# NOPT042 Constraint programming: Tutorial 12 - Planning

```
In [1]: %load_ext ipicat
```

Picat version 3.7

# The planning problem

Abstractly, planning refers to a class of problems where we are given:

- · an initial state,
- final states (a set of them, or perhaps a property that makes a state final),
- a set of possible actions (how they transforming a state to another state),

and our task is to find a sequence of actions transforming the inital state to the final state, possibly optimizing some objective, and satisfying some resource limits. This problem appears in many practical applicatios (e.g. logistics, robotics), as well as in logical puzzles (e.g. the 15 puzzle) or games (e.g. Sokoban).

On an abstract level, we are trying to find a path (from the initial state to some final state) in the state graph (aka state transition diagram). The state graphis not explicitly constructed, as it is typically huge.

Several (all?) of the exercises from the last tutorial (on Tabling) can be seen as planning problems, generally following this pseudocode (the book):

```
table (+,-,min)

path(S,Plan,Cost),final(S) =>
    Plan=[],Cost=0.

path(S,Plan,Cost) =>
    action(S,NextS,Action,ACost),
    path(NextS,Plan1,Cost1),
    Plan = [Action|Plan1],
    Cost = ACost+Cost1.
```

# The planner module

For planning problems, Picat provides the module planner which implements essentially the above pseudocode. It is enough to define the predicates

- final(S),
- action(S, NextS, Action, ACost),

and provide an initial state SInit. The ACost must be nonnegative. For example, if we only care about the number of steps, we set it to 1. The actions are tried in the order in which they are defined (as rules defining the predicate action). Remember that if there are multiple actions, all but the last rule must be backtrackable (?=>).

The above pseudocode implements *depth-unbounded search*. The module planner also implements *depth-bounded search* (e.g. *iterative deepening* and *branch and bound*) and heuristic search.

See the guide, Chapter 8 for more details. (The module is not big, only  $\sim$ 300 LOC.)

```
In [2]: !wget http://picat-lang.org/download/planner.pi
        !cat planner.pi
       --2024-10-02 14:52:34-- http://picat-lang.org/download/planner.pi
       Resolving picat-lang.org (picat-lang.org)...
       64.71.35.59
       Connecting to picat-lang.org (picat-lang.org)|64.71.35.59|:80...
       connected.
      HTTP request sent, awaiting response...
      200 OK
       Length: 13806 (13K)
       Saving to: 'planner.pi.1'
       planner.pi.1
                            0%[
                                                          0 --.-KB/s
       planner.pi.1
                          100%[==========] 13.48K --.-KB/s
                                                                         in 0s
       2024-10-02 14:52:35 (117 MB/s) - 'planner.pi.1' saved [13806/13806]
```

