
Smallest font
Welcome
Calibration slide
Stand by
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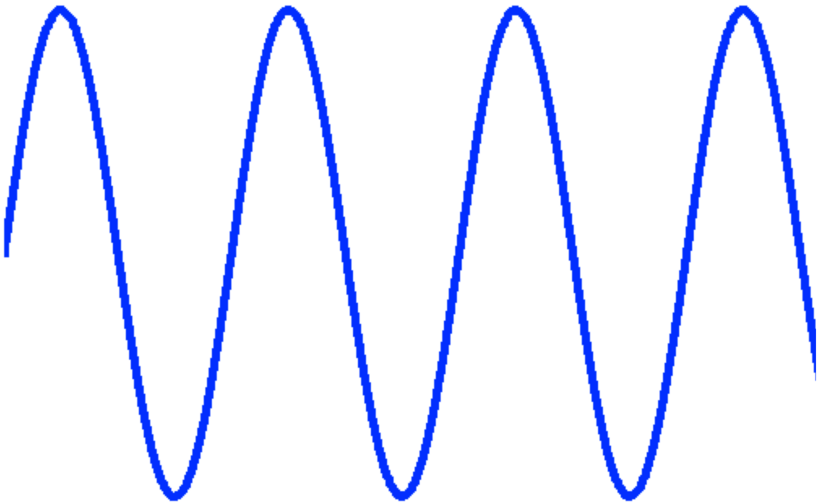
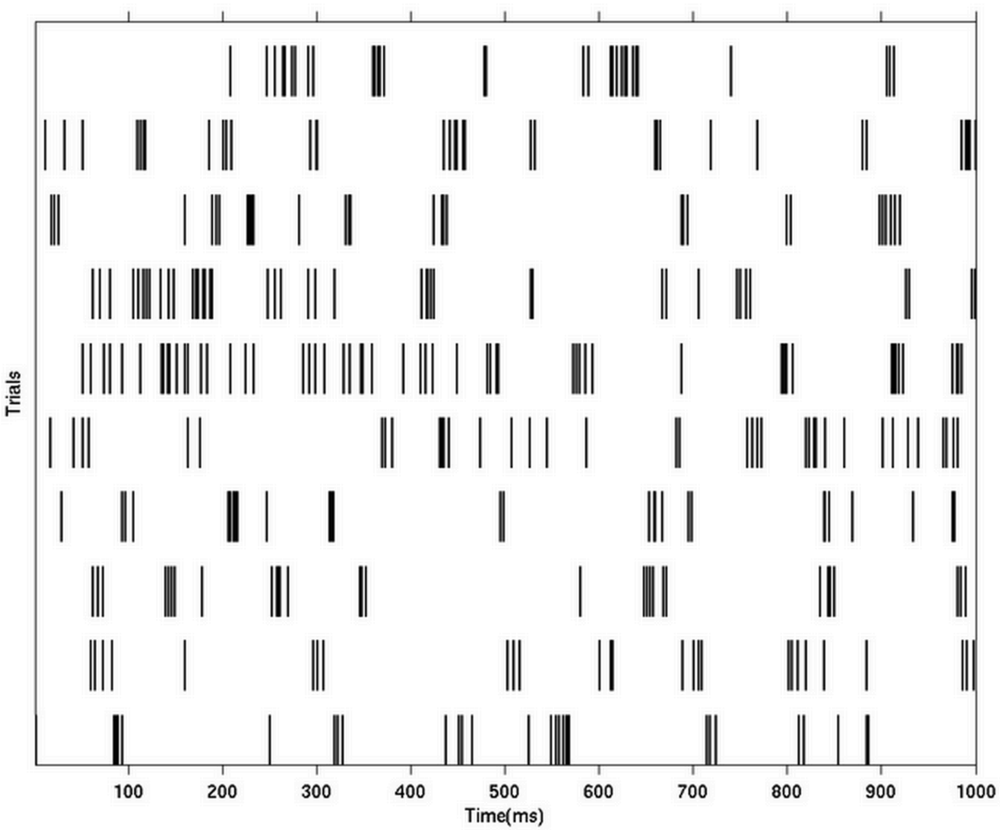
Scientific Programming and Computing for the Behavioral Sciences

