

# NOPT042 Constraint programming: Tutorial 2 – Intro to CP

## Constraint programming aka modeling

Discrete ('combinatorial', as opposed to 'continuous') optimization, constraint satisfaction

- a form of decision making, many everyday problems
- solve Sudoku
- schedule classes
- schedule trains
- coordinate multi-facility production
- logistics of product transportation
- ...

Assign values to variables subject to **constraints**, satisfy/optimize.

## We will learn to...

- solve complex problems "without even knowing how"
- state the problem in a high-level language, use a constraint solver to "automagically" solve it (magic explained in the lectures)
- techniques and tricks to build efficient constraint models
- best practices, testing and debugging

## Why constraint programming?

- the 'holy grail' of programming: tell the computer what you want, not how to do it
- an order of magnitude easier than programming algorithms
- huge engineering investment in constraint solvers, highly optimized,
- often faster than your own algorithm would be (especially in "mixed" NP-complete problems), heuristic approach
- easier for molecular biologists to learn to specify their problems in a formal language, than for programmers to learn molecular biology

## History and (folk) etymology

- prográphō ("I set forth as a public notice"), from pró ("towards") + gráphō ("I write")

- program of a political movement
- program of a concert, broadcast programming, tv program
- computer program (1940s) Independently:
- U.S. Army operational programs (1940s)
- "linear programming" (1946) Maximize  $\mathbf{c}^T \mathbf{x}$  (objective function) subject to  $A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0$  (constraints).
- integer programming (1964),
- logic programming (late 1960s)
- constraint logic programming (1987)
- constraint programming (early 1990s)
- Modeling (USA) vs. Modelling (Commonwealth)

## Why Picat?

From a high-level point of view, all constraint programming languages are quite similar. Picat is:

- modern, simple yet powerful
- easier syntax and better utils than SICStus Prolog
- more flexible than MiniZinc
- fast: won several CP competitions (see [Picat homepage](#))

## Recall the basics of constraint programming:

- **decision variables** vs. **parameters**
- **domains**
- **constraints**
- **solution**
- **constraint propagation**
- **search space**
- **decision** vs. **optimization**
- modeling strategies

TODO, see the [slides](#) (pages 27-34).

```
In [1]: %load_ext ipicat
```

```
Picat version 3.5#5
```

## Structure of a constraint program

Typically, the structure of a constraint program looks like this (code inspired by the [tutorial](#)):

```

import cp.

problem(Variables) =>
    declare_variables(Variables),
    post_constraints(Variables).

main =>
    problem(Variables),
    solve(Variables).

```

Instead of `cp` we can use another solver, e.g. `mip` or `sat`. If the problem is parametrized, then we can pass the `Parameters` as an argument, e.g. `problem(Variables, Parameters)` or we can have a separate function `get_instance(Parameters) = Variables`.

## Introductory examples

### Example: Chinese remainder

After an indecisive battle, general Han Xin wanted to know how many soldiers of his 42000-strong army remained. In order to prevent enemy spies hidden among his soldiers to learn the number, he decided to use modular algebra: He ordered his soldiers to form rows of 5 and 3 soldiers remained. Then rows of 7; 2 remained. Then rows of 9; 4 remained. Then rows of 11; 10 remained. Finally, rows of 13; 1 remained.

- What are the **parameters** of our problem?
- Identify the decision variables: type, domains (as small as possible).
- What are the constraints? (Are there some implicit constraints?)
- Is this a satisfaction or an optimization problem?
- Find a solution using Picat.
- Is the last condition (rows of 13) needed?

```

In [2]: %%picat -n chinese
import cp.

chinese(X) =>
    % variables
    X :: 1..42000,

    % constraints
    X mod 5 #= 3,
    X mod 7 #= 2,
    X mod 9 #= 4,

    X mod 11 #= 10.

```

```
%    X mod 11 #= 10,  
%    X mod 13 #= 1.
```

```
In [3]: %%picat  
main =>  
    chinese(X),  
  
    % solve  
    solve(X),  
  
    % output  
    println(X).
```

373

```
In [4]: %%picat  
main =>  
    chinese(X),  
    solve_all(X) = Solutions,  
    foreach (Solution in Solutions)  
        println(Solution)  
    end.
```

