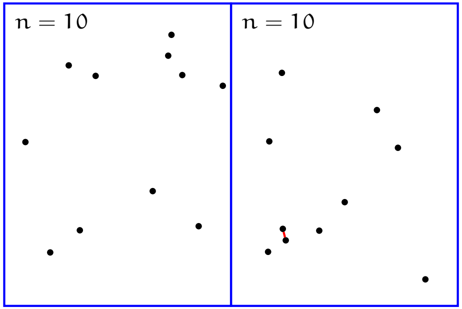
- Combine: pick the minimum of the individual solutions



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- Gain is higher when n is larger, and we divide further

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Divide and conquer
summary

- This is probably the most common pattern
- Divide and conquer does not always yield good results, the cost of merging should be less than the gain from the division(s)
- Many of the important algorithms fall into this category:
  - merge sort and quick sort (coming soon)
  - integer multiplication
  - matrix multiplication
  - fast Furrier transform (FFT)

Greedy algorithms

- An algorithm is greedy if it optimizes a local constraint
- For some problems, greedy algorithms result in correct solutions
- In others they may result in ‘good enough’ solutions
- If they work, they are efficient
- An important class of graph algorithms fall into this category (e.g., finding shortest paths, scheduling)

Greedy algorithms

a simple example: 'change making'

- We want to produce minimum number of coins for a particular sum s
  1. Pick the largest coin $c <= s$
  2. set $s = s - c$
  3. repeat 1 & 2 until $s = 0$

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- Is this algorithm correct?
- Think about coins of 10, 30, 40 and apply the algorithm for the sum value of 60
- Is it correct if the coin values were limited Euro coins?

Dynamic programming

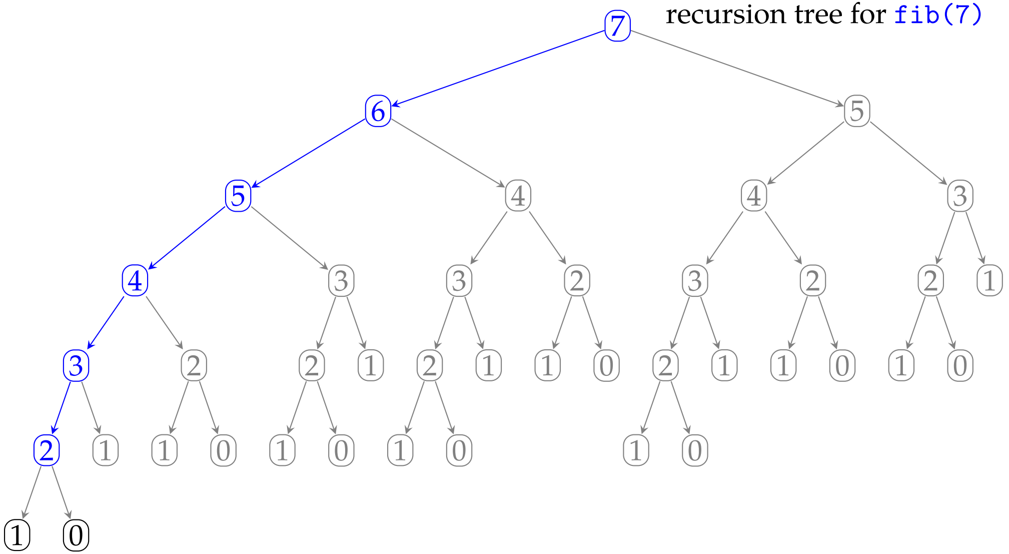
- Dynamic programming is a method to save earlier results to reduce computation
- It is sometimes called memoization (it is not a typo)
- Again, a large number of algorithms we use fall into this category, including common parsing algorithms

Dynamic programming
example: Fibonacci

```python
def memofib(n, memo = {0: 0, 1:1}):
    if n not in memo:
        memo[n] = memofib(n-1) + memofib(n-2)
    return memo[n]
```

- We save the results calculated in a dictionary,
- if the result is already in the dictionary, we return without recursion
- Otherwise we calculate recursively as before
- The difference is big, but there is also a ‘neater’ solution without (explicit) memoization

Complexity of Fibonacci algorithm with dynamic pogramming



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Next:

- Sorting
- Reading: Goodrich, Tamassia, and Goldwasser (2013, chapter 12)

Nearest neighbors
an exercise

- Define and implement a divide-and-conquer algorithm for nearest neighbor problem, which divides the input into two until the solution becomes trivial
- Analyze your algorithm and compare to the naive version sketched above (an implementation was provided in the previous lecture)

Linear search

a little bit of optimization

```
def rl_search(seq, val, i=0):
    if not seq:
        return None
    if val == seq[0]:
        return i
    else:
        return rl_search(seq[1:], val,
        ↪  i+1)
```

```
def rl_search2(seq, val, i=0):
    if i >= len(seq):
        return None
    if val == seq[i]:
        return i
    else:
        return rl_search2(seq, val, i
        ↪  + 1)
```

Which one is faster, and why?

Better solutions for Fibonacci numbers

```python
def fib2(n):
    if n <= 1:
        return (n, 0)
    a, b = fib2(n - 1)
    return (a+b, a)
```

```python
def fib3(n):
    if n <= 1:
        return n
    a, b = 0, 1
    for i in range(0, n):
        a, b = b, a + b
    return a
```

Which one is faster/better?

Segmentation

without `yield`

```python
def segment_r(seq):
    segs = []
    if len(seq) == 1:
        return [seq]
    for seg in segment_r(seq[1:]):
        segs.append([seq[0]] + seg)
        segs.append([seq[0] + seg[0]] + seg[1:])
    return segs
```

Acknowledgments, credits, references

- Some of the slides are based on the previous year's course by Corina Dima.

Goodrich, Michael T., Roberto Tamassia, and Michael H. Goldwasser (2013). *Data Structures and Algorithms in Python*. John Wiley & Sons, Incorporated. ISBN: 9781118476734.

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