A Derivation of the Formula The following lemmas and theorem derive the formula in the leaky noisy OR-gate model.

Lemma 5.1 *Given the assumptions and notation shown above for the leaky noisy OR-gate model,*

$$p_0 = p'_H \times P(H = yes).$$

Proof. *Owing to Equality 5.3, we have*

Law of Total Probability

$$\begin{aligned} p_0 &= P(Y = yes|X_1 = no, X_2 = no, \ldots X_n = no) \\ &= P(Y = yes|X_1 = no, X_2 = no, \ldots X_n = no, H = yes)P(H = yes) + \\ &\quad P(Y = yes|X_1 = no, X_2 = no, \ldots X_n = no, H = no)P(H = no) \\ &= p'_H \times P(H = yes) + 0 \times P(H = no). \end{aligned}$$

This completes the proof. ■

Lemma 5.2 *Given the assumptions and notation shown above for the leaky noisy OR-gate model,*

$$1 - p'_i = \frac{1 - p_i}{1 - p_0}.$$

Proof. *Owing to* definition of p_i we have

$1 - p_i$

Law of Total Probability

Lemma 5.1

$$\begin{aligned} &= P(Y = no|X_1 = no, \ldots X_i = yes, \ldots X_n = no) \\ &= P(Y = no|X_1 = no, \ldots X_i = yes, \ldots X_n = no, H = yes)P(H = yes) + \\ &\quad P(Y = no|X_1 = no, \ldots X_i = yes, \ldots X_n = no, H = no)P(H = no) \\ &= (1 - p'_i)(1 - p'_H)P(H = yes) + (1 - p'_i)P(H = no) \\ &= (1 - p'_i)(1 - p'_H \times P(H=yes)) \\ &= (1 - p'_i)(1 - p_0). \end{aligned}$$

The last equality is due to Lemma 5.1. This completes the proof. ■

Theorem 5.2 *Given the assumptions and notation shown above for the leaky noisy OR-gate model,*

$$P(Y = no|\mathsf{X}) = (1 - p_0) \prod_{i \text{ such that } X_i \in \mathsf{X}} \frac{1 - p_i}{1 - p_0}.$$

Proof. *We have*

Law of Total Probability.

$$
\begin{aligned}
P(Y = no|\mathsf{X}) &= P(Y = no|\mathsf{X}, H = yes)P(H = yes) + \\
&\quad P(Y = no|\mathsf{X}, H = no) \cdot P(H = no) \\
&= P(H = yes)(1 - p'_H) \prod_{i \text{ such that } X_i \in \mathsf{X}} (1 - p'_i) + \\
&\quad P(H = no) \prod_{i \text{ such that } X_i \in \mathsf{X}} (1 - p'_i) \\
&= (1 - p_0) \prod_{i \text{ such that } X_i \in \mathsf{X}} (1 - p'_i) \\
&= (1 - p_0) \prod_{i \text{ such that } X_i \in \mathsf{X}} \frac{1 - p_i}{1 - p_0}.
\end{aligned}
$$

Lemma 5.1.

Lemma 5.2.

The second to the last equality is due to Lemma 5.1, and the last is due to Lemma 5.2. ■