

# From Anomalies to Forecasts: Toward a Descriptive Model of Decisions Under Risk, Under Ambiguity, and From Experience

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Experimental studies of choice behavior document distinct, and sometimes contradictory, deviations from maximization. For example, people tend to overweight rare events in 1-shot decisions under risk, and to exhibit the opposite bias when they rely on past experience. The common explanations of these results assume that the contradicting anomalies reflect situation-specific processes that involve the weighting of subjective values and the use of simple heuristics. The current article analyzes 14 choice anomalies that have been described by different models, including the Allais, St. Petersburg, and Ellsberg paradoxes, and the reflection effect. Next, it uses a choice prediction competition methodology to clarify the interaction between the different anomalies. It focuses on decisions under risk (known payoff distributions) and under ambiguity (unknown probabilities), with and without feedback concerning the outcomes of past choices. The results demonstrate that it is not necessary to assume situation-specific processes. The distinct anomalies can be captured by assuming high sensitivity to the expected return and 4 additional tendencies: pessimism, bias toward equal weighting, sensitivity to payoff sign, and an effort to minimize the probability of immediate regret. Importantly, feedback increases sensitivity to probability of regret. Simple abstractions of these assumptions, variants of the model *Best Estimate and Sampling Tools* (BEAST), allow surprisingly accurate ex ante predictions of behavior. Unlike the popular models, BEAST does not assume subjective weighting functions or cognitive shortcuts. Rather, it assumes the use of sampling tools and reliance on small samples, in addition to the estimation of the expected values.

**Keywords:** experience-description gap, out-of-sample predictions, St. Petersburg paradox, prospect theory, random forest

Behavioral decision research is often criticized on the grounds that it highlights interesting choice anomalies, but rarely supports clear forecasts. The main reason for the difficulty in deriving clear predictions is that the classical anomalies are explained with several descriptive models, and in many cases these models sug-

gest contradicting behavioral tendencies. Thus, it is not easy to predict the joint effect of the different tendencies. Nobel laureate Alvin E. Roth (see Erev, Ert, Roth et al., 2010) clarified this critique by asking authors of articles that explain some of the anomalies to add a 1–800 (toll-free) phone number and be ready to answer questions concerning the conditions under which their model applies.

In a seminal article, Kahneman and Tversky (1979) attempted to address this problem by first identifying four of the most important deviations from maximization (defined here as violations of the assumption that people maximize expected return), replicating them in one experimental paradigm, and finally proposing prospect theory, a model that captures the joint effect of all these phenomena and thus allows clear predictions. Specifically, Kahneman and Tversky replicated—and prospect theory addresses—the certainty effect (Allais paradox; Allais, 1953; see row 1 in Table 1), the reflection effect (following Markowitz, 1952; see row 2 in Table 1), overweighting of rare events (following Friedman & Savage, 1948; see row 3 in Table 1), and loss aversion (following Samuelson, 1963; see row 4 in Table 1). Importantly, Kahneman and Tversky's replications, and prospect theory, focused on a very specific choice environment: choice between gambles with “at most two non-zero outcomes” (Kahneman & Tversky, 1979, p. 275).

This article was published Online First March 9, 2017.

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This research was supported by the I-CORE program of the Planning and Budgeting Committee and the Israel Science Foundation (grant 1821/12 for Ido Erev, and grant 1739/14 for Eyal Ert). We thank Alvin E. Roth, Asen Kochov, Eric Schulz, and Michael Sobolev for their useful comments. We also thank members of 25 teams for participation in the competition, and Shier Cohen-Amin, Shani Haviv, Liron Honen, and Yaara Nussinovitch for their help in data collection.

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Table 1  
*Examples of Fourteen Phenomena and Their Replications in the Current Study*

Phenomenon	Classical demonstration		Current replication	
	Problems	%B Choice	Problems	%B Choice
Phenomena observed in studies of decisions without feedback				
1. Certainty effect/Allais paradox (Kahneman & Tversky, 1979, following Allais, 1953)	A: 3,000 with certainty B: 4,000, .8; 0 otherwise	20%	A: 3 with certainty B: 4, .8; 0 otherwise	42%
	A': 3,000, .25; 0 otherwise B': 4,000, .20; 0 otherwise	65%	A': 3, .25; 0 otherwise B': 4, .20; 0 otherwise	61%
2. Reflection effect (Kahneman & Tversky, 1979)	A: 3,000 with certainty B: 4,000, .8; 0 otherwise	20%	A: 3 with certainty B: 4, .8; 0 otherwise	42%
	A': -3,000 with certainty B': -4,000, .8; 0 otherwise	92%	A': -3 with certainty B': -4, .8; 0 otherwise	49%
3. Over-weighting of rare events (Kahneman & Tversky, 1979)	A: 5 with certainty B: 5,000, .001; 0 otherwise	72%	A: 2 with certainty B: 101, .01; 1 otherwise	55%
4. Loss aversion (Ert & Erev, 2013, following Kahneman & Tversky, 1979)	A: 0 with certainty B: -100, .5; 100 otherwise	22%	A: 0 with certainty B: -50, .5; 50 otherwise	34%
5. St. Petersburg paradox/risk aversion (Bernoulli, 1954)	A fair coin will be flipped until it comes up heads. The number of flips will be denoted by the letter $k$ . The casino pays a gambler $2^k$ . What is the maximum amount of money that you are willing to pay for playing this game?	Modal response: less than 8	A: 9 with certainty B: 2, 1/2; 4, 1/4; 8; 1/8; 16, 1/16; 32, 1/32; 64, 1/64; 128, 1/128; 256 otherwise	38%
6. Ellsberg paradox/Ambiguity aversion (Einhorn & Hogarth, 1986, following Ellsberg, 1961)	Urn K contains 50 red and 50 white balls. Urn U contains 100 balls, each either red or white, with unknown proportions. Choose between: A: 100 if a ball drawn from K is red; 0 otherwise B: 100 if a ball drawn from U is red; 0 otherwise C: Indifference	47% 19% 34%	A: 10 with probability .5; 0 otherwise B: 10 with probability 'p'; 0 otherwise ('p' unknown constant)	37%
7. Low magnitude eliminates loss aversion (Ert & Erev, 2013)	A: 0 with certainty B: -10, .5; 10 otherwise	48%	A: 0 with certainty B: -1, .5; 1 otherwise	49%
8. Break-even effect (Thaler & Johnson, 1990)	A: -2.25 with certainty B: -4.50, .5; 0 otherwise	87%	A: -1 with certainty B: -2, .5; 0 otherwise	58%
	A': -7.50 with certainty B': -5.25 .5; -9.75 otherwise	77%	A': -2 with certainty B': -3 .5; -1 otherwise	48%
9. Get-something effect (Ert & Erev, 2013, following Payne, 2005)	A: 11, .5; 3 otherwise B: 13, .5; 0 otherwise	21%	A: 1 with certainty B: 2, .5; 0 otherwise	35%
	A': 12, .5; 4 otherwise B': 14, .5; 1 otherwise	38%	A': 2 with certainty B': 3 .5; 1 otherwise	41%
10. Splitting effect (Birnbaum, 2008, following Tversky & Kahneman, 1986)	A: 96; .90; 14, .05; 12, .05 B: 96; .85; 90, .05; 12, .10	73%	A: 16 with certainty B: 1, .6; 50, .4 A': 16 with certainty B': 1, .6; 44, .1; 48, .1; 50, .2	49.9% 50.4%

Table 1 (continued)

Phenomenon	Classical demonstration		Current replication	
	Problems	%B Choice	Problems	%B Choice
Phenomena observed in studies of repeated decisions with feedback				
11. Under-weighting of rare events (Barron & Erev, 2003)	A: 3 with certainty B: 32, .1; 0 otherwise	32%	A: 1 with certainty B': 20, .05; 0 otherwise	29%
	A': -3 with certainty B': -32, .1; 0 otherwise	61%	A': -1 with certainty B': -20 .05; 0 otherwise	64%
12. Reversed reflection (Barron & Erev, 2003)	A: 3 with certainty B: 4, .8; 0 otherwise	63%	A: 3 with certainty B: 4, .8; 0 otherwise	65%
	A': -3 with certainty B': -4, .8; 0 otherwise	40%	A': -3, with certainty B': -4, .8; 0 otherwise	40%
13. Payoff variability effect (Erev & Haruvy, 2016, following Busemeyer & Townsend, 1993)	A: 0 with certainty B: 1 with certainty	96%	A: 2 with certainty B: 3 with certainty	100%
	A': 0 with certainty B': -9, .5; 11 otherwise	58%	A': 6 if E; 0 otherwise B': 9 if not E; 0 otherwise P(E) = .5	84%
14. Correlation effect (Grosskopf, Erev, & Yechiam, 2006, following Diederich & Busemeyer, 1999)	A: 150 + N <sub>1</sub> if E; 50 + N <sub>1</sub> otherwise B: 160 + N <sub>2</sub> if E'; 60 + N <sub>2</sub> otherwise	82%	A: 6 if E; 0 otherwise B: 9 if not E; 0 otherwise	84%
	A': 150 + N <sub>1</sub> if E; 50 + N <sub>1</sub> otherwise B': 160 + N <sub>2</sub> if E; 60 + N <sub>2</sub> otherwise N <sub>i</sub> ~ N(0, 20), P(E) = P(E') = .5	98%	A': 6 if E; 0 otherwise B': 8 if E; 0 otherwise P(E) = .5	98%

*Note.* The notation  $x, p$  means payoff of  $x$  with probability  $p$ . In the classical demonstrations, choice rates are for one-shot decisions in the no-feedback phenomena and for mean of the final 100 trials (of 200 or 400) in the with-feedback phenomena. In current replications, choice rates are for five consecutive choices without feedback in the no-feedback phenomena and for the last five trials (of 25) in the with-feedback phenomena.

Tversky and Kahneman (1992; and see Wakker, 2010) presented a refined version of prospect theory (cumulative prospect theory, CPT) that clarifies the assumed processes and their relationship to the computations required to maximize expected value (EV). To compute the EV of a prospect, the decision maker should weight each monetary outcome by its objective probability. For example, the EV of a prospect that provides “4,000 with probability 0.8, 0 otherwise” is  $4,000 \cdot (.8) + 0 \cdot (.2) = 3200$ . CPT assumes modified weighting. The subjective value of each outcome is weighted by its subjective weight. For example, the attractiveness of the prospect “4,000 with probability 0.8, 0 otherwise” under CPT is  $V(4,000) \cdot \pi(.8) + V(0) \cdot (1 - \pi(0.8))$ , where  $V(\cdot)$  is a subjective value function that is assumed to reflect diminishing sensitivity and loss aversion, and  $\pi(\cdot)$  is a subjective weighting function that is assumed to reflect oversensitivity to extreme outcomes.

The success of prospect theory—and later of its successor CPT—has triggered three lines of follow-up studies that attempt to reconcile this model with other choice anomalies. One line of research (e.g., Rieger & Wang, 2006; Tversky & Bar-Hillel, 1983; Tversky & Fox, 1995; Wakker, 2010) focuses on classical choice anomalies originally observed under experimental paradigms not addressed by the original model: the St. Petersburg paradox (Bernoulli, 1954; see row 5 in Table 1) and the Ellsberg paradox (Ellsberg, 1961; see row 6 in Table 1). This line of research suggests that extending prospect theory or CPT to address these classical anomalies generally requires additional nontrivial assumptions and/or parameters. For example, it is difficult to capture

the four original anomalies and the St. Petersburg paradox with one set of parameters (e.g., Blavatsky, 2005; Rieger & Wang, 2006).

A second line of research focuses on newly observed choice anomalies documented within the limited setting of choice between simple fully described gambles, which prospect theory was developed to address. These studies highlight new anomalies that emerge in this setting but cannot be captured with CPT. For example, Ert and Erev (2013; see row 7 in Table 1) showed that low stakes eliminate the tendency to exhibit loss aversion; Thaler and Johnson (1990) documented a “break-even” effect (more risk seeking when only the risky choice can prevent losses; see row 8 in Table 1); Payne (2005) documented a “get-something” effect (less risk seeking when only the safe prospect guarantees a gain; see row 9 in Table 1); and Birnbaum (2008) documented a splitting effect (splitting a possible outcome into two less desirable outcomes can increase its attractiveness; see row 10 in Table 1). Against the background of the difficulty in reconciling CPT with these new anomalies, an alternative approach has suggested that some anomalies might be more naturally explained as a reflection of simple heuristics rather than as a reflection of subjective weighting processes. For example, the get-something effect can be the product of the use of a *Pwin* heuristic, which implies choosing the option that maximizes the probability of gaining (e.g., Venkatraman, Payne, & Huettel, 2014). Brandstätter, Gigerenzer, and Hertwig (2006) show that it is also possible to find a simple heuristic that can explain the anomalies that motivated prospect theory in the first place (see rows 1 to 4 in Table 1).

A third line of follow-up research focuses on the effects of experience. Thaler, Tversky, Kahneman, and Schwartz (1997) and Fox and Tversky (1998) presented natural generalizations of prospect theory to situations in which agents have to rely on their past experience, but subsequent research (Hertwig, Barron, Weber, & Erev, 2004) highlights robust phenomena that cannot be captured with these generalizations. This line of research has proven to be particularly difficult to reconcile with prospect theory, as it questions the generality of the very anomalies that motivate it. The availability of feedback was found to reverse some of the anomalies considered above. The clearest examples for the effects of feedback include underweighting of rare events (Barron & Erev, 2003; see row 11 in Table 1), a reversed reflection effect (Barron & Erev, 2003; see row 12 in Table 1), a payoff variability effect (Busemeyer & Townsend, 1993; see row 13 in Table 1), and a correlation effect (Diederich & Busemeyer, 1999; see row 14 in Table 1). Once again, for generalizations of prospect theory or CPT to capture anomalies in this line of research, additional nontrivial assumptions are necessary (see, e.g., Fox & Hadar, 2006; Glöckner, Hilbig, Henninger, & Fiedler, 2016). Congruently, the leading explanations of the phenomena observed in decisions involving feedback assume a different underlying process, according to which decision makers tend to rely on small samples of their past experiences (Erev & Barron, 2005; Hertwig et al., 2004).

The different studies that aimed to develop descriptive models that capture subsets of the 14 phenomena we have just described and are summarized in Table 1, have led to many useful insights. However, they also suffer from a major shortcoming. Different modifications of prospect theory and models that assume other processes address different well-studied domains of problems. Thus, it is not clear how to use them outside the boundaries of these domains. For example, according to Erev, Ert, Roth et al. (2010), the best models that capture behavior in decisions under risk assume very different processes than the best models that capture behavior in repeated decisions with feedback. But which type of model should we use if our goal is, for example, to design an incentive mechanism to be implemented in in-vehicle data recorders aimed at promoting safe driving? The drivers of a car equipped with such devices would be informed of the incentives and would also gain experience using the system. In other words, which model should be used to predict the choice between fully described gambles following a few trials with feedback? And what if the gambles include many possible outcomes (as in the St. Petersburg paradox) or if one of these gambles is ambiguous (as in the Ellsberg paradox)? The existing models shed only limited light on the conditions that trigger different behavioral tendencies, so better understanding of these conditions is required in order to address Roth's 1–800 critique.

The current research attempts to improve our understanding of the conditions that trigger the different anomalies by extending the focus of the analysis. Rather than focusing on a subset of the 14 anomalies presented in Table 1 (specifically, Kahneman and Tversky focused on a subset of size 4), we try to capture all 14 anomalies with a single model. Unlike the previous attempts to extend Kahneman and Tversky's (1979) analysis, discussed above, we build on their method rather than on their model. Like Kahneman and Tversky (1979), we start by trying to replicate the target anomalies in a single experimental paradigm, and then develop a single model that captures the behavioral results.

## The Problem of Overfitting and the Current Project

As noted above, previous research suggests that choice behavior is possibly affected by three very different types of cognitive processes: processes that weigh subjective values by subjective functions of their probabilities; those that assume simple heuristics; and those that assume sampling from memory. It is also possible that one of these processes captures behavior better than the others, but it requires making different assumptions in different settings. An attempt to capture the interaction between several unobserved processes or to identify the boundaries of the settings in which different assumptions are necessary involves a high risk of overfitting the data. There are many feasible abstractions of the possible processes and their interactions, and with so many degrees of freedom, it is not too difficult to find abstractions that fit all 14 anomalies.

The current research takes four measures to reduce this risk. The most important measure is the focus on predictions, rather than on fitting. Models are estimated here based on one set of problems, and then compared based on their ability to predict the behavior in a second, initially unobserved, set of problems. A second measure involves the replication of the classical anomalies in one standard paradigm (Hertwig & Ortmann, 2001; like Kahneman & Tversky, 1979). This replication eliminates the need to estimate paradigm-specific parameters. A third measure involves the study of randomly selected problems (in addition to the problems that demonstrate the interesting anomalies). This serves to increase the amount of the data used to estimate and evaluate the models. A fourth measure involves the organization of a choice prediction competition (see Arifovic, McKelvey, & Pevnitskaya, 2006; Erev, Ert, Roth et al., 2010). The first three coauthors of the current article (Erev, Ert, and Plonsky; hereinafter EEP) first presented the best model they could find, and then challenged other researchers to find a better model. The competition methodology is expected to reduce the risk of overfitting the data caused by focusing on a small set of models considered by a small group of researchers. That is, rather than putting forth the set of contender models themselves, EEP asked the research community to provide the contenders.

In the first part of the current project, EEP developed a “standard” paradigm (Hertwig & Ortmann, 2001) and identified an 11-dimensional space of experimental tasks wide enough to replicate all 14 behavioral phenomena described above and illustrated in Table 1. Next, EEP conducted a replication study consisting of 30 carefully selected choice tasks. The replication study shows that all 14 behavioral phenomena emerge in this 11-dimensional space. Yet, their magnitude tends to be smaller than it was in the original demonstrations.

The second part of the current project includes a calibration study in which 60 additional problems, randomly selected from the same 11-dimensional space of tasks that includes the replication problems, were investigated. The results clarify the robustness and the boundaries of the distinct behavioral phenomena. Based on the results of these 90 problems (30 in the replication study and 60 in the calibration study), EEP developed a “baseline” model that presents their best attempt to capture behavior in the wide space of choice tasks. This model assumes that two components drive choice. The first is the option's expected value (and not a weighting of subjective values, as modeled by expected utility theory and CPT). The second is the outcome of four distinct tendencies that are products of sampling “tools.”



The article concludes with the presentation of the choice prediction competition. The organizers (EEP) posted the results of the first two experiments, as well as a description of the baseline model, on the web (<http://departments.agri.huji.ac.il/cpc2015>). EEP then challenged other researchers (with e-mails to members of the popular scientific organizations in psychology, decision science, behavioral economics, and machine learning) to develop a better model. The competition focused on the prediction of the results of a third (“test”) experimental study that was run after posting the baseline model. The call for participation in the competition was posted in January 2015, and the competition study was run in April 2015 (its results were published only after the submission deadline in May 2015). A famous Danish proverb (commonly attributed to Niels Bohr) states “it is difficult to make predictions, especially about the future.” Running the competition study only after everyone submitted their models ensured that the competition participants actually dealt with this difficulty of predicting the future, and could not satisfy with fitting data that is already known.

Researchers from five continents responded to the prediction competition challenge, submitting a total of 25 models. The submissions included three models that are variants of prospect theory with situation-specific parameters, 14 models that are similar to the baseline model and assume that behavior is driven by the expected return and four additional behavioral tendencies, and seven models that do not try to directly abstract the underlying process, but rely primarily on statistical methods (like machine learning algorithms). All 12 highest ranked submissions were variants of the baseline. The “prize” for the winners was coauthorship of this article; the last two authors (Cohen and Cohen) submitted the winning model.

### Space of Choice Problems

The previous studies that demonstrated the behavioral phenomena summarized in Table 1 used diverse experimental paradigms. For example, the Allais paradox/certainty effect was originally demonstrated in studies that examined choice among fully described gambles (Allais, 1953; Kahneman & Tversky, 1979), while the Ellsberg paradox was originally demonstrated in studies that focused on bets on the color of a ball drawn from partially described urns (Einhorn & Hogarth, 1986; Ellsberg, 1961). In addition, within the same experimental paradigm, different payoff distributions give rise to different behavioral phenomena. In other words, the differences among the various demonstrations of the behavioral phenomena in Table 1 involve multiple dimensions, such as the framing manipulation, the number of possible outcomes, and the shape of the payoff distributions. Thus, it is possible to think of the classical demonstrations in Table 1 as points in a multidimensional space of “choice tasks.” This abstraction clarifies the 1–800 critique against behavioral decision research. The critique rests on the observation that the leading models were designed to capture specific sections (typically involving interesting anomalies) in this space of choice problems. Thus, different models address different points in the space, and the models’ boundaries are not always clear. Consequently, it is not clear which model should be used to predict behavior in a new choice task.

