Zad. 5.

$i = 0, 1, 2.$

$i=0$

$$\phi_0(x) = \frac{1}{h^3} \cdot \begin{cases} (x - x_{-2})^3 & \text{dla } x \in \langle x_{-2}, x_{-1}) \\ (x - x_{-2})^3 - 4(x - x_{-1})^3 & x \in \langle x_{-1}; x_0) \\ (x_2 - x)^3 - 4(x_1 - x)^3 & x \in \langle x_0; x_1) \\ (x_2 - x)^3 & x \in \langle x_1; x_2) \\ 0 & x \in R - \langle x_{-2}; x_2) \end{cases}$$

$i=1$.

$$\phi_1(x) = \frac{1}{h^3} \cdot \begin{cases} (x - x_{-1})^3 & \text{dla } x \in \langle x_{-1}; x_0) \\ (x - x_{-1})^3 - 4(x - x_0)^3 & x \in \langle x_0; x_1) \\ (x_3 - x)^3 - 4(x_2 - x)^3 & x \in \langle x_1; x_2) \\ (x_3 - x)^3 & x \in \langle x_2; x_3) \\ 0 & x \in R - \langle x_{-1}; x_3) \end{cases}$$

$i=2$

$$\phi_2(x) = \frac{1}{h^3} \cdot \begin{cases} (x - x_0)^3 & \text{dla } x \in \langle x_0; x_{+1}) \\ (x - x_0)^3 - 4(x - x_1)^3 & x \in \langle x_1; x_2) \\ (x_4 - x)^3 - 4(x_3 - x)^3 & x \in \langle x_2; x_3) \\ (x_4 - x)^3 & x \in \langle x_3; x_4) \\ 0 & x \in R - \langle x_0; x_4) \end{cases}$$

$i=-1$

$$\phi_{-1}(x) = \frac{1}{h^3} \cdot \begin{cases} (x - x_{-3})^3 & \text{dla } x \in \langle x_{-3}; x_{-2}) \\ (x - x_{-3})^3 - 4(x - x_{-2})^3 & x \in \langle x_{-2}; x_{-1}) \\ (x_1 - x)^3 - 4(x_0 - x)^3 & x \in \langle x_{-1}; x_0) \\ (x_1 - x)^3 & x \in \langle x_0; x_1) \\ 0 & x \in R - \langle x_{-3}; x_1) \end{cases}$$

$i=3$

$$\phi_3(x) = \frac{1}{h^3} \cdot \begin{cases} (x - x_1)^3 & \text{dla } x \in \langle x_1; x_2) \\ (x - x_1)^3 - 4(x - x_2)^3 & x \in \langle x_2; x_3) \\ (x_5 - x)^3 - 4(x_4 - x)^3 & x \in \langle x_3; x_4) \\ (x_5 - x)^3 & x \in \langle x_4; x_5) \\ 0 & x \in R - \langle x_1; x_5) \end{cases}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ 1 & 4 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -2{,}79 + \frac{0{,}1}{3} \cdot 4{,}02 \\ -2{,}26 \\ -1{,}99 - \frac{0{,}1}{3}(-2{,}62) \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ 1 & 4 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -2{,}656 \\ -2{,}26 \\ \cancel{-1{,}8127} \end{bmatrix} -1{,}9026$$

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 0 \\ 1 & 4 & 1 \\ 0 & 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -2{,}656 \\ -2{,}26 \\ \cancel{-1{,}8127} \end{bmatrix} -1{,}9026$$

$$\det \begin{bmatrix} 4 & 2 & 0 \\ 1 & 4 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{matrix} 4 & 2 \\ 1 & 4 \\ 0 & 2 \end{matrix} = 4^3 + 0 + 0 - 0 - 2 \cdot 4 - 2 \cdot 4 = 64 - 16 = 48$$