The Fourier transform

Genuine frequency analysis

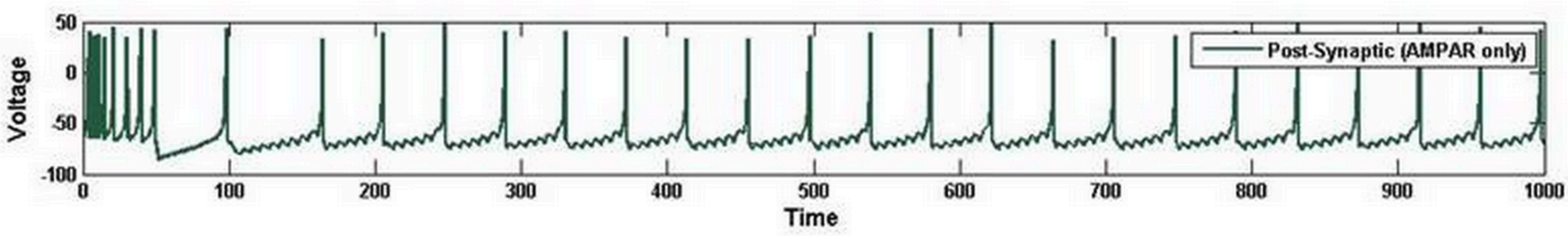
- Deals with *periodic* signals.
- Unlike spike trains (not in Hz).
- Spiking is a point process, not a periodic signal.





Frequency space

- Basic idea: The signals we measure are usually sampled in the time domain.



- Periodic signals and their properties are more aptly described in frequency space.
- The Fourier Transform allows to convert time based signals into a frequency space representation.
- All signals, no matter how aperiodic looking they might be can be expressed as a series of aptly summed frequency components.

