Answer-Set Programming

Basics, Combinations with Ontologies, and Debugging

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We deal with *answer-set programming* (*ASP*)

- Programming paradigm

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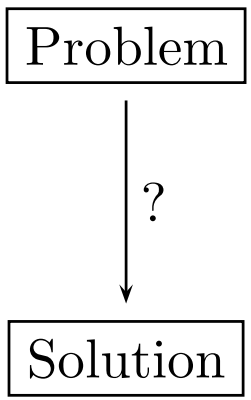

Declarative vs. Procedural Programming

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- General Questions:
  - What *steps* do I need to take to solve the problem?
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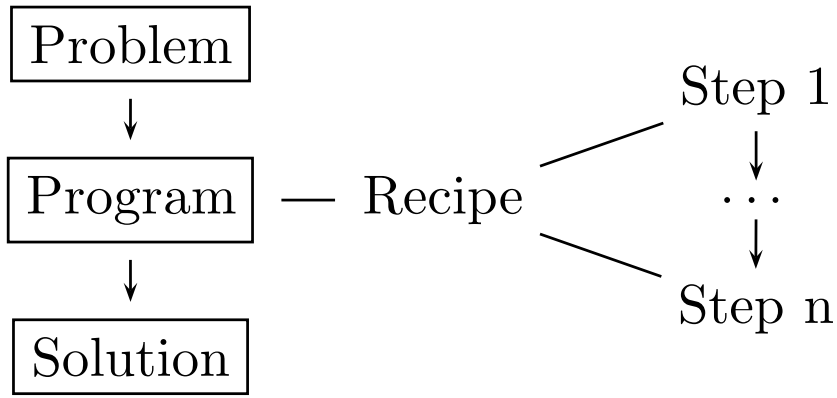

Declarative vs. Procedural Programming (ctd.)

In *declarative programming*, a problem is solved by writing a program that corresponds to a *description* of the problem.

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  - *What* is the problem?

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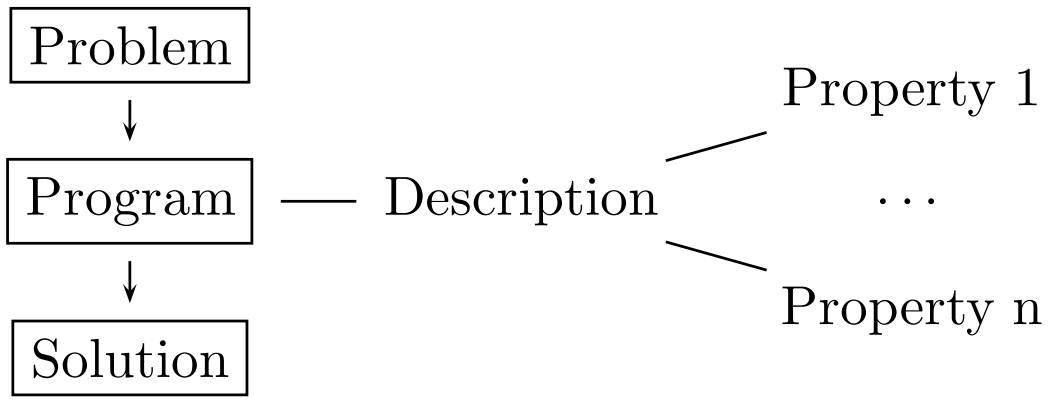

- Control flow is *not* provided in the program.
- *No algorithm* is given for solving the problem.

ASP Big Picture

In *Answer Set Programming (ASP)*, the problem is encoded in terms of a logic theory - typically using logic programming rules.

- Answer-Set Solver computes dedicated models of the theory.
- They are called *answer sets*.

