Tarea 2 **February 8, 2022**
Métodos computacionales 2 Presentado por:
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1 1.1 Series de Fourier

Dado

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(nw_0 t) + b_n \sin(nw_0 t)] \tag{1}$$

Se puede diferenciar término por término para obtener.

$$f'(t) = \sum_{n=1}^{\infty} nw_0[-a_n \sin(nw_0 t) + b_n \cos(nw_0 t)] \tag{2}$$

De la misma manera, dado que es continuo en todo el dominio se puede integrar término por termino, donde:

$$\int_{t_1}^{t_2} \frac{a_0}{2} dt = \frac{1}{2} a_0 (t_2 - t_1)$$

$$\int_{t_1}^{t_2} a_n \cos(nw_0 t) dt = \frac{a_n}{nw_0} \sin(nw_0 t_2) - \frac{a_n}{nw_0} \sin(nw_0 t_1)$$

$$\int_{t_1}^{t_2} b_n \sin(nw_0 t) dt = \frac{-b_n}{nw_0} \cos(nw_0 t_2) + \frac{b_n}{nw_0} \cos(nw_0 t_1)$$

De esta manera:

$$\int_{t_1}^{t_2} f(t) dt = \frac{1}{2} a_0 (t_2 - t_1) + \sum_{n=1}^{\infty} \frac{1}{nw_0} \left[a_n(\sin(nw_0 t_2) - \sin(nw_0 t_1)) - b_n(\cos(nw_0 t_2) - \frac{b_n}{nw_0} \cos(nw_0 t_1) \right] \tag{3}$$

2 1.2 Presentación de funciones

Sea $f(t) = t$ En el intervalo $(-\pi, \pi)$ y con un periodo de 2π

$$a_0 = \frac{2}{T} \int_{T/2}^{-T/2} f(t) dt = \frac{1}{\pi} \int_{-\pi}^{\pi} t \; dt = \frac{1}{\pi} \left[\frac{t^2}{2} \right]_{-\pi}^{\pi} = 0$$

$$a_n = \frac{2}{T} \int_{T/2}^{-T/2} f(t) \cos\left(\frac{2n\pi}{T} t \right) dt = \frac{1}{\pi} \int_{T/2}^{-T/2} t \cos(nt) \, dt$$

Usando integración por partes se obtiene

$$a_n = \frac{1}{\pi} \left(\frac{1}{n^2} \left[nt \sin(nt) + \cos(nt) \right] \right)_{-\pi}^{\pi} = 0$$

$$b_n = \frac{2}{T}\int_{T/2}^{-T/2} f(t)\sin\left(\frac{2n\pi}{T}t\right)dt = frac1\pi \int_{T/2}^{-T/2} t\sin{(nt)}\,dt$$

Usando integración por partes se obtiene

$$b_n = \frac{1}{\pi}\left(\frac{1}{n^2}\sin(nt) - \frac{1}{n}t\cos(nt)\right)_{-\pi}^{\pi} = \frac{2(-1)^{n-1}}{n}$$

Obteniendo así

$$f(t) = 2\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\sin(nt) \tag{4}$$

3 1.3 Función ζ de Rienmann

3.1 1

Integrar analíticamente la serie de fourier de $f(t) = t^2$ en el intervalo $-\pi \leq t \leq \pi$ y $f(t+2\pi) = f(t)$.

$$a_0 = \frac{2}{T}\int_{T/2}^{-T/2} f(t)dt = \frac{1}{\pi}\int_{-\pi}^{\pi} t^2\ dt = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{T}\int_{T/2}^{-T/2} f(t)\cos\left(\frac{2n\pi}{T}t\right)dt = \frac{1}{\pi}\int_{T/2}^{-T/2} t^2\cos{(nt)}\,dt = \frac{4\pi(-1)^n}{n^2}$$

$$b_n = \frac{2}{T}\int_{T/2}^{-T/2} f(t)\cos\left(\frac{2n\pi}{T}t\right)dt = frac1\pi \int_{T/2}^{-T/2} t^2\sin{(nt)}\,dt = 0$$

De esta manera:

$$f(t) = t^2 = \frac{1}{2\pi}\frac{2\pi^3}{3} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{4\pi(-1)^n}{n^2}\cos(nt) \tag{5}$$

Integrando la seria de Fourier se obtiene qué

$$\int_{-\pi}^{\pi} f(t)dt = \frac{2\pi^3}{3} = \frac{2\pi^3}{3} + 2\sum_{n=1}^{\infty}\frac{4\pi(-1)^n}{n^2}\frac{\sin(n\pi)}{n} = \frac{2\pi^3}{3} \tag{6}$$

Usando la identidad de Parseval se obtiene que:

$$\frac{1}{\pi}\int_{-\pi}^{\pi} t^4 dt = \frac{2}{\pi}\frac{\pi^5}{5} = \frac{1}{2}\left(\frac{2\pi^3}{3}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{4\pi(-1)^n}{n^2}\right)^2$$

$$\frac{2\pi^4}{5} = \frac{2\pi^6}{9} + 16\pi^2\sum_{n=1}^{\infty}\frac{(-1)^{2n}}{n^4}$$

$$\left(\frac{2\pi^4}{5} - \frac{2\pi^6}{9}\right) = 16\pi^2\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{18\pi^4 - 10\pi^6}{45}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{9\pi^2 - 5\pi^3}{360} \tag{7}$$