NOPT042 Constraint programming: Tutorial 8 - Global constraints

In [1]: `%load_ext ipicat`

Picat version 3.7

Example: Magic sequence

A magic sequence of length n is a sequence of integers $x_0, \ldots, x_{n-1}$ between 0 and $n - 1$, such that for all $i \in \{0, \ldots, n - 1\}$, the number i occurs exactly x_i-times in the sequence. For instance, 6,2,1,0,0,0,1,0,0,0 is a magic sequence since 0 occurs 6 times in it, 1 occurs twice, etc.

(Problem from the book.)

Let's maximize the sum of the numbers in the sequence.

In [2]: `!time picat magic-sequence/magic-sequence.pi 8`

CPU time 0.001 seconds. Backtracks: 20

[4,2,1,0,1,0,0,0]

```
real    0m0.032s
user    0m0.011s
sys     0m0.001s
```

The constraint `global_cardinality`

```
global_cardinality(List, Pairs)
```

Let `List` be a list of integer-domain variables `[X1, . . ., Xd]`, and `Pairs` be a list of pairs `[K1-V1, . . ., Kn-Vn]`, where each key `Ki` is a unique integer, and each `Vi` is an integer-domain variable. The constraint is true if every element of List is equal to some key, and, for each pair `Ki-Vi`, exactly `Vi` elements of `List` are equal to `Ki`. This constraint can be defined as follows:

```
global_cardinality(List,Pairs) =>
    foreach($Key-V in Pairs)
        sum([B : E in List, B#<=>(E#=Key)]) #= V
    end.
```

---from the guide

In [3]: `!cat magic-sequence/magic-sequence.pi`

```
/***********************************************************
   Adapted from
   magic_sequence.pi
   from Constraint Solving and Planning with Picat, Springer
   by Neng-Fa Zhou, Hakan Kjellerstrand, and Jonathan Fruhman
 ***********************************************************/
import cp.

main([N]) =>
  N := N.to_int,
  magic_sequence(N,Sequence),
  println(Sequence).

magic_sequence(N, Sequence) =>
  Sequence = new_list(N),
  Sequence :: 0..N-1,

  % create list: [0-Sequence[1], 1-Sequence[2], ...]
  Pairs = [$I-Sequence[I+1] : I in 0..N-1],
  global_cardinality(Sequence,Pairs),

  time2(solve(Sequence)).
```

In [4]: `!time picat magic-sequence/magic-sequence2.pi 64`
`!time picat magic-sequence/magic-sequence2.pi 400`

```
CPU time 0.007 seconds. Backtracks: 7

[60,2,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0]

real    0m0.038s
user    0m0.021s
sys     0m0.009s
CPU time 0.216 seconds. Backtracks: 7

[396,2,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0]

real    0m6.294s
user    0m6.139s
sys     0m0.151s
```

```
In [5]: !cat magic-sequence/magic-sequence2.pi
```

```
/***********************************************************
  Adapted from
  magic_sequence.pi
  from Constraint Solving and Planning with Picat, Springer
  by Neng-Fa Zhou, Hakan Kjellerstrand, and Jonathan Fruhman
***********************************************************/
import cp.

main([N]) =>
  N := N.to_int,
  magic_sequence(N,Sequence),
  println(Sequence).

magic_sequence(N, Sequence) =>
  Sequence = new_list(N),
  Sequence :: 0..N-1,

  % % create list: [0-Sequence[1], 1-Sequence[2], ...]
  Pairs = [$I-Sequence[I+1] : I in 0..N-1],
  global_cardinality(Sequence,Pairs),

  % extra/redudant (implicit) constraints to speed up the model
  N #= sum(Sequence),
  Integers = [I : I in 0..N-1],
  scalar_product(Integers, Sequence, N),

  time2(solve([ff], Sequence)).
```

```
In [6]: !time picat magic-sequence/magic-sequence3.pi 400
        !time picat magic-sequence/magic-sequence3.pi 1024
```

```
CPU time 0.09 seconds. Backtracks: 7

[396,2,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0]

real    0m0.276s
user    0m0.218s
sys     0m0.050s
```

```
CPU time 0.497 seconds. Backtracks: 7

[1020,2,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0]

real    0m3.561s
user    0m3.300s
sys     0m0.260s
```