module planner.

```
% best plan(S,Limit,Plan) => best plan(S,Limit,Plan).
% best plan(S,Limit,Plan,PlanCost) => best plan(S,Limit,Plan,PlanCost).
% best plan(S,Plan,PlanCost) => best plan(S,Plan,PlanCost).
% best plan(S,Plan) => best plan(S,Plan).
% best plan bin(S,Limit,Plan) => best plan bin(S,Limit,Plan).
% best plan bin(S,Limit,Plan,PlanCost) => best plan bin(S,Limit,Plan,PlanCos
t).
% best plan bin(S,Plan,PlanCost) => best plan bin(S,Plan,PlanCost).
% best plan bin(S,Plan) => best plan bin(S,Plan).
% best_plan_bb(S,Limit,Plan) => best_plan bb(S,Limit,Plan).
% best plan bb(S,Plan,PlanCost) => best plan bb(S,Plan,PlanCost).
% best plan bb(S,Limit,Plan,PlanCost) => best plan bb(S,Limit,Plan,PlanCos
t).
% best plan bb(S,Plan) => best plan bb(S,Plan).
% best plan nondet(S,Limit,Plan) => best plan nondet(S,Limit,Plan).
% best plan nondet(S,Plan,PlanCost) => best plan nondet(S,Plan,PlanCost).
% best plan nondet(S,Limit,Plan,PlanCost) => best plan nondet(S,Limit,Plan,P
lanCost).
% best plan nondet(S,Plan) => best plan nondet(S,Plan).
% best plan unbounded(S,Limit,Plan) => best plan unbounded(S,Limit,Plan).
% best plan unbounded(S,Plan,PlanCost) => best plan unbounded(S,Plan,PlanCos
t).
% best plan unbounded(S,Limit,Plan,PlanCost) => best plan unbounded(S,Limit,
Plan, PlanCost).
% best plan unbounded(S,Plan) => best plan unbounded(S,Plan).
% current plan() = current plan().
% current resource() = current resource().
% current resource plan(Amount,Plan) => current resource plan(Amount,Plan).
% current resource plan cost(Amount,Plan,Cost) => current resource plan cost
```

```
(Amount, Plan, Cost).
% insert state list(StateL,Elm) = insert state list(StateL,Elm).
% is tabled state(S) => is tabled state(S).
% new state list(List) = new state list(List).
% plan(S,Limit,Plan) => plan(S,Limit,Plan).
% plan(S,Plan,PlanCost) => plan(S,Plan,PlanCost).
% plan(S,Limit,Plan,PlanCost) => plan(S,Limit,Plan,PlanCost).
% plan(S,Plan) => plan(S,Plan).
% plan unbounded(S,Limit,Plan) => plan unbounded(S,Limit,Plan).
% plan unbounded(S,Plan,PlanCost) => plan unbounded(S,Plan,PlanCost).
% plan unbounded(S,Limit,Plan,PlanCost) => plan unbounded(S,Limit,Plan,PlanC
ost).
% plan unbounded(S,Plan) => plan unbounded(S,Plan).
% A state-list is an ordered list, where the ordering of symbols is determin
% by the addresses of the symbols in the symbol table, not by the characters
that
% constitute the symbols. This ordering allows for efficient ordering of sym
% Note that this ordering is different from lexicographical ordering, which
is used by sort().
new state list(List) = SList => new state list aux(List,[],SList).
new stte list aux([],SList0,SList) => SList = SList0.
new state list aux([E|List],SList0,SList) =>
    bp.b INSERT STATE LIST ccf(SList0,E,SList1),
    new state list aux(List,SList1,SList).
insert state list(SList0,Elm) = SList => bp.b INSERT STATE LIST ccf(SList0,E
```

```
lm,SList).
%%%
current resource() = Amount =>
    bp.b PLANNER CURR RPC fff(Amount, Plan, Cost).
current plan() = Plan =>
    bp.b PLANNER CURR RPC fff( Amount, Plan, Cost).
current resource plan(Amount,Plan) =>
    bp.b PLANNER CURR RPC fff(Amount,Plan, Cost).
current resource plan cost(Amount,Plan,Cost) =>
    bp.b PLANNER CURR RPC fff(Amount, Plan, Cost).
%%%
plan(S,Plan),var(Plan) =>
    bp.picat_plan(S,268435455,Plan,_). % the limit is assumed to be 268435
455
plan( S,Plan) =>
    throw plan arg error(1,Plan, ,plan).