373  
3838  
7303  
10768  
14233  
17698  
21163  
24628  
28093  
31558  
35023  
38488  
41953

```
In [5]: !picat chinese-remainder/chinese-remainder.pi  
!picat chinese-remainder/chinese-remainder.pi 5 [2,3] [4]  
!picat chinese-remainder/chinese-remainder.pi 42000 [5,7,9,11] [3,2,4,10]
```

```
35023
*** illegal_arguments
```

```
373
3838
7303
10768
14233
17698
21163
24628
28093
31558
35023
38488
41953
```

```
In [6]: !cat chinese-remainder/chinese-remainder.pi
```

```
import cp.
```

```
chinese(X, Parameters) =>
  % parameters
  [Max, Primes, Moduli] = Parameters,

  % here we could test input data
  if Primes.length != Moduli.length then
    throw(illegal_arguments)
  end,

  % variables
  X :: 1..Max,

  % constraints
  foreach(I in 1..Primes.length)
    X mod Primes[I] #= Moduli[I]
  end.

solve_and_output(Parameters) =>
  chinese(X, Parameters),
  solve_all(X) = Solutions,
  foreach (Solution in Solutions)
    println(Solution)
  end.

main =>
  Parameters = [42000, [5,7,9,11,13], [3,2,4,10,1]],
  solve_and_output(Parameters).

main(Parameters_as_Strings) =>
  Parameters = map(parse_term, Parameters_as_Strings),
  solve_and_output(Parameters).
```

## Example: SEND + MORE = MONEY

Solve the crypt-arithmetic puzzle (each letter represents a different base-10 digit, S and M are nonzero):

SEND + MORE = MONEY

```
In [7]: !picat send-more-money/send-more-money.pi
```

```
[9,5,6,7,1,0,8,2]
```

```
In [8]: !cat send-more-money/send-more-money.pi
```

```
import cp.

send_more_money(Digits) =>
    Digits = [S,E,N,D,M,O,R,Y],
    Digits :: 0..9,
    S #!= 0,
    M #!= 0,

    % digits are all different: naive
    % foreach(I in 1..Digits.length, J in I+1..Digits.length)
    %     Digits[I] #!= Digits[J]
    % end,

    % digits are all different: using a global constraint (much better propagation!)
    all_different(Digits),

    % arithmetic
    1000 * S + 100 * E + 10 * N + D
    +
    1000 * M + 100 * O + 10 * R + E
    #= 10000 * M + 1000 * O + 100 * N + 10 * E + Y.

main =>
    send_more_money(Digits),
    solve(Digits),
    println(Digits).
```

All constraint languages are somewhat similar, see the included models in `send-more-money/models-in-other-languages/` : one in the C++-based solver Gecode, two in SICStus Prolog, and two in the high-level modeling language MiniZinc:

```
In [9]: !ls send-more-money/models-in-other-languages/
        #!cat send-more-money/models-in-other-languages/*
```

```
send-more-money2.mzn  send-more-money.cpp  send-more-money.pl
send-more-money2.pl   send-more-money.mzn
```

## Exercises

## Exercise: Pythagorean triples

1. Generate all Pythagorean triples up to a given parameter, i.e. positive integers such that  $a^2 + b^2 = c^2$ , where  $a \leq b$  (an example of [symmetry breaking](#)).
2. Modify your program to accept the flag `-c` to output the number of solutions.

```
picat pythagorean.pi 42
picat pythagorean.pi -c 42
```

## Exercise: Send more carry bits

Write a better constraint model for the SEND+MORE=MONEY crypt-arithmetic puzzle based on carry bits.

Why is it better?

Some letters can be computed from other letters and invalidity of the constraint can be checked before all letters are known. (from R. Barták's tutorial in Prolog, see the code)

If we don't study the mistakes of the future, we're bound to repeat them for the first time. (Ken M)

## Homework: crypt-arithmetic

1. Write a program that accepts a general crypt-arithmetic puzzle on the input and outputs a solution as a string where letters are replaced by digits, e.g.

```
picat crypt-arithmetic.pi SEND MORE MONEY
```

should output `9567 + 1085 = 10652` (since this is the only solution). Don't forget to include the spaces.

2. Ignore case, e.g. accept also:

```
picat crypt-arithmetic.pi Donald Gerald Robert
picat crypt-arithmetic.pi baijjajiihfcfebbjea dhfgabccidbiffagfeje
gjegacddhfafjbfiheef
```

(source of the last instance: [Hakan Kjellerstrand's library](#), may run for a long time)

2. Modify your program to accept the flag `-c` to output the number of solutions instead, e.g.

```
picat crypt-arithmetic.pi -c send more money  
picat crypt-arithmetic.pi -c aa aa bb
```

should output **1** and **4** .

Your model must be reasonably efficient, e.g. use the carry bit implementation.

See the assignment on GitHub Classroom. You will find there code that you can use to parse the input.