$$\begin{bmatrix} 4 & 2 & 0 \\ 1 & 4 & 1 \\ 0 & 2 & 4 \end{bmatrix}^{D} = \begin{bmatrix} \begin{vmatrix} 4 & 1 \\ 2 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} \\ -\begin{vmatrix} 2 & 0 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 4 & 0 \\ 0 & 4 \end{vmatrix} & -\begin{vmatrix} 4 & 2 \\ 0 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 2 \\ 1 & 4 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} 14 & -4 & 2 \\ -8 & 16 & -8 \\ 2 & -4 & 14 \end{bmatrix}^T = \begin{bmatrix} 14 & -8 & 2 \\ -4 & 16 & -4 \\ 2 & -8 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \cdot A^D \quad \rightarrow \quad \begin{bmatrix} 4 & 2 & 0 \\ 1 & 4 & 1 \\ 0 & 2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{14}{48} & -\frac{8}{48} & \frac{2}{48} \\ -\frac{4}{48} & \frac{16}{48} & -\frac{4}{48} \\ \frac{2}{48} & -\frac{8}{48} & \frac{14}{48} \end{bmatrix} = \begin{bmatrix} \frac{7}{24} & -\frac{1}{6} & \frac{1}{24} \\ -\frac{1}{12} & +\frac{1}{3} & -\frac{1}{12} \\ \frac{1}{24} & -\frac{1}{6} & \frac{7}{24} \end{bmatrix}$$

$$\begin{bmatrix} \frac{7}{24} & -\frac{1}{6} & \frac{1}{24} \\ -\frac{1}{12} & +\frac{1}{3} & -\frac{1}{12} \\ \frac{1}{24} & -\frac{1}{6} & \frac{7}{24} \end{bmatrix} \cdot \begin{bmatrix} -2,656 \\ -2,26 \\ -1,9026 \\ \cancel{-1,8127} \end{bmatrix} = \begin{bmatrix} \frac{7}{24}(-2,656) - \frac{1}{6}(-2,26) - \frac{1}{24} \cdot \overset{1,9026}{\cancel{1,8127}} \\ \frac{2,656}{12} - \frac{2,26}{3} + \frac{\cancel{1,8127}\ 1,9026}{12} \\ -\frac{2,656}{24} + \frac{2,26}{6} - \frac{\cancel{7\ 1,8127}\ 1,9026}{24} \end{bmatrix} =$$

$$\approx \begin{bmatrix} -0,4772275 \\ -0,37345 \\ -0,288925 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$$

$$c_{-1} = c_1 - \frac{h}{3}\alpha = (-0,37345) - \frac{\cancel{0,1}\ 0,1}{3} \cdot 4,02 = \underline{\overset{-0,50745}{\cancel{-0,38685}}}$$

$$c_3 = c_{n+1} + \frac{h}{3}\beta = \underset{-0,37345}{(\cancel{-0,288925})} + \frac{0,1}{3} \cdot (-2,62) = \underline{\underset{-0,460783}{\cancel{-0,376258}}}$$

x_{-3}	x_{-2}	x_{-1}	x_0	x_1	x_2	x_3	x_4
$-0,2$	$-0,1$	0	$0,1$	$0,2$	$0,3$	$0,4$	$0,5$

$$0,23 \in \langle x_1 , x_2 \rangle$$

$$\phi_{-1}(0,23) = 0$$

$$\phi_0(0,23) = \frac{1}{0,1^3} \cdot (0,3 - 0,23)^3 = 0,343$$

$$\phi_1(0,23) = \frac{1}{0,1^3} \cdot \left[(0,4-0,23)^3 - 4(0,3-0,23)^3\right] = \cancel{0,0}3,541$$

$$\phi_2(0,23) = \frac{1}{0,1^3} \cdot \left[0,23-0,1)^3 - 4(0,23-0,2)^3\right] = 2,089$$

$$\phi_3(0,23) = \frac{1}{0,1^3} \cdot (0,23 - 0,2)^3 = 0,027$$

$$S_3(x) = \sum_{i=-1}^{n+1} \left(c_i \, \phi_i(x)\right)$$

$$S_3(0,23) = 0 + \cancel{3,4}\, 0,343\,(-0,477275) + 3,541(-0,37345) +$$

$$+ 2,089(-0,288925) + 0,027(-0,460783) = -2,1020972241 =$$

$$\approx \underline{\underline{-2,10}}$$

odp.