plan(S,Limit,Plan),var(Plan),integer(Limit),Limit>=0 =>
    bp.picat plan(S,Limit,Plan, ).
plan(S,Plan,PlanCost),var(Plan),var(PlanCost) =>
    bp.picat plan(S,268435455,Plan,PlanCost).
plan(_S,Limit,Plan) =>
    throw_plan_arg_error(Limit,Plan,_,plan).
```

```
plan(S,Limit,Plan,PlanCost),var(Plan),var(PlanCost),integer(Limit),Limit>=0
    bp.picat_plan(S,Limit,Plan,PlanCost).
plan( S,Limit,Plan,PlanCost) =>
    throw plan arg error(Limit, Plan, PlanCost, plan).
plan(S,Limit,Plan,PlanCost,FinS),var(Plan),var(PlanCost),integer(Limit),Limi
t>=0 =>
    bp.picat plan(S,Limit,Plan,PlanCost,FinS).
plan( S,Limit,Plan,PlanCost, FinS) =>
    throw plan arg error(Limit, Plan, PlanCost, plan).
%%%
plan unbounded(S,Plan),var(Plan) =>
    bp.picat_plan_unbounded(S,268435455,Plan,_). % the limit is assumed to
be 268435455
plan unbounded( S,Plan) =>
    throw plan arg error(1,Plan, ,plan unbounded).
plan unbounded(S,Limit,Plan),var(Plan),integer(Limit),Limit>=0 =>
    bp.picat plan unbounded(S,Limit,Plan, ).
plan unbounded(S,Plan,PlanCost),var(Plan),var(PlanCost) =>
    bp.picat plan unbounded(S,268435455,Plan,PlanCost).
plan unbounded( S,Limit,Plan) =>
    throw plan arg error(Limit, Plan, ,plan unbounded).
plan unbounded(S,Limit,Plan,PlanCost),var(Plan),var(PlanCost),integer(Limi
t),Limit>=0 =>
    bp.picat plan unbounded(S,Limit,Plan,PlanCost).
plan unbounded( S,Limit,Plan,PlanCost) =>
```

```
throw plan arg error(Limit, Plan, PlanCost, plan unbounded).
```

```
%%%
%% iterative deepening
best plan(S,Plan),var(Plan) =>
    best plan downward(S,0,268435455,Plan,,).
best plan( S,Plan) =>
    throw plan arg error(1,Plan, ,best plan).
best plan(S,Limit,Plan),var(Plan),integer(Limit),Limit>=0 =>
    best plan downward(S,0,Limit,Plan, , ).
best plan(S,Plan,PlanCost),var(Plan),var(PlanCost) =>
    best plan downward(S,0,268435455,Plan,PlanCost, ).
best_plan(_S,Limit,Plan) =>
    throw plan arg error(Limit, Plan, , best plan).
best plan(S,Limit,Plan,PlanCost),var(Plan),var(PlanCost),integer(Limit),Limi
t>=0 =>
    best plan downward(S,0,Limit,Plan,PlanCost, ).
best plan( S,Limit,Plan,PlanCost) =>
    throw plan arg error(Limit, Plan, PlanCost, best plan).
best plan(S,Limit,Plan,PlanCost,FinS),var(Plan),var(PlanCost),integer(Limi
t),Limit>=0 =>
    best plan downward(S,0,Limit,Plan,PlanCost,FinS).
best_plan(_S,Limit,Plan,PlanCost,_FinS) =>
    throw plan arg error(Limit, Plan, PlanCost, best plan).
```

```
best plan downward(S,Level, Limit, Plan, PlanCost, FinS) ?=>
    (bp.global_get('_$picat_log',0,1) -> printf("% Searching with the bound
%d\n",Level); true),
    call picat plan(S, Level),
    fail.
best plan downward(S, Level, Limit, Plan, PlanCost, FinS),
    Map = get_table_map(' $planner'),
    Map.has key($current best plan(S))
    Map.get($current best plan(S)) = Plan,
    Map.get($current best plan cost(S)) = PlanCost,
    Map.get($current best plan fin state(S)) = FinS.
best plan downward(S,Level,Limit,Plan,PlanCost,FinS) =>
    bp.global get(' $planner explored depth',0,Depth),
    (Depth == 268435455 -> Level1 = Level+1; Level1 = Depth),
        Level1 =< Limit,</pre>
    best plan downward(S,Level1,Limit,Plan,PlanCost,FinS).
%% iterative deeping and binary search
best plan bin(S,Plan),var(Plan) =>
    best plan downward bin(S,0,268435455,Plan, , ).
best plan bin( S,Plan) =>
    throw plan arg error(1,Plan, ,best plan).
best plan bin(S,Limit,Plan),var(Plan),integer(Limit),Limit>=0 =>