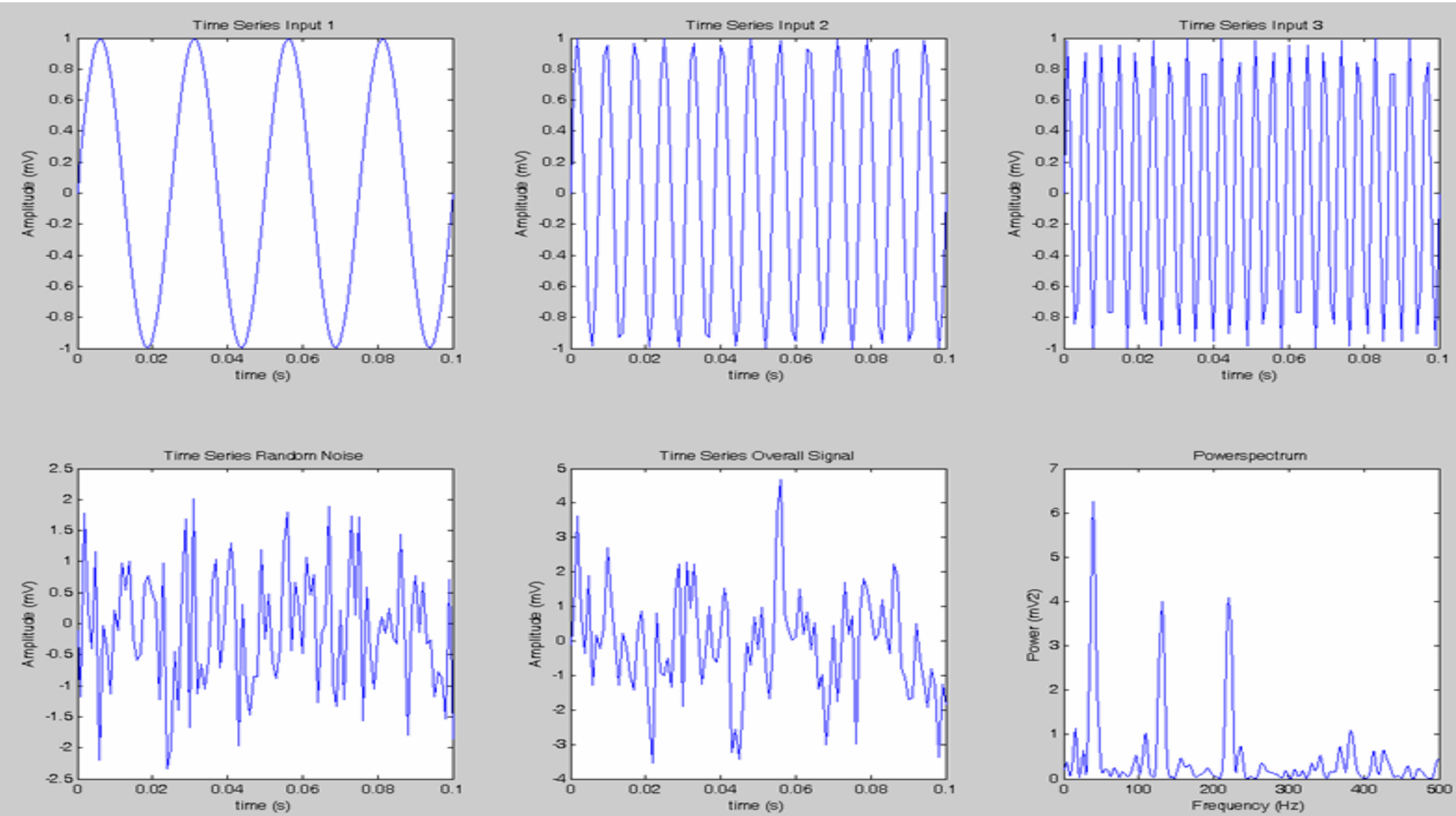
Time vs. frequency domains

- **Time domain**: When is a signal happening (and what is it)?
- **Frequency domain**: How often is a signal repeating (and how strong is it)?

How to transform signals from the time domain to the frequency domain?

- Figured out by Joseph Fourier.
- Studying periodic signals.
- Not to be confused with Charles Fourier.

Different representations emphasize different properties



The Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-i 2\pi f t} \, dt$$

Scary!

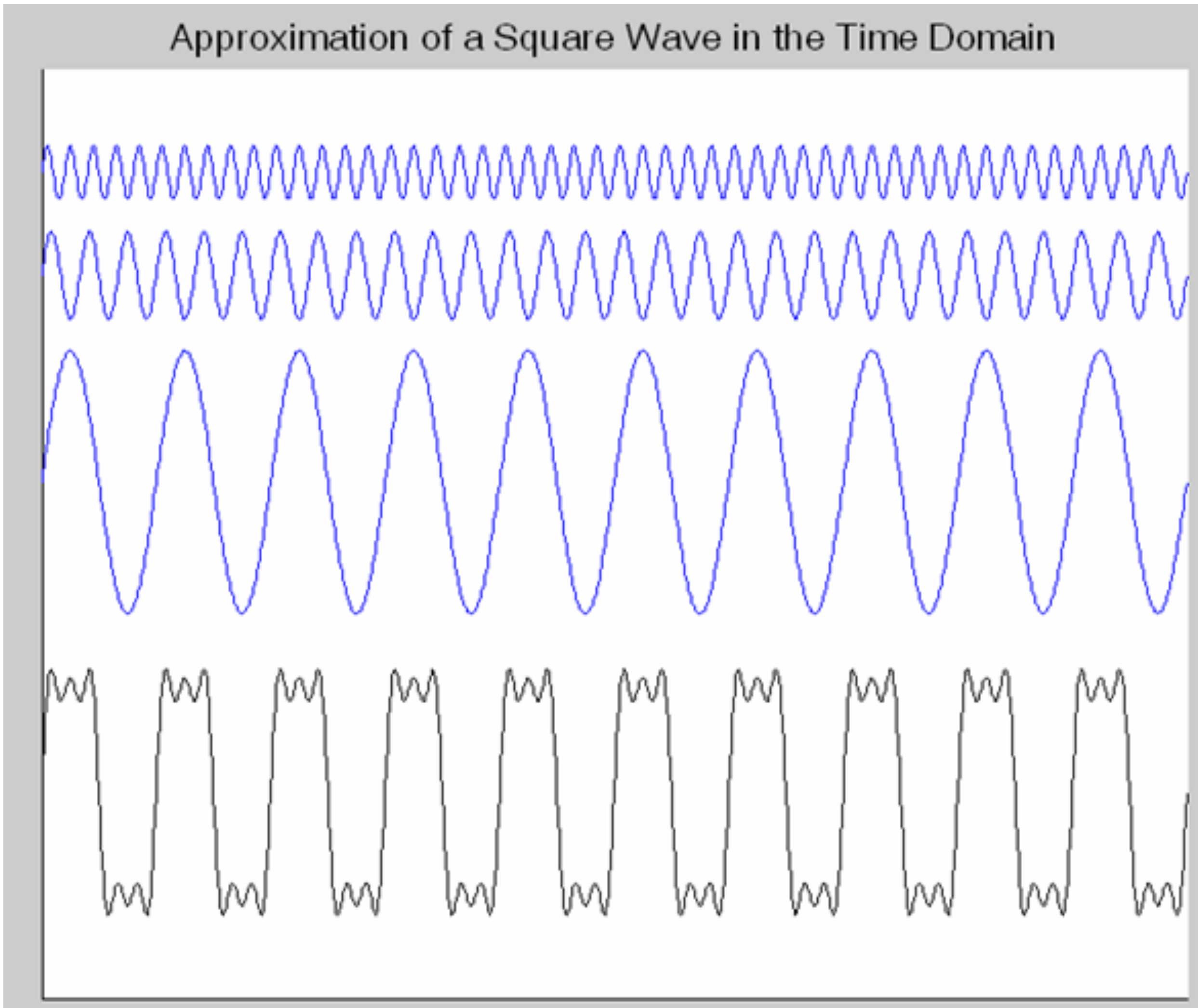
Easy to screw up if one doesn't understand it well

