Modelling for Science and Engineering
Data Visualitzation and Modelling

BAYESIAN NETWORKS

Excercises Block 1

1. Suppose the FCB plays the match of the final of the “Copa del Rey” against R. Madrid and I assess the probability that FCB win to be 0.6. I also feel there is a 0.9 probability there will be a big crowd celebrating at my favorite “bar” that night if they do win. However, even if they lose, I feel there might be a big crowd because a lot of people may show up to lick their wounds. So, I assign a probability of 0.3 to a big crowd if they lose.

   Suppose that on match day I work all day, drive straight to my favorite “bar” without finding out the result, and see a big crowd overflowing into the parking lot. Compute my conditional probability FCB did indeed win.

   Is the evidence of the big crowd in favor of or against FCB to win?

2. A forgetful nurse is supposed to give Mr. Smith a pill each day. The probability that she will forget to give the pill on a given day is 0.3. If he receives the pill, the probability he will die is 0.1, but if he does not receive the pill, the probability he will die is 0.8. Mr. Smith died today.

   Compute the probability the nurse forgot to give him the pill.

   Is the evidence that Mr. Smith died in favor or against the nurse had a lethal neglect?

3. Let $V = \{X, Y, Z\}$, all the three variables taking values T=true and F=false. Assume that

$$
\begin{aligned}
&P(X = T) = 0.2 \\
&P(Y = T \,/\, X = T) = 0.3, \quad P(Y = T \,/\, X = F) = 0.4 \\
&P(Z = T \,/\, X = T) = 0.1, \quad P(Z = T \,/\, X = F) = 0.5
\end{aligned}
$$

   (a) Define a joint probability distribution P of X, Y and Z as the product of these values.

   (b) Show that the values in this joint probability distribution sum 1.

   (c) Show further that $I_P(Z, Y/X)$.

   (d) Create a DAG Γ such that (Γ, P), with P given by (a), be a Bayesian Network (which result ensures that indeed it is?).

4. Consider Example 1 of theory notes, and the DAG $\Gamma = (V, E)$ with $V = \{L, S, C\}$ and $E = \{(L, C), (C, S)\}$.

   (a) Show that (Γ, P) satisfies the Markov condition, with P being the probability that assigns $1/13$ to each of the 13 elements of Ω.

(b) Therefore, based on Theorem 1, that probability distribution must be equal to the product of the corresponding conditional distributions. Show this fact by direct computation.

5. In Example 9 of theory notes, prove that although variables A and B are independent (why? with which probability P?), if we instantiate variable C, then they become dependent.

6. (**Exercise to deliver individually**) Dr. Ann Nicholson spends 60 % of her work time in her office. The rest of her work time is spent elsewhere. When Ann is in her office, half the time her light is off (when she is trying to hide from students and get research done). When she is not in her office, she leaves her light on only 5 % of the time. 80 % of the time she is in her office, Ann is logged onto the computer. Because she sometimes logs onto the computer from home, 10 % of the time she is not in her office, she is still logged onto the computer.

   (a) Construct a Bayesian network to represent the scenario just described.

   (b) Suppose a student checks Dr. Nicholson's login status and sees that she is logged on. What effect does this have on the student's belief that Dr. Nicholson's light is on?

   (c) Is the evidence that Dr. Nicholson's is logged on in favor of or against her light is on?