    best plan downward bin(S,0,Limit,Plan, , ).
best plan bin(S,Plan,PlanCost),var(Plan),var(PlanCost) =>
```

```
best plan downward bin(S,0,268435455,Plan,PlanCost, ).
best plan bin( S,Limit,Plan) =>
    throw plan arg error(Limit,Plan, ,best plan).
best plan bin(S,Limit,Plan,PlanCost),var(Plan),var(PlanCost),integer(Limit),
Limit>=0 =>
    best plan downward bin(S,0,Limit,Plan,PlanCost, ).
best plan bin( S,Limit,Plan,PlanCost) =>
    throw plan arg error(Limit, Plan, PlanCost, best plan).
best plan bin(S,Limit,Plan,PlanCost,FinS),var(Plan),var(PlanCost),integer(Li
mit),Limit>=0 =>
    best plan downward bin(S,0,Limit,Plan,PlanCost,FinS).
best plan bin( S,Limit,Plan,PlanCost, FinS) =>
    throw plan arg error(Limit, Plan, PlanCost, best plan).
%%%
best plan downward bin(S,Level,Limit,Plan,PlanCost,FinS) =>
    Map = get table map(' $planner'),
    loop best plan downward bin(S,Level,Limit,Plan,PlanCost,FinS,Map).
loop best plan downward bin(S,Level,Limit, Plan, PlanCost, FinS, Map) ?=>
    Level =< Limit,
    (bp.global get(' $picat log',0,1) -> printf("%% Searching with the bound
%d\n",Level); true),
    call picat plan(S,Level),
    fail.
loop best plan downward bin(S, Level, Limit,Plan,PlanCost, FinS,Map),
    Map.has key($current best plan(S))
```

```
Lower = Map.get($current lower bound($),0),
    Upper = Map.get($current best plan cost(S))-1,
    loop best plan bin(S,Map,Lower,Upper,Plan,PlanCost).
loop best plan downward bin(S,Level,Limit,Plan,PlanCost,FinS,Map) =>
    bp.global get(' $planner explored depth',0,Depth),
    writeln(depth=Depth),
    (Depth == 268435455 -> Lower = Level+1; Lower = Depth),
    Map.put($current lower bound(S),Lower),
    Lower < Limit,
    Level1 = 2*Lower,
    NewLevel = cond(Level1>Limit, Limit, Level1),
    loop best plan downward bin(S,NewLevel,Limit,Plan,PlanCost,FinS,Map).
% binary search
loop best plan bin(S,Map,Lower,Upper,Plan,PlanCost),
    Lower =< Upper
    NewLimit = Lower + (Upper-Lower) div 2,
    (bp.global get(' $picat log',0,1) -> printf("% Searching with the bound
%d\n",NewLimit); true),
    if call picat plan(S, NewLimit) then
        NewUpper = Map.get($current best plan cost(S))-1,
        loop best plan bin(S,Map,Lower,NewUpper,Plan,PlanCost)
    else
        NewLower = NewLimit+1,
        loop best plan bin(S,Map,NewLower,Upper,Plan,PlanCost)
    end.
loop_best_plan_bin(S,Map,_Lower,_Upper,Plan,PlanCost) =>
```

```
Map.has key($current best plan(S)),
    Map.get($current best plan(S)) = Plan,
    Map.get($current best plan cost(S)) = PlanCost.
%%%
best plan nondet(S,Plan),var(Plan) =>
    best plan nondet aux(S,268435455,Plan, ).
best plan nondet( S,Plan) =>
    throw_plan_arg_error(1,Plan,_,best_plan_nondet).
best plan nondet(S,Limit,Plan),var(Plan),integer(Limit),Limit>=0 =>
    best plan nondet aux(S,Limit,Plan, ).
best plan nondet(S,Plan,PlanCost),var(Plan),var(PlanCost) =>
    best plan nondet aux(S,268435455,Plan,PlanCost).
best plan nondet( S,Limit,Plan) =>
    throw plan arg error(Limit, Plan, , best plan nondet).
best plan nondet(S,Limit,Plan,PlanCost),var(Plan),var(PlanCost),integer(Limi
t),Limit>=0 =>
    best plan nondet aux(S,Limit,Plan,PlanCost).
best plan nondet( S,Limit,Plan,PlanCost) =>
    throw plan arg error(Limit, Plan, PlanCost, best plan nondet).
best plan nondet aux(S,Limit,Plan,PlanCost) =>
    not not (M = get global map(),
             M.put(' first best plan',[]),
