Nombres: Laura Thomas & Diana Sáenz

1. Derivada finita.

1 $f'(x_i) \approx \dfrac{f(x_{j+1}) - f(x_{j-1})}{2h} = j(x_i)$

$f''(x_i) = [f'(x_i)]' = g'(x_i) \approx \dfrac{g(x_{i+1}) - g(x_{i-1})}{2h}$

$\rightarrow g(x_{i+1}) = \dfrac{f(x_{i+2}) - f(x_i)}{2h}$ $\rightarrow g(x_{i-1}) = \dfrac{f(x_i) - f(x_{i-2})}{2h}$

$\Rightarrow g'(x_i) = \dfrac{\frac{1}{2h}\left(f(x_{i+2}) - f(x_i)\right) - \frac{1}{2h}\left(f(x_i) - f(x_{i-2})\right)}{2h}$

$\frac{1}{2h}\left(f(x_{i+2}) \overbrace{- f(x_i) - f(x_i)}^{-2f(x_i)} + f(x_{i-2})\right) \cdot \frac{1}{2h}$

$= \left(f(x_{i+2}) - 2f(x_i) + f(x_{i-2})\right) \cdot \frac{1}{4h^2}$

2.a. $h=0,05$

$f'(-10) = 2,57676.. E-17$ $f'(-9) = 1,26642\ E-13$

$f'(-8) = 2,28974.. E-10$ $f'(-7) = 1,523\ ..E-7$

$f'(-6) = 0,00003...$ $f'(-5) = 0,00335...$

$f'(-4) = 0,11112...$ $f'(-3) = 1,34448...$

$f'(-2) = 5,29703..$ $f'(-1) = 8,41219..$

$f'(0) = 13,73615...$ $f'(1) = 16,98072...$

$f'(2) = 18,54957.$ $f'(3) = 19,90965..$

$f'(4) = 19,99938$ $f'(5) = 19,99999.$

$f'(6) = 19,9999$ $f'(7) = 20$

$f'(8) = 20$ $f'(9) = 20$

$f'(10) = 20$

b. $\sigma(Df(-10)) = 2,57676\times10^{-17} + 1,9128\times10^{-21} = 2,57576\times10^{-12}$

$6(Df(-9)) = 1,26642\times10^{-13} + 2,3190\times10^{-17} = 1,2597\times10^{-13}$

$6(Df(-8)) = 2,28974\times10^{-10} + 1,013133\times10^{-13} = 2,2886\times10^{-10}$

$6(Df(-7)) = 1,523\times10^{-7} + 1,60281\times10^{-10} = 1,524\times10^{-7}$

$6(Df(-6)) = 0{,}00003 + 9{,}1379\times10^{-8} = 3{,}009138\times10^{-5}$

$6(Df(-5)) = 0{,}00335 + 1{,}86332\times10^{-5} = 3{,}368633\times10^{-3}$

$6(Df(-4)) = 0{,}11112 + 0{,}001341 = 0{,}112461$

$6(Df(-3)) = 1{,}34448 + 0{,}033320 = 1{,}37812$

$6(Df(-2)) = 5{,}29703 + 0{,}26340 = 5{,}56043$

$6(Df(-1)) = 8{,}41219 + 0{,}379123 = 8{,}791313$

$6(Df(0)) = 13{,}73615 = 13{,}73615$

$6(Df(1)) = 8{,}41219 - 0{,}379123 = 8{,}033067$

$6(Df(2)) = 5{,}29703 - 0{,}26340 = 5{,}03363$

$6(Df(3)) = 1{,}34448 - 0{,}033320 = 1{,}31116$

$6(Df(4)) = 0{,}11112 - 0{,}001341 = 0{,}109779$

$6(Df(5)) = 0{,}00335 - 1{,}86332\times10^{-5} = 3{,}331366\times10^{-3}$

$6(Df(6)) = 0{,}00003 - 9{,}1379\times10^{-8} = 2{,}9908621\times10^{-5}$

$6(Df(7)) = 1{,}523\times10^{-7} - 1{,}60281\times10^{-10} = 1{,}52139\times10^{-7}$

$6(Df(8)) = 2{,}28974\times10^{-10} - 1{,}013133\times10^{-13} = 2{,}288726\times10^{-10}$

$6(Df(9)) = 1{,}26642\times10^{-13} - 2{,}3190\times10^{-17} = 1{,}26596\times10^{-13}$

$6(Df(10)) = 2{,}57676\times10^{-17} - 1{,}9128\times10^{-21} = 2{,}57656\times10^{-17}$

5. $$f''(x_i) \cong \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2} + \underbrace{\Theta(h^2)}_{\text{Por ahora lo dejo de lado}} = g(x_i)$$

$$g''(x_i) \cong \left(g(x_{i+1}) - 2g(x_i) + g(x_{i-1})\right) \cdot \frac{1}{h^2}$$

$$g(x_{i+1}) = \left(f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)\right) \cdot \frac{1}{h^2}$$

$$g(x_i) = \left(f(x_{i+1}) - 2f(x_i) + f(x_{i-1})\right) \cdot \frac{1}{h^2}$$

$$g(x_{i-1}) = \left(f(x_i) - 2f(x_{i-1}) + f(x_{i-2})\right) \cdot \frac{1}{h^2}$$

$$g''(x_i) = \frac{\left(\frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}\right) - 2\left(\frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}\right) + \left(\frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2}\right)}{h^2}$$

factor común

$$g''(x_i) = \frac{1}{h^2}\left(\begin{array}{l} f(x_{i+2}) - 2f(x_{i+1}) + f(x_i) - 2f(x_{i+1}) + 4f(x_i) - 2f(x_{i-1}) \\ + f(x_i) - 2f(x_{i-1}) + f(x_{i-2}) \end{array}\right) \cdot \frac{1}{h^2}$$

$$g''(x_i) = \frac{1}{h^2}\left(f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})\right) \cdot \frac{1}{h^2}$$

$$g''(x_i) = \frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4}$$

Para $(\mathcal{O}(h^K))$

$$f''(x_i) \cong \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2} + \mathcal{O}(h^2) = g(x)$$

$$g''(x) \cong \frac{(g(x_{i+1}) - 2g(x_i) + g(x_{i-1})}{h^2} + \mathcal{O}(h^2)$$

$$g''(x) \cong \frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4} \underbrace{+ \mathcal{O}(h^2) + \mathcal{O}(h^2)}_{2\,\mathcal{O}(h^2)}$$

$\Rightarrow K = 2.$