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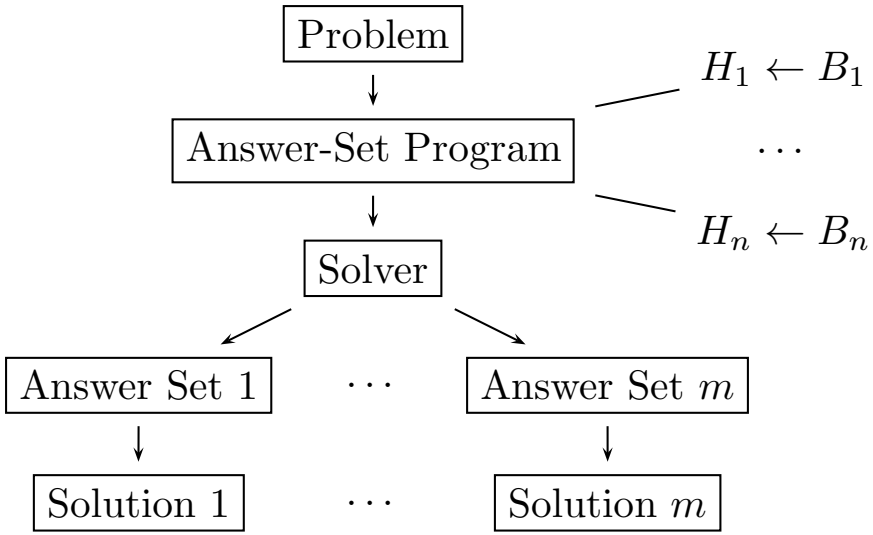
- Answer-Set Solver computes dedicated models of the theory.
- They are called *answer sets*.
- Correspond one-to-one to the (multiple) solutions of the problem.



Answer Sets

Multiple Answer Sets:

- Non-monotonic reasoning (use of default negation)
- Ability to express non-determinism (guessing)
- Sometimes useful: Preference relations between answer sets
- Most used semantics:
  - Stable model semantics ($\simeq$ Answer-Set semantics)
- Different modes of reasoning:
  - Model generation: compute 1, n, or all answer sets
  - Cautious reasoning: query holds in all answer sets
  - Brave reasoning: there is an answer set for which query holds

Syntax

- We deal with *disjunctive logic programs* (*DLPs*) under the answer-set semantics, which are sets of rules of form:

$$a_1 \vee \cdots \vee a_l \leftarrow a_{l+1}, \ldots, a_m, \mathit{not}\ a_{m+1}, \ldots, \mathit{not}\ a_n$$

  - it must hold that $n \geq 1$
  - all a_i are literals over some fixed function-free first-order language $\mathscr{L}$
  - "*not*" denotes *default negation* (negation as failure)
  - a (strong) literal is either an atom or an atom preceded by *strong negation*, $-$.

Syntax (ctd.)

- We deal with rules of form:

$$a_1 \vee \cdots \vee a_l \leftarrow a_{l+1}, \ldots, a_m, \mathit{not}\ a_{m+1}, \ldots, \mathit{not}\ a_n$$

- We distinguish the following parts of rule r of the form above:
  - the head $\mathrm{H}(r) = \{a_1, \cdots, a_l\}$,
  - the body $\mathrm{B}(r) = \{a_{l+1}, \ldots, a_m, \mathit{not}\ a_{m+1}, \ldots, \mathit{not}\ a_n\}$,
  - the positive body $\mathrm{B}^+(r) = \{a_{l+1}, \ldots, a_m\}$, and
  - the negative body $\mathrm{B}^-(r) = \{a_{m+1}, \ldots, a_n\}$.

- A rule r is
  - a *fact* if $\mathrm{B}(r) = \emptyset$,
  - a *constraint* if $\mathrm{H}(r) = \emptyset$,
  - *normal* if $|\mathrm{H}(r)| \leq 1$,
  - *positive* if $\mathrm{B}^-(r) = \emptyset$, or
  - *Horn* if it is both, normal and positive.

Semantics

- The answer-set semantics for disjunctive logic programs was proposed by Gelfond and Lifschitz (1991).
- Definition based on ground programs:
  - Rules do not contain variables
- Semantics for a program P with variables based on the program's grounding $gr(P)$:
  - Replace each non-ground rule r by all ground rules that can be obtained from r by substituting variables by constants in the Herbrand Universe of the program.

Semantics (ctd.)

- An *interpretation* is a consistent set I of ground literals
  - a is true iff $a \in I$.

