

Finite state automata

Data Structures and Algorithms for Computational Linguistics III
(ISCL-BA-07)

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Seminar für Sprachwissenschaft

Winter Semester 2021/22

version: 2021.04.02.0022.00.00

Why study finite-state automata?

- Unlike some of the abstract machines we discussed, *finite-state automata* are efficient models of computation
- There are many applications
 - Electronic circuit design
 - Workflow management
 - Games
 - Pattern matching
 - ...
- But more importantly >):
 - Tokenization, stemming
 - Morphological analysis
 - Spell checking
 - Shallow parsing/chunking
 - ...

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Finite-state automata (FSA)

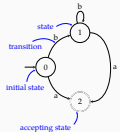
- A finite-state machine is in one of a finite-number of states in a given time
 - The machine changes its state based on its input
 - Every regular language is generated/recognized by an FSA
 - Every FSA generates/recognizes a regular language
 - Two flavors:
 - Deterministic finite automata (DFA)
 - Non-deterministic finite automata (NFA)
- Note: the NFA is a superset of DFA.

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FSA as a graph

- An FSA is a directed graph
- States are represented as nodes
- Transitions are labeled edges
- One of the states is the *initial state*
- Some states are accepting states



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DFA: formal definition

Formally, a finite state automaton, M , is a tuple $(\Sigma, Q, q_0, F, \Delta)$ with

Σ is the alphabet, a finite set of symbols

Q a finite set of states

q_0 is the start state, $q_0 \in Q$

F is the set of final states, $F \subseteq Q$

Δ is a function that takes a state and a symbol in the alphabet, and returns another state ($\Delta: Q \times \Sigma \rightarrow Q$)

At any state and for any input,
a DFA has a single well-defined action to take.

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DFA: formal definition

an example

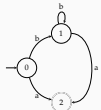
$\Sigma = \{a, b\}$

$Q = \{q_0, q_1, q_2\}$

$q_0 = q_0$

$F = \{q_2\}$

$\Delta = \{(q_0, a) \rightarrow q_2, (q_0, b) \rightarrow q_1, (q_1, a) \rightarrow q_2, (q_1, b) \rightarrow q_1\}$



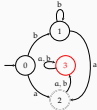
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Another note on DFA

error or sink state

- Is this FSA deterministic?
- To make all transitions well-defined, we can add a sink (or error) state
- For brevity, we skip the explicit error state
 - In that case, when we reach a dead end, recognition fails



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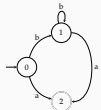
DFA: the transition table

transition table

| | symbol | |
|-------|--------|-----|
| | a | b |
| start | →0 | 2 1 |
| | 1 | 2 1 |
| | *2 | ∅ ∅ |

→ marks the start state

* marks the accepting state(s)



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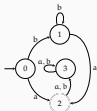
DFA: the transition table

transition table

| | symbol | |
|-------|--------|-----|
| | a | b |
| start | →0 | 2 1 |
| | 1 | 2 1 |
| | *2 | 3 3 |
| | 3 | 3 3 |

→ marks the start state

* marks the accepting state(s)

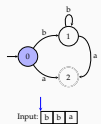


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DFA recognition

- Start at q_0
- Process an input symbol, move accordingly
- Accept if in a final state at the end of the input

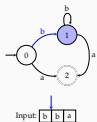


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DFA recognition

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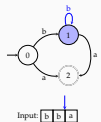


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DFA recognition

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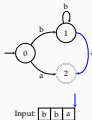


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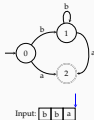
DFA recognition

1. Start at q_0
2. Process an input symbol, move accordingly
3. Accept if in a final state at the end of the input



DFA recognition

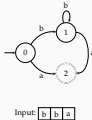
1. Start at q_0
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DFA recognition

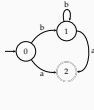
1. Start at q_0
2. Process an input symbol, move accordingly
3. Accept if in a final state at the end of the input