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-i 2\pi f t}\, dt$$

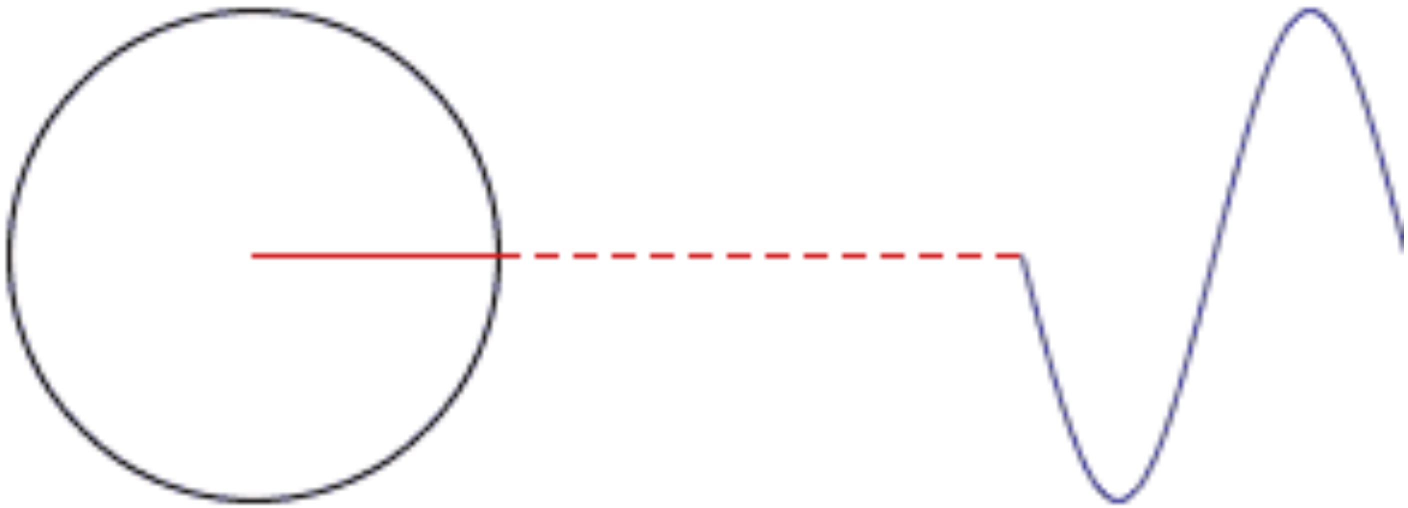
$$\omega = 2\pi f$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t}\, dt$$