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- An *interpretation* is a consistent set I of ground literals
  - a is true iff $a \in I$.
- For a ground rule r,
  - $I \models \mathrm{B}(r)$ iff $\mathrm{B}^+(r) \subseteq I$ and $\mathrm{B}^-(r) \cap I = \emptyset$;
  - $I \models \mathrm{H}(r)$ iff $\mathrm{H}(r) \cap I \neq \emptyset$;
  - $I \models r$ iff ($I \models \mathrm{B}(r)$ implies $I \models \mathrm{H}(r)$)
- I is a model of a ground program P iff $I \models r$ for all $r \in P$.

Semantics (ctd.)

- The *reduct* of a ground program P w.r.t. I is the program

$$P^I := \{\mathrm{H}(r) \leftarrow \mathrm{B}^+(r) \mid r \in P \text{ and } I \cap \mathrm{B}^-(r) = \emptyset\}$$

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- I is an *answer set* of a program P if it is the minimal model of $gr(P)^I$
- By $AS(P)$, we denote the set of all answer sets of P.

Semantics (ctd.)

Many alternative definitions

- By means of propositional formulas (Lin and Zhao, 2004)
- In terms of equilibrium logic and logic of here-and-there (Pearce, 1996)
- Using the FLP-reduct P^I_{FLP} instead of P^I (Faber, Leone, and Pfeifer, 2004):
  - I is an *answer set* of a program P if it is a minimal model of the rules $r \in gr(\Pi)$ such that $I \models \mathrm{B}(r)$.
  - Allows for intuitive handling of recursive aggregates.
- ...

Small example

$$
\begin{aligned}
P = \{ \quad a(X) &\leftarrow not\ e(X), d(X), \\
e(X) &\leftarrow not\ \ a(X), d(X), \\
c(X,Y) &\leftarrow a(X), a(Y), not\ -b(X), \\
d(1) &\leftarrow, \\
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\end{aligned}
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The grounding of P is given by

$$
\begin{aligned}
gr(P) = \{ \quad & a(1) \leftarrow not\ e(1), d(1), & & a(2) \leftarrow not\ e(2), d(2), \\
& e(1) \leftarrow not\ \ a(1), d(1), & & e(2) \leftarrow not\ \ a(2), d(2), \\
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Consider the interpretation $I_1 = \{d(1), d(2)\}$

$$
\begin{aligned}
gr(P)^{I_1}_{\mathrm{FLP}} = \{ \quad & a(1) \leftarrow not\ e(1), d(1), && a(2) \leftarrow not\ e(2), d(2), \\
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I_1 is not a model of $gr(P)^{I_1}_{\mathrm{FLP}}$.

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gr(P)^{I_2}_{\mathrm{FLP}} = \{ \quad & \cancel{a(1) \leftarrow not\ e(1), d(1),} && \cancel{a(2) \leftarrow not\ e(2), d(2),} \\
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I_3 is a minimal model of $gr(P)^{I_3}_{\mathrm{FLP}}$, hence I_3 is an answer set of P.

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P has the following answer sets:

