NOPT042 Constraint programming: Tutorial 9 - Implicit constraints

In [1]:
```
%load_ext ipicat
```

```
Picat version 3.7
```

Example: Seesaw

Adam, Boris, and Cecil want to sit on a 10-feet long seesaw such that they are at least 2 feet apart and the seesaw is balanced. Adam weighs 36 lbs, Boris 32 lbs, and Cecil 16 lbs. Write a general model. You can assume that the length is even, the distance is integer, and that they can only sit at integer points.

(Problem from Marriott & Stuckey "Programming with Constraints", page 257. Instance from R. Barták's tutorial.)

In [2]:
```
!cat seesaw/instance1.pi
```

```
% sample instance
instance(NumPeople, Length, Distance, Weights) =>
    NumPeople = 3,
    Length = 10,
    Distance = 2,
    Weights = [36, 32, 16].
```

Possible decision variables?

- Position on the seesaw for each person.
- Distances between persons, position of the first person, and order of persons.
- Person or empty for each position on the seesaw.

Global constraints? Symmetry breaking? Multiple modeling? Search strategies?

In [3]:
```
!ls seesaw/
```

```
instance1.pi  instance3.pi  instance5.pi  seesaw2.pi  seesaw4.pi
instance2.pi  instance4.pi  seesaw1.pi    seesaw3.pi
```

In [4]:
```
!time picat seesaw/seesaw1.pi instance4.pi
!time picat seesaw/seesaw2.pi instance4.pi
!time picat seesaw/seesaw3.pi instance4.pi
!time picat seesaw/seesaw4.pi instance4.pi
```

```
[-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,6,8,7,9,14,15,10,16,1
1,12,13]

real	0m23.235s
user	0m23.176s
sys	0m0.019s
[-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,6,8,7,9,14,15,10,16,1
1,12,13]

real	0m23.166s
user	0m23.156s
sys	0m0.007s
[-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,6,8,7,9,14,15,10,16,1
1,12,13]

real	0m30.443s
user	0m30.433s
sys	0m0.007s
[-8,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,-7,-6,-5,-4,-3,-9,-10,-13,-12,-11,-16,-
14]

real	0m0.014s
user	0m0.009s
sys	0m0.004s
```

Example: Golomb's ruler

A Golomb's ruler is an imaginary ruler with n marks such that the distance between every two marks is different. Find the shortest possible ruler for a given n.

(The solution for N=28 was announced on Nov 23, 2022! The length is 585.)

- What length are you able to solve in reasonable time?
- Add suitable implicit constraints. (We will discuss this in class.)

Redundant (implicit) constraints

Redundant constraints do not restrict the solution set but rather express properties of a solution from a different viewpoint. This can lead to

- faster domain reduction,
- a significant boost in propagation,
- improved communication between variables.

We have already seen one example last week in the Magic sequence problem: adding the `scalar_product` constraint.

Implicit constraints based on the following:

$$dist[i,j] = dist[i,i+1] + dist[i+1,i+2] + \ldots + dist[j-1,j]$$

Now estimate distances by 1, sum from i to j:

```
foreach(I in 1..N-1, J in I+1..N)
    Distances[I,J-I] #>= (J-I)*(J-I+1) div 2,
    Distances[I,J-I] #<= Length - (N-J+I-1)*(N-J+I) div 2
end
```

In [5]:
```
!picat golomb/golomb.pi 10
```

```
CPU time 96.734 seconds. Backtracks: 14554575

length = 55
[0,1,6,10,23,26,34,41,53,55]
```

In [6]:
```
!picat golomb/golomb-improved 11
```

```
CPU time 20.051 seconds. Backtracks: 1224484

length = 72
[0,1,4,13,28,33,47,54,64,70,72]
```

In [7]:
```
!picat golomb/golomb-improved 11
```

```
CPU time 20.041 seconds. Backtracks: 1224484

length = 72
[0,1,4,13,28,33,47,54,64,70,72]
```