- What is the complexity of the algorithm?
- How about inputs:
 - bbbb
 - aa



A few questions

- What is the language recognized by this FSA?
- Can you draw a simpler DFA for the same language?
- Draw a DFA recognizing strings with even number of 'a's over $\Sigma = \{a, b\}$



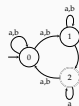
Non-deterministic finite automata

Formal definition

A non-deterministic finite state automaton, M , is a tuple $(\Sigma, Q, q_0, F, \Delta)$ with

- Σ is the alphabet, a finite set of symbols
- Q a finite set of states
- q_0 is the start state, $q_0 \in Q$
- F is the set of final states, $F \subseteq Q$
- Δ is a function from (Q, Σ) to $P(Q)$, power set of Q ($\Delta: Q \times \Sigma \rightarrow P(Q)$)

An example NFA



| transition table | | | |
|------------------|--------|-----|-----|
| state | symbol | | |
| | a | b | |
| | →0 | 0,1 | 0,1 |
| 1 | 1 | 1,2 | 1 |
| *2 | 0,2 | 0 | |

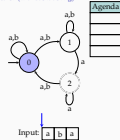
- We have nondeterminism, e.g., if the first input is a, we need to choose between states 0 or 1
- Transition table cells have sets of states

Dealing with non-determinism

- Follow one of the links, store alternatives, and backtrack on failure
- Follow all options in parallel

NFA recognition

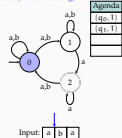
as search (with backtracking)



1. Start at q_0
2. Take the next input, place all possible actions to an agenda
3. Get the next action from the agenda, act
4. At the end of input
 - Accept if in an accepting state
 - Reject not in accepting state & agenda empty
 - Backtrack otherwise

NFA recognition

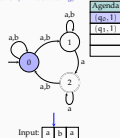
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NFA recognition

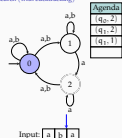
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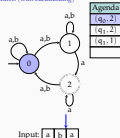
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NFA recognition

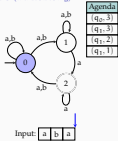
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NFA recognition

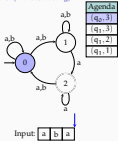
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NFA recognition

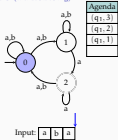
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NFA recognition

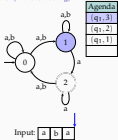
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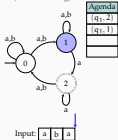
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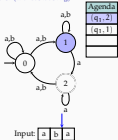
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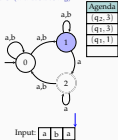
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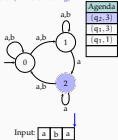
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NFA recognition

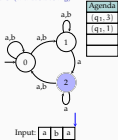
as search (with backtracking)



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NFA recognition

as search (with backtracking)



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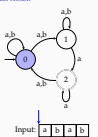
NFA recognition as search

summary

- Worst time complexity is exponential
 - Complexity is worse if we want to enumerate all derivations
- We used a stack as *agenda*, performing a depth-first search
- A queue would result in breadth-first search
- If we have a reasonable heuristic A^* search may be an option
- Machine learning methods may also guide finding a fast or the best solution

NFA recognition

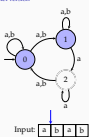
parallel version



- Start at q_0
- Take the next input, mark all possible next states
- If an accepting state is marked at the end of the input, accept

NFA recognition

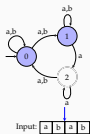
parallel version



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NFA recognition

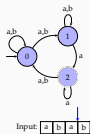
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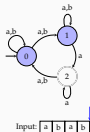
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NFA recognition

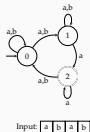
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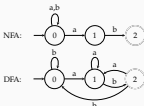


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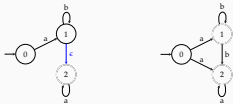
Note: the process is *deterministic*, and *finite-state*.