$$\{d(1), d(2), a(1), a(2), c(1,1), c(1,2), c(2,1), c(2,2)\},$$
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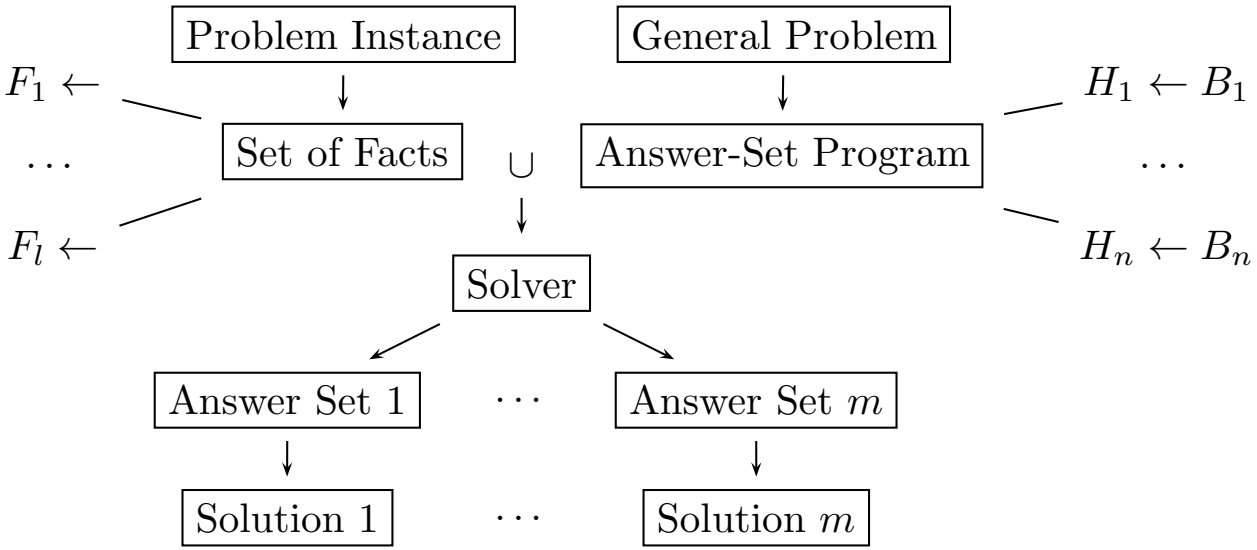
Uniform Problem Encoding

- Typically, answer-set programs are *uniform problem encodings*.
- The program itself encodes the general problem,
- whereas individual problem instances represented by facts can be joined with the program on demand.



Graph Colouring

Uniform problem encoding:

$$P_1 = \{ another\_col(V, C) \leftarrow vertex(V), col(C), col(D), col\_of(V, D), C \neq D,$$

$$col\_of(V, C) \leftarrow vertex(V), col(C), not\ another\_col(V, C),$$

$$\leftarrow vertex(U), vertex(V), edge(U, V), col\_of(U, C), col\_of(V, C) \}.$$

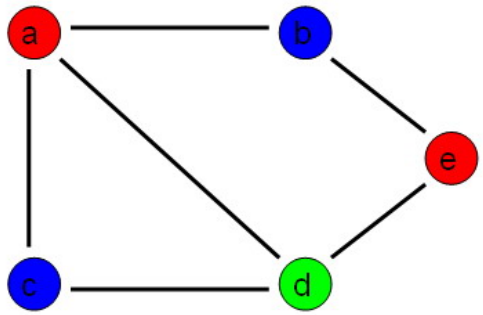
Input instance as set of facts:

$$F_1 = \{ col(red), col(green), col(blue), vertex(a), vertex(b), vertex(c), vertex(d), vertex(e), edge(a, b), edge(a, c), edge(a, d), edge(b, e), edge(c, d), edge(d, e) \}.$$

Graph Colouring (ctd.)

- The answer sets of $P_1 \cup F_1$ correspond to the solutions of the problem instance.

$$\{col\_of(a, red), col\_of(b, blue), col\_of(c, blue), col\_of(d, green), col\_of(e, red), \dots\}$$



Use

- Complexity captured by uniform problem encodings:
  - normal logic programs: NP-hard problems
  - disjunctive logic programs: Σ_2^P-hard problems
- Applications of ASP
  - configuration, computer-aided verification, software testing, health care, bio informatics, planning, music composition, database repair, *Semantic-Web reasoning*, ...
- Strengths:
  - Concise representation, elaboration tolerance, non-determinism, use of recursion
  - Efficient solvers:
    - Clasp (University of Potsdam)
    - DLV (University of Calabria, Vienna University of Technology)

ASP Language Extensions

- Many extensions have been proposed, partly motivated by applications
- Some are syntactic sugar, other strictly add expressiveness
- Incomplete list:
  - optimisation: minimize/maximize statements, weak constraints
  - aggregates, cardinality constraints
  - templates (for macros)
  - function symbols (undecidability)
  - Frame Logic syntax
  - KR frontends (diagnosis, planning,...) in DLV
  - higher-order features (variables as predicates)
  - *external atoms*

Formalisms combining Ontologies and Rules

- Hybrid formalisms - Ontologies / Logical rules.
  - loosely coupled approaches
    - DL-programs
    - F-Logic#
  - tightly coupled approaches
    - SWRL and restrictions (DLP, Carin, DL-safe rules, Desription Logic Rules)
    - R-hybrid KBs
    - F-hybrid KBs
  - Embedding approaches
    - hybrid MKNF KBs
    - Quantified Equilibrium Logic (QEL)

Ontologies and Rules in the Semantic Web

Why a "Semantic" Web?