             best plan downward(S,0,Limit,Plan0,PlanCost0, ), % use table
d search to find the first best plan
             M.put(' first best plan',(Plan0,PlanCost0))),
```

```
get global map().get(' first best plan') = (Plan0,PlanCost),
    (
         Plan = Plan0
        bp.picat best plan nondet nontabled search(S,Plan,PlanCost),
        Plan0 != Plan
    ).
%% Branch-and-Bound
best plan bb(S,Plan),var(Plan) =>
    loop best plan bb(S,268435455,Plan, ).
best plan bb( S,Plan) =>
    throw plan arg error(1,Plan, ,best plan bb).
best plan bb(S,Limit,Plan),var(Plan),integer(Limit),Limit>=0 =>
    loop best plan bb(S,Limit,Plan, PlanCost).
best plan bb(S,Plan,PlanCost),var(Plan),var(PlanCost) =>
    loop best plan bb(S,268435455,Plan,PlanCost).
best plan bb( S,Limit,Plan) =>
    throw plan arg error(Limit,Plan, ,best plan bb).
best plan bb(S,Limit,Plan,PlanCost),var(Plan),var(PlanCost),integer(Limit),L
imit>=0 =>
    loop best plan bb(S,Limit,Plan,PlanCost).
best plan bb( S,Limit,Plan,PlanCost) =>
    throw plan arg error(Limit, Plan, PlanCost, best plan bb).
loop_best_plan_bb(S,Limit,Plan,PlanCost),
    (bp.global_get('_$picat_log',0,1) -> printf("% Searching with the bound
%d\n",Limit); true),
```

```
call picat plan(S,Limit)
=>
    get_table_map('_$planner').get($current_best_plan_cost(S)) = PlanCost1,
    loop best plan bb(S,PlanCost1-1,Plan,PlanCost).
loop best plan bb(S,_Limit,Plan,PlanCost) =>
    Map = get table map(' $planner'),
    Map.has key($current best plan(S)),
    Map.get($current best plan(S)) = Plan,
    Map.get($current best plan cost(S)) = PlanCost.
call picat plan(S,Limit) =>
    bp.global set(' $planner explored depth',0,268435455),
    not not call picat plan aux(S,Limit). % discard exception catchers crea
ted by picat plan
call picat plan aux(S,Limit) =>
    bp.picat plan(S,Limit,Plan,PlanCost,FinS),
    Map = get table map(' $planner'),
    Map.put($current best plan(S),Plan),
    Map.put($current best plan cost(S),PlanCost),
    Map.put($current best plan fin state(S),FinS).
%%%
best plan unbounded(S,Plan),var(Plan) =>
    bp.picat best plan unbounded(S,Plan, ).
best plan unbounded( S,Plan) =>
    throw_plan_arg_error(1,Plan,_,best_plan_unbounded).
best plan unbounded(S,Limit,Plan),var(Plan),integer(Limit),Limit>=0 =>
```

```
bp.picat best plan unbounded(S,Plan,PlanCost),
    PlanCost=<Limit.
best plan unbounded(S,Plan,PlanCost),var(Plan),var(PlanCost) =>
    bp.picat best plan unbounded(S,Plan,PlanCost).
best plan unbounded( S,Limit,Plan) =>
    throw plan arg error(Limit, Plan, , best plan unbounded).
best plan unbounded(S,Limit,Plan,PlanCost),var(Plan),var(PlanCost),integer(L
imit),Limit>=0 =>
    bp.picat best plan_unbounded(S,Plan,PlanCost),
    PlanCost =< Limit.
best plan unbounded( S,Limit,Plan,PlanCost) =>
    throw plan arg error(Limit, Plan, PlanCost, best plan unbounded).
%%%
is tabled state(S) =>
    bp.b IS PLANNER STATE c(S).
throw plan arg error( Limit,Plan, PlanCost,Source),nonvar(Plan) =>
    handle exception($var expected(Plan),Source).
throw plan arg error( Limit, Plan,PlanCost,Source),nonvar(PlanCost) =>
    handle exception($var expected(PlanCost),Source).
throw plan arg error(Limit, Plan, PlanCost, Source), integer(Limit) =>
    handle exception($nonnegative integer expected(Limit),Source).
throw_plan_arg_error(Limit,_Plan,_PlanCost,Source) =>
    handle exception($integer expected(Limit), Source).
```

