KEEP IT SIMPLE

1) $P(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$

$$\to \int_a^b P(x)\,dx = \int_a^b \left(\frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)\right) dx$$

$$= \int_a^b \frac{x-b}{a-b} f(a)\,dx + \int_a^b \frac{x-a}{b-a} f(b)\,dx$$

$$\cdot \int_a^b \frac{x-b}{a-b} f(a)\,dx = \frac{1}{a-b} f(a) \int_a^b (x-b)\,dx = \frac{f(a)}{a-b}\left(\int_a^b x\,dx - \int_a^b b\,dx\right)$$

$$= \frac{f(a)}{a-b}\left(\frac{1}{2}x^2\Big|_a^b - bx\Big|_a^b\right) = \frac{f(a)}{a-b}\left(\frac{b^2-a^2}{2} - (b^2-ab)\right)$$

$$= \frac{f(a)}{a-b}\left(\frac{b^2-a^2}{2} + ab - b^2\right) = f(a)\left(\frac{\cancel{(b-a)}(b+a)}{-2\cancel{(b-a)}} + \frac{b\cancel{(a-b)}}{\cancel{a-b}}\right)$$

$$= f(a)\left(\frac{-(a+b)}{2} + b\right) = f(a)\left(\frac{-a-b}{2} + \frac{2b}{2}\right) = \frac{2b-a-b}{2} f(a) = \frac{b-a}{2} f(a)$$

$$\cdot \int_a^b \frac{x-a}{b-a} f(b)\,dx = \frac{1}{b-a} f(b) \int_a^b (x-a)\,dx = \frac{f(b)}{b-a}\left(\int_a^b x\,dx - \int_a^b a\,dx\right)$$

$$= \frac{f(b)}{b-a}\left(\frac{1}{2}x^2\Big|_a^b - ax\Big|_a^b\right) = \frac{f(b)}{b-a}\left(\frac{b^2-a^2}{2} - (ab-a^2)\right)$$

$$= f(b)\left(\frac{(b+a)\cancel{(b-a)}}{2\cancel{(b-a)}} - \frac{a\cancel{(b-a)}}{\cancel{b-a}}\right) = f(b)\left(\frac{b+a}{2} - a\right) = f(b)\left(\frac{b+a}{2} - \frac{2a}{2}\right)$$

$$= \frac{a+b-2a}{2} f(b) = \frac{b-a}{2} f(b)$$

$$\to \int_a^b P(x)\,dx = \frac{b-a}{2} f(a) + \frac{b-a}{2} f(b) = \frac{b-a}{2}\left(f(a) + f(b)\right)$$

3) $P_2(x) = \frac{(x-b)(x-m)}{(a-b)(a-m)} f(a) + \frac{(x-a)(x-b)}{(m-a)(m-b)} f(m) + \frac{(x-a)(x-m)}{(b-a)(b-m)} f(b)$

$\rightarrow m = \frac{a+b}{2} \rightarrow$ punto medio (partición equiespaciada) : $b - m = m - a = \frac{b-a}{2}$

$$\left.\begin{array}{l} \cdot\ b-m = \frac{2b}{2} - \frac{a+b}{2} = \frac{2b-a-b}{2} = \frac{b-a}{2} \\ \cdot\ m-a = \frac{a+b}{2} - \frac{2a}{2} = \frac{a+b-2a}{2} = \frac{b-a}{2} \end{array}\right\} b-m = m-a = \frac{b-a}{2} = h \rightarrow \boxed{h = \frac{b-a}{2}}$$

$$\int_a^b P_2(x)\,dx = \int_a^m P_2(x)\,dx + \int_m^b P_2(x)\,dx = \frac{h}{2}\big(f(a)+f(m)\big) + \frac{h}{2}\big(f(m)+f(b)\big)$$

$$= \frac{h}{2}\big(f(a)+f(b)\big) + h\sum_{i=1}^{n-1} f(x_i) = \frac{h}{2}\big(f(a)+f(b)\big) + h f(m)$$

$$= \frac{h}{2}\big(f(a)+f(b)\big) + \frac{2h}{2} f(m) = \frac{h}{2}\big(f(a)+f(b)\big) + \frac{h}{2}\big(2f(m)\big)$$

$$= \frac{h}{2}\big(f(a)+f(b)+2f(m)\big) = \frac{h}{2}\big(f(a)+2f(m)+f(b)\big)$$