- Undoubtedly, the World Wide Web is one of the most significant technical innovations of the last decades,
- The WWW will be (and already is) strongly impacting, and changing, our living, culture, and society.

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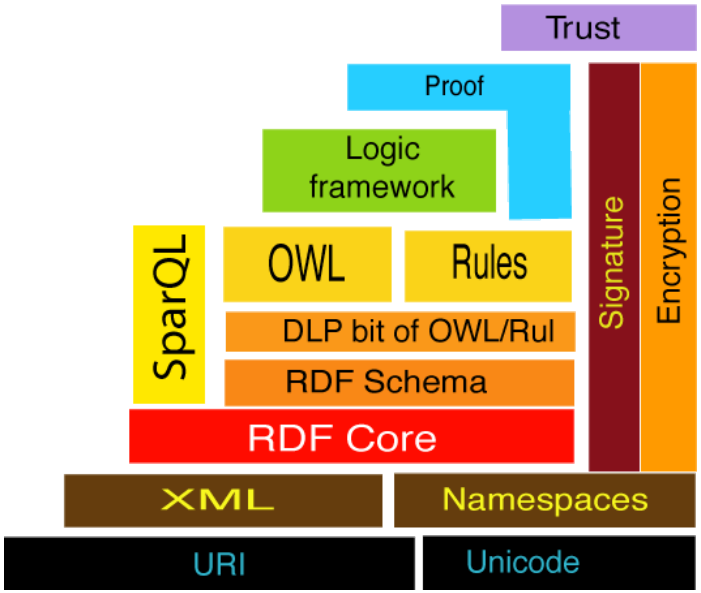
- The WWW has been designed for human users, not for machines
- To process the data and information out on the Web, semantic annotation and description is needed.
- The Semantic Web is the vision of such an enriched, future generation Web
- Logic and logic-based formalisms (should/might) play an important role in this endeavor.

Building the Semantic Web (T. Berners-Lee, 04/2005)



- RDF is the data model of the Semantic Web
- RDF Schema semantically extends RDF by simple taxonomies and hierarchies
- OWL is a W3C standard, which builds on Description Logics
- Rule languages: *Rule Interchange Format (RIF)* WG of W3C

ASP vs. Ontologies

- ASP and OWL/DLs have related yet different underlying settings
- At the heart, the difference is between LP and Classical logic
- **Main Differences:**
  - Closed vs. Open World Assumption (CWA/OWA)
  - Negation as failure vs. classical negation
  - Strong negation vs. classical negation
  - Unique names, equality

LP / Classical Logic: CWA vs. OWA

- LP aims at building a single model, by closing the world

  Reiter's CWA:

  If $T \not\models A$, *then conclude* $\neg A$, for ground atom A

- FO logic / description logics keep the world open
- In the Semantic Web, this is often reasonable
- However, taking the agnostic stance of OWA may be not helpful for drawing rational conclusions under incomplete information

LP / Classical Logic: NAF vs. Classical Negation

$$P: \quad wine(X) \leftarrow whiteWine(X).$$
$$nonWhite(X) \leftarrow not\ \ whiteWine(X).$$
$$wine(myDrink).$$

$$T: \quad \forall X.(whiteWine(X) \supset wine(X)) \wedge$$
$$\forall X.(\neg whiteWine(X) \supset nonWhite(X)) \wedge$$
$$wine(myDrink).$$

LP / Classical Logic: NAF vs. Classical Negation

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- Query $nonWhite(myDrink)$?

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- Query $nonWhite(myDrink)$?
  - Conclude $nonWhite(myDrink)$ from P.