A relatively simple idea

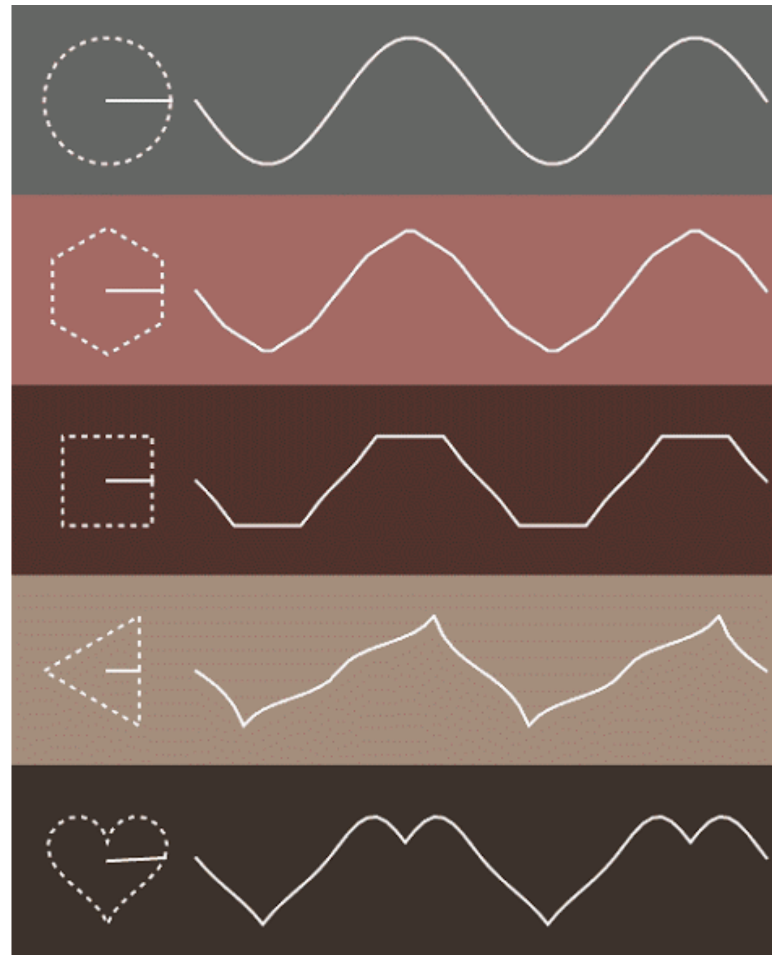


Making it visceral

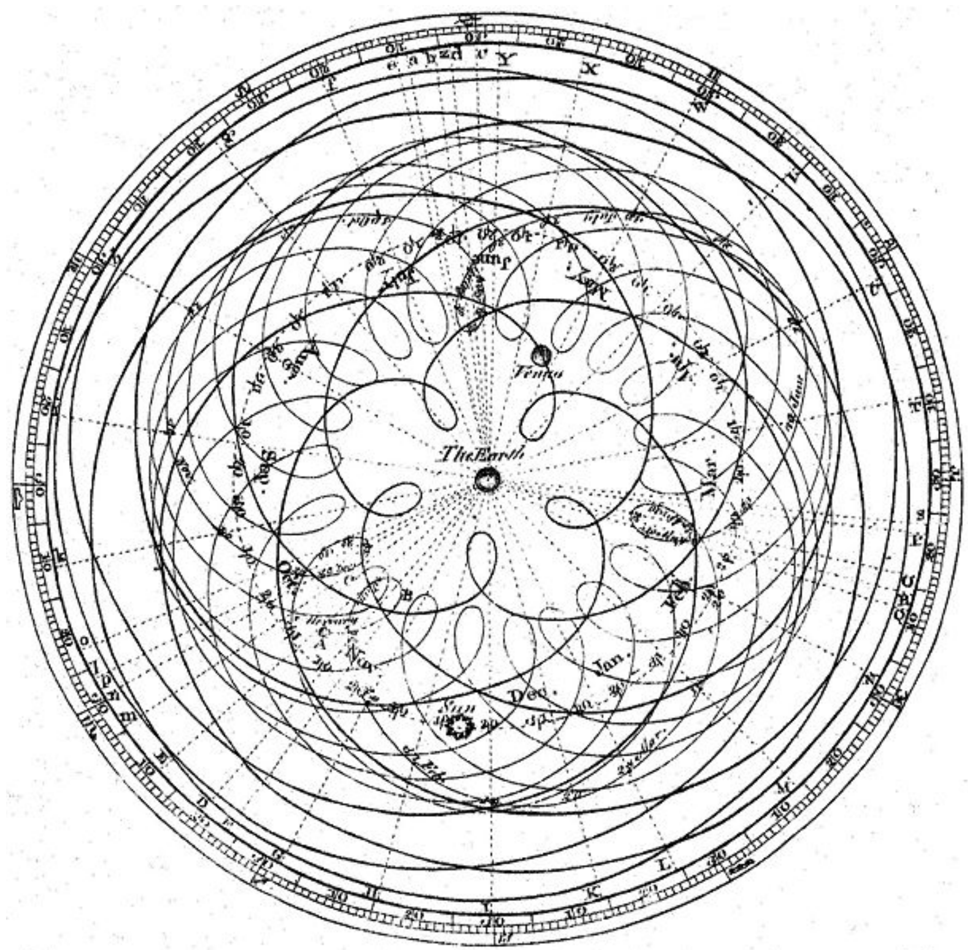


What is special about sine waves?

- You could use other functions to go around periodically in a “circle”.
- But adding sine waves is simpler than most other functions.
- Also, one is tracing out circles, which have important properties.