An exercise

Construct an NFA and a DFA for the language over $\Sigma = \{a, b\}$ where all sentences end with ab .

One more complication: ϵ transitions

- An extension of NFA, ϵ -NFA, allows moving without consuming an input symbol, indicated by an ϵ -transition (sometimes called a λ -transition)
- Any ϵ -NFA can be converted to an NFA

 ϵ -transitions need attention

- How does the (depth-first) NFA recognition algorithm we described earlier work on this automaton?
- Can we do without ϵ transitions?

NFA-DFA equivalence

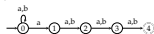
- The language recognized by every NFA is recognized by some DFA
- The set of DFA is a subset of the set of NFA (a DFA is also an NFA)
- The same is true for ϵ -NFA
- All recognize/generate regular languages
- NFA can automatically be converted to the equivalent DFA

Why do we use an NFA then?

- NFA (or ϵ -NFA) are often easier to construct
 - Intuitive for humans (cf. earlier exercise)
 - Some representations are easy to convert to NFA rather than DFA, e.g., regular expressions
- NFA may require less memory (fewer states)

A quick exercise – and a not-so-quick one

1. Construct (draw) an NFA for the language over $\Sigma = \{a, b\}$, such that 4th symbol from the end is an a



2. Construct a DFA for the same language

Summary

- FSA are efficient tools with many applications
- FSA have two flavors: DFA, NFA (or maybe three: ϵ -NFA)
- DFA recognition is linear, recognition with NFA may require exponential time
- Reading suggestion: Hopcroft and Ullman (1979, Ch. 2&3) (and its successive editions), Jurafsky and Martin (2009, Ch. 2)

Next:

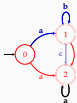
- FSA determinization, minimization
- Reading suggestion: Hopcroft and Ullman (1979, Ch. 2&3) (and its successive editions), Jurafsky and Martin (2009, Ch. 2)

Acknowledgments, credits, references

- [B] Hopcroft, John E. and Jeffrey D. Ullman (1979). *Introduction to Automata Theory, Languages, and Computation*. Addison-Wesley Series in Computer Science and Information Processing. Addison-Wesley. [sac: 9780201029688](https://doi.org/10.2307/98688).
- [B] Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*, second edition. Pearson Prentice Hall. [sac: 978-0-13-904196-3](https://doi.org/10.2307/98688).

 ϵ removal

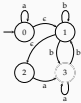
- We start with finding the ϵ -closure of all states
 - $\epsilon\text{-closure}(q_0) = \{q_0\}$
 - $\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$
 - $\epsilon\text{-closure}(q_2) = \{q_2\}$
- Replace each arc to each state with $\text{arc}(s)$ to all states in the ϵ -closure of the state



ε removal

a(nother) solution with the transition table

| state | symbol | | | | |
|-------|--------|-----|---|-----|-------|
| | a | b | ε | ε* | |
| | →0 | 0 | ∅ | 1 | 0,1,2 |
| 1 | ∅ | 1,3 | 2 | 1,2 | |
| 2 | 3 | ∅ | ∅ | 2 | |
| *3 | 3 | 1 | ∅ | 3 | |



ε removal

a(nother) solution with the transition table

| state | symbol | | | | | → | symbol | | |
|-------|--------|-----|---|-----|-------|---|--------|-------|-------|
| | a | b | ε | ε* | | | a | b | |
| | →0 | 0 | ∅ | 1 | 0,1,2 | | →0 | 0,1,2 | 1,3 |
| 1 | ∅ | 1,3 | 2 | 1,2 | | | 1 | 1,2,3 | 1,2,3 |
| 2 | 3 | ∅ | ∅ | 2 | | | 2 | 3 | ∅ |
| *3 | 3 | 1 | ∅ | 3 | | | *3 | 3 | 1,2 |