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$$wine(myDrink).$$

- Query $nonWhite(myDrink)$?
  - Conclude $nonWhite(myDrink)$ from P.
  - Do not conclude $nonWhite(myDrink)$ from T.

LP / Classical Logic: Strong vs. Classical Negation

P : $wine(X) \leftarrow whiteWine(X).$
$-wine(myDrink).$

T : $\forall X.(WhiteWine(X) \supset Wine(X)) \wedge$
$\neg Wine(myDrink).$

LP / Classical Logic: Strong vs. Classical Negation

P : $wine(X) \leftarrow whiteWine(X).$
$-wine(myDrink).$

T : $\forall X.(WhiteWine(X) \supset Wine(X)) \wedge$
$\neg Wine(myDrink).$

- Conclude $\neg WhiteWine(myDrink)$ from T;

LP / Classical Logic: Strong vs. Classical Negation

P : $wine(X) \leftarrow whiteWine(X).$
$-wine(myDrink).$

T : $\forall X.(WhiteWine(X) \supset Wine(X)) \wedge$
$\neg Wine(myDrink).$

- Conclude $\neg WhiteWine(myDrink)$ from T;
- Do not conclude $-whiteWine(myDrink)$ from P
- Note: no contraposition in LP!

$$-whiteWine(X) \leftarrow -wine(X).$$

is not equivalent to

$$wine(X) \leftarrow whiteWine(X).$$

LP / Classical Logic: Unique Names, Equality

- In LP, usally we have *Unique Names Assumption (UNA)*:
  Syntactically different ground terms are different objects.
- Thus, usually only Herbrand interpretations are considered in LP
- Ontology languages like OWL don't make UNA, and allow to link objects (owl:sameAs)
- OWL considers also non-Herbrand interpretations
- Further, related problems with existential quantifiers:

$$T: \quad \forall X \exists Y . (Wine(X) \supset hasColor(X, Y))$$

(in DL Syntax, $Wine \sqsubseteq \exists hasColor$)

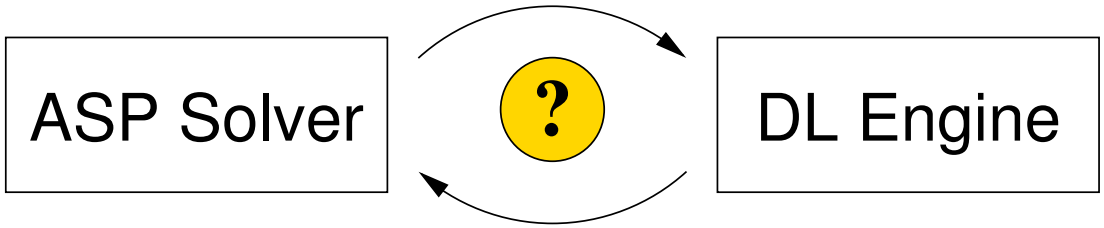
Simple skolemisation does not work in general

DL-programs

- Recently, much effort has been put into the study of combinations of *rules* and *ontologies* in the context of the Semantic Web.
- *DL-programs* have been proposed as a fruitful approach for combining logic programs (LP) under the answer-set semantics and description logic (DL) ontologies [Eiter et al. 2004,2008].
- The interface beetween LP and DL is based on *loose coupling*.

Loose Coupling

- The two components do not rely on a shared language or common logical models.
- Instead, they interact via dedicated atoms, called *DL-atoms*, in the LP.
- A DL-atom can be seen as a *query to the ontology*.
- Bi-directional flow of information:
  - before the query is evaluated, predicates from the LP can be temporarily imported to the DL.
  - this way, the DL-reasoner takes information into account that has been derived by rules in the LP.