- Sine waves have other important mathematical properties (“eigenfunctions” of linear systems)



Epicycles



The proper combination of cycles can reconstruct *any* orbit, no matter how convoluted. Also reach any point in the space. So any signal in space/time can be represented as a combination of cycles with properly picked radii and phases.

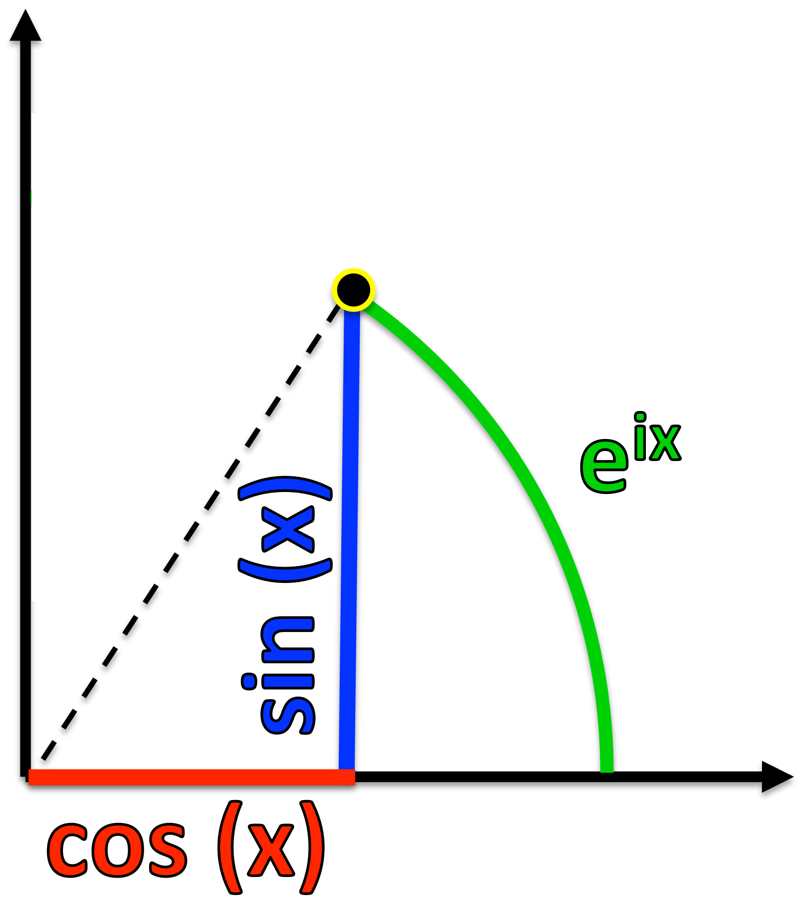
A demonstration

Still scary?

$$\Im\{x(t)\} = X(f) = \int_{-\infty}^{\infty} x(t)e^{-i2\pi ft}\,dt$$

Euler's identity

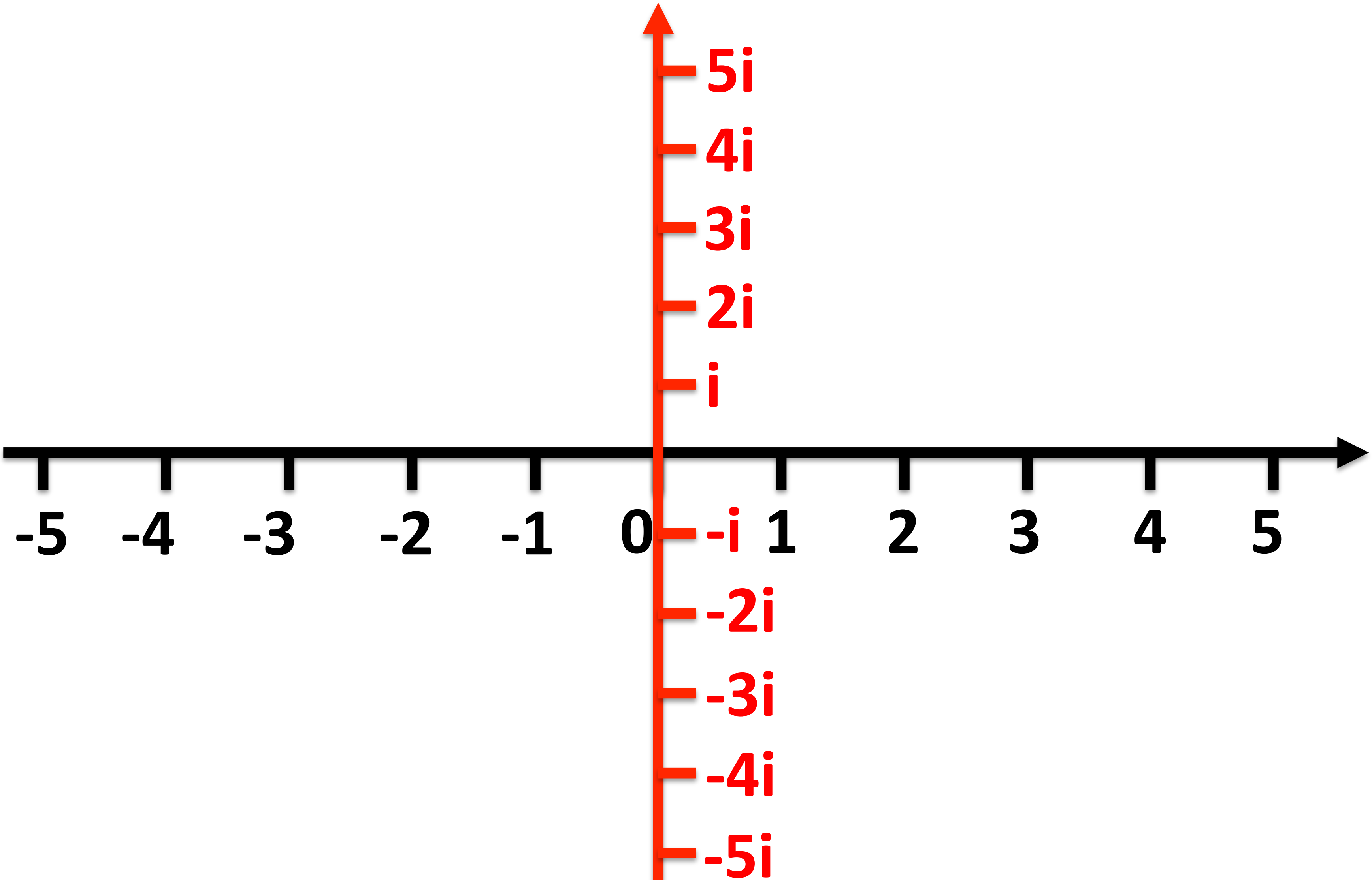
$$e^{ix} = \cos x + i \sin x$$



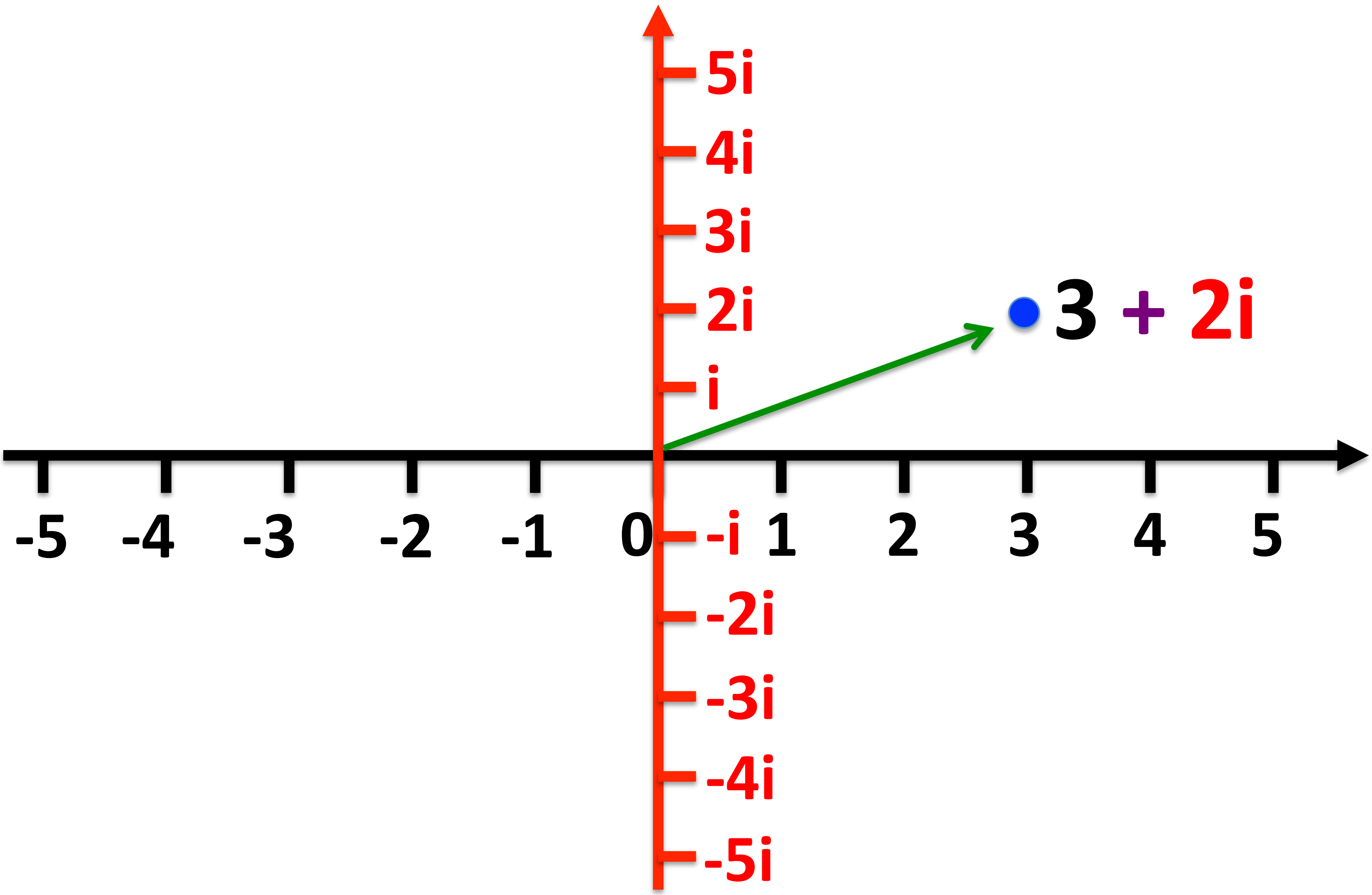
Complex numbers

- Complex numbers have a real part and an imaginary part.
- C = R + I, e.g.
- C = 5 + 2i
- i = sqrt(-1)
- Allows to represent the magnitude and the phase of something in a single number.
- Inherently a vector.

The complex plane



Numbers are inherently vectors



The complex conjugate