In [7]: `!cat magic-sequence/magic-sequence3.pi`

```
/*********************************************************
  Adapted from
  magic_sequence.pi
  from Constraint Solving and Planning with Picat, Springer
  by Neng-Fa Zhou, Hakan Kjellerstrand, and Jonathan Fruhman
*********************************************************/
import cp.

main([N]) =>
  N := N.to_int,
  magic_sequence(N,Sequence),
  println(Sequence).

magic_sequence(N, Sequence) =>
  Sequence = new_list(N),
  Sequence :: 0..N-1,

  % extra/redudant (implicit) constraints to speed up the model
  N #= sum(Sequence),
  Integers = [I : I in 0..N-1],
  scalar_product(Integers, Sequence, N),

  % % create list: [0-Sequence[1], 1-Sequence[2], ...]
  Pairs = [$I-Sequence[I+1] : I in 0..N-1],
  global_cardinality(Sequence,Pairs),

  time2(solve([ff], Sequence)).
```

The order of constraints

The order might matter to the solver, the above model is an example. When the
`cp` solver parses a constraint, it tries to reduce their domains. The implicit
(redundant) constranint using `scalar_product` can reduce the model a lot,
which is much better to do before parsing `global_cardinality` . (Note: e.g.
MiniZinc doesn't preserve the order of constraints during compilation, the
behaviour is a bit unpredictable.)

(Heuristic: easy constraints and constraints that are strong [in reducing the
search space] should go first??)

Example: Knight tour

Given an integer N, plan a tour of the knight on an $N \times N$ chessboard such that
the knight visits every field exactly once and then returns to the starting field.
You can assume that N is even.

Hint: For a matrix `M` is a matrix, use `M.vars()` to extract its elements into a
list.

```
In [8]:   !picat knight-tour/knight-tour.pi 8
```

CPU time 0.0 seconds. Backtracks: 0

x = {{11,12,13,10,15,21,22,14},{3,20,26,18,23,4,30,6},{2,1,29,5,27,32,8,7},
{19,9,37,34,35,40,16,38},{43,17,50,53,52,28,24,46},{51,25,49,61,60,36,62,3
1},{59,33,57,58,63,64,45,39},{42,41,44,54,55,56,48,47}}
X:
```
 11 12 13 10 15 21 22 14
  3 20 26 18 23  4 30  6
  2  1 29  5 27 32  8  7
 19  9 37 34 35 40 16 38
 43 17 50 53 52 28 24 46
 51 25 49 61 60 36 62 31
 59 33 57 58 63 64 45 39
 42 41 44 54 55 56 48 47
```

Tour:
```
  1 62  5 10 13 24 55  8
  4 11  2 63  6  9 14 23
 61 64 35 12 25 56  7 54
 34  3 26 59 36 15 22 57
 39 60 37 18 27 58 53 16
 30 33 40 43 46 17 50 21
 41 38 31 28 19 48 45 52
 32 29 42 47 44 51 20 49
```

The `circuit` constraint

The constraint `circuit(L)` requires that the list L represents a permutation of $1, \ldots, n$ consisting of a single cycle, i.e., the graph with edges $i \to L[i]$ is a cycle. A similar constraint is `subcircuit(L)` which requires that elements for which $L[i] \neq i$ form a cycle.

```
In [9]:   !cat knight-tour/knight-tour.pi
```

```
/***********************************************************
  Adapted from
  knight_tour.pi
  from Constraint Solving and Planning with Picat, Springer
  by Neng-Fa Zhou, Hakan Kjellerstrand, and Jonathan Fruhman
 ***********************************************************/
import cp.

main([N]) =>
  N := N.to_int,
  knight(N,X),
  println(x=X),
  println("X:"),
  print_matrix(X),
  extract_tour(X,Tour),
  println("Tour:"),
  print_matrix(Tour).

% Knight's tour for even N*N.
knight(N, X) =>
  X = new_array(N,N),
  X :: 1..N*N,
  XVars = X.vars(),
  % restrict the domains of each square
  foreach (I in 1..N, J in 1..N)
      D = [-1,-2,1,2],
      Dom = [(I+A-1)*N + J+B : A in D, B in D,
               abs(A) + abs(B) == 3,
               member(I+A,1..N), member(J+B,1..N)],
      Dom.length > 0,
      X[I,J] :: Dom
  end,
  circuit(XVars),
  time2(solve([ff,split],XVars)).

extract_tour(X,Tour) =>
  N = X.length,
  Tour = new_array(N,N),
  K = 1,
  Tour[1,1] := K,
  Next = X[1,1],
  while (K < N*N)
    K := K + 1,
    I = 1+((Next-1) div N),
    J = 1+((Next-1) mod N),
    Tour[I,J] := K,
    Next := X[I,J]
  end.

print_matrix(M) =>
  N = M.length,
  V = (N*N).to_string().length,
  Format = "% " ++ (V+1).to_string() ++ "d",
  foreach(I in 1..N)
      foreach(J in 1..N)
          printf(Format,M[I,J])
```

```
        end,
        nl
    end,
    nl.
```