Syntax

- A *DL-atom* $a(\mathbf{t})$ has the form

$$\mathrm{DL}[S_1\ op_1\ p_1, \ldots, S_m\ op_m\ p_m; Q](\mathbf{t}), \text{ where}$$

  - $m \geq 0$,
  - each S_i is either a DL concept or a role predicate
  - $op_i \in \{\uplus, \cup, \cap\}$,
  - p_i is a unary, resp. binary, LP predicate symbol, and
  - $Q(\mathbf{t})$ is a DL-query
- We call $\gamma = S_1\ op_1\ p_1, \ldots, S_m\ op_m\ p_m$ the *input signature*.
- Intuitively,
  - $op_i = \uplus$ increases S_i by the extension of p_i,
  - $op_i = \cup$ increases $\neg S_i$ by the extension of p_i, and
  - $op_i = \cap$ constrains S_i to p_i.

Syntax (ctd.)

- A *DL-rule* r is a normal rule of form:

  $$a \leftarrow b_1, \ldots, b_k, \mathit{not}\ b_{k+1}, \ldots, \mathit{not}\ b_m,\ m \geq k \geq 0, \text{ where}$$

  - a is a strong literal, and
  - $b_1, \ldots, b_m$ are strong literals or DL-atoms.
  - "*not*" denotes *default negation* (negation as failure).
  - $\mathrm{H}(r) = a$, $\mathrm{B}^+(r) = \{b_1, \ldots, b_k\}$, $\mathrm{B}^-(r) = \{b_{k+1}, \ldots, b_m\}$, $\mathrm{B}(r) = \{b_1, \ldots, b_k, \mathit{not}\ b_{k+1}, \ldots, \mathit{not}\ b_m\}$

- A *DL-program* $KB = (T, P)$ consists of a DL ontology T and a finite set P of DL-rules.
- The signatures of T and P have the same set F of (0-ary) function symbols.
- The grounding $gr(P)$ of P is the set of all ground rules that can be obtained by replacing variables of rules in P by constants occurring in F.

Semantics

- Let $KB = (T, P)$ be a DL-program.
- An *interpretation* for KB is a consistent set of strong literals from $gr(P)$.
- A ground strong literal a is *true* under interpretation I ($I \models^T a$) iff $a \in I$.

Semantics (ctd.)

- A ground DL-atom $a = DL[S_1 op_1\ p_1, \ldots, S_m op_m p_m; Q](\mathbf{c})$ is *true* under DL T ($I \models^T a$) iff
  - $T \cup \tau_I(a) \models Q(\mathbf{c})$, where
  - the *extension* of a under I is $\tau_I(a) = \bigcup_{i=1}^{m} A_i(I)$ such that
    - $A_i(I) = \{S_i(\mathbf{e}) \mid p_i(\mathbf{e}) \in I\}$, for $op_i = \uplus$;
    - $A_i(I) = \{\neg S_i(\mathbf{e}) \mid p_i(\mathbf{e}) \in I\}$, for $op_i = \cup$;
    - $A_i(I) = \{\neg S_i(\mathbf{e}) \mid p_i(\mathbf{e}) \notin I\}$, for $op_i = \cap$.
- $I \models^T \mathrm{B}^+(r)$ iff $I \models^T a$ for all $a \in \mathrm{B}^+(r)$.
- $I \models^T \mathrm{B}^-(r)$ iff $I \not\models^T a$ for all $a \in \mathrm{B}^-(r)$.
- $I \models^T \mathrm{B}(r)$ iff $I \models^T \mathrm{B}^+(r)$ and $I \models^T \mathrm{B}^-(r)$.
- $I \models^T r$ iff $I \models^T \mathrm{H}(r)$ whenever $I \models^T \mathrm{B}(r)$.
- I is a *model* of KB ($I \models KB$) iff $I \models^T r$ for all $r \in gr(P)$.

Semantics (ctd.)

- Answer sets defined analogously to answer-set programs using the FLP-reduct
  - ☞ remember: introduced for an intuitive handling of nonmonotone aggregates in ASP (Faber, Leone, Pfeifer 2004).
- The *FLP-reduct* KB^I_{FLP} of DL-program $KB = (T, P)$ relative to I is the set of rules $r \in gr(P)$ such that $I \models^T \mathrm{B}(r)$.

- Definition

An interpretation I is an *answer set* of KB if it is a minimal model of KB^I_{FLP}.

- The set of all answer sets of KB is denoted by $AS(KB)$.

