6) Función de costo n puntos y modelo lineal:

$$X^2(a_0, a_1) = \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i))^2$$
$$= \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

minimizar $X^2(a_0, a_1)$:

- $\frac{\partial X^2}{\partial a_0} = -2(y_i - a_0 - a_1 x_i) = 0 \rightarrow y_i = a_0 + a_1 x_i$
- $\frac{\partial X^2}{\partial a_1} = -2x_i(y_i - a_0 - a_1 x_i) = 0 \rightarrow x_i y_i - a_0 x_i - a_1 x_i^2 = 0$

$\hookrightarrow x_i y_i = a_0 x_i + a_1 x_i^2$

modelo lineal $\rightarrow$ recta de la forma $y = mx + b$

$\hookrightarrow \sum_{i=1}^{n} y_i = a_0 + a_1 \sum_{i=1}^{n} x_i \rightarrow \bar{y} = a_1 \bar{x} + a_0$, donde a_0 es el corte y a_1 la pendiente del modelo.

($\bar{x}$ y $\bar{y}$: valores medios)

- $a_0 = \bar{y} - a_1 \bar{x}$

$\rightarrow \sum_{i=1}^{n} x_i y_i = a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2 \rightarrow \Sigma xy = a_0 \Sigma x + a_1 \Sigma x^2$

$a_1 = \frac{\Sigma xy - a_0 \Sigma x}{\Sigma x^2}$, $a_0 = \bar{y} - a_1 \bar{x} = \frac{\Sigma y}{n} - a_1 \frac{\Sigma x}{n}$,

$\hookrightarrow a_1 = \left(\frac{\Sigma y}{n} - a_0\right) \div \frac{\Sigma x}{n}$

$$a_1 = \frac{\Sigma xy - a_0 \Sigma x}{\Sigma x^2} = \frac{\frac{\Sigma y}{n} - a_0}{\frac{\Sigma x}{n}} \rightarrow \frac{\Sigma xy - a_0 \Sigma x}{\Sigma x^2} - \frac{\frac{\Sigma y}{n} - a_0}{\frac{\Sigma x}{n}} = 0$$

$$\rightarrow \cancel{a_0} \left(\frac{\Sigma xy - \Sigma x}{\Sigma x^2} - \frac{\frac{\Sigma y}{n} + 1}{\frac{\Sigma x}{n}} \right) = 0$$

$$\frac{\Sigma xy - \Sigma x}{\Sigma x^2} - \frac{\frac{\Sigma y}{n} + 1}{\frac{\Sigma x}{n}} = a_1$$

$$\frac{\Sigma xy - \Sigma x - \frac{(\Sigma x)^2}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} - \frac{-\Sigma x - \frac{(\Sigma x)^2}{n} - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = a_1$$

$$a_1 = \frac{\Sigma xy - \cancel{\Sigma x} - \cancel{\frac{(\Sigma x)^2}{n}} + \cancel{\Sigma x} + \cancel{\frac{(\Sigma x)^2}{n}} - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$\cdot\ a_1 = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

Función de costo n puntos y modelo cuadrático:

$$X^2(a_0, a_1, a_2) = \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2$$

$$= \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i - a_2 x_i^2)^2$$

minimizar $X^2(a_0, a_1, a_2)$:

$$\cdot\ \frac{\partial X^2}{\partial a_0} = \sum_{i=1}^{n} \left[2(y_i - a_0 - a_1 x_i - a_2 x_i^2)(-1) = 0\right]$$

$$= \sum_{i=1}^{n} \left[y_i - a_0 - a_1 x_i - a_2 x_i^2 = 0\right]$$

$$= \sum_{i=1}^{n} \left[y_i = a_0 + a_1 x_i + a_2 x_i^2\right]$$

- $$\frac{\partial \chi^2}{\partial a_1} = \sum_{i=1}^{n} \left[2(-x_i)(y_i - a_0 - a_1 x_i - a_2 x_i^2) = 0 \right]$$

$$= \sum_{i=1}^{n} \left[x_i y_i - a_0 x_i - a_1 x_i^2 - a_2 x_i^3 = 0 \right]$$

$$= \sum_{i=1}^{n} \left[x_i y_i = a_0 x_i + a_1 x_i^2 + a_2 x_i^3 \right]$$

- $$\frac{\partial \chi^2}{\partial a_2} = \sum_{i=1}^{n} \left[2(-x_i^2)(y_i - a_0 - a_1 x_i - a_2 x_i^2) = 0 \right]$$

$$= \sum_{i=1}^{n} \left[x_i^2 y_i - a_0 x_i^2 - a_1 x_i^3 - a_2 x_i^4 = 0 \right]$$

$$= \sum_{i=1}^{n} \left[x_i^2 y_i = a_0 x_i^2 - a_1 x_i^3 - a_2 x_i^4 \right]$$