### Depth-unbounded search

The module planner implements the following two predicates:

- plan\_unbounded(S, Limit, Plan, PlanCost): find any plan where
   PlanCost <= Limit</li>
- best\_plan\_unbounded(S, Limit, Plan, PlanCost): find the best plan

The arguments PlanCost and Limit can be omitted.

#### Resource-bounded search

The module planner also implements the following predicates:

- plan(S, Limit, Plan, PlanCost): perform resource-bounded search (i.e., keep a resource amount, do not explore a state if the resource amount is negative or if the state has previously failed with the same or more resource), if the resource is plan length (number of actions), this is depth-bounded search.
- best\_plan(S, Limit, Plan, PlanCost): finds the lowest-cost plan, using
   iterative deepening; calls plan/4 setting the initial cost to 0 and then
   iteratively increasing.
- best\_plan\_bb(S, Limit, Plan, PlanCost): first find any plan using plan/4, then branch and bound lowering the limit.

And we can use the function current resource() = Limit which returns the resource of the last call of plan; this can be used to implement a heuristic to prune the search (e.g. in 01-knapsack, if taking all of the remaining items, ignoring weight, won't give us sufficient total value, better than best so far).

# Example: 15 puzzle

We will use the planner module to solve the 15 puzzle. Before checking the solution, think about what are the states and actions.

See this paper for a solution and more examples.

```
right
right
down
left
up
left
down
down
left
up
right
down
down
right
right
up
.
left
down
left
left
up
right
up
right
down
right
up
up
left
down
left
left
up
```