Originally, two different semantics have been defined (Eiter, Lukasiewicz,Schindlauer, and Tompits 2004):

- Strong Answer-Set semantics
- Weak Answer-Set semantics

Example

- A computer store obtains hardware from several vendors.
- DL knowledge base T_{ex} contains information about the product range provided by each vendor.
- For some parts, a shop may already be contracted as supplier.
- T_{ex} T-Box

$$\geq 1\ \mathit{provides} \sqsubseteq \mathit{shop};\quad \top \sqsubseteq \forall \mathit{provides}.\mathit{part};$$

- T_{ex} A-Box

$$\mathit{provides}(s_1, \mathit{cpu});\quad \mathit{provides}(s_1, \mathit{case});\quad \mathit{provides}(s_2, \mathit{cpu});$$
$$\mathit{provides}(s_3, \mathit{harddisk});\quad \mathit{provides}(s_3, \mathit{case});$$
$$\mathit{supplies}(s_3, \mathit{case});$$

Example (ctd.)

- DL-program $KB_{ex} = (T_{ex}, P_{ex})$ not-deterministically chooses a vendor for each needed part.
- LP part P_{ex}:

$$
\begin{aligned}
needed(X) \leftarrow & \mathrm{DL}[; part](X);\\
alreadyContracted(P) \leftarrow & \mathrm{DL}[; supplies](S,P), needed(P);\\
offer(S,P) \leftarrow & \mathrm{DL}[; provides](S,P), needed(P),\\
& \quad not\ alreadyContracted(P);\\
chosen(S,P) \leftarrow & offer(S,P), not\ notChosen(S,P);\\
notChosen(S,P) \leftarrow & offer(S,P), not\ chosen(S,P);\\
supplier(S,P) \leftarrow & \mathrm{DL}[supplies \uplus chosen; supplies](S,P), needed(P);\\
anySupplied(P) \leftarrow & supplier(S,P);\\
\leftarrow & not\ needed(P), not\ anySupplied(P).
\end{aligned}
$$

Example (ctd.)

- KB_{ex} has two answer sets, I_1 and I_2
- both containing the same atoms of predicates *needed*, *offer*, *alreadyContracted*, and *anySupplied*:

$$I' = \{needed(cpu), needed(harddisk), needed(case), alreadyContracted(case), offer(s_1, cpu), offer(s_2, cpu), offer(s_3, harddisk), anySupplied(cpu), anySupplied(harddisk), anySupplied(case)\}$$

- The remaining atoms of I_1 are given by

$$I_1 \setminus I' = \{chosen(s_1, cpu), chosen(s_3, harddisk), notChosen(s_2, cpu), supplier(s_1, cpu), supplier(s_3, harddisk), supplier(s_3, case)\}$$

- The remaining atoms of I_2 are given by

$$I_2 \setminus I' = \{chosen(s_2, cpu), chosen(s_3, harddisk), notChosen(s_1, cpu), supplier(s_2, cpu), supplier(s_3, harddisk), supplier(s_3, case)\}$$

Complexity of DL-programs

Theorem

Given and a dl-program $KB = (T, P)$ with P in SHIF($\mathbf{D}$), deciding whether KB has an answer set is complete for NEXP *in the general case, and complete for* EXP *when KB is positive or stratified.*

Tractable DL-programs

- Tractable Reasoning with DL-programs (Heymans, Eiter, and Xiao 2010)
  - Datalog-rewritable Description Logics
  - DL-program is translated to a "pure" logic program
  - Approximation of answer-set semantics: well-founded semantics
  - new DL defined: LDL^+
  - OWL2 fragments:
    - OWL 2 RL: fully datalog rewritable (strict subset of LDL^+)
    - OWL 2 EL: partly d. r. (neg. and existential qu. in axiom r.h.s.)
    - OWL 2 QL: partly d. r. (neg. and existential qu. in axiom r.h.s.)

Conclusion

- Answer-set programming is a formalism for declarative problem solving
- Solvers become more and more efficient and expressive.
- Combinations of logical rules and ontologies: e.g., DL-programs
- An approach for debugging ASP programs has been presented.
- Future work:
  - Debugging of DL-programs and HEX-programs.