1. If you change the initial guesses the Solution does Change very very slightly almost negligibly.

Varying through a wide range of initial guesses though the algorithm still tended to converge around the same single value 4.093.

This is expected as numerical Solutions were used to approximate the minimum so the values will be ever so slighly different with each different Initial guess.

2. $b \in \mathbb{R}^n \quad A \in \mathbb{R}^{n \times n}$

$$f(x) = b^T x + x^T A x$$

a.) gradient and Hessian

$$\boxed{\nabla_x f(x) = b + (A + A^T) x \in \mathbb{R}^n}$$

$$\boxed{H = 0 + (A + A^T) \quad \in \mathbb{R}^{n \times n}}$$

b) 1st order Taylor at $x_0 = 0$

$$f(x) = f(x_0) + \nabla f(x_0)^T \cdot (x - x_0)$$

$$f(x) = 0 + [b + \cancel{(A + A^T)(0)}][x - x_0]$$

$$f(x) = b^T x$$

2nd order

$$f(x) = f(x_0) + \nabla f(x_0)^T (x - x_0) + \frac{1}{2}(x - x_0)^T H(x_0)(x - x_0)$$

$$f(x) = 0 + b^T x + \frac{1}{2} x^T [A + A^T] x$$

$$\boxed{f(x) = b^T x + \frac{1}{2} x^T (A + A^T) x}$$

* This approximation is exact since the function is 2nd order.

3. Let $A \in R^{n \times n}$ be a saure matrix

a.) In order for A to be positive definite all its eigen values must be positive.
neccassory and sufficient

b.) For A to have full rank
It is neceassary that its rows/columns are linearly independent.
It is sufficient that its determinant is non zero.

c.) For the eqation $Ax = b$ to have a solution.

b must be ortogonal to y
and y must be non zero

4.) n = food types m = nutrition types

a_{ij} → nutrition type j in food type i

c_i → cost of food

b_j → required amount of nutrition per week per type j

minimize the cost $\sum_{i=1}^{n} c_i x_i$

constraints!

nutrition per week $\sum_{i=1}^{n} a_{ij} x_i \geq b_j \quad j = 1 \to m$

and x_i cannot be negative $x_i \geq 0$

food types x_i $n=2$ nutrition types a_{ij} $m=3$

Food type 1 x_1: $c_1 = \$2$ nutrition $a_{11}=3$ $a_{12}=2$ $a_{13}=1$

food type 2 x_2: $c_2 = \$3$ nutrition $a_{21}=1$ $a_{22}=2$ $a_{23}=2$

nutrition requirements ? $b_1=6$ $b_2=8$ $b_3=4$

minimize cost $C = \sum_{i=1}^{n} c_i x_i = 2x_1 + 3x_2$

constraints $3x_1 + x_2 \geq 6$, $2x_1 + 2x_2 \geq 8$

$x_1 + 2x_2 \geq 4$ $x_1 \geq 0$ $x_2 \geq 0$

optimize $x_1 = x_2 = 2$ cost = \$10