Zad. 2.

$$P = \frac{R^2 i^2}{C+R} \quad [W]$$

$C = 1,1\ [mF] \pm 5\% = 1,1 \cdot 10^{-3} \pm 5\%\ [F]$

$R = 1\ [k\Omega] \pm 2\% = 1 \cdot 10^{3} \pm 2\%\ [\Omega]$

$i = 1,2\ [mA] \pm 1\% = 1,2 \cdot 10^{-3} \pm 1\%\ [A]$

$$\varepsilon = \frac{\Delta x}{|x|} \cdot 100\% \rightarrow \Delta x = \frac{\varepsilon |x|}{100\%}$$

1. Błędy bezwzględne pomiarów

$\Delta C = 0,05 \cdot 1,1 \cdot 10^{-3} = 5,5 \cdot 10^{-5}\ [F]$

$\Delta R = 0,02 \cdot 1 \cdot 10^{3} = 20\ [\Omega]$

$\Delta i = 0,01 \cdot 1,2 \cdot 10^{-3} = 1,2 \cdot 10^{-5}\ [A]$

2. Błąd bezwzględny funkcji 3 zmiennych

$$\Delta P = |f_x(x,y,z)|\Delta x + |f_y(x,y,z)|\Delta y + |f_z(x,y,z)|\Delta z =$$

$$= |f_C(C,R,i)|\Delta C + |f_R(C,R,i)|\Delta R + |f_i(C,R,i)|\Delta i$$

$$\frac{\partial P}{\partial C} = \left| \frac{\partial}{\partial C}\left(\frac{a}{f}\right) = -a\,\frac{\frac{\partial}{\partial C}(f)}{f^2} \right|_{C,R,i} = -R^2 i^2\,\frac{\frac{\partial}{\partial C}(C+R)}{(C+R)^2} =$$

$$= \left| \frac{\partial}{\partial C}(f+g) = \frac{\partial f}{\partial C} + \frac{\partial g}{\partial C} \right|_{C,R,i} = -R^2 i^2\,\frac{\frac{\partial}{\partial C}C + \frac{\partial}{\partial C}R}{(C+R)^2} =$$

$$= -R^2 i^2\,\frac{1+0}{(C+R)^2} = -\frac{R^2 i^2}{(C+R)^2} = -\frac{(10^3)^2 \cdot (1,2 \cdot 10^{-3})^2}{((1,1 \cdot 10^{-3}) + 1 \cdot 10^3)^2} \approx -1,44 \cdot 10^{-6}$$

$$\frac{\partial P}{\partial R} = \frac{\frac{\partial}{\partial R}(i^2R^2)(C+R) - (i^2R^2)\frac{\partial}{\partial R}(C+R)}{(C+R)^2} =$$

$$= \frac{i^2 \cdot 2R \cdot (C+R) - i^2R^2 \frac{\partial}{\partial R}(C+R)}{(C+R)^2} =$$

$$= \frac{2Ri^2}{C+R} - \frac{i^2R^2}{(C+R)^2} = \frac{2(10^3)(1{,}2\cdot10^{-3})^2}{(1{,}1\cdot10^{-3})+(1\cdot10^3)} - \frac{(1{,}2\cdot10^{-3})^2(10^3)^2}{(1{,}1\cdot10^{-3}+10^3)^2} =$$

$$\approx 1{,}44\cdot10^{-6}$$

$$\frac{\partial P}{\partial i} = \frac{R^2}{C+R}\cdot\frac{\partial}{\partial i}(fi^2) = \frac{R^2}{C+R}\cdot 2i = \frac{2iR^2}{C+R} = \frac{2(1{,}2\cdot10^{-3})(10^3)^2}{(1{,}1\cdot10^{-3})+10^3} \approx 2{,}4C$$

$$\Delta P = -1{,}44\cdot10^{-6}\cdot 5{,}5\cdot10^{-5} + 1{,}44\cdot10^{-6}\cdot 20 + 2{,}40\cdot10^{-5}\cdot 1{,}2 =$$

$$= 6{,}05\cdot10^{-5}$$

3. Błąd względny P

$$P = \frac{(10^3)^2\cdot(1{,}2\cdot10^{-3})^2}{(1{,}1\cdot10^{-3})+(1\cdot10^3)} \approx 1{,}44\cdot10^{-3}$$

$$\delta P = \frac{\Delta P}{P} = \frac{6{,}05\cdot10^{-5}}{1{,}44\cdot10^{-3}} \approx 0{,}0420$$

$$\delta P = 0{,}0420\cdot100\% = 4{,}2\%$$