In [4]: !cat puzzle15/puzzle15.pi

```
/************************
  15 puzzle.pi
  from Constraint Solving and Planning with Picat, Springer
  by Neng-Fa Zhou, Hakan Kjellerstrand, and Jonathan Fruhman
import planner.
main =>
   InitS = [(1,2),(2,2),(4,4),(1,3),
            (1,1),(3,2),(1,4),(2,4),
            (4,2),(3,1),(3,3),(2,3),
            (2,1),(4,1),(4,3),(3,4)],
    best plan(InitS,Plan),
    foreach (Action in Plan)
      println(Action)
   end.
final(State) \Rightarrow State=[(1,1),(1,2),(1,3),(1,4),
                      (2,1),(2,2),(2,3),(2,4),
                      (3,1),(3,2),(3,3),(3,4),
                      (4,1),(4,2),(4,3),(4,4)].
action([P0@(R0,C0)|Tiles],NextS,Action,Cost) =>
    Cost = 1,
    (R1 = R0-1, R1 >= 1, C1 = C0, Action = up;
    R1 = R0+1, R1 = < 4, C1 = C0, Action = down;
    R1 = R0, C1 = C0-1, C1 >= 1, Action = left;
    R1 = R0, C1 = C0+1, C1 = 4, Action = right),
    P1 = (R1,C1),
    slide(P0,P1,Tiles,NTiles),
    current resource() > manhattan dist(NTiles),
    NextS = [P1|NTiles].
% slide the tile at P1 to the empty square at P0
slide(P0,P1,[P1|Tiles],NTiles) =>
    NTiles = [P0|Tiles].
slide(P0,P1,[Tile|Tiles],NTiles) =>
    NTiles=[Tile|NTilesR],
    slide(P0,P1,Tiles,NTilesR).
manhattan dist(Tiles) = Dist =>
    final([ |FTiles]),
    Dist = sum([abs(R-FR)+abs(C-FC) :
               {(R,C),(FR,FC)} in zip(Tiles,FTiles)]).
```

## Exercise: 01-Knapsack

Implement the 01-knapsack problem using the planner module. (Every CSP can be viewed as a planning problem where states are partial assignments and actions represent the choice of value for a variable. We are looking for a path from the root of the search tree to one of the leaves.)

```
instance(ItemNames, Capacity, Values, Weights) =>
        ItemNames = {"tv", "desktop", "laptop", "tablet", "vase", "bottle", "jac
ket"},
        Capacity = 23,
        Values = {500,350,230,115,180,75,125},
        Weights = {15,11,5,1,7,3,4}.

In [6]: !cd knapsack && picat knapsack.pi instance.pi
        (take,tv)
        (leave,desktop)
        (take,laptop)
        (take,tablet)
        (leave,vase)
        (leave,bottle)
        (leave,jacket)
In [7]: !cat knapsack/knapsack.pi
```

```
import planner.
main([Filename]) =>
    cl(Filename),
    instance(ItemNames, TotalCapacity, Values, Weights),
    AllItems = [(ItemNames[I], Values[I], Weights[I]) : I in 1..ItemNames.le
ngth],
    % state: S@(RemainingItems, RemainingCapacity)
    InitialState = (AllItems, TotalCapacity),
    % PlanCost is the value of items we did not take
    best plan(InitialState, Plan, PlanCost),
    foreach (Action in Plan)
       println(Action)
    end.
% take the current item
action(CurrentState@(Items, Capacity), NextState, Action, Cost) ?=>
    Items = [Item | RemainingItems],
    Item = (ItemName, ItemValue, ItemWeight),
    Action = (take, ItemName),
    % taking an item costs nothing
    Cost = 0,
    % is this action valid?
    Capacity >= ItemWeight,
    % take the item, lower capacity
    NextState = (RemainingItems, Capacity - ItemWeight).
% leave the current item
action(CurrentState@(Items, Capacity), NextState, Action, Cost) =>
    Items = [Item | RemainingItems],
    Item = (ItemName, ItemValue, ItemWeight),
    Action = (leave, ItemName),
    % leaving an item costs its value
    Cost = ItemValue,
    % leave the item, capacity does not change
    NextState = (RemainingItems, Capacity).
% finish if no remaining items
final(S@(Items, Capacity)) => Items = [].
```

## Exercise: Jugs

Solve the Three Jugs Problem (exercise 3.12/8 in the book):

There are 3 water jugs. The first jug can hold 3 liters of water, the second jug can hold 5 liters, and the third jug is an 8-liter container that is full of water. At the start, the first and second jugs are empty. The goal is to get exactly 4 liters of water in one of the containers. (We are not allowed to spill water).

Generalize to any number of jugs with arbitrary maximum and intial volumes, and any target volume, e.g.: