The Cost of Accumulating Evidence

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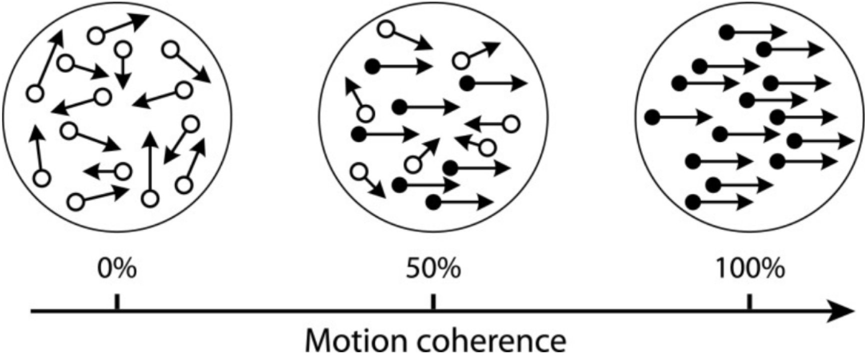
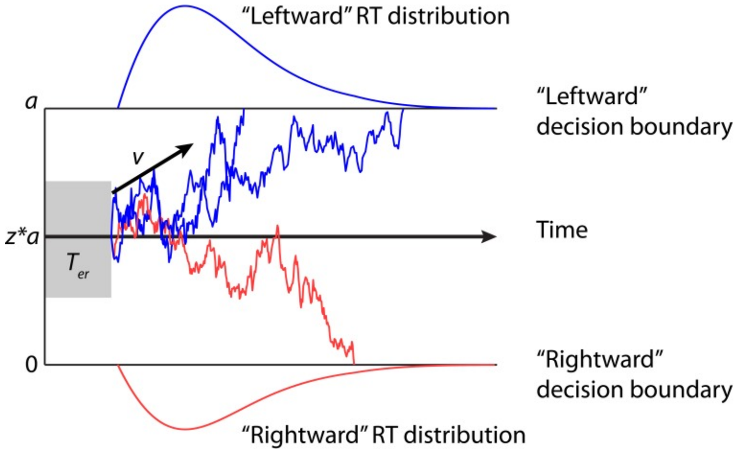
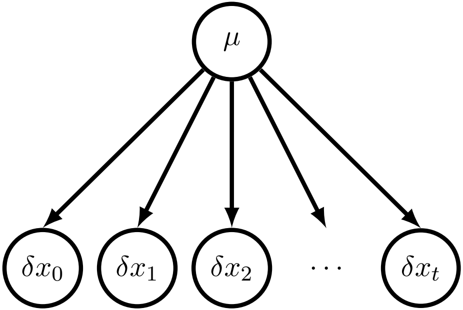
Introduction to the Task



Drift Diffusion Model



Step 1: Generative Model



$$p(\mu) = \mathcal{N}(\mu; 0, \sigma_\mu^2)$$

$$p(\delta x_{0\ldots t}|\mu) = \mathcal{N}(\delta x_{0\ldots t}; \mu\delta t, \delta t)$$

Inference of Interests

- sign(μ)
  - T_t: trial time
  - t_i: intertrial time
  - t_p: time penalty for incorrect answers
  - R_{ij}: reward for choosing hypothesis i when true was j.
- $c(t)$: Momentary cost function: $C(T_d) = \int_0^{T_d} c(t)dt$

$$\rho = \frac{< R > - < C(T_d) >}{< T_t > + < t_i > + < t_p >}$$

- $g(t)$: Momentary belief function: $p(H_1|\delta x_{0...t}) = p(\mu \geq 0|\delta x_{0...t})$

Step 2: Inference and Decision Rule

- Posterior

$$\boxed{p(\mu|\delta x_{0\ldots t})} \propto \mathcal{N}(\mu; 0, \sigma_\mu^2) \prod_n \boxed{\mathcal{N}(\delta x_n; \mu\delta t, \delta t)}$$

- Prior
- Likelihood

Step 2: Inference and Decision Rule

- Posterior

$$\boxed{p(\mu|\delta x_{0\ldots t})} \propto \mathcal{N}(\mu; 0, \sigma_\mu^2) \prod_n \boxed{\mathcal{N}(\delta x_n; \mu\delta t, \delta t)}$$

- Prior
- Likelihood

$$p(\mu|\delta x_{0\ldots t}) = \mathcal{N}\left(\mu; \frac{x(t)}{t+\sigma_\mu^{-2}}, \frac{1}{t+\sigma_\mu^{-2}}\right)$$

where, $x(t)=\sum_n \delta x_n$, and $t=\sum_n \delta t$

Decision Rule

- The decision rule is more complex that normal because we must determine not only what choice the optimal decider would choose but also when they would choose that option.

$$\tilde{V}(g,t) = max \begin{Bmatrix} gR_{11} + (1-g)R_{12} - (< t_i > +(1-g)\bar{t_p})\rho, \\ (1-g)R_{22} + gR_{21} - (< t_i > +g\bar{t_p})\rho, \\ < \tilde{V}(g(t+\delta t), t+\delta t)|g,t >_{g(t+\delta t)} -c(t)\delta t - \rho\delta t \end{Bmatrix}$$

* *We will remove the extra degrees of freedom and identify the optimal ρ by setting $\tilde{V}(\frac{1}{2}, 0) = 0$.*

Model Predictions: Accuracy

- This model makes the prediction that participants accuracy will be derived by there belief function at the time of the decision. Thus, if we use the true drift rate, we will be able to obtain optimal accuracy for a given stimulus strength.
- If have symmetric time varying bounds $\theta(t)$, we can solve for optimal accuracy with the following.

$$\theta(t) = \sqrt{t + \sigma_\mu^{-2}}\Phi^{-1}(g_\theta(t))$$

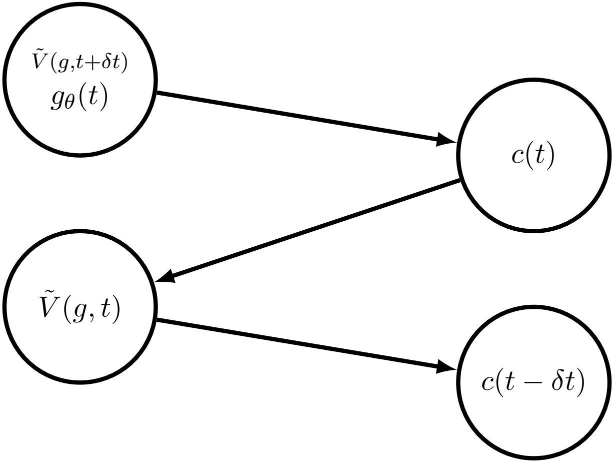
$$p(x = \theta(t) | x = \pm\theta(t), t, \mu = \mu_0) = g(t, \mu_0) = \frac{1}{1 + e^{2\theta(t)\mu_0}}$$

Cost Function

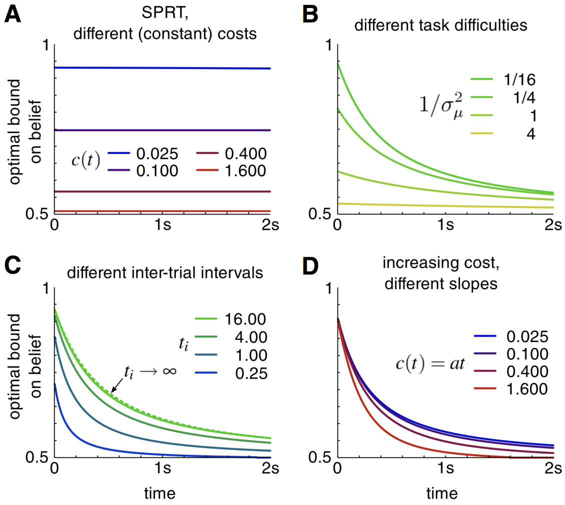
- One quintessential addition to this model above the DDM, is that we have a time varying momentary cost function $c(t)$. This function is unique to the individual and can be solved with the following formula.

$$c(t) = \frac{1}{\delta t}\begin{pmatrix} < \tilde{V}(g(t+\delta t), t+\delta t)|g,t >_{g(t+\delta t)} - \\ gR_{11} - (1-g)R_{12} - \\ (< t_i > + (1-g_\theta(t)) < t_p > + \delta t)\rho \end{pmatrix}$$

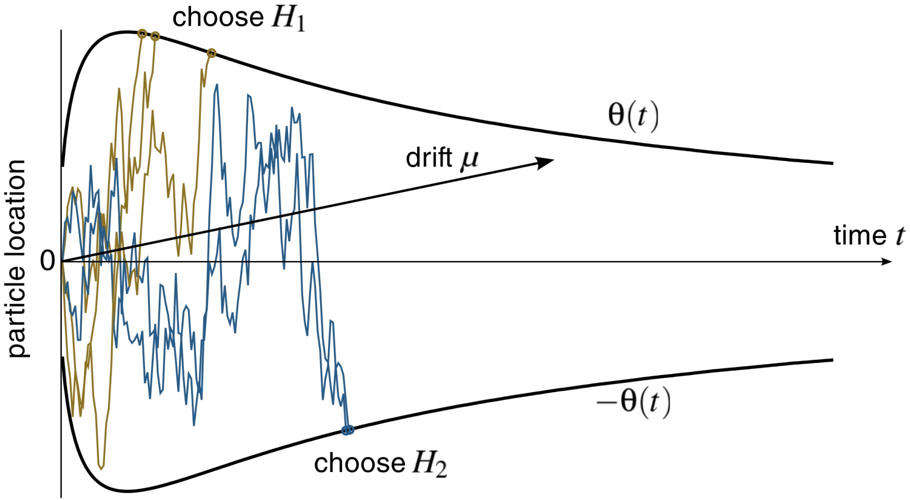
Cascading Solutions



Changing Parameters



Dynamic decision boundaries



Model Fit

Table 1. Fit quality per subject

	Trials	Coefficient of determination, R^2			AIC	AIConst	KS
		Chron	Psych	Avg			
Palmer et al. (2005)							
AH	554	0.948	0.974	0.961	−167.85 (±4.16)	33.90	1 ($p < 0.01$)
EH	560	0.952	0.944	0.948	245.11 (±3.71)	347.41	1 ($p < 0.05$)
JD	567	0.897	0.974	0.936	−558.39 (±57.68)	516.36	3 ($p < 0.01$)
JP	555	0.976	0.998	0.987	−330.01 (±5.28)	2754.84	2 ($p < 0.05$)
MK	573	0.973	0.993	0.983	−504.24 (±4.60)	2041.61	2 ($p < 0.01$)
MM	564	0.928	0.978	0.953	−262.25 (±3.65)	107.83	0
Avg	562.2 (±3.3)	0.946 (±0.013)	0.977 (±0.008)	0.961 (±0.009)			
Roitman and Shadlen (2002)							
B	2615	0.985	0.989	0.987	−809.16 (±18.99)	26,436.32	0
N	3534	0.983	0.991	0.987	620.34 (±33.69)	48,005.64	3 ($p < 0.05$)

The table shows, for each subject, the number of trials that were fitted; the coefficient of determination (R^2) for the chronometric function (Chron), the psychometric function (Psych), and averaged over both (Avg); the goodness of fit of our model according to the AIC (smaller is better), and the comparison goodness of fit of a diffusion model with constant bound (AIConst); the number of conditions (out of 12) for which the Kolmogorov–Smirnov test revealed a statistical significance between the reaction time distribution featured by the subject and that predicted by the model, together with the level of significance (KS). For the human dataset, we also provide the mean across subjects (±1 SEM) for the number of trials and the coefficient of determination. The AIC measure for our model is computed separately for 500 posterior samples, and here we provide mean ± 1 SD.

QUESTIONS?

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