Our research attempts to address this critique by facilitating the study of a space of choice tasks wide enough to give rise to all 14 phenomena summarized in Table 1. We began by trying to identify

the critical dimensions of this multidimensional space. Our effort suggests that the main properties of the problems in Table 1 include at least 11 dimensions. Nine of the 11 dimensions can be described as parameters of the payoff distributions. These parameters include:  $L_A$ ,  $H_A$ ,  $pH_A$ ,  $L_B$ ,  $H_B$ ,  $pH_B$ ,  $LotNum$ ,  $LotShape$ , and  $Corr$ . In particular, each problem in the space is a choice between Option A, which provides  $H_A$  with probability  $pH_A$  or  $L_A$  otherwise (with probability  $1 - pH_A$ ), and Option B, which provides a lottery (that has an expected value of  $H_B$ ) with probability  $pH_B$ , and provides  $L_B$  otherwise (with probability  $1 - pH_B$ ). The distribution of the lottery around its expected value ( $H_B$ ) is determined by the parameters  $LotNum$  (which defines the number of possible outcomes in the lottery) and  $LotShape$  (which defines whether the distribution is symmetric around its mean, right-skewed, left-skewed, or undefined if  $LotNum = 1$ ), as explained in Appendix A. The  $Corr$  parameter determines whether there is a correlation (positive, negative, or zero) between the payoffs of the two options.

The tenth parameter, ambiguity ( $Amb$ ), captures the precision of the initial information the decision maker receives concerning the probabilities of the possible outcomes in Option B. We focus on the two extreme cases:  $Amb = 1$  implies no initial information concerning these probabilities (they are described with undisclosed fixed parameters), and  $Amb = 0$  implies complete information and no ambiguity (as in Allais, 1953; Kahneman & Tversky, 1979).

The eleventh dimension in the space is the amount of feedback the decision maker receives after making a decision. As Table 1 shows, some phenomena emerge in decisions without feedback (i.e., *decisions from description*), and other phenomena emerge when the decision maker can rely on feedback (i.e., *decisions from experience*). We studied this dimension within problem. That is, decision makers faced each problem first without feedback, and then with full feedback (i.e., realization of the obtained and forgone outcomes following each choice).

The main hypothesis of the replication exercise described below is that this 11-dimensional space is sufficiently large to give rise to all the behavioral phenomena from Table 1.<sup>1</sup> That is, we examine whether all these phenomena can be replicated within the abstract framing of choice between gambles (used, e.g., by Allais, 1953; Kahneman & Tversky, 1979), although some were originally demonstrated in different experimental paradigms, such as urns or coin-tosses. To facilitate this examination, we considered, in addition to the 11 dimensions, two framing manipulations (“coin-toss” and “accept/reject”) that were suggested by previous studies as important to two of the phenomena (to the St. Petersburg

<sup>1</sup> Notice that the 11 dimensions were selected to ensure that certain value combinations would imply choice tasks likely to give rise to the 14 phenomena. The first six dimensions are necessary to allow for the Allais pattern. The  $LotNum$  and  $LotShape$  dimensions are necessary to allow for the St. Petersburg paradox and splitting pattern. The ambiguity dimension is necessary to allow for the Ellsberg paradox. The correlation dimension is necessary to allow for the regret/correlation effect. And the feedback dimension is necessary to allow for the experience phenomena. We also had to limit the range of values that each of the 11 dimensions could take, which inevitably added technical constraints to the space of problems we actually studied. For example, the manner in which the lottery parameters define its distribution limits the possible lottery distributions in the space (see Appendix A). Hence, the genuine hypothesis of the replication study is that even the limited 11-dimensional space is sufficiently large to replicate all the classical phenomena from Table 1.

paradox, see Erev, Glozman, & Hertwig, 2008; and to loss aversion, see Ert & Erev, 2013, respectively). Under the “accept/reject framing,” Option B is presented as the acceptance of a gamble, and Option A as the status quo (rejecting the gamble). Under the “coin-toss framing,” the lottery is described as a coin-toss game similarly to Bernoulli’s (1954) description in his illustration of the St. Petersburg paradox. Hence, each of the 30 problems studied in our replication study (see Appendix B) is uniquely defined by specific values in each of the 10 dimensions described above in addition to a framing manipulation (and, as noted, the eleventh dimension, feedback, is studied within the problems).

Initially, we also planned to consider the role of the difference between hypothetical and real monetary payoffs. We chose to drop this dimension and focus on real payoffs, following a pilot study in which more than 30% of the subjects preferred the hypothetical gamble “−1,000 with probability .1; +1 otherwise” over the status quo (zero with certainty). This pilot study reminded us that the main effect of the study of hypothetical problems is an increase in choice variance (Camerer & Hogarth, 1999).

### Replications of Behavioral Phenomena

The current investigation was designed to undertake the following objectives: (a) to explore whether the current 11-dimensional space is wide enough to replicate the 14 choice phenomena summarized on the left-hand side of Table 1; (b) to clarify the boundaries and relative importance of these phenomena; and (c) to test the robustness of two of the phenomena to certain framing manipulations, which, according to previous research, matter. To achieve these goals, we studied the 30 choice problems detailed in Appendix B.

### Method

One hundred and twenty-five students (63 male,  $M_{\text{Age}} = 25.5$ ) participated in the experimental condition of the replication study, 60 at the Hebrew University of Jerusalem (HUJI), and 65 at the Technion. Each participant faced each of the 30 decision problems presented in Appendix B for 25 trials (i.e., each participant made 750 decisions). The order of the 30 problems was random. Participants were told that they were going to play several games, each for several trials, and their task in each trial was to choose one of the two options on the screen for real money. The participants were also told that at the end of the study, one of the trials would be randomly selected and that their obtained outcome in that trial would be realized as their payoff (see examples of the experimental screen and a translation of the instructions in Appendix C). Notice that this payment rule excludes potential “wealth effects.” It implies that the participants could not “build portfolios,” for example, by taking risks in some trials and compensating for losses in others. Furthermore, both rational considerations and the isolation effect (Kahneman & Tversky, 1979) imply independence between the choices in the problems without ambiguity.

In the first five trials of each problem, the participants did not receive feedback after each choice, so they had to rely solely on the description of the payoff distributions. Starting at Trial 6, participants were provided with full feedback (the outcomes of each prospect) after each choice; that is, in the last 19 trials, the participants could rely on the description of the payoff distribu-

tions and on feedback concerning the outcomes of previous trials.<sup>2</sup> The final payoff (including show-up fee) ranged from 10 to 110 shekels ( $M = 41.9$ , approximately \$11).<sup>3</sup>

In addition to the experimental condition, we ran two control conditions that used the same participant recruitment and incentive methods as the main condition. The first, referred to as “Single Choice,” used Kahneman and Tversky’s (1979) paradigm. Each of 60 participants faced each of the 30 problems only once and without any feedback. The second control, referred to as “FB from 1st”, was identical to the main replication study except that all choices, from the very first trial, were followed by feedback. This control condition included 29 participants. The results of both conditions are reported in the Control Conditions and Robustness Checks section.

### Results and Discussion

The main results of the experimental condition, the mean choice rates of Option B per block of five trials and by feedback type (i.e., no-FB or with-FB) for each of the 30 problems are presented in Appendix B. The raw data (nearly 94,000 lines) is available online (see <https://zenodo.org/record/321652#.WK2oPG996UI>). In short, the results show that (a) all the 14 phenomena described in Table 1 emerge in our setting, but most description phenomena are eliminated or even reversed after few trials with feedback; and (b) feedback increases maximization when the best option leads to the best payoff in most trials, but can impair maximization when this condition does not hold. Below we clarify the implications of the results for the 14 behavioral phenomena summarized in Table 1.

**The Allais paradox/certainty effect.** The Allais paradox (Allais, 1953) is probably the clearest and most influential counterexample to expected utility theory (Von Neumann & Morgenstern, 1944). Kahneman and Tversky (1979) show that the psychological tendency that underlies this paradox can be described as a certainty effect: safer alternatives are more attractive when they provide gain with certainty. Figure 1 summarizes our investigation of this effect using variants of the problems used by Kahneman and Tversky (1979) (row 1 in Table 1) to replicate Allais’ common ratio version of the paradox. Analysis of Block 1 (first five trials,

<sup>2</sup> In addition, we compared two order conditions in a between-subjects design. Sixty participants (30 in each location) were assigned to the “by problem” (ByProb) order: They faced each problem for one sequence of 25 trials before facing the next problem. The other participants were assigned to the “by feedback” (ByFB) order condition. This condition was identical to the ByProb condition, with one exception: The participants first performed the five no-feedback trials in each of the 30 problems (in one sequence of 150 trials), and then faced the remaining 20 trials with feedback of each problem (in one sequence of 600 trials, and in the same order of problems they played in the no-feedback trials). Our analyses suggested almost no differences between the two conditions, therefore we chose to focus on the choice patterns across conditions, and report these subtle differences in the section Effects of Location and Order.

<sup>3</sup> The show-up fee was determined for each participant individually such that the minimal possible compensation for the experiment was 10 shekels. This was the maximum between 30 shekels and the sum of 10 shekels and the maximal possible loss in the problem that was randomly selected to determine the payoff. For example, if Problem 12 was selected, the show-up fee was 60 shekels, but if Problem 1 was selected the show-up fee was 30 Shekels. This procedure was never disclosed to participants and they only knew in advance their expected total payoff. Specifically, there was no deception.

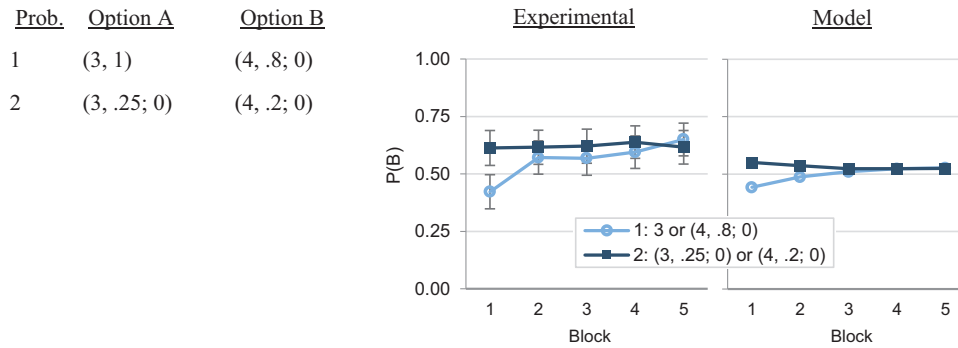


Figure 1. Problems that test the allais paradox/certainty effect in the replication study. The notation  $(x, p; y)$  refers to a prospect that yields a payoff of  $x$  with probability  $p$  and  $y$  otherwise. Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. (The right-hand plot presents the prediction of the baseline model described below). See the online article for the color version of this figure.

without feedback, or "no-FB") shows the robustness of the certainty effect in decisions from description. The safer prospect (A) was more attractive in Problem 1 when it provided a positive payoff with certainty (A-rate of 58%, B-rate of 42%,  $SD = 0.42$ ), than in Problem 2 when it involved some uncertainty (A-rate of 39%, B-rate of 61%,  $SD = 0.44$ ). The difference between the two rates is significant,  $t(124) = -3.69, p < .001$ .<sup>4</sup> However, feedback reduced this difference. The difference between the two problems across the four with-FB blocks (B-rate of 60%,  $SD = 0.37$  in Problem 1, and B-rate of 62%,  $SD = 0.39$  in Problem 2) is insignificant:  $t(124) = -0.59$ .

**The reflection and reversed reflection effects.** A comparison of Problem 3 and Problem 4 in Figure 2 demonstrates the reflection effect (Kahneman & Tversky, 1979, and row 2 in Table 1) in the no-FB block: risk aversion in the gain domain (B-rate of 35%,  $SD = 0.42$  in Problem 4) and risk seeking in the loss domain (B-rate of 58%,  $SD = 0.42$  in Problem 3). The difference is significant,  $t(124) = -4.99, p < .001$ . Feedback reduces this effect. The B-rate across the four with-FB blocks (2 to 5) is 52% ( $SD = 0.37$ ) in Problem 4 and 59% ( $SD = 0.35$ ) in Problem 3. This difference is insignificant,  $t(124) = -1.59$ .

A comparison of Problem 1 with Problem 5 reveals a weaker indication of the reflection effect. The results in the no-FB block show risk aversion in the gain domain (B-rate of 42%,  $SD = 0.42$  in Problem 1) and risk neutrality in the loss domain (B-rate of 49%,  $SD = 0.42$  in Problem 5); the difference is insignificant,  $t(124) = -1.24$ . Feedback reverses the results and leads to lower risk-taking rate in the loss domain (B-rate of 40%,  $SD = 0.37$  in Problem 5) than in the gain domain (B-rate of 60%,  $SD = 0.37$  in Problem 1). This reversed reflection pattern (Barron & Erev, 2003; see row 12 in Table 1) across the four with-FB blocks, which suggests learning to maximize EV, is significant,  $t(124) = 3.90, p < .001$ .

**The weighting of rare events.** Figure 3 summarizes our investigation of the weighting of rare events. An analysis of the no-FB block shows that the modal choice in Problem 10 reflects overweighting of the rare (probability .01) event. Participants tend to prefer the long-shot gamble. Yet, the magnitude of this effect is not large; the mean rate (55%,  $SD = 0.44$ ) is not significantly different from 50%,  $t(124) = 1.15$ . Moreover, Problems 7, 8, and

9 show no indication of initial overweighting of rare events. One explanation for the difference between these findings and Kahneman and Tversky's (1979, and row 3 in Table 1) strong indications for overweighting of rare events in decisions from description focuses on the probabilities. The classical demonstrations examined a 1/1,000 event, while we studied 1/20 and 1/100 events. It is possible that the tendency to overweight rare events increases with their rarity. This explanation is supported by the observation that our results for the positive rare outcomes reveal higher B-rate in the 1/100 case (mean of 51% in Problems 9 and 10,  $SD = 0.38$ ) than in the 1/20 case (B-rate of 39%,  $SD = 0.43$ , in Problem 8). This difference is significant,  $t(124) = 3.32, p = .001$ .

Another explanation for the weaker indication of overweighting of rare events in our no-FB block involves the possibility that the expectation that choice will be repeated (in the current paradigm) reduces the weighting of rare events. An experiment that evaluates (and rejects) this "expected repetitions" hypothesis is presented in the section Control Conditions and Robustness Checks below.

Figure 3 also shows that the emergence of underweighting of rare events in decisions with feedback is robust (Barron & Erev, 2003; Lejarraaga & Gonzalez, 2011; and see row 11 in Table 1). Experience reduces sensitivity to the rare event in all five problems. In the four equal expected value problems (Problems 7, 8, 9, and 10), the choice rate following feedback reflects a clear indication of underweighting of rare events: the choice rate of the prospect that leads to the best payoff most of the time is 63% ( $SD = 0.37$ ), 67% ( $SD = 0.39$ ), 61% ( $SD = 0.42$ ), and 57% ( $SD = 0.44$ ) in Problems 7, 8, 9, and 10, respectively (and all four values are significantly greater than 50%,  $t(124) = 3.82, 4.77, 2.90, 1.72$ , respectively). Problem 11 highlights one boundary of the underweighting of rare events. When the difference in expected value is sufficiently large (19 vs. 16), experience does not eliminate the tendency to prefer the high expected value option over the risky

<sup>4</sup> We report significance tests to clarify the robustness of each finding in our setting, and use a weaker criterion to define replication. A phenomenon is considered to be "replicated" if the observed choice rates are in the predicted direction.

Prob.	Option A	Option B
3	(-1, 1)	(-2, .5; 0)
4	(1, 1)	(2, .5; 0)

Prob.	Option A	Option B
5	(-3, 1)	(-4, .8; 0)
1	(3, 1)	(4, .8; 0)

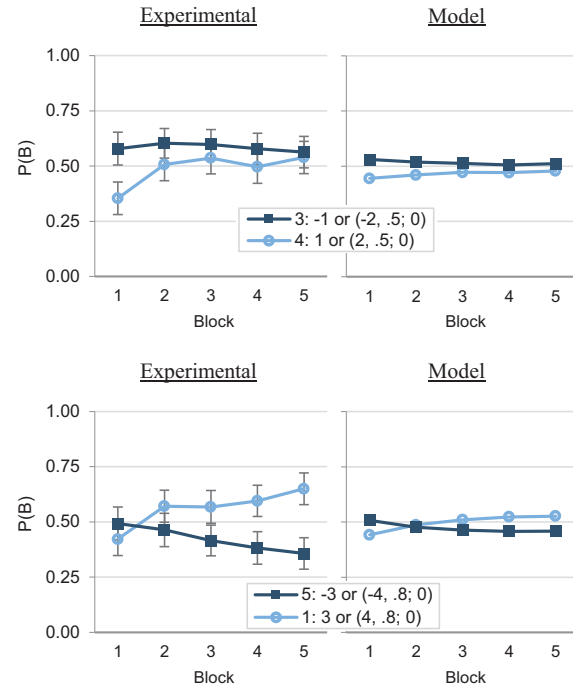


Figure 2. Problems that test the reflection effect in the replication study. The notation (x, p; y) refers to a prospect that yields a payoff of x with probability p and y otherwise. Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. See the online article for the color version of this figure.

alternative that leads to better payoff most of the time (90% of the trials).

**Loss aversion and the magnitude effect.** The loss aversion hypothesis implies a preference of the status quo over a symmetric fair gamble (e.g., a gamble that provides equal probability of winning or losing x, row 4 in Table 1). Figure 4 summarizes our investigation of this hypothesis. Evaluation of the no-FB block in Problem 12 shows that the status quo was preferred over equal chances to win or lose 50 in 66% ( $SD = 0.43$ , significantly more than 50%,  $t(124) = 4.25$ ,  $p < .001$ ) of the cases. Problem 13 focuses on the same objective task as Problem 12, with a different framing. The results show that in the current setting, the difference between the accept/reject and the abstract framing is small: 64% ( $SD = 0.42$ , significantly more than 50%,  $t(124) = 3.66$ ,  $p < .001$ ) of the choices reflect rejection of the gamble in Problem 13, similar to the rates observed in Problem 12.<sup>5</sup> Problem 14 replicates the finding that low stakes eliminate the initial loss aversion bias (Ert & Erev, 2013; Harinck, Van Dijk, Van Beest, & Mersmann, 2007; and see row 5 in Table 1); the gamble was selected in 49% ( $SD = 0.44$ ) of the cases. The difference between Problems 14 and 12 in the no-FB block is significant,  $t(124) = -3.64$ ,  $p < .001$ . The results for the last four blocks show that feedback eliminated the magnitude effect, but not the general tendency to select the status quo over the fair gamble.

We added Problems 15, 16, 17, and 18 (lower panel in Figure 4) to study one boundary of loss aversion, the observation that the addition of small losses to a dominant (EV-wise) option can increase its attractiveness (Yechiam & Hochman, 2013). Our results do not reveal this so-called "loss attention" pattern. Rather, they show similar sensitivity to the expected values in all cases.

**St. Petersburg paradox.** Our experimental paradigm differs from the St. Petersburg problem (row 5 in Table 1) in many ways. Most importantly, we study choice rather than bidding, and avoid the study of hypothetical tasks (and for that reason cannot examine a problem with unbounded payoffs). Nevertheless, the robustness of the main behavioral tendency demonstrated by the St. Petersburg paradox, risk aversion in the gain domain, can be examined in our setting. Figure 5 summarizes our investigation. We studied two framings of a bounded variant of the St. Petersburg problem. In Problem 19, the participants were asked to choose between 9 with certainty, and a coin-toss game with the same expected value. In Problem 20, the game's possible outcomes and their objective probabilities were listed on the screen. The results reveal a tendency to avoid the game that was slightly increased by experience. The B-rates in the no-FB Block were 36% ( $SD = 0.42$ ), and 38% ( $SD = 0.43$ ) in the "coin-toss" (St. Petersburg) and the "abstract" variants, respectively. Both rates are significantly lower than 50%,  $t(124) = -3.61$  and  $-3.22$ , both  $p < .001$ . In addition, the results across all five blocks show slightly lower B-rates in the coin format (34% vs. 37%). This difference is in the direction of the mere presentation hypothesis suggested by Erev, Glozman, and Hertwig (2008), but the difference in the current setting is insignificant.

<sup>5</sup> Ert and Erev (2008, 2013) observed stronger support for loss aversion in the accept/reject framing manipulation than in the abstract presentation. We believe that the lack of difference here reflects the fact that our subjects were faced with many abstract problems, and this experience eliminated the format effect.



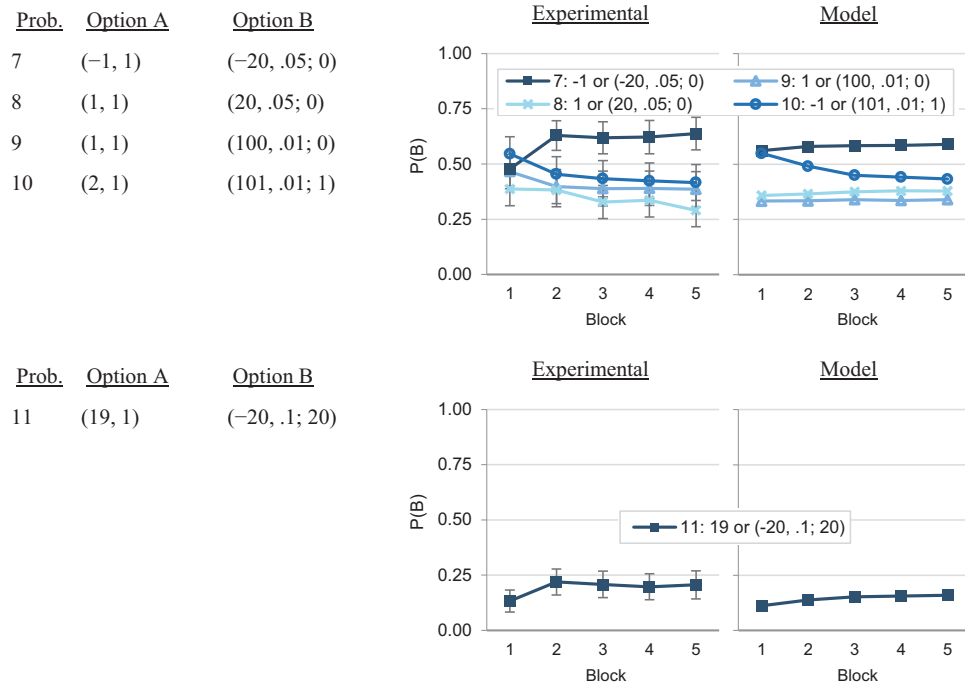


Figure 3. Problems that test the weighting of rare events in the replication study. The notation (x, p; y) refers to a prospect that yields a payoff of  $x$  with probability  $p$  and  $y$  otherwise. Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. See the online article for the color version of this figure.

**Ambiguity aversion/Ellsberg paradox.** Ellsberg (1961, see row 6 in Table 1) shows a violation of subjective expected utility theory that can be described as an indication of ambiguity aversion. Figure 6 summarizes our analysis of this phenomenon. The first block in Problem 21 reveals ambiguity aversion: the typical choice (63%,  $SD = 0.41$ ) favors the prospect "10, .5; 0" over the ambiguous prospect "10 or 0 with unknown probabilities." This value is significantly greater than 50%,  $t(124) = 3.49$ ,  $p < .001$ . Problem 22 reveals that when gaining in the nonambiguous option (A) occurs with low probability, people favor the ambiguous option (ambiguity rate of 82%,  $SD = 0.30$ ). Problem 23 shows a strong tendency to avoid the ambiguous option when gaining in the nonambiguous option is associated with high probability (ambiguity rate of 15%,  $SD = 0.30$ ). Both rates are significantly different from 0.5,  $t(124) = 12.0$  and  $-13.1$ , both  $p < .001$ , and are in line with previous findings of studies in decisions in uncertain settings without feedback (e.g., Camerer & Weber, 1992). Evaluation of the effect of experience reveals that feedback eliminates these attitudes toward ambiguity (see Ert & Trautmann, 2014, for similar findings).<sup>6</sup> The average choice rate of the ambiguous option over the four with-FB blocks in these problems was 49%.

**The break-even effect.** Thaler and Johnson (1990, see row 8 in Table 1) noticed that people are more likely to take a risk in the loss domain when this risk can cover all their losses and lead to a break-even outcome. The results, summarized in Figure 7, document the break-even effect in the no-FB block. Our participants took significantly more risk in Problem 3 (B-rate of 58%,  $SD = 0.42$ ) and Problem 5 (B-rate 49%,  $SD = 0.42$ ) when the risk could eliminate the loss, than in Problem 24 (B-rate of 48%,  $SD = 0.44$ )

and Problem 6 (B-rate 38%,  $SD = 0.42$ ) when the loss could not be avoided,  $t(124) = -2.01$ ,  $p = .047$  and  $t(124) = -2.12$ ,  $p = .036$ , respectively. Feedback did not eliminate this difference in the first pair (3 and 24), but did eliminate it in the second (5 and 6). The B-rates over the four with-FB blocks are 59% ( $SD = 0.35$ ) in Problem 3 and 48% ( $SD = 0.37$ ) in Problem 24, and the difference is significant,  $t(124) = -2.76$ ,  $p = .007$ . However, in both Problems 5 and 6, the B-rates over the four with-FB blocks are 41%. The elimination of the break-even effect in the latter case can be a reflection of an emergence, with feedback, of underweighting of the relatively rare (20%) attractive no-loss outcome in Problem 5.

**The get-something effect.** Payne (2005) shows that people are more likely to take action that increases the probability of positive outcome than action that does not affect this probability (row 9 in Table 1). Our analysis of this tendency, summarized in Figure 8, focuses on the comparison of Problem 4 with Problem 25 and the comparison of Problem 9 with Problem 10. Both comparisons reveal that in the no-FB block, our participants took less risk when only the safer prospect (A) guaranteed a gain (B-rate of 35%,  $SD = 0.42$  in Problem 4, and B-rate of 47%  $SD = 0.44$  in Problem 9) than they did in problems in

<sup>6</sup> Note that the description informed the subjects that the probabilities are fixed throughout the choice task. Thus, the outcome observed in the early trials reduces the objective ambiguity. As previously noted (e.g., Epstein & Schneider, 2007; Maccheroni & Marinacci, 2005) there are situations, which go beyond the scope of our space (e.g., when the probabilities can change), in which experience cannot eliminate ambiguity.

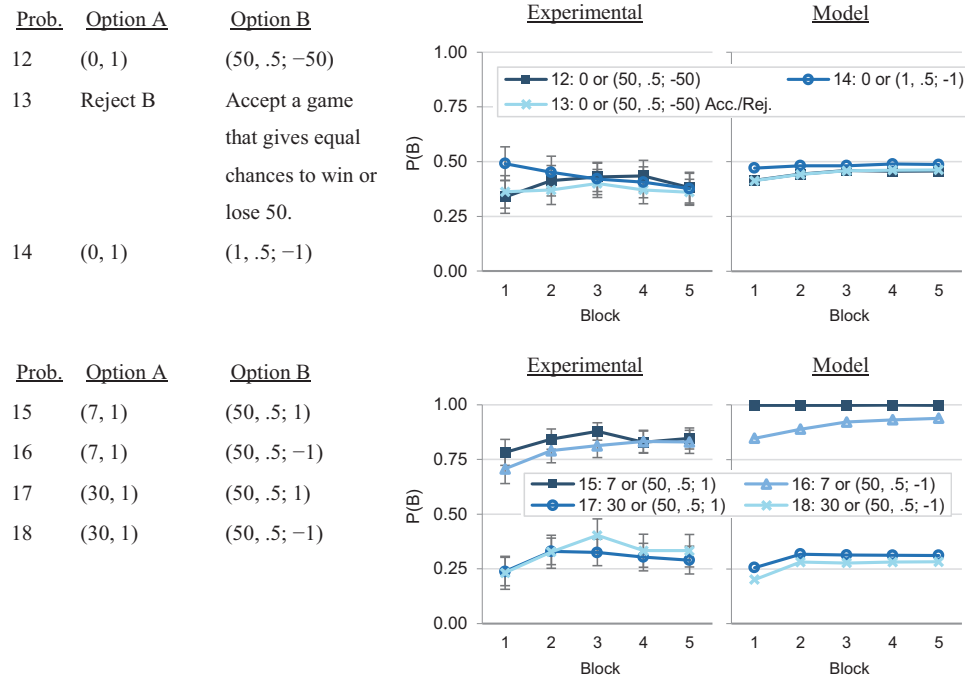


Figure 4. Problems that test loss aversion and magnitude effects in the replication study. The notation  $(x, p; y)$  refers to a prospect that yields a payoff of  $x$  with probability  $p$  and  $y$  otherwise. Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. See the online article for the color version of this figure.

which both options guaranteed a gain (B-rate of 41%,  $SD = 0.44$  in Problem 25, and B-rate of 55%,  $SD = 0.44$  in Problem 10). The effect is not large, but the difference between the two pairs is significant in a one-tailed test,  $t(124) = -1.87$ ,  $p = .032$ . Feedback eliminated this effect.

**The splitting effect.** Studies of decisions under risk show that splitting an attractive outcome into two distinct outcomes can increase the attractiveness of a prospect even when it reduces its

expected value (see Birnbaum, 2008; Tversky & Kahneman, 1986; and see row 10 in Table 1). Figure 9 summarizes our effort to replicate this effect in our paradigm. Specifically, we examine the effect of replacing the outcome 50 (in Problem 26) with the outcomes 44, 48, and 50 (in Problem 27). The results of the no-FB block show a slight increase in the predicted direction, from 49.9% ( $SD = .42$ ) to 50.4% ( $SD = .42$ ). This difference is insignificant, but it should be noted that the expected value of the riskier option

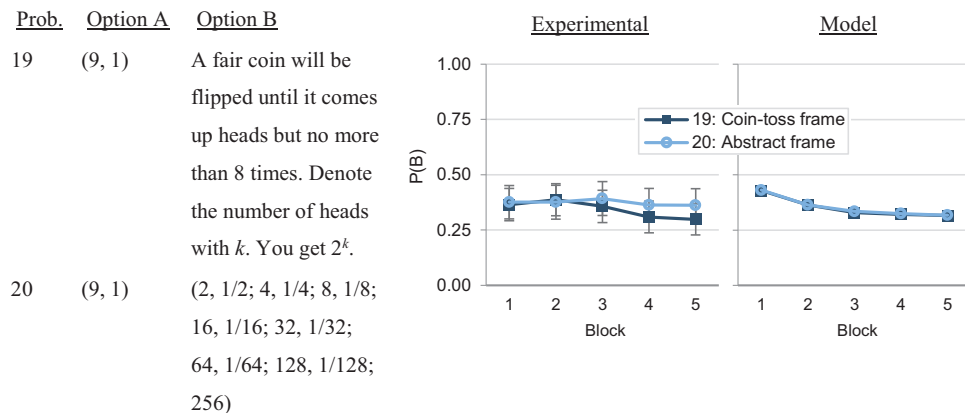


Figure 5. Problems that test the St. Petersburg paradox in the replication study. The notation  $(x_1, p_1; x_2, p_2; \dots; y)$  refers to a prospect that yields a payoff of  $x_1$  with probability  $p_1$ , a payoff of  $x_2$  with probability  $p_2$ ,  $\dots$ , and  $y$  otherwise. Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. See the online article for the color version of this figure.

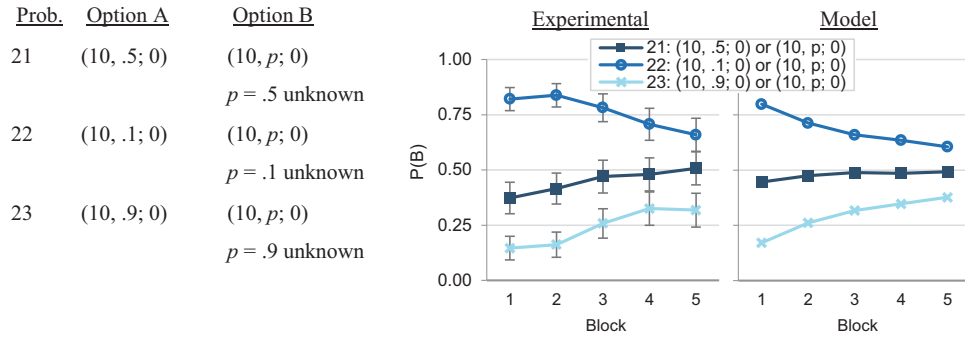


Figure 6. Problems that test ambiguity attitudes in the replication study. The notation  $(x, p; y)$  refers to a prospect that yields a payoff of  $x$  with probability  $p$  and  $y$  otherwise. In these problems, the probabilities of the outcomes in Option B are undisclosed to participants (an ambiguous problem). Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. See the online article for the color version of this figure.

decreased, while its choice rate increased slightly. Feedback reverses the effect and moves behavior toward maximization.

**The payoff variability and correlation effects.** Studies of decisions from experience demonstrate that payoff variability moves behavior toward random choice (Busemeyer & Townsend, 1993, see row 13 in Table 1), and positive correlation between the payoffs of the different alternatives reduces the payoff variability effect and facilitates learning (Diederich & Busemeyer, 1999, see row 14 in Table 1). Figure 10 summarizes our effort to replicate these effects in the current setting. A comparison of Problem 28

with Problem 29 documents the payoff variability effect: lower maximization rate in the high variability problem, although the expected benefit from maximization is higher in this problem. This difference was observed in the no-FB block (max-rate of 91%,  $SD = 0.21$  in Problem 28 in comparison with max-rate of 97%,  $SD = 0.15$  in Problem 29) and *intensified* in the with-FB blocks (max-rate of 85%,  $SD = 0.19$  in Problem 28 in comparison with max-rate of 99%,  $SD = 0.06$  in Problem 29). Both reflections of the payoff variability effect are significant,  $t(124) = 3.56$ , and  $8.56$ , both  $p < .001$ .

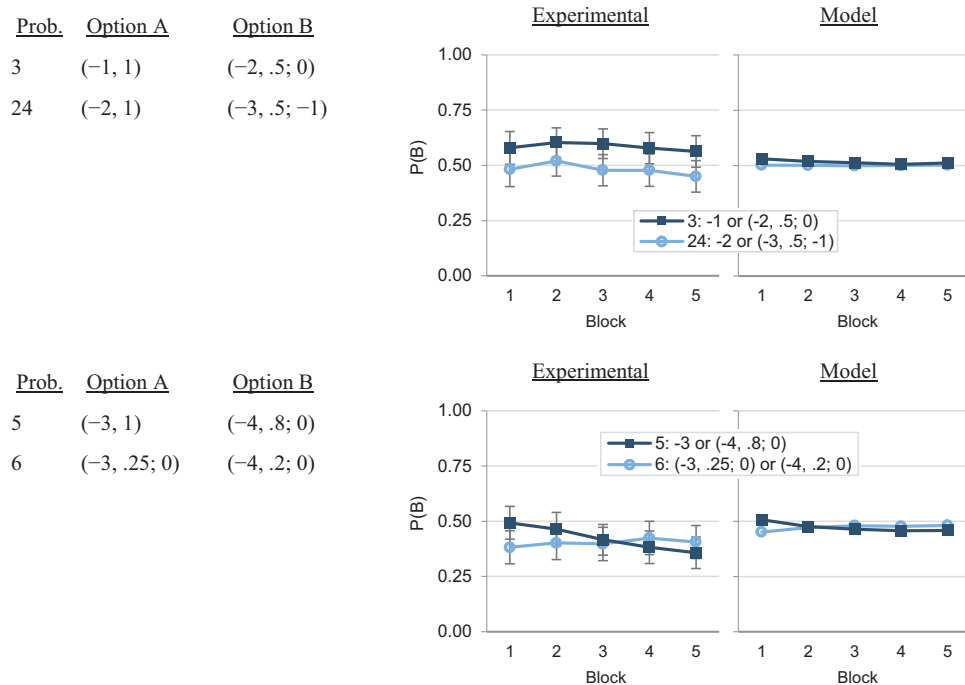
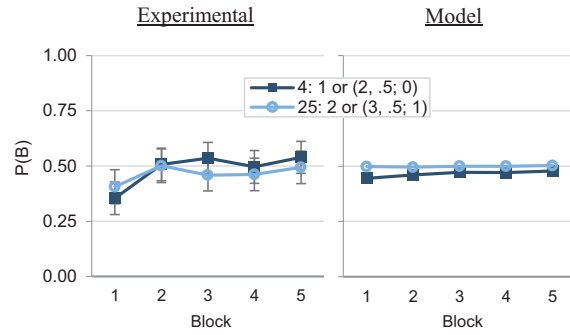
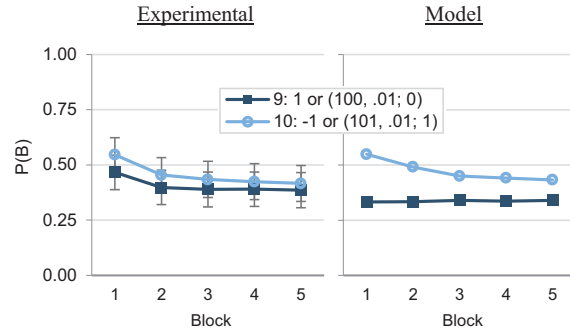


Figure 7. Problems that test the break-even effect in the replication study. The notation  $(x, p; y)$  refers to a prospect that yields a payoff of  $x$  with probability  $p$  and  $y$  otherwise. Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. See the online article for the color version of this figure.

Prob.	Option A	Option B
4	(1, 1)	(2, .5; 0)
25	(2, 1)	(3, .5; 1)



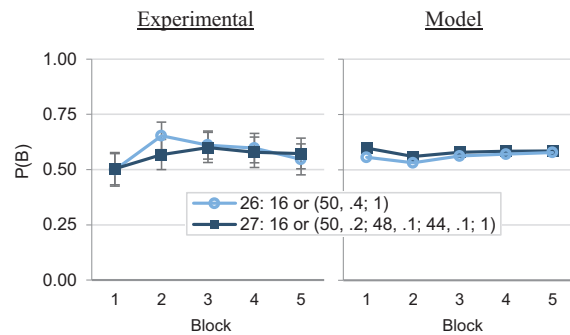
Prob.	Option A	Option B
9	(1, 1)	(100, .01; 0)
10	(2, 1)	(101, .01; 1)



**Figure 8.** Problems that test the get-something effect in the replication study. The notation  $(x, p; y)$  refers to a prospect that yields a payoff of  $x$  with probability  $p$  and  $y$  otherwise. Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. See the online article for the color version of this figure.

A comparison of Problems 28 and 30 highlights the significance of the correlation effect. The positive correlation between the payoffs significantly increased the maximization rate in the with-FB blocks from 85% in Problem 28 to 97% ( $SD = 0.10$ ) in Problem 30,  $t(124) = 7.39$ ,  $p < .001$ . It should be noted that the correlation effect leads to the pattern predicted by regret theory (Loomes & Sugden, 1982). The negative correlation that impairs maximization implies regret in 50% of the trials. The current results suggest that feedback intensifies the impact of regret.

Prob.	Option A	Option B
26	(16, 1)	(50, .4; 1)
27	(16, 1)	(50, .2; 48, .1; 44, .1; 1)



**Figure 9.** Problems that test the splitting effect in the replication study. The notation  $(x_1, p_1; x_2, p_2; \dots; y)$  refers to a prospect that yields a payoff of  $x_1$  with probability  $p_1$ , a payoff of  $x_2$  with probability  $p_2$ ,  $\dots$ , and  $y$  otherwise. Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. See the online article for the color version of this figure.

## Control Conditions and Robustness Checks

In order to disentangle specific features of the replication experiment that might have an effect on the results, we ran two additional control conditions and robustness checks. This section reports briefly on each of the additional measures and their results.

**Single-choice condition.** The analysis of "decisions under risk" in the current design focuses on behavior across the first five trials with no feedback. While the results replicated most of the



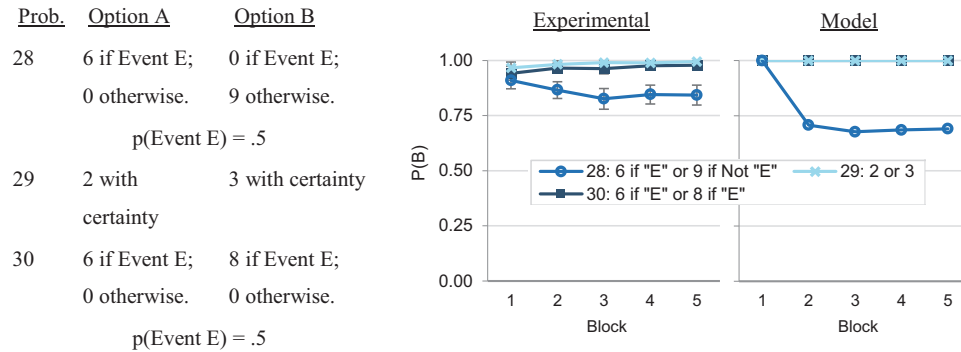


Figure 10. Problems that test the payoff variability and correlation effects in the replication study. The notation  $(x, p; y)$  refers to a prospect that yields a payoff of  $x$  with probability  $p$  and  $y$  otherwise. Option B's choice proportions are shown in five blocks of five trials each (Block 1: "no-FB;" Blocks 2–5: "with-FB"). The experimental results are given with 95% CI for the mean. See the online article for the color version of this figure.

behavioral phenomena from the previous studies, we chose to run an additional condition, using more common experimental design of decisions under risk. This condition used Kahneman and Tversky's paradigm. Each of the 60 participants faced each of the 30 problems only once and without any feedback. Thus, they made one-shot decisions with no feedback for real money. A comparison of the results of this condition with the results of the first block of the replication experiment reveals very similar behavioral patterns. In particular, as noted above, we did not find stronger evidence for overweighting of rare events in the single choice condition. Appendix D shows the mean choice rates for each of the 30 problems in this condition.

**Repetition, feedback, and the "Feedback from 1st" condition.** The clearest effects of experience, described above (Figures 1, 2, 3, and 7), can be summarized with the assertion that experience reduces the weighting of rare events. This effect of experience, in turn, can be either the product of the repetition of the choice process, or the product of the feedback, or both. We compared these interpretations of the results by focusing on learning within the 25 trials. The light curve in Figure 11 shows the proportions of choices that reflect overweighting of rare events (in the nine full information problems, with up to two outcomes, in which the probability of the most extreme outcome is lower than .25; Problems 1, 2, 5, 6, 7, 8, 9, 10, and 11) across the 125 participants of the experimental condition. The results reveal an increase in the weighting of rare events during the five initial no-FB trials and a decrease during the 20 with-FB trials. Interestingly, the unpredicted increasing linear trend in the no-FB trials is significant,  $t(124) = 2.22, p = .028$ .

To clarify this pattern, we ran another condition, "FB from 1st," which was identical to the experimental condition except that the feedback was provided after each choice starting from the very first trial. This condition was run at HUJI and included 29 participants. The proportions of choices consistent with overweighting of rare events in this condition are presented by the dark curve in Figure 11. The results show that in the "FB from 1st" condition, the decrease in overweighting of rare events begins immediately after the first trial. The difference in the linear trends during the first five trials between the "FB from 1st" condition and the

experimental condition is significant,  $t(152) = -2.99, p = .003$  over all subjects, and  $t(87) = -2.41, p = .018$  across the HUJI participants). These results suggest that the effect of experience documented above is driven by the feedback. For some reason, repetition without feedback yields the opposite effect. Appendix D exhibits the mean choice rates for all the 30 problems in this control condition too.

**Individual differences.** Analysis of individual differences (see Appendix E) reveals that the proportion of participants who exhibit the distinct anomalies is larger than the rate expected under random (as well as under maximizing) choice in all 14 cases. In addition, the results reveal relatively low correlations between the different anomalies. For example, the correlation between overweighting of rare events and the Allais pattern is 0.0008, and the

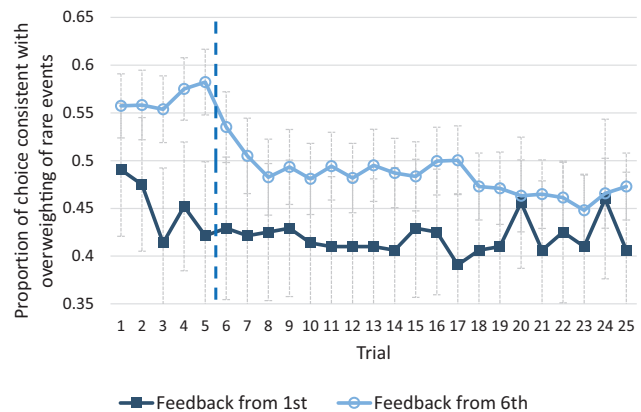


Figure 11. The choice rates that reflect overweighting of rare events in the nine "2-outcome, full information, with rare events" problems. The light curve shows the results across the 125 participants in the replication study, in which feedback was given starting in Trial 6 (the vertical dashed line marks the transition from the No-FB to the With-FB trials in this study). The dark curve shows the results of the 29 participants in Condition "FB from 1st," in which feedback was given starting in Trial 1. Data is shown with 95% CI for the mean. See the online article for the color version of this figure.

correlation between loss aversion and risk aversion in the St. Petersburg problem is only 0.07 ( $p = .46$ ). Most of the large correlations could be the product of a “same choices bias” (the same choice rates are used to estimate two anomalies). The largest correlation free of the same choice bias involves the negative correlation ( $r = -0.39$ ) between overweighting of rare events and risk aversion in the St. Petersburg problem, suggesting that the attitude toward positive rare events reflects a relative stable individual characteristic.

**Effects of location and order.** Recall that the experimental condition was run in two locations, the Technion and HUJI, under two orders (as explained in Footnote 2). Differences were found to be minor. The correlations between HUJI and the Technion and between the two orders were 0.92 or higher. Moreover, for the purposes of the current study, any existing differences were of little interest, as all the behavioral phenomena from Table 1 were reproduced in both locations and most emerged in both locations and order conditions.<sup>7</sup>

### Calibration: Randomly Selected Problems

As noted above, the replication study focuses on 30 carefully selected points in an 11-dimensional space of choice tasks. The results demonstrate that our space is wide enough to replicate the classical choice anomalies. However, our analysis also highlights the fact that the classical problems are a small nonrandom sample from a huge space. Thus, the attempt to develop a model based on the results of the replication study can lead to overfitting the classical anomalies. The current study is designed to reduce this risk of overfitting by studying 60 new problems selected randomly from the space of problems described above. Because the two framing manipulations did not reveal interesting effects in the replication study, the current study focuses only on the abstract representation. Appendix F shows the problem-selection algorithm. This algorithm implies an inverse relationship between risks and rewards (correlation of  $-0.6$ ), a characteristic of many natural environments (Pleskac & Hertwig, 2014). Appendix G details the 60 problems selected.

### Method

One hundred and sixty-one students (81 male,  $M_{\text{Age}} = 25.6$ ) participated in the calibration study.<sup>8</sup> Each participant faced one set of 30 problems from Appendix G: 81 participants faced Problems 31 through 60 and the rest faced Problems 61 through 90. The experiment was run both at the Technion ( $n = 81$ ) and at HUJI. The apparatus and design were similar to those of the replication study. In particular, participants faced each problem for 25 trials, the first five trials without feedback (no-FB), and the rest with full (including the forgone outcome) feedback (with-FB). Participants were paid for one randomly selected trial in one randomly selected problem in addition to a show-up fee (determined as in the replication study). The final payoff ranged between 10 and 144 shekels ( $M = 47.7$ ).

### Results and Discussion

The mean choice rates per block and by feedback type (i.e., no-FB or with-FB) for each of the 60 problems are summarized in

Appendix G. The raw data is available at <https://zenodo.org/record/321652#.WK2oPG996U1>. Below we summarize the main results.

**Full information problems.** Analysis of the 46 full information (nonambiguous) problems (i.e., those that do not involve ambiguity,  $Amb = 0$ ) in which the two options had different expected values (EV) shows a preference for the option with the higher EV. The maximization rate (i.e., choice rate of the higher-EV option) in the no-FB trials was 64% ( $SD = 0.18$ ). In 26 problems, this maximization rate differed significantly from 50% (at .05 significance level, corrected for multiple comparisons according to the procedure by Hochberg, 1988), and in 24 of these, this rate was higher than 50%. In only two problems (Problem 44 and Problem 61, see Figure 12) the maximization rate in the no-FB trials was significantly lower than 50%. The initial deviation from maximization in both problems may reflect overweighting of rare events. Figure 12 shows that feedback reduced these deviations, but did not reverse them.

Feedback increased maximization. Total maximization rate in the with-FB trials was 67% ( $SD = 0.21$ ). In 10 of the 11 problems in which choice rates in the no-FB and with-FB trials significantly differed; maximization rates were higher in the with-FB trials. Analysis of these 10 problems reveals they have a property in common. In all 10 problems, the option that maximizes expected value also minimized probability of immediate regret by providing the higher payoff most of the time (see Erev & Roth, 2014). Congruently, in the only problem in which feedback significantly decreased maximization rates (Problem 39, see Figure 14), the maximizing option provided a better payoff in only 20% of the trials. Figure 13 confirms the generality of this finding. It examines all 46 relevant problems and shows that the higher the proportion of better payoffs generated by the maximizing option (i.e., the lower the probability of regretting a maximizing choice), the larger the increase in maximization rates brought about by feedback (and vice versa). The correlation between the two is 0.65, 95% CI [0.45, 0.79]. In other words, the addition of feedback increased the choice rate of the prospect that minimizes the probability of the regret a decision maker experiences after observing he or she selected the option that generated the lower payoff in a particular trial (see related ideas in Hart, 2005; Loomes & Sugden, 1982; Sarver, 2008).

<sup>7</sup> Some behavioral phenomena, such as the splitting effect and loss aversion, were more common under the ByFB condition, and other phenomena, such as the break-even effect and the reversed reflection effect, were more common under the ByProb condition. The largest effect of the order was observed in Trial 6. The ByProb subjects faced this trial immediately after Trial 5, and exhibited similar behavior to their behavior in that trial. The ByFB subjects faced many other tasks between Trial 5 and Trial 6 (they first completed the No-FB block in all problems, and typically also played some problems, 15 on average, with feedback). This gap was associated with less overweighting of rare events by the ByFB group in Trial 6. Yet, in general, the qualitative differences are minor.

<sup>8</sup> Due to an experimenter error at the Technion, several participants of the current study also participated in the replication study and a few participated in the current study twice. Because this error was only revealed late into the competition, the published data includes these participants. However, we ran robustness checks to make sure that the results are unaffected by these participants: We compared the published mean choice rates with the rates that would be obtained had we excluded their second participation and found virtually no differences between the two.

Prob.	Option A	Option B
44	(23, 1)	(-33, .01; 24)
	[EV=23]	[EV= 23.43]
61	(25, .75; 26)	(23, .025; 24, .95; 25, .0125; 29, .00625; 37, .00313; 53, .00156; 85, .00078; 149, .00078)
	[EV= 25.25]	[EV= 24.25]

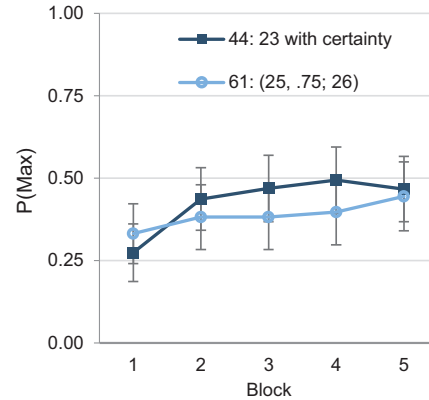


Figure 12. Problems with initial low maximization rates in the calibration study. The notation  $(x_1, p_1; x_2, p_2; \dots; y)$  refers to a prospect that yields a payoff of  $x_i$  with probability  $p_i$ , a payoff of  $x_2$  with probability  $p_2, \dots$ , and  $y$  otherwise. Maximization rates are shown in five blocks of five trials each (Block 1: “no-FB;” Blocks 2–5: “with-FB”), and are given with 95% CI for the mean. See the online article for the color version of this figure.

Figure 14 demonstrates this observation in four problems: two problems (36 and 70) in which feedback moved participants toward maximization and two problems (39 and 52) in which it moved participants away from maximization. The effect of experience in all four problems is consistent with the hypothesis that feedback increases the tendency to select the option that is better most of the time and minimizes the probability of immediate regret. Note, in addition, that the initial maximization rates in three of these problems are in the direction predicted by several phenomena given in Table 1 (loss aversion in Problems 36, 52, and 70; break-even effect in Problem 52; and the certainty effect in Problems 36 and 70).

Figure 15 summarizes the results for the three full information problems that include two options with identical EVs. All three

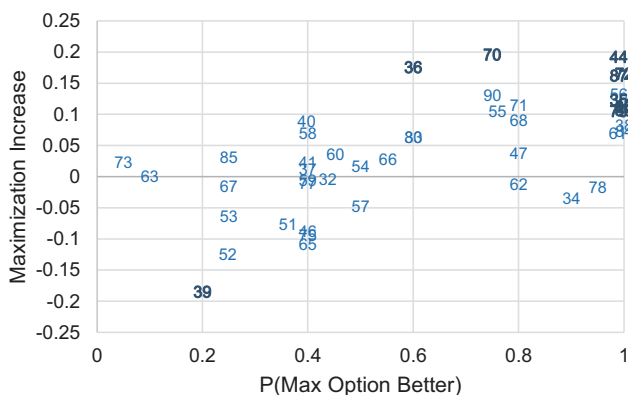


Figure 13. Increase in choice of the maximizing option between with-FB and no-FB trials as a function of the probability that the maximizing option provides higher payoff than the low EV option in a random trial in the calibration study. Each data point represents one problem and is marked with the number of that problem (see Appendix G). The bold dark markers represent problems with maximization increase significantly different from zero. The correlation is .65. See the online article for the color version of this figure.

problems involve choice between a safe gain and a multialternative symmetric-distribution gamble with the same expected value. The observed choice rates in the no-FB trials (Block 1) suggest risk neutrality; the gamble's choice rates were 50%, 54%, and 44% for Problems 45, 49, and 50, respectively ( $SD = 0.45, 0.46$ , and  $0.44$ ). In none of the problems is the difference from 50% statistically significant:  $t(80) = -0.07, 0.75$ , and  $-1.3$ , respectively. Moreover, in two of the three problems, feedback increased risk taking. These results differ from the common observation of risk aversion in the gain domain (e.g., Kahneman & Tversky, 1979) and are consistent with recent studies that demonstrated that feedback can lead to risk seeking in the gain domain (Ludvig & Spetch, 2011; Tsetsos, Chater, & Usher, 2012). The initial risk neutrality could be the product of the multioutcome symmetric distribution used here. Another feasible explanation involves the fact that in Problems 45 and 49 (where we observe the higher risk-seeking rates) the worst possible outcome from the gamble is high relative to the safe alternative.

Another contribution of the full information calibration problems is the suggestion that the loss aversion bias may be less robust to feedback than suggested by Problems 12, 13, and 14 (see Figure 4) of the replication study. For example, in Problem 75 (“13 with certainty” or “50, .6; -45”), experience increased the choice rate of the counterproductive mixed gamble (EV of 12) from 35% to 50%.

**Ambiguous problems.** Results of the 11 ambiguous problems replicate the main findings from the replication study. Specifically, the results show that the initial behavior (no-FB trials) reflects some pessimism and a tendency to maximize expected return assuming all outcomes of the ambiguous option are equally likely. Feedback tends to increase maximization; the choice rates of the ambiguous option tend to decrease when the ambiguous option is objectively inferior to the alternative and increase when the opposite is true.

### Capturing the Joint Effect of the 14 Behavioral Phenomena

The main goal of this section is to propose a model that can capture the joint effect of the 14 phenomena discussed above and the interactions between them. Specifically, the competition's or-

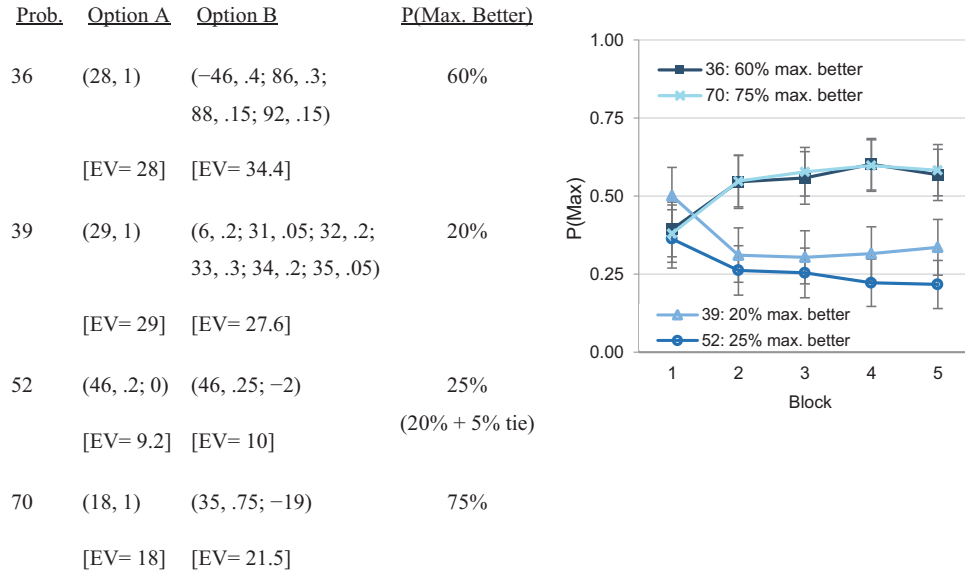


Figure 14. Problems that demonstrate the typical effect of feedback in the calibration study. The notation  $(x_1, p_1; x_2, p_2; \dots; y)$  refers to a prospect that yields a payoff of  $x_1$  with probability  $p_1$ , a payoff of  $x_2$  with probability  $p_2$ ,  $\dots$ , and  $y$  otherwise. P(Max. Better) is the probability that the maximizing option provides better payoff than the low EV option in a random trial. Maximization rates are shown in five blocks of five trials each (Block 1: “no-FB;” Blocks 2–5: “with-FB”) and are given with 95% CI for the mean. See the online article for the color version of this figure.

ganizers (EEP) attempted to develop a simple model that could reproduce the initial choice rates and the effects of feedback in all 90 problems from the replication and calibration studies, and serve as a baseline model for a choice prediction competition.

The relatively high maximization rates led EEP to assume the model should include high sensitivity to the expected value (EV) rule and focus their model development on the deviations from the pre-

scription of this rule. This analysis shows that nearly all of the anomalies can be described as the product of (at least one of) the following four tendencies: (a) *Equal weighting*, a tendency toward the option expected to lead to the best payoff assuming that all the outcomes are equality likely. This tendency captures the Allais paradox, overweighting of rare events, and the splitting effect. (b) *Payoff sign*, a tendency toward the option that maximizes the probability of

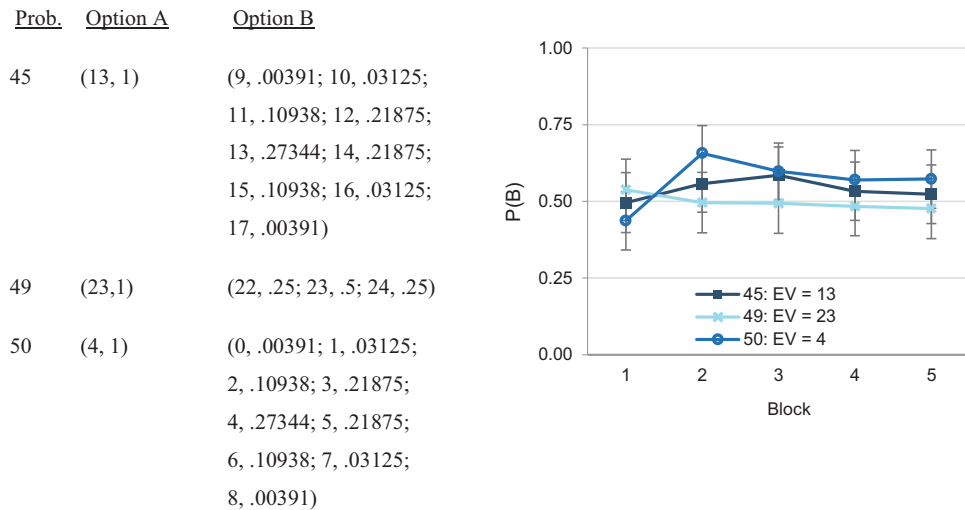


Figure 15. Problems with identical expected values in the calibration study. The notation  $(x_1, p_1; x_2, p_2; \dots; y)$  refers to a prospect that yields a payoff of  $x_1$  with probability  $p_1$ , a payoff of  $x_2$  with probability  $p_2$ ,  $\dots$ , and  $y$  otherwise. Proportions of the riskier choice are shown in five blocks of five trials each (Block 1: “no-FB;” Blocks 2–5: “with-FB”), and are given with 95% CI for the mean. See the online article for the color version of this figure.



the best possible payoff sign. This tendency captures the reflection effect, the break-even effect, and the get-something effect. (c) *Pessimism*, a tendency to assume the worst. This tendency explains the Allais paradox, loss aversion, risk aversion in the St. Petersburg problem, and the Ellsberg paradox. (d) *Minimization of regret*, a tendency to select the option that minimizes the probability of immediate regret. This tendency captures the experience phenomena: underweighting of rare events, reversed reflection, the payoff variability effect, and the correlation effect. The results also suggest that the tendency to favor the option that minimizes the probability of regret increases with feedback.

### Integrating Expected Value and the Four Tendencies

The observation that the main experimental results can be captured with the assertion that decision makers (a) are highly sensitive to the expected value and (b) exhibit four behavioral tendencies, naturally raises the question of how these elements should best be integrated into one model. In answering this question, EEP made three main modeling decisions.

The first decision involves the basic question of how the four tendencies should be implemented. The most popular approach for capturing behavioral tendencies of this sort assumes they are reflections of hard-wired subjective functions of values and/or probabilities. Expected utility theory and prospect theory are two prominent examples of this approach. For instance, prospect theory captures the tendency to prefer the status quo over a mixed gambles with the same EV (see Problem 12) by assuming asymmetric subjective value function, whereas a tendency to overweight rare events and underweight medium-probability events is captured by assuming an S-shape weighting function. As discussed in preceding text, however, the main shortcoming of this popular approach is that it is difficult to find functions that capture all the anomalies addressed here with one set of assumptions and parameters. This shortcoming led EEP to follow an alternative approach for implementing the four tendencies. They assumed that the distinct tendencies reflect the use of specific cognitive strategies, or “tools,” which may be particularly useful in certain settings (Gigerenzer, Todd, & ABC Group, 1999). For example, a tendency to behave pessimistically can be useful when decision makers face adversarial settings. Yet, the use of the tool in settings in which it is inappropriate leads to the behavioral anomalies.

A second modeling decision EEP made involves the output of the decision tools. Assuming that the tools are used to simplify the decision making process, it is natural to use their output to determine the final choice (i.e., prescribe choice of Option A, choice of Option B, or indifference; as in Dhami, 2003; Ert, Erev, & Roth, 2011; Gigerenzer et al., 1999; Payne, Bettman, & Johnson, 1993) Under an alternative abstraction, the output of the tools is “just another estimate” of the expected benefit from a specific choice, and this estimate is weighted with other estimates. EEP chose the just-an-estimate abstraction. Their choice reflects the observation that the results reveal sensitivity to the magnitude (and not only the direction) of the differences in expected values. For example, consider Problem 11 (“19 with certainty” or “.20, .9; −20”, EV = 16) and Problem 75 (“13 with certainty” or “.50, .6; −45”, EV = 12). Both problems are similar in the sense that the tendency to minimize the probability of regret favors the gamble and contradicts the prescription of the EV rule (and the other three tendencies agree with the EV rule). The results show stronger deviations from maximization in Problem 75 (50% in the last

block), than in Problem 11 (21% in the last block). This difference suggests that the impact of the tendency to minimize probability of regret decreases with the expected cost of this behavior.

The final modeling decision EEP made involves the question of whether the values the tools generate are deterministic or depend on the option’s payoff variability. Analysis of the current data shows that the latter possibility is the more reasonable one. For example, compare Problem 23 (“2 with certainty” or “3 with certainty”) with Problem 54 (“18 with certainty” or “.64, .5; −33”, EV = 15.5). All four tendencies agree with the EV rule in both problems. Yet, the results show a large difference: high maximization rates in Problem 23 (97% without and 99% with feedback), and much lower rates in Problem 54 (68% without and 70% with feedback). Thus, the results suggest more choice variability in the problem with higher payoff variability (see a similar observation by Busemeyer, 1985). To address this and similar observations, EEP abstracted the processes that underlie the four tendencies as “sampling tools” (rather than deterministic tools, which are similar to simple heuristics). Specifically, the use of a “sampling tool” implies that the decision maker mentally samples outcomes from payoff distributions that correspond to the underlying tendency. This abstraction is explained in more detail in the next section, which describes the baseline model.

Clearly, making the above three modeling decisions does not define a unique model. There are many possible abstractions that assume high sensitivity to the expected value in addition to four tendencies implemented as the product of sampling tools. We now turn to present the best such model EEP were able to find dubbed “Best Estimate and Sampling Tools” (BEAST).

### Baseline Model: Best Estimate and Sampling Tools (BEAST)

BEAST assumes that the attractiveness of each prospect is the sum of the best estimate of its EV (estimated pessimistically in ambiguous gambles) and the mean value generated by the use of sampling tools that correspond to the four behavioral tendencies. Quantitatively, BEAST assumes that Option A is strictly preferred over Option B, after  $r$  trials, if and only if:

$$[BEV_A(r) - BEV_B(r)] + [ST_A(r) - ST_B(r)] + e(r) > 0 \quad (1)$$

where  $BEV_A(r) - BEV_B(r)$  is the advantage of A over B based on the best estimation of their expected values,  $ST_A(r) - ST_B(r)$  is the advantage of A over B based on the use of sampling tools, and  $e(r)$  is an error term.<sup>9</sup> In trivial choices, when one of the options dominates the other,  $e(r) = 0$ .<sup>10</sup> In all other cases,  $e(r)$  is drawn from a normal distribution with a mean of 0 and standard deviation of  $\sigma_i$  (a property of agent  $i$ ).

When the payoff distributions are known (as in the nonambiguous problems in our study), the best estimations of the expected values are the actual objective ones. That is,  $BEV_j(r)$  equals the expected value of option  $j$ ,  $EV_j$  (for all  $r$ ). The value of option  $j$  based on the use of the sampling tools,  $ST_j(r)$ , equals the average

<sup>9</sup> When the left-hand side of Inequality (1) equals exactly zero, we assume random choice between the options.

<sup>10</sup> In dominance, we mean either deterministic dominance or first-order stochastic dominance. In the first 90 problems (i.e., the replication and calibration studies), the trivial problems are Problems 28, 29, 30, 38, 43, 72, 74, 81, 83, 84, and 89.

of  $\kappa_i$  (a property of  $i$ ) outcomes that are each generated by using one sampling tool.<sup>11</sup>

There are four possible sampling tools. Sampling tool *unbiased* was introduced to capture the tendency to minimize immediate regret that implies a preference for the option that produces a higher outcome most of the time (in one random trial, rather than on average). It can be described as random and unbiased mental draws, either from the options' described distributions or from the options' observed history of outcomes. Before obtaining feedback (decisions in Trials 1 to 6), the draws are taken from the objective distributions using a *luck-level* procedure. First, the agent draws a luck-level, a uniform number between zero and one. Then, for each prospect, the agent uses the same luck-level as a percentile in the prospect's cumulative distribution function and draws the outcome that fits that percentile.<sup>12</sup> When the agents can rely on feedback (Trials 7 to 25), they first sample one of the previous trials (all with-FB trials are equally likely to be sampled), and the drawn outcomes for both options are those observed in that trial.

The other three sampling tools are "biased." They can be described as a mental draw from distributions that differ from the objective distributions. The probability of choosing one of the biased tools,  $P_{Bias}$ , decreases when the agent receives feedback. Specifically,  $P_{Bias}(t) = \beta_i / (\beta_i + 1 + t^{\theta_i})$ , where  $\beta_i > 0$  captures the magnitude of the agent's initial tendency to use one of the biased tools,  $t$  is the number of trials with feedback, and  $\theta_i > 0$  captures agent  $i$ 's sensitivity to feedback.<sup>13</sup> The assumption that the probability of using the unbiased tool increases with feedback was introduced to capture the observation that the main deviations from maximization after obtaining feedback—the four "experience" phenomena in Table 1—suggest increased sensitivity to the probability of regret.

The three biased tools are each used with equal probability,  $P_{Bias}(t)/3$ . The sampling tool *uniform* yields each of the possible outcomes with equal probability (see a related idea by Birnbaum, 2008) using the luck-level procedure described above (the draws are made from the uniform cumulative distribution function even after feedback is obtained). This tool corresponds to the tendency "equal weighting" and therefore helps the model capture the Allais paradox, overweighting of rare events, and the splitting effect.

The sampling tool *contingent pessimism* is similar to the priority heuristic (Brandstätter et al., 2006); it depends on the sign of the best possible payoff (*SignMax*) and the ratio of the minimum payoffs (*RatioMin*). When  $SignMax > 0$  and  $RatioMin \leq \gamma_i$  ( $0 < \gamma_i < 1$  is a property of  $i$ ), this tool yields the worst possible payoffs for each option ( $MIN_A$  and  $MIN_B$ ). It corresponds to the tendency to be pessimistic, and helps the model capture loss aversion, the certainty effect, and risk aversion in the St. Petersburg paradox. When one of the two conditions is not met, the current tool implies random choice among the possible payoffs (identically to the uniform tool). *RatioMin* is computed as:

*RatioMin* =

$$\begin{cases} 1, & \text{if } MIN_A = MIN_B \\ \frac{\min(|MIN_A|, |MIN_B|)}{\max(|MIN_A|, |MIN_B|)}, & \text{if } MIN_A \neq MIN_B \text{ and} \\ 0, & \text{otherwise} \end{cases} \quad \text{sign}(MIN_A) = \text{sign}(MIN_B) \quad (2)$$

For example, *RatioMin* = 0 in Problem 9 ("1" or "100, .01; 0"), and 0.5 in Problem 10 ("2" or "101, .01; 1"). The contingencies

capture two regularities. The sensitivity to *SignMax* implies less pessimism (less risk aversion) in the loss domain, hence the reflection effect. The second, *RatioMin* contingency, implies less pessimism when the minimal outcomes appear similar (have the same sign and are close in magnitude). This implies that the addition of a constant to all the payoffs decreases risk aversion in the gain domain. In addition, it implies higher sensitivity to rare events in problems like Problem 10 and Problem 61 (large *RatioMin*) than in problems like Problem 9 and Problem 25 (small *RatioMin*).

The sampling tool *sign* implies high sensitivity to the payoff sign (Payne, 2005). It is identical to the tool *unbiased*, with one important exception: positive drawn values are replaced by  $R$ , and negative outcomes are replaced by  $-R$ , where  $R$  is the payoff range (the difference between the best and worst possible payoffs in the current problem; e.g., 100 in Problem 9 and Problem 10).<sup>14</sup> It enables the model to capture the reflection effect, the break-even effect and the get-something effect.

When the probabilities of the different outcomes are unknown (as in the problems with ambiguous Option B), they are initially estimated with a pessimistic bias (Gilboa & Schmeidler, 1989). The initial expected value of the ambiguous option is estimated as a weighted average of three terms:  $EV_A$ ,  $MIN_B$ , and  $UEV_B$ , the latter being the estimated EV from Option B, under the assumption that all the possible outcomes are equally likely. The model assumes the same weighting for  $EV_A$  and  $UEV_B$  and captures the weighting of  $MIN_B$  with  $0 \leq \varphi_i \leq 1$ , an ambiguity aversion trait of agent  $i$ . That is,

$$BEV_B(0) = (1 - \varphi_i)(UEV_B + EV_A)/2 + \varphi_i \cdot MIN_B, \quad (3)$$

For example, assuming  $\varphi_i = 0.05$ ,  $BEV_B(0)$  in Problem 22 ("10, .1; 0" or "10,  $p$ ; 0") equals  $.95(5 + 1)/2 + .05(0) = 2.85$ . In decisions made prior to receiving feedback (Trials 1 to 6) the probabilities of the  $m$  possible outcomes are estimated under the assumption that the subjective probability of the worst outcome,  $SP_{MINB}$ , is higher than  $1/m$ , and each of the other  $m - 1$  subjective probabilities equals  $(1 - SP_{MINB})/(m - 1)$ . Specifically,  $SP_{MINB}$  is computed as the value that minimizes the difference between  $BEV_B(0)$  and the estimated expected value from Option B based on the subjective probabilities:  $SP_{MINB} \cdot MIN_B + (1 - SP_{MINB}) \cdot U_{Bh}$ , where  $U_{Bh} = (m \cdot UEV_B - MIN_B)/(m - 1)$  denotes the average of the best  $m - 1$  outcomes. This assumption implies that

<sup>11</sup> For example, consider an agent with  $\kappa_i = 3$  who faces Problem 17 ("30" or "50, .5; -1") based on the following sampling tools results {30, 50}, {30, 50}, and {30, -1} and the error term  $e(r) = -2$ . The left-hand side of Inequality (1) yields  $(30 - 24.5) + (90/3 - 99/3) - 2 = 0.5$ . Thus, the model implies an A choice.

<sup>12</sup> That is, the outcome drawn is the result of  $F^{-1}(x)$ , where  $x$  is the luck-level and  $F^{-1}$  is the prospect's inverse cumulative distribution function. For example, in Problem 2 ("3, .25; 0" or "4, .2; 0"), a luck level of .67 yields the draw {0, 0}, a luck level of .77 yields the draw {3, 0}, and a luck level of .87 yields the draw {3, 4}.

<sup>13</sup> For example, assuming  $\beta_i = 3$ , and  $\theta_i = .5$ , the probability of using one of the biased tools in each of the  $\kappa_i$  simulations is  $3/(3 + 1) = .75$  when  $t = 0$  (Trials 1 to 6),  $3/(3 + 1 + 1) = .6$  when  $t = 1$  (Trial 7), and  $3/(3 + 1 + 3.36) = .407$  when  $t = 19$  (Trial 25).

<sup>14</sup> For example, in Problem 9 ("1" or "100, .01; 0"), all the positive outcomes are replaced by +100 (the value of  $R$ ), and the 0 remains 0.

$$SP_{MINB} = \begin{cases} 0, & \text{if } BEV_B(0) > U_{Bh} \\ 1, & \text{if } BEV_B(0) < MIN_B \\ \frac{U_{Bh} - BEV_B(0)}{U_{Bh} - MIN_B}, & \text{otherwise} \end{cases} \quad (4)$$

That is, in Problem 22 With  $\varphi_i = 0.05$ ,  $SP_{MINB} = (10 - 2.85)/(10 - 0) = 0.715$ .

Each trial with feedback in the ambiguous problems moves  $BEV_B(t)$  toward  $EV_B$ . Specifically,

$$BEV_B(t+1) = (1 - 1/T) \cdot BEV_B(t) + (1/T) \cdot O_B(r) \quad (5)$$

where  $T$  is the expected number of trials with feedback (20 in the current setting) and  $O_B(r)$  is the observed payoff generated from the ambiguous Option B at trial  $r$ .<sup>15</sup>

The six properties of each agent are assumed to be drawn from uniform distributions between 0 and the model's parameters:  $\sigma_i \sim U(0, \sigma)$ ,  $\kappa_i \sim (1, 2, 3, \dots, \kappa)$ ,  $\beta_i \sim U(0, \beta)$ ,  $\theta_i \sim U(0, \theta)$ ,  $\gamma_i \sim U(0, \gamma)$ , and  $\varphi_i \sim U(0, \varphi)$ . That is, the model has six free parameters:  $\sigma$ ,  $\kappa$ ,  $\beta$ ,  $\gamma$ ,  $\varphi$ , and  $\theta$ . Note that only four of these parameters ( $\sigma$ ,  $\kappa$ ,  $\beta$ , and  $\gamma$ ) are needed to capture decisions under risk without feedback (the class of problems addressed by prospect theory). The parameter  $\varphi$  captures attitude toward ambiguity, and  $\theta$  abstracts the reaction to feedback.

BEAST's parameters were estimated using the mean squared deviation (MSD) measure and 14 additional constraints that correspond to the 14 qualitative phenomena summarized in Table 1. Specifically, we used a grid search procedure to find the set of parameters that minimizes the MSD over the 450 B-rates (90 problems  $\times$  5 blocks) and also reproduces the 14 qualitative phenomena. Best fit was obtained with the parameters  $\sigma = 7$ ,  $\kappa = 3$ ,  $\beta = 2.6$ ,  $\gamma = .5$ ,  $\varphi = .07$ , and  $\theta = 1$ . The MSD score is 0.0073. The right-hand graphs in Figures 1 through 10 present the predictions of BEAST with these parameters.

**BEAST, individual differences, and inertia.** Notice that under BEAST, each choice is affected by at least  $6 + \kappa_i$  independent draws from the uniform distribution  $[0, 1]$ , and one draw from a normal distribution. The first six draws determine the agent's properties (the values  $\sigma_i$ ,  $\kappa_i$ ,  $\beta_i$ ,  $\theta_i$ ,  $\gamma_i$ ,  $\varphi_i$ ),  $\kappa_i$  uniform draws determine the sampling tools used (and, in most cases, the outcomes generated by the use of these tools are determined by additional independent uniform draws, e.g., for the luck levels), and the normal draw is the error term. The timing of these draws does not affect the model's aggregate predictions, but it does affect the predicted individual differences and inertia statistics. The basic version of BEAST assumes that the six properties are drawn before the experiment starts (when the virtual agent is "born"), and the other draws are taken before each choice. Appendix E shows that with this assumption the model underpredicts the magnitude of the correlations among the different phenomena. Additional analysis shows that the basic version of BEAST also underpredicts the inertia level (the tendency to repeat the last choice). The observed inertia level is 0.87 before feedback and 0.83 with feedback, and BEAST predictions are below 0.65.

These results remind us that BEAST can be improved, and also suggest a natural way to improve it. Stronger individual differences and higher inertia rates can easily be captured by changing the assumptions concerning the timing of the random draws (e.g., the sampling tools used or luck levels) that underlie the predictions of BEAST. Because changes of this type cannot affect the aggregate

predictions of BEAST, which are the focus of the current article, we chose to leave the refinement of the abstraction of the individual difference statistics to future research.

## A Choice Prediction Competition

The experimental results summarized above suggest that the main deviations from maximization presented in Table 1 can be reliably observed in our 11-dimensional space of problems. In addition, the above analysis suggests that the coexistence of distinct deviations in contradicting directions (e.g., over- and underweighting of rare events) can be captured with a single quantitative model that assumes sensitivity to the expected return and four additional behavioral tendencies (the product of using sampling tools), one of which, regret minimization, becomes more prominent when decision makers can use feedback.

The main shortcoming of this analysis is the fact that BEAST, the quantitative model EEP proposed, lives up to its name: it is not elegant in the sense that the exact implementation of the different sampling tools includes some post hoc assumptions that were introduced to capture the current results. Thus, it is possible that it overfits the data, and better models exist or can be easily developed. Moreover, in developing BEAST, EEP made three major modeling decisions that narrowed the space of possible models they considered, potentially ignoring better descriptive models. In particular, the fact that EEP did not find it easy to develop a useful and elegant model using "subjective functions" of values and probabilities (like prospect theory, and most other decision making models) does not mean that it is impossible to find one. To explore these possibilities, EEP organized a choice prediction competition (see Arifovic et al., 2006; Erev, Ert, Roth et al., 2010; Erev, Ert, & Roth, 2010; Ert et al., 2011; and a recent review of this approach in Spiliopoulos & Ortmann, 2014) using a generalization criterion (Bussemeyer & Wang, 2000). Specifically, EEP ran a third, test study, using the design of the calibration study, and challenged other researchers to participate in an open competition that focuses on the prediction of the results of this experiment. It should be noted that the use of a generalization criterion (which is similar to cross-validation, except that the validation is tested on new experimental designs) for model comparisons greatly reduces the risk of choosing an overly complex model because it "puts the models on equal footing in the generalization stage, despite the fact that the more complex model has an unfair advantage in the calibration stage" (Bussemeyer & Wang, 2000, p. 179).

The participants of the prediction competition were asked to send the organizers a model implemented in a computer program that reads the 10 parameters of each problem as input and provides the predicted mean B-rates in five blocks of five trials as output. The choice problems of the test set were only revealed, together with the main results, a day after the submission deadline. Yet, it was common knowledge that the problems would be sampled from the same space of problems studied in the replication and calibration studies, and that subjects would be drawn from the same student population (Technion and HUJI). The potential competition participants knew that the winning model would be the one with the lowest mean squared deviation score over the five blocks

<sup>15</sup> For example, in Problem 22 with  $\varphi_i = 0.05$ , observing  $O_B(6) = 0$  implies that  $BEV_B(1) = (1 - 1/20) \cdot 2.85 + (1/20) \cdot 0 = 2.707$ .



of trials in the test set. Additionally, to facilitate accumulation of knowledge, the submitted models had to replicate the 14 phenomena from Table 1. Moreover, to facilitate the models' usability for future research they had to be accompanied by a clear, concise verbal description. This latter requirement signifies a distinction between our competition (and most competitions organized in the social sciences) and most tournaments or prediction markets aimed at practical applications (primarily of interest in computer science), which are not interested in interpretability of the successful models (Spiliopoulos & Ortmann, 2014). The winning participants (Cohen and Cohen) were invited to coauthor the current article. Appendix H presents the call for the competition and its detailed requirements.

### Competition Criterion: Mean Squared Deviation (MSD)

The current competition focused on the prediction of the mean B-rates in each of five blocks of trials for each choice problem. As in previous competitions (Erev, Ert, Roth et al., 2010; Erev et al., 2010; Ert et al., 2011), the accuracy of the predictions was evaluated using MSD scores. We first computed the squared difference between the observed and predicted rates in each block of five trials for each of the 60 problems, and then computed the mean across the 300 scores.

The MSD criterion has several advantages over other model estimation criteria (e.g., likelihood criteria). In particular, the MSD score underlies traditional statistical methods (like regression and the  $t$  test), is a proper scoring rule (Brier, 1950; Selten, 1998), and can be translated to the intuitive Equivalent Number of Observations (ENO) score explained below. Note that the use of a proper scoring rule is particularly important in prediction competitions such as ours, and serves to incentivize participants to nominate "truthful" models rather than biased ones, from which it is more difficult to derive theoretical insights.

### Relationship to Previous Competitions

The current competition addresses the critique of previous choice prediction competitions (Spiliopoulos & Ortmann, 2014), which asserted that since competitions are typically run as single implementations, they might be susceptible to auxiliary assumptions. Therefore "before running a tournament it is important to very carefully select the implementation details based on prior studies and knowledge" (Spiliopoulos & Ortmann, 2014, p. 243). The starting point of the current competition, which focused on the replication of 14 well-known behavioral phenomena, follows this proposition. Furthermore, we believe that the requirement that submitted models be clear in their description so other researchers can easily use them should facilitate the models' parsimony and usability.

Another relationship to previous competitions involves the similarity of BEAST to the model that won the learning-in-games competition (Erev et al., 2010). This model (Chen, Liu, Chen, & Lee, 2011) assumes high sensitivity to the estimated expected value (average payoff), some initial biased tendencies, and reliance on small samples of past experiences that implies immediate regret minimization. The main difference between BEAST and the 2010 competition winner is the nature of the initial biased tendencies. Hence, these tendencies appear to be situation-specific.

### Competition Submissions

Fifty-three teams of researchers registered for the competition. Twenty-five of these teams, with members from five continents, submitted models. The submissions can be classified into three main categories. The first class, which contains three submissions, builds primarily on the traditional research approach assuming behavior can be well described by subjective functions of values and probabilities. Specifically, the typical model in this class uses a variant of prospect theory that assumes that the parameters of the underlying subjective functions are situation-specific.

The second class, which consists of 14 submissions, includes models that can be described as variants of BEAST. In particular, all these models assume high sensitivity to the expected value and four behavioral tendencies that can be captured by assuming the use of four sampling tools. Moreover, all but one of these models also share with BEAST the three modeling decisions EEP made and are discussed above.

The third class, which includes seven submissions, involves models that do not attempt to directly identify the underlying processes, but rather use machine learning or similar statistical approaches. Three of the models in this class rely on theoretical insights taken from the decision making literature as input (explanatory variables, or "features") for the models. The four other models in this class can be considered theory-free in the sense that the features used in these models include primarily the structure of the task (the dimensions that define it), in the hope that the model can predict the choice rates based on these basic features.

One submission does not fit in any of these three categories. It is a variant of instance-based learning (IBL; Gonzalez, Lerch, & Lebiere, 2003) that uses the described payoff distributions to create mental instances prior to obtaining feedback and then applies an IBL model as usual.

### The Test Set

As noted above, the test set includes 60 problems selected randomly from the space we study, using the algorithm described in Appendix F. As implied by the wide space of problems and the random drawing mechanism, the test set included different problems than those examined in the replication and calibration studies. The selected problems are shown in Appendix I.

**Method.** One-hundred and sixty (72 male,  $M_{\text{Age}} = 24.5$ ) students from the Technion ( $n = 80$ ) and HUJI participated in the competition study. Half the participants faced Problems 91 through 120 and the others faced Problems 121 through 150 (see Appendix I). The apparatus and design were identical to those of the calibration study. The final payoff ranged between 10 and 149 shekels ( $M = 38.9$ ).

**Results.** The main experimental results are summarized in Appendix I. The results appear to be similar to those of the calibration study. First, where relevant, the maximization rate of the full information problems was 68% ( $SD = 0.17$ ) in the no-FB trials and 72% ( $SD = 0.19$ ) in the with-FB trials. Second, Figure 16 (cf. Figure 13) shows the robustness of participants' tendency to minimize experienced regret by learning to choose the option that is better most of the time. The correlation between the measures in Figure 16 is 0.71, 95% CI [0.55, 0.82]. Third, in the only nonambiguous problem in which both options had identical EVs



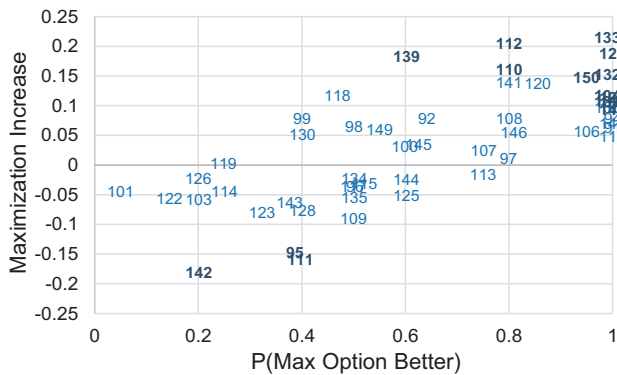


Figure 16. Increase in choice of the maximizing option between with-FB and no-FB trials as a Function of the probability that the maximizing option provides higher payoff than the low EV option in a random trial in the competition study. Each data-point represents one problem and is marked with the number of that problem (see Appendix I). The bold dark markers represent problems with maximization increase significantly different from zero. The correlation is .71. See the online article for the color version of this figure.

(Problem 129), participants seem to exhibit risk neutrality: B-rates are 46% in the no-FB trials and 51% in the with-FB trials. The difference from 50% is insignificant in every block of trials. Finally, behavior in the ambiguous problems reflects an initial tendency to assume uniform probabilities with some ambiguity aversion, and learning toward maximization.

### Performance of the Baseline Model, BEAST

Figure 17 presents the correlation between the observed B-rates (Y-axis) and the predictions of the baseline model BEAST (X-axis) in the no-FB and in the with-FB blocks. Each point presents one of the 60 problems (using the problem numbers, see Appendix I). The correlations are 0.95 in both the no-FB and with-FB blocks. Correlations in each block separately are above 0.94 each. The figure also presents the problems in which BEAST deviates most from the observed rates. Notice that the failure of BEAST in Problem 122 implies less overweighting of rare events than predicted by BEAST. We discuss the other large misses (Problems 93, 126, and 137) below.

The MSD score of BEAST, across the five blocks and 60 problems, is 0.0098. Although this value is larger than the corresponding value in Studies 1 and 2 (0.0073), considering the fact that the parameters were estimated to fit the replication and calibration studies, the difference is not large. To clarify the implication of the MSD score, we computed the implied ENO score (Erev, Roth, Slonim, & Barron, 2007). The ENO of a model is an estimation of the number of subjects that has to be run until the mean choice rate in each problem provides a better prediction for the behavior of the next subject than the prediction of the model. The ENO of BEAST in the current setting is 12.07. That is, the average error of BEAST, over the 60 problems, is smaller than the average error when predicting the next subject based on the mean choice rates of the first 12 subjects.

### The Winning Model and the Performance of Other Submissions

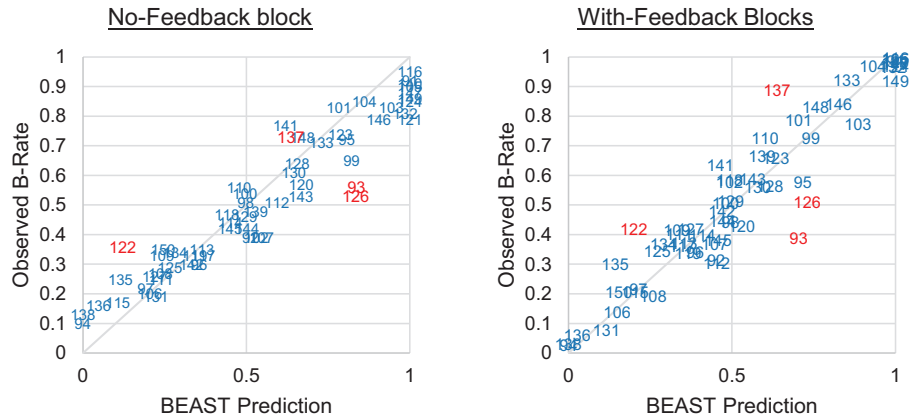
Twenty-five models were submitted to the competition, and were tested together with the baseline model, BEAST. Four models failed to replicate at least one of the phenomena in Table 1 (i.e., failed to comply with the “replication” criterion, detailed in the competition’s website, as explained in Appendix H).<sup>16</sup> The other 21 models were ranked according to their predictions’ MSD on the test set. All 12 top models were variants of the BEAST model. These models’ MSDs ranged between 0.0089 and 0.0114. Seven of these models had better MSDs than the baseline BEAST. The other nine models achieved MSDs ranging between 0.0124 and 0.0501.

The winning model, referred to as Cohen’s BEAST (see Appendix J for details) is a version of BEAST that includes a tendency to avoid “complex prospects” that include multiple outcomes and/or ambiguity when the payoff range is large, as in Problems 93 and 126. The model also includes a tendency to favor such complex prospects when the payoff range is narrow, as in Problem 137. Cohen’s BEAST achieved MSD of 0.0089 and ENO of 13.51. It also outperformed all other variants of BEAST in fitting the calibration data (MSD of 0.0059).

To examine the robustness of the competition’s ranking, we performed a bootstrap analysis on the submitted models’ scores compared to the competition’s winner. Specifically, we simulated 1,000 sets of 60 test problems each by randomly sampling from the original test set (with replacement) and computed the MSD of each submitted model in each simulated set. Then, we computed the 1,000 differences between each model’s MSDs and the winner’s MSDs. By removing the 25 smallest MSD differences and the 25 largest MSD differences, we constructed for each model a 95% confidence interval around its performance relative to the competition’s winner. This procedure allows for a more robust estimate of the relative model performance, reducing the dependency on the exact (random) selection of the test set.

The results of this exercise show that the performance of 12 submitted models does not differ significantly from the performance of the winner: Those ranked 2 through 9 and 11 through 14 in the competition. (The model ranked 10 was similar to Cohen’s BEAST in many problems, but consistently less accurate in many other problems.) These results merit additional examination of the similarities and differences among these 12 models and the winner. Our examination reveals that these 13 models share more similarities than differences. Specifically, 11 of the 13 models (including the winner) are close variants of BEAST, whereas the two models that are not close variants of BEAST (ranked 13 and 14 in the competition) share many of BEAST’s assumptions. In particular, all 13 models assume relatively high sensitivity to the difference between the actual expected values (or their best estimates). Moreover, all 13 models assume sensitivity to the probability that one option generates a better outcome than the other. That is, they assume a tendency to minimize immediate regret by preferring the option better most of the time. Finally, all 13 models also reflect additional “biased” behavioral tendencies that BEAST aims to abstract, such as pessimism.

<sup>16</sup> Nevertheless, we tested these models on the test set as well. The results revealed that had these models participated in the competition, they would have been ranked relatively low compared to the other submissions.



Problem	Option A	Option B	B-rate	
			No-FB	With-FB
93	5 with certainty	-9, $p_1$ ; 92, $p_2$ ; 100, $p_3$ ; 104, $p_4$ ; 106, $p_5$ ( $p_1 = .9$ , $p_2 = .0125$ , $p_3 = .0125$ , $p_4 = .025$ , $p_5 = .05$ unknown)	.56	.39
122	-14, .95; 68, .05	-36, .1; -11, .9	.36	.42
126	-8 with certainty	-18, .8; 77, .00313; 78, .0188, 79, .0469; 80, .0625; 81, .0469; 82, .0188; 83, .00313	.53	.51
137	3 with certainty	2, $p_1$ ; 3, $p_2$ ; 4, $p_3$ ; 5, $p_4$ ; 6, $p_5$ ( $p_1 = .025$ , $p_2 = .7$ , $p_3 = .15$ , $p_4 = .1$ , $p_5 = .025$ unknown)	.73	.89

Figure 17. BEAST predictions versus observed B-rates by problem in the competition study. Each data point represents one problem and is marked with the number of that problem (see Appendix I). The diagonal is the best possible prediction. The red markers represent four problems in which BEAST deviates the most from the observed choice rates. The table below details these problems and their observed choice rates. See the online article for the color version of this figure.

The main differences among the 13 models are much subtler. In particular, most of the BEAST-like successful models only differ by assuming different implementation details of the sampling tools, different likelihoods of using the various sampling tools, or one additional behavioral tendency or cognitive strategy. Importantly, only three of the 13 models assume sensitivity to a subjective expected utility construct similar to that assumed by CPT (in addition to assuming sensitivity to the actual EV). Incidentally, these three models were ranked at the bottom of the successful models list (12, 13, and 14 in the competition).<sup>17</sup> These three models include the two most successful non-BEAST variants, both of which use statistical approaches that integrate theoretical insights in order to choose the underlying features. Appendix K provides more details regarding the 12 successful models that did not win the competition.

### Machine Learning and the Abstraction of the Process

Of the seven submissions that did not aim to identify the underlying processes but relied on statistical or machine learning methods, four were ranked lowest of all 25 submissions according to their prediction MSD, whereas two did not predict significantly worse than the competition's winner. We believe that the main

difference between the successful and the less successful submissions in this category is the choice of building blocks (features, x-variables) that the machine learning algorithm or statistical method uses to derive predictions. Indeed, many of the features used by the more successful submissions were similar to the building blocks assumed by BEAST.

In order to evaluate this hypothesis, we performed a post hoc analysis focused around the importance of the features a machine learning algorithm is supplied with. In this analysis, we used one of the most successful machine learning algorithms available, Random Forest (Breiman, 2001; and see Strobl, Malley, & Tutz, 2009 for a review), trained it on the 90 replication and calibration problems and then tested its predictive performance on the 60 test problems. This procedure was conducted twice, and each time the

<sup>17</sup> We also considered a stochastic expected utility model as a benchmark. It assumes that the utility from each outcome  $x$  is:  $U(x) = (\alpha + x)^\beta$ , where  $\alpha$  is a free parameter that captures initial wealth, and  $\beta$  is the risk aversion parameter. In addition, this model assumes a noisy term as in BEAST. The prediction MSD of this model is 0.0183. Next, we examined the statistical significance of this benchmark relative to other models (using a bootstrap analysis), and found that all 13 top models (and the baseline BEAST) significantly outperform it.

algorithm was supplied with a different set of features. First, we supplied it with the dimensions that define the problems (i.e., without any theory-grounded features). The algorithm then implies extremely poor performance (it would have come out last in the competition).

In the second run, we supplied the random forest algorithm with features that capture the main psychological constructs underlying the baseline BEAST. Specifically, we designed 13 features that capture the assertion that choice is driven by sensitivity to the expected return and four behavioral tendencies that are the result of sampling tools (see [Appendix L](#) for details). The prediction MSD implied by this analysis was found to be identical to that of BEAST. Therefore, supplying the machine learning algorithm with theory-based features is fundamental to facilitating its performance.

It should be noted that, although both the random forest algorithm and BEAST use the same underlying psychological constructs, the observation that they imply similar performance is nontrivial. The random forest algorithm structures these constructs in a complex manner, aiming at pure prediction. That is, random forest simultaneously examines many possible complex interactions among the assumed BEAST components and produces the best dynamics that the algorithm can find to account for the training data. Notably, it does not assume a cognitive and/or a learning process. Rather, it produces predictions based on the observed choice rates of similar problems from the training data (and the complex interaction among the constructs defines similarity). BEAST, in contrast, assumes a very specific interpretable interaction among the psychological constructs, one that can potentially shed light on the underlying process.

Of course, the fact that both methods imply the same predictive performance in the current data does not mean that they produce the same model dynamics (in fact, this is highly unlikely). However, it does suggest that it is not easy to find a model that significantly outperforms BEAST, at least not without adding more (or using different) “building blocks.” The competition results corroborate this statement by suggesting that the many attempts to amend BEAST in some way did not result in significantly better performance.

The current method of using random forest with BEAST-inspired features also makes it possible to test whether removal of some of BEAST’s underlying psychological constructs impairs performance meaningfully. Specifically, we ran the random forest algorithm with various subsets of the 13 features that BEAST assumes. [Appendix L](#) provides details of this exercise. The main results show that removal of the sensitivity to the best estimates of the expected values, as well as removal of the features that capture the tendency to minimize regret lead to poor predictive performance. In contrast, removal of features that capture each of the other three behavioral tendencies assumed by BEAST only slightly detracts from the performance in the current data (but recall they are necessary for replication of known anomalies). Yet, removal of all features that capture these three tendencies also significantly impairs performance.

### General Discussion

Experimental studies of human decision making reveal robust deviations from maximization that appear to suggest contradictory biases. For example, people tend to overweight rare events in

decisions under risk ([Kahneman & Tversky, 1979](#)), but to underweight rare events in decisions from experience ([Hertwig et al., 2004](#)). Previous research addressed the contradictory results by presenting different models to capture different experimental paradigms. For example, prospect theory ([Kahneman & Tversky, 1979](#)) focuses on four choice anomalies that emerge in one-shot decisions under risk (no feedback) among numerically described prospects with up to two nonzero outcomes. Similarly, the choice prediction competitions presented in [Erev, Ert, and Roth \(2010\)](#) favor very different models for decisions from description and for decisions from experience.

The effort to find the best model for each paradigm led to many interesting insights, but it also entails important shortcomings. It sheds limited light on the relationship between the different behavioral phenomena and the distinct underlying processes. Thus, it cannot provide clear predictions of behavior in situations that fall between the well-studied paradigms (e.g., cases where people receive some description and have some experience, such as the safer driving vehicle system mentioned above), and, for that reason, cannot resolve Roth’s 1–800 critique.

The current research aimed to address this critique by facilitating the development and comparison of models that address larger sets of situations and phenomena. Specifically, we used Kahneman and Tversky’s method (“replicate several behavioral phenomena in one paradigm and then capture them with a descriptive model”), but increased the number of anomalies we try to capture from 4 to 14. To reduce the risk of overfitting the data, the current project also studied randomly selected problems, focused on predictions, and used a choice prediction competition methodology. The results of this exercise highlight the following observations.

### The Interaction Between the Different Behavioral Biases

The current results demonstrate that the existence of contradictory deviations from maximization does not imply that the different deviations cancel each other out and/or that it is impossible to develop a descriptive model with high predictive value. Indeed, the main interactions between the different biases are not complex. For example, when people can use both description and experience, they initially exhibit overweighting of rare events, but feedback leads them to exhibit the opposite bias. The results suggest high sensitivity to the expected values and four additional tendencies: pessimism, a bias toward the option that maximizes the probability of the best payoff sign, a bias toward the option that leads to the best payoff assuming that all the outcomes are equally likely, and an effort to minimize the probability of immediate regret. In addition, the results show an increase in the tendency to select the option that minimizes the probability of immediate regret with experience.

### Subjective Weighting, Simple Heuristics, and Sampling Tools

The leading models of human decision-making generalize the expected value rule by assuming maximization of weighted subjective values. For example, prospect theory ([Kahneman & Tversky, 1979](#); [Wakker, 2010](#)) assumes the weighting of subjective values (utilities) by a subjective function of their objective probabilities. The current research highlights one shortcoming of this approach. It shows that it

is not easy to find subjective function models that can capture the classical deviations from maximization with a single set of parameters. The attempt to submit models of this type to our competition revealed that they need different parameters to reproduce the 14 anomalies, and that they do not provide good predictions.

Most previous attempts to present alternatives to the subjective functions approach focused on the role of simple heuristics (Brandstätter et al., 2006; Payne et al., 1993). The basic assumption of this research is that people simplify the decision task by using simple rules that lead to satisficing choices. The main shortcoming of this approach is the observation that people behave as if they are weighing the expected value considerations with other considerations. Thus, assuming decision makers indeed use rules, the outputs of these rules are abstracted better as “just-additional-estimates” than as “simple determinants of final choices.”

These shortcomings of the popular subjective functions, and simple heuristics approaches led EEP to consider a third theoretical approach. Specifically, they assumed that decisions makers weigh the best estimate of the expected values with the output of several sampling tools. The results of the current competition suggest that this approach outperforms the more popular approaches. The main observations (high sensitivity to expected return and four tendencies) are naturally abstracted with models like BEAST that assume best estimate and sampling tools.

### The Experience–Description Gap

The current results show that the existence of a large difference between the deviations from maximization in decisions from description and decisions from experience (e.g., Barron & Erev, 2003; Hertwig & Erev, 2009; Weber, Shafir, & Blais, 2004) does not necessarily imply a large difference between the underlying cognitive processes. The coexistence of overweighting of rare events in decisions from description and the opposite bias in decisions from experience can be captured with the hypothesis that the availability of feedback increases the tendency to rely on unbiased draws from the relevant payoff distributions. This hypothesis implies that feedback increases sensitivity to the probability of immediate regret.

### Overgeneralization, Overdiscrimination, and Skinner’s Critique

Skinner (1985) has criticized the early study of judgment and decision making on the ground that the popular cognitive explanations are descriptions of specific generalizations, and are not likely to capture robust features of human behavior. Under Skinner’s critique, organisms always generalize from similar past experiences (behavior is selected by the contingencies of reinforcements), and the classical deviations from maximization are reflections of situation specific overgeneralizations that emerge when the subjects face confusing new problems.

We believe that the current research takes two steps toward addressing this critique. First, it shows that it is not necessary to assume different generalizations to capture each of the classical deviations from maximization. In particular, the four behavioral tendencies assumed by BEAST can be described as generalizations that capture behavior in wide sets of situations, and their abstraction allows predictions of the initial overgeneralizations in new settings. This interpretation of BEAST’s sampling tools assumes that they approximate

common generalizations of past experiences that occurred before the beginning of our experiment. For example, contingent pessimism could reflect generalizations from descriptions provided by salespersons that were found to be too optimistic in retrospect (see Ert & Erev, 2008). Because the descriptions in our studies were accurate, this reasonable tendency implies an overgeneralization in the current context.

In addition, our analysis demonstrates that not all the deviations from maximization reflect overgeneralizations that are eliminated by experience. Specifically, our analysis suggests that feedback increases the tendency to behave as if relying on small samples of experiences in the current task (which is abstracted by BEAST as an increased use of the unbiased sampling tool). This observation is naturally described as overdiscrimination among relevant trials in the current task. According to this account, feedback leads the subjects to rely on small samples of past experiences in the current task because they attempt to respond to patterns (Plonsky, Teodorescu, & Erev, 2015). That is, they believe that the state of nature changes from trial to trial, and select the option that was found to be the best in (the small set of) past experiences that are most similar to the current trial. This attempt approximates the optimal strategy in dynamic settings, but leads to a bias toward the option that minimizes the probability of immediate regret in the current setting.

An alternative, and perhaps simpler, explanation for an increased use of unbiased sampling (and less use of biased tools) with feedback involves the observation that the unbiased sampling tool is likely to be the most effective of all tools. Although it still implies certain deviations from maximization (such as underweighting of rare events), in most cases feedback reinforces its use. Specifically, a strategy selection learning (SSL; Rieskamp & Otto, 2006) framework that assumes the selection of cognitive strategy (or tool) is based on reinforcement learning would predict, in the current setting, transition from the biased tools to the unbiased tool with feedback.

### The Potential and Limitations of Choice Prediction Competitions

The current project highlights the potential of the choice prediction competition methodology, but also reveals some of its shortcomings. The potential lies in the fact that it facilitates studies that focus on wide spaces of problems, can clarify the relationship between different phenomena and different theoretical approaches, and reduce the risk of overfitting the data. Competitions can also help satisfy the goal, commonly attributed to Albert Einstein, that “everything should be made as simple as possible, but not simpler.” Specifically, we believe that a paper that presented the current investigation and baseline model (BEAST) without the competition would be criticized on the grounds that the model is too complex. The competition demonstrates that it is not easy to find a simpler model that allows useful predictions. At least 25 teams have tried already with no success.

One shortcoming of the prediction competition methodology is the observation that the winning model is not significantly better than 12 of the other models. This observation suggests that the main contribution of the current analysis is the clarification of the common features of the leading models, rather than the identification of a single “correct” model. Yet, it should also be noted that the relative high proportion of successful models among those submitted does not mean that it is easy to find a successful model. The set of submitted models suffers from a selection bias. Specifically, models that were found by their developers to perform poorly on the calibration data are



not likely to be submitted in the first place. Indeed, 28 additional groups of researchers registered for the competition but did not submit, and it is very possible that some of them indeed attempted to develop a successful model, but found this task more challenging than they first thought it would be.

Throughout the article we have discussed the risk of overfitting data in the social sciences and the need of studying a large space of tasks. A related risk, which received less focus in the current examination, might be overgeneralizing results obtained from a specific sample of participants (Israeli undergraduates in our example) to broader populations. As such, future competitions should examine diverse populations of interest to increase confidence in the generalizability of their results.

## Summary

The current analysis questions the existence of qualitative differences between decisions from description and decisions from experience. It shows that both classes of decisions can be captured with a model that assumes a single process. Yet, it also shows that the quantitative effect of experience can be very large. Importantly, feedback changes behavior in predictable ways, even when the decision makers can rely on complete description of the incentive structure, and the feedback does not add relevant information. Most of the well-known deviations from maximization in decisions from description, examined here, are eliminated or reversed by feedback.

In addition, our analysis questions the value of assuming that choice behavior reflects the weighting of subjective values by subjective functions of their probabilities, and/or situation-specific cognitive shortcuts. It suggests that the initial (reaction to description) deviations can be captured as reflections of four tendencies: pessimism, maximizing payoff sign, equal waiting, and minimizing the probability of immediate regret. Experience was found to decrease the first three tendencies, and increase the impact of the probability of regret. The results also show high sensitivity to the expected values. This pattern clarifies the conditions under which people are likely to respond to economic incentives: Before gaining experience, high maximization rate is predicted when all four tendencies agree with the prescription of the EV rule. Reliable increase in maximization with experience is predicted when the prospect that maximizes expected return also minimizes the probability of immediate regret (that is, leads to the best ex post payoff most of the time).

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## Appendix A

### Derivation of Lotteries in Multioutcome Problems

When *LotNum* is larger than 1, Option B includes more than two possible outcomes. Simple, but longer, descriptions of the payoff distributions in these cases (list of possible outcomes and their probabilities) appear in the figures of the main text and in <http://departments.agri.huji.ac.il/cpc2015>. The derivation of the exact distributions follows the following algorithm: When *LotNum* > 1 the high outcome in Option B ( $H_B$ ) implies a lottery with *LotNum* outcomes. There are three types of lottery distributions (defined by *LotShape*). “Symm,” “R-skew,” and “L-skew,” and all have expected value equal to  $H_B$  (i.e., the lottery maintains the original expected value of Option B).

In problems with *LotShape* = “Symm,” the lottery’s possible outcomes are generated by adding the following terms to  $H_B$ :  $-k/2$ ,  $-k/2 + 1$ ,  $\dots$ ,  $k/2 - 1$ , and  $k/2$ , where  $k = \text{LotNum} - 1$  (hence the lottery includes exactly *LotNum* possible outcomes). The lottery’s distribution around  $H_B$  is binomial, with parameters  $k$  and  $1/2$ . In other words, the lottery’s distribution is a form of discretization of a normal distribution with mean  $H_B$ . Formally, if in a particular trial the lottery (rather than  $L_B$ ) is drawn (which happens with probability  $pH_B$ ), Option B’s generated outcome is:

$$\begin{cases} H_B - \frac{k}{2}, & \text{with probability } \binom{k}{0} \left(\frac{1}{2}\right)^k \\ H_B - \frac{k}{2} + 1, & \text{with probability } \binom{k}{1} \left(\frac{1}{2}\right)^k \\ \vdots \\ H_B - \frac{k}{2} + k, & \text{with probability } \binom{k}{k} \left(\frac{1}{2}\right)^k \end{cases}$$

In problems with *LotShape* = “R-skew,” the possible outcomes are generated by adding the following terms to  $H_B$ :  $C^+ + 2^1$ ,  $C^+ + 2^2$ ,  $\dots$ ,  $C^+ + 2^n$ , where  $n = \text{LotNum}$  and  $C^+ = -n - 1$ . In problems with *LotShape* = “L-skew,” the possible outcomes are generated by adding the following terms to  $H_B$ :  $C^- - 2^1$ ,  $C^- - 2^2$ ,  $\dots$ ,  $C^- - 2^n$ , where  $C^- = n + 1$  (and  $n = \text{LotNum}$ ). Note that  $C^+$  and  $C^-$  are constants that keep the lottery’s distribution mean at  $H_B$ . In both cases (*R-skew* and *L-skew*), the lottery’s distribution around  $H_B$  is a truncated geometric distribution with the parameter  $1/2$  (with the last term’s probability adjusted up such that the distribution is well-defined). That is, the distribution is skewed: very large outcomes in *R-skew* and very small outcomes in *L-skew* are obtained with small probabilities. For example, if *LotShape* = “R-skew” and *LotNum* = 5 (in which case,  $C^+ = -6$ ), the lottery’s implied distribution is:

$$\begin{cases} H_B - 6 + 2, & \text{with probability } 1/2 \\ H_B - 6 + 4, & \text{with probability } 1/4 \\ H_B - 6 + 8, & \text{with probability } 1/8 \\ H_B - 6 + 16, & \text{with probability } 1/16 \\ H_B - 6 + 32, & \text{with probability } 1/16 \end{cases}$$

If *LotShape* = “L-skew” and *LotNum* = 5 (i.e.  $C^- = 6$ ), the implied distribution is:

$$\begin{cases} H_B + 6 - 2, & \text{with probability } 1/2 \\ H_B + 6 - 4, & \text{with probability } 1/4 \\ H_B + 6 - 8, & \text{with probability } 1/8 \\ H_B + 6 - 16, & \text{with probability } 1/16 \\ H_B + 6 - 32, & \text{with probability } 1/16 \end{cases}$$

(Appendices continue)

## Appendix B

### The Choice Problems and Main Results in the Replication Study

Prob.	Option A			Option B			Lottery		Corr.	Amb	B-rate				
											No-FB	With-FB			
	H	pH	L	H	pH	L	Num	Shape				1	2	3	4
1	3	1	3	4	.8	0	1	—	0	0	.42	.57	.57	.60	.65
2	3	.25	0	4	.2	0	1	—	0	0	.61	.62	.62	.64	.62
3	−1	1	−1	0	.5	−2	1	—	0	0	.58	.60	.60	.58	.56
4	1	1	1	2	.5	0	1	—	0	0	.35	.51	.54	.50	.54
5	−3	1	−3	0	.2	−4	1	—	0	0	.49	.46	.42	.38	.36
6	0	.75	−3	0	.8	−4	1	—	0	0	.38	.40	.40	.42	.41
7	−1	1	−1	0	.95	−20	1	—	0	0	.48	.63	.62	.62	.64
8	1	1	1	20	.05	0	1	—	0	0	.39	.38	.33	.34	.29
9	1	1	1	100	.01	0	1	—	0	0	.47	.40	.39	.39	.39
10	2	1	2	101	.01	1	1	—	0	0	.55	.45	.43	.42	.42
11	19	1	19	20	.9	−20	1	—	0	0	.13	.22	.21	.20	.21
12	0	1	0	50	.5	−50	1	—	0	0	.34	.41	.43	.44	.38
13 <sup>a</sup>	0	1	0	50	.5	−50	1	—	0	0	.36	.37	.40	.37	.36
14	0	1	0	1	.5	−1	1	—	0	0	.49	.45	.42	.41	.38
15	7	1	7	50	.5	1	1	—	0	0	.78	.84	.88	.83	.85
16	7	1	7	50	.5	−1	1	—	0	0	.71	.79	.81	.83	.83
17	30	1	30	50	.5	1	1	—	0	0	.24	.33	.33	.30	.29
18	30	1	30	50	.5	−1	1	—	0	0	.23	.33	.40	.33	.33
19 <sup>b</sup>	9	1	9	9	1	9	8	R-skew	0	0	.37	.39	.36	.31	.30
20	9	1	9	9	1	9	8	R-skew	0	0	.38	.38	.39	.36	.36
21	10	.5	0	10	.5	0	1	—	0	1	.37	.42	.47	.48	.51
22	10	.1	0	10	.1	0	1	—	0	1	.82	.84	.78	.71	.66
23	10	.9	0	10	.9	0	1	—	0	1	.15	.16	.26	.33	.32
24	−2	1	−2	−1	.5	−3	1	—	0	0	.48	.52	.48	.48	.45
25	2	1	2	3	.5	1	1	—	0	0	.41	.50	.46	.46	.49
26	16	1	16	50	.4	1	1	—	0	0	.50	.65	.61	.60	.55
27 <sup>c</sup>	16	1	16	48	.4	1	3	L-skew	0	0	.50	.57	.60	.58	.57
28	6	.5	0	9	.5	0	1	—	−1	0	.91	.87	.83	.85	.84
29	2	1	2	3	1	3	1	—	0	0	.97	.98	.99	.99	1.0
30	6	.5	0	8	.5	0	1	—	1	0	.94	.97	.96	.98	.98

*Note.* B-rates are mean choice rates for Option B, presented in five blocks of five trials each: no-FB is the block without feedback, and with-FB are the blocks with feedback. Simpler but longer descriptions of the payoff distributions appear in Figures 1 through 10 of the main text and in <http://departments.agri.huji.ac.il/cpc2015>.

<sup>a</sup> An accept/reject type problem (the problem is replaced with a proposal to accept or reject a game of chance with Option B's outcomes). <sup>b</sup> A coin-toss type problem (Option B is construed as a game of chance similar to that used by Bernoulli, 1954). Its implied payoff distribution is described in Row 6 of Table 1 in the main text. <sup>c</sup> The implied payoff distribution is described in Row 10 of Table 1 in the main text.

(Appendices continue)



## Appendix C

### Translated Instructions and Examples of the Experimental Screen

The instructions for participants under the ByProb condition (see Footnote 2 in main text) were:

This experiment consists of many games, which you will play one after the other.

In every game, there are multiple trials and in every trial you will have to choose between two options presented on the screen. The choice will be made by clicking on the button that corresponds with the option you have selected, which will be located below that option.

Following some of the trials, there will appear on the selected button the outcome you obtained by selecting that option (this outcome will appear in black font).

On the other button, there will appear the outcome you could have obtained had you selected the other option (the forgone outcome will appear in dull font).

At the end of the experiment, one trial will be selected at random from all the experiment's trials and your obtained outcome in that trial will be your payoff for the performance in the experiment. Trials in which outcomes did not appear on the screen may also be selected to count as your payoff.

Please note: The more trials you have with larger obtained outcomes, the greater the chance you will receive a larger sum of money at the end of the experiment.

The initial instructions for participants under the ByFB condition were:

This experiment consists of many games, which you will play one after the other.

In every game there are multiple trials, and in every trial you will have to choose between two options presented on the screen. The choice will be made by clicking on the button that corresponds with the option you have selected, which will be located below that option.

At the end of the experiment, one trial will be selected at random from all the experiment's trials and your obtained outcome in that trial will be your payoff for the performance in the experiment.

Please note: The more trials you have with larger obtained outcomes, the greater the chance you will receive a larger sum of money at the end of the experiment.

After completing all (150) no-FB trials (five per problem), the participants in the ByFB condition were shown the following instructions:

The first part of the experiment is over.

In the second part of the experiment, there will appear on the selected button the outcome you obtained by selecting that option (this outcome will appear in black font).

On the other button, there will appear the outcome you could have obtained had you selected the other option (the forgone outcome will appear in dull font).

The rest of the instructions remain unchanged.

Screenshot examples of the experimental paradigm appear in Figures C1 through C5. Figure C1 demonstrates a problem with abstract representation and  $Amb = 0$ ; Figure C2 demonstrates a problem with abstract representation and  $Amb = 1$ ; Figure C3 demonstrates the coin-toss framing manipulation; Figure C4 demonstrates the accept/reject framing manipulation; and Figure C5 demonstrates the feedback given to participants following a choice. Note that the location of each option on the screen was counterbalanced and the information regarding correlation between options (bottom row on the screen) only appeared if both options had more than one possible outcome (i.e., when correlation information was relevant).

*(Appendices continue)*

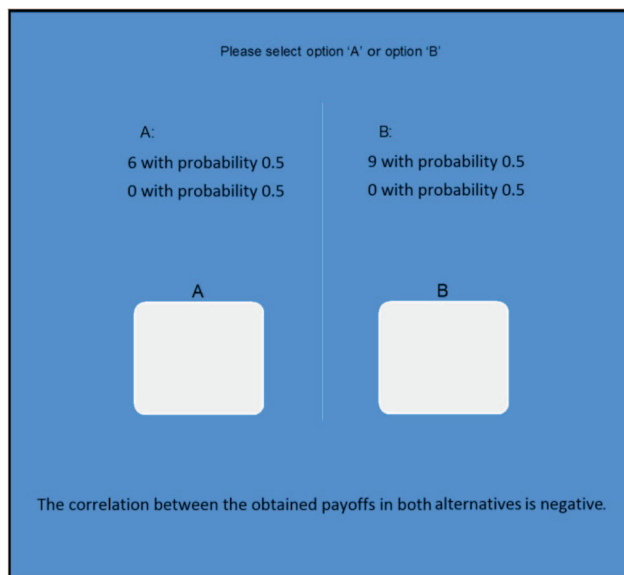


Figure C1. Example of a translated experimental screen in an abstract problem with  $Amb = 0$ . See the online article for the color version of this figure.

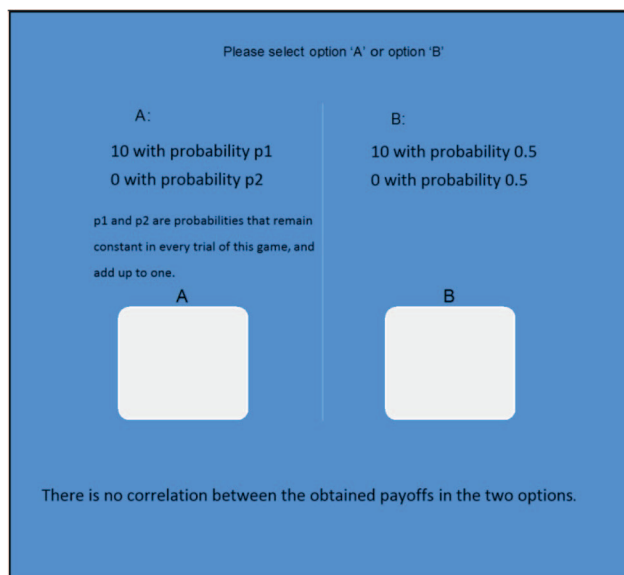


Figure C2. Example of a translated experimental screen in an abstract problem with  $Amb = 1$  (ambiguous problem). See the online article for the color version of this figure.

(Appendices continue)

Please select option 'A' or option 'B'

A:

Playing the following game: a fair coin (i.e., with equal chance for heads or tails) will be flipped consecutively until it comes up heads, but not more than 8 times. Your payoff will be 2 to the power of actual flips of the coin.

B:

9 with certainty (probability 1)

A

B

Figure C3. Example of a translated experimental screen in a problem framed with a “coin-toss” manipulation. See the online article for the color version of this figure.

Are you willing to take the following gamble?

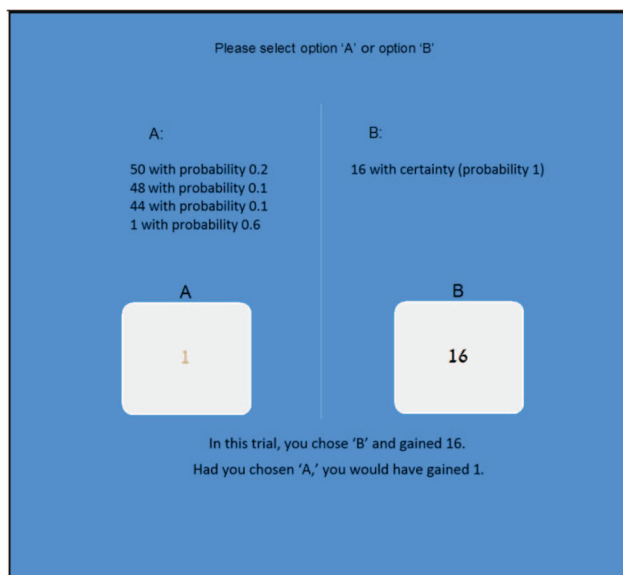
50 with probability 0.5  
-50 with probability 0.5

Yes, I am willing to take the gamble.

No thanks.

Figure C4. Example of a translated experimental screen in a problem framed with an “accept/reject” manipulation. See the online article for the color version of this figure.

(Appendices continue)



*Figure C5.* Example of a translated experimental screen when full feedback is given (Blocks 2–5). The participant here chose Option B. See the online article for the color version of this figure.

*(Appendices continue)*



## Appendix D

## The Main Results in the Control Conditions for the Replication Study

Prob.	Option A			Option B			Lottery		Corr.	Amb	Condition					
	H	pH	L	H	pH	L	Num	Shape			Single choice ( <i>n</i> = 60)	FB from 1st ( <i>n</i> = 29)				
												1	2	3	4	5
1	3	1	3	4	.8	0	1	—	0	0	.48	.54	.61	.66	.63	.66
2	3	.25	0	4	.2	0	1	—	0	0	.52	.56	.56	.58	.66	.62
3	−1	1	−1	0	.5	−2	1	—	0	0	.63	.64	.52	.46	.48	.57
4	1	1	1	2	.5	0	1	—	0	0	.47	.50	.50	.50	.40	.44
5	−3	1	−3	0	.2	−4	1	—	0	0	.62	.46	.31	.28	.31	.31
6	0	.75	−3	0	.8	−4	1	—	0	0	.43	.37	.32	.28	.34	.35
7	−1	1	−1	0	.95	−20	1	—	0	0	.42	.71	.74	.77	.73	.68
8	1	1	1	20	.05	0	1	—	0	0	.35	.17	.19	.18	.20	.24
9	1	1	1	100	.01	0	1	—	0	0	.38	.30	.28	.28	.24	.23
10	2	1	2	101	.01	1	1	—	0	0	.43	.34	.37	.32	.28	.32
11	19	1	19	20	.9	−20	1	—	0	0	.20	.15	.23	.21	.22	.23
12	0	1	0	50	.5	−50	1	—	0	0	.37	.46	.43	.46	.43	.46
13 <sup>a</sup>	0	1	0	50	.5	−50	1	—	0	0	.33	.50	.39	.39	.39	.35
14	0	1	0	1	.5	−1	1	—	0	0	.72	.44	.38	.36	.31	.32
15	7	1	7	50	.5	1	1	—	0	0	.82	.69	.72	.68	.72	.70
16	7	1	7	50	.5	−1	1	—	0	0	.82	.68	.74	.70	.72	.74
17	30	1	30	50	.5	1	1	—	0	0	.25	.32	.37	.30	.31	.26
18	30	1	30	50	.5	−1	1	—	0	0	.25	.37	.37	.37	.38	.39
19 <sup>b</sup>	9	1	9	9	1	9	8	R-skew	0	0	.42	.58	.48	.35	.48	.43
20	9	1	9	9	1	9	8	R-skew	0	0	.33	.51	.46	.44	.36	.36
21	10	.5	0	10	.5	0	1	—	0	1	.27	.43	.50	.55	.48	.50
22	10	.1	0	10	.1	0	1	—	0	1	.87	.77	.68	.60	.61	.59
23	10	.9	0	10	.9	0	1	—	0	1	.15	.23	.26	.22	.21	.26
24	−2	1	−2	−1	.5	−3	1	—	0	0	.60	.51	.46	.35	.48	.45
25	2	1	2	3	.5	1	1	—	0	0	.53	.50	.53	.54	.54	.48
26	16	1	16	50	.4	1	1	—	0	0	.55	.48	.53	.57	.59	.52
27 <sup>c</sup>	16	1	16	48	.4	1	3	L-skew	0	0	.40	.60	.57	.50	.50	.53
28	6	.5	0	9	.5	0	1	—	−1	0	.97	.85	.81	.74	.79	.79
29	2	1	2	3	1	3	1	—	0	0	.97	.98	.97	.98	.99	.98
30	6	.5	0	8	.5	0	1	—	1	0	.93	.88	.92	.93	.97	.98

*Note.* Values in the six right-most columns are mean choice rates of Option B. In the single choice condition, participants made one-shot decisions in each problem. In the FB from 1st condition, participants made 25 repeated decisions of each problem with full feedback following each choice. The choice rates in this condition are presented in five blocks of five trials each. Simpler but longer descriptions of the payoff distributions appear in Figures 1 through 10 of the main text and in <http://departments.agri.huji.ac.il/cpc2015>.

<sup>a</sup> An accept/reject type problem. <sup>b</sup> A coin-toss type problem. <sup>c</sup> The implied payoff distribution is described in Row 10 of Table 1 in the main text.

(Appendices continue)

Appendix E

Observed and Predicted Correlations Among Individual Tendencies to Exhibit 14 Behavioral Phenomena

Phenomenon	1	2	3	4	5	6	7	8	9	10	11	12	13	14	<i>M</i>
1. Allais paradox	—	.39 <sup>†</sup>	-.07	.00	.03	.02	-.13	.00	.00	-.07	.12	.02	-.01	-.14	.33
2. Reflection effect	.32 <sup>†</sup>	—	-.25 <sup>†</sup>	.07	.08	.17	.10	.08	.05	-.10	-.03	.15	.11	.11	.59
3. Reversed reflection	-.04	-.05	—	.08	.04	-.15	-.13	-.18*	.06	.09	-.05	-.04	-.03	.15	.66
4. Overweighting of rare events	-.03	.03	-.08*	—	-.48 <sup>†</sup>	.10	.09	-.39*	-.08	.06	.04	.03	-.13	.16	.54
5. Underweighting of rare events	.02	-.01	.04	-.15 <sup>†</sup>	—	.04	.05	.27*	.08	-.17	.08	-.08	.04	-.15	.58
6. Loss aversion	.01	.03	.04	.04	-.01	—	.51 <sup>†</sup>	.07	.14	.04	-.02	.10	.24*	-.03	.67
7. Magnitude effect of LA	.03	.01	.03	.01	.01	.32 <sup>†</sup>	—	-.08	-.04	.08	-.22*	.11	-.13	.06	.60
8. Risk aversion in St. Petersburg	.00	-.03	.05*	-.09*	.06*	-.01	-.01	—	.20*	-.09	-.01	.06	.05	-.17	.61
9. Ambiguity aversion	.02	-.02	.04	-.01	.05*	.00	.01	.05*	—	.10	-.04	.06	.20*	-.14	.63
10. Break-even effect	.04	.04	.03	.00	.01	.00	.00	-.01	-.01	—	-.10	.00	.05	.05	.56
11. Get-something	.02	.01	.00	-.01	.03	-.01	-.02	.01	.01	.03	—	-.04	-.09	-.02	.52
12. Splitting effect	-.02	.01	.01	.03	.01	-.01	-.03	-.01	.01	.01	.01	—	.20*	.08	.54
13. Payoff variability	.01	.00	.04	-.12*	.09*	.00	-.03	.06	.03	.01	.01	-.02	—	.07	.91
14. Correlation	.05*	.02	.05*	.06*	.07*	-.00	-.00	.08	.03	.04	.03	.00	.1	—	.74
<i>M</i>	.31	.55	.56	.54	.62	.66	.55	.61	.55	.53	.54	.55	.71	.65	

*Note.* Observed intercorrelations are presented above the diagonal, and predicted intercorrelations are presented below the diagonal. Means (*M*) represent the proportion of agents who exhibit the behavioral phenomenon: The observed proportions are shown in the vertical column, and the predicted proportions are shown in the horizontal row. In all cases, agents who exhibit the behavior consistent with the phenomenon are coded as 1, agents who exhibit the opposite behavior are coded as 0, and agents who do not exhibit any bias in their behavior are coded as .5. Predictions are based on 2500 virtual agents of the baseline model BEAST.

<sup>†</sup>  $p < .05$ , but the two measures are based on the same problems. \*  $p < .05$ .

(Appendices continue)

## Appendix F

### The Problem Selection Algorithm

The 60 problems in the calibration study were generated according to the following algorithm. (This algorithm was also used to determine the problems in the competition study.)

1. Draw randomly  $EV_A' \sim \text{Uni}(-10, 30)$  (a continuous uniform distribution).
2. Draw number of outcomes for Option A,  $N_A$ : 1 With Probability .5; 2 Otherwise.
  - 2.1. If  $N_A = 1$  then set  $L_A = H_A = \text{Round}(EV_A')$ ;  $pH_A = 1$
  - 2.2. If  $N_A = 2$  then draw  $pH_A$  uniformly from the set  $\{.01, .05, .1, .2, .25, .4, .5, .6, .75, .8, .9, .95, .99, 1\}$ 
    - 2.2.1. If  $pH_A = 1$  then set  $L_A = H_A = \text{Round}(EV_A')$
    - 2.2.2. If  $pH_A < 1$  then draw an outcome  $temp \sim \text{Triangular}[-50, EV_A', 120]$ 
      - 2.2.2.1. If  $\text{Round}(temp) < EV_A'$  then set  $L_A = \text{Round}(temp)$ ;
 
$$H_A = \text{Round}([EV_A' - L_A(1 - pH_A)]/pH_A)$$
      - 2.2.2.2. If  $\text{Round}(temp) > EV_A'$  then set  $H_A = \text{Round}(temp)$ ;
 
$$L_A = \text{Round}[(EV_A' - H_A \cdot pH_A)/(1 - pH_A)]$$
      - 2.2.2.3. If  $H_A > 150$  or  $L_A < -50$  then stop and start the process over
3. Draw difference in expected values between options,  $DEV$ :  $DEV = \frac{1}{5} \sum_{i=1}^5 U_i$ , where  $U_i \sim \text{Uni}[-20, 20]$
4. Set  $EV_B' = EV_A + DEV$ , where  $EV_A$  is the real expected value of Option A.
5. Draw  $pH_B$  uniformly from the set  $\{.01, .05, .1, .2, .25, .4, .5, .6, .75, .8, .9, .95, .99, 1\}$ 
  - 5.1. If  $pH_B = 1$  then set  $L_B = H_B = \text{Round}(EV_B')$
  - 5.2. If  $pH_B < 1$  then draw an outcome  $temp \sim \text{Triangular}[-50, EV_B', 120]$ 
    - 5.2.1. If  $\text{Round}(temp) < EV_B'$  then set  $L_B = \text{Round}(temp)$ ;
 
$$H_B = \text{Round}([EV_B' - L_B(1 - pH_B)]/pH_B)$$
    - 5.2.2. If  $\text{Round}(temp) > EV_B'$  then set  $H_B = \text{Round}(temp)$ ;
 
$$L_B = \text{Round}[(EV_B' - H_B \cdot pH_B)/(1 - pH_B)]$$
    - 5.2.3. If  $H_B > 150$  or  $L_B < -50$  then stop and start the process over
6. Set lottery (see [Appendix A](#))
  - 6.1. With probability 0.5 the lottery is degenerate. Set  $LotNum = 1$  and  $LotShape = \text{"-"}$
  - 6.2. With probability 0.25 the lottery is skewed. Draw  $temp$  uniformly from the set  $\{-7, -6, \dots, -3, -2, 2, 3, \dots, 7, 8\}$ 
    - 6.2.1. If  $temp > 0$  then set  $LotNum = temp$  and  $LotShape = \text{"R-skew"}$
    - 6.2.2. If  $temp < 0$  then set  $LotNum = -temp$  and  $LotShape = \text{"L-skew"}$
  - 6.3. With probability 0.25 the lottery is symmetric. Set  $LotShape = \text{"Symm"}$  and draw  $LotNum$  uniformly from the set  $\{3, 5, 7, 9\}$
7. Draw  $Corr$ : 0 with probability .8; 1 with probability .1;  $-1$  with probability .1
8. Draw  $Amb$ : 0 with probability .8; 1 otherwise.

In addition, in the following cases the generated problem was not used for technical reasons: (a) there was a positive probability for an outcome larger than 256 or an outcome smaller than  $-50$ ; (b) options were indistinguishable from participants' perspectives (i.e., had the same distributions and  $Amb = 0$ ); (c)  $Amb = 1$ , but Option B had only one possible outcome; and (d) at least one option had no variance, but the options were correlated.

(Appendices continue)

### Appendix G

#### The Choice Problems and Main Results in the Calibration Study

Prob.	Option A			Option B			Lottery		Corr	Amb	B-rate						
	H	pH	L	H	pH	L	Num	Shape			No-FB	With-FB					
											B1	B2	B3	B4	B5	All	
31	4	1	4	40	.6	−44	1	—	0	1	.23	.31	.34	.41	.37	.36	
32	24	.75	−4	82	.25	3	1	—	0	0	.68	.68	.67	.67	.69	.68	
33	−3	1	−3	14	.4	−22	1	—	0	0	.33	.28	.31	.25	.22	.26	
34	7	1	7	27	.1	4	3	Symm	0	0	.39	.45	.43	.41	.40	.42	
35	−5	1	−5	47	.01	−15	1	—	0	0	.18	.09	.04	.03	.05	.05 <sup>a</sup>	
36	28	1	28	88	.6	−46	4	R-skew	0	0	.39	.55	.56	.60	.57	.57 <sup>a</sup>	
37	23	.9	0	64	.4	−7	1	—	0	0	.38	.41	.42	.37	.37	.39	
38	24	1	24	34	.05	28	1	—	0	0	.91	.98	.99	1.0	1.0	.99	
39	29	1	29	33	.8	6	5	Symm	0	0	.50	.69	.70	.68	.66	.68 <sup>a</sup>	
40	3	.8	−37	79	.4	−46	7	L-skew	0	0	.49	.54	.61	.60	.56	.58	
41	29	1	29	44	.4	21	5	Symm	0	0	.68	.73	.74	.68	.68	.71	
42	−6	1	−6	54	.1	−21	1	—	0	1	.61	.48	.25	.23	.20	.29 <sup>a</sup>	
43	14	1	14	12	.9	9	1	—	0	0	.13	.04	.00	.00	.01	.01 <sup>a</sup>	
44	23	1	23	24	.99	−33	1	—	0	0	.27	.44	.47	.49	.47	.47 <sup>a</sup>	
45	13	1	13	13	1	13	9	Symm	0	0	.50	.56	.59	.53	.52	.55	
46	37	.01	9	30	.6	−37	1	—	0	0	.20	.27	.27	.31	.30	.29	
47	11	1	11	57	.2	−5	6	L-skew	0	0	.22	.20	.21	.16	.15	.18	
48	−2	1	−2	24	.5	−24	1	—	0	1	.39	.49	.52	.45	.42	.47	
49	23	1	23	23	1	23	3	Symm	0	0	.54	.50	.49	.48	.48	.49	
50	4	1	4	4	1	4	9	Symm	0	0	.44	.66	.60	.57	.57	.60	
51	42	.8	−18	68	.2	23	1	—	0	0	.79	.71	.74	.72	.70	.72	
52	46	.2	0	46	.25	−2	1	—	0	0	.36	.26	.25	.22	.22	.24	
53	28	1	28	42	.75	−22	1	—	0	0	.36	.43	.42	.42	.42	.42	
54	18	1	18	64	.5	−33	1	—	0	0	.32	.30	.31	.32	.29	.30	
55	43	.2	19	22	.25	17	9	Symm	−1	0	.21	.16	.09	.07	.07	.10	
56	−8	1	−8	−5	.99	−34	1	—	0	0	.76	.88	.91	.90	.89	.90	
57	49	.5	−3	33	.95	17	9	Symm	−1	0	.77	.71	.70	.73	.75	.72	
58	85	.4	−7	40	.25	24	1	—	0	0	.60	.51	.52	.54	.56	.53	
59	17	.25	16	43	.4	2	1	—	0	0	.51	.52	.52	.49	.49	.51	
60	51	.1	21	38	.6	1	1	—	0	0	.37	.38	.34	.29	.30	.33	
61	26	.25	25	29	.05	24	7	R-skew	0	0	.67	.62	.62	.60	.56	.60	
62	25	1	25	45	.2	17	1	—	0	0	.32	.32	.35	.34	.34	.34	
63	17	1	17	60	.1	15	5	Symm	0	0	.68	.70	.67	.66	.69	.68	
64	52	.1	−8	5	.9	−43	1	—	0	1	.35	.55	.70	.72	.68	.66 <sup>a</sup>	
65	12	.4	−16	−5	1	−5	1	—	0	0	.33	.40	.44	.45	.45	.43	
66	45	.6	2	54	.1	20	5	L-skew	0	0	.43	.35	.40	.44	.43	.40	
67	85	.25	4	54	.25	11	1	—	1	0	.45	.47	.46	.48	.43	.46	
68	12	1	12	102	.2	−14	1	—	0	0	.39	.27	.29	.32	.31	.30	
69	49	.5	11	31	.95	21	3	Symm	0	1	.39	.29	.37	.40	.45	.38	
70	18	1	18	35	.75	−19	1	—	0	0	.38	.55	.58	.60	.58	.58 <sup>a</sup>	
71	13	.6	−20	76	.2	−26	1	—	0	0	.38	.25	.29	.23	.28	.26	
72	−9	1	−9	13	.25	−8	1	—	0	0	.82	.96	1.0	1.0	1.0	.99 <sup>a</sup>	
73	2	1	2	51	.05	0	7	Symm	0	0	.37	.38	.39	.39	.41	.39	
74	44	.05	16	14	.9	10	3	Symm	0	1	.13	.05	.02	.00	.00	.02 <sup>a</sup>	
75	13	1	13	50	.6	−45	1	—	0	0	.35	.44	.42	.44	.50	.45	
76	35	.01	16	20	.5	13	5	Symm	0	1	.68	.71	.71	.68	.64	.68	
77	1	1	1	38	.4	−9	1	—	0	0	.65	.66	.65	.60	.63	.64	
78	19	1	19	44	.05	9	1	—	0	0	.11	.12	.11	.14	.12	.12	
79	32	.01	19	65	.01	9	1	—	0	0	.14	.07	.04	.02	.02	.03 <sup>a</sup>	
80	3	1	3	50	.4	−36	1	—	0	0	.47	.37	.41	.41	.43	.40	
81	10	.25	2	−1	.9	−32	1	—	0	1	.14	.04	.01	.01	.01	.02 <sup>a</sup>	
82	25	1	25	26	.01	25	7	Symm	0	1	.55	.72	.77	.81	.82	.78 <sup>a</sup>	

(Appendices continue)



Appendix G (continued)

											B-rate						
Option A			Option B			Lottery					No-FB	With-FB					
Prob.	H	pH	L	H	pH	L	Num	Shape	Corr	Amb	B1	B2	B3	B4	B5	All	
83	9	1	9	64	.01	9	1	—	0	0	.87	.96	.98	.98	.99	.98 <sup>a</sup>	
84	27	1	27	22	.99	−7	1	—	0	0	.08	.02	.00	.00	.00	.01	
85	20	1	20	70	.25	6	1	—	0	0	.43	.45	.49	.46	.44	.46	
86	71	.5	−11	61	.75	−49	1	—	0	1	.13	.23	.30	.32	.25	.28 <sup>a</sup>	
87	−2	1	−2	4	.99	−34	7	Symm	0	0	.81	.96	.98	.96	.98	.97 <sup>a</sup>	
88	17	.05	−7	13	.25	−15	1	—	0	1	.68	.57	.51	.44	.37	.47 <sup>b</sup>	
89	17	1	17	44	.1	17	1	—	0	0	.88	.96	.99	1.0	1.0	.99 <sup>a</sup>	
90	10	1	10	31	.75	−49	1	—	0	0	.42	.55	.53	.56	.55	.55	

*Note.* B-rates are mean choice rates for Option B, presented according to blocks of five trials each or according to availability of feedback: no-FB (no feedback) or with-FB (with feedback). The rightmost column shows the mean B-rate across all four with-FB blocks. Values in bold (in the no-FB and all-with-FB columns) are significantly different from .5 at .05 significance level (corrected for multiple testing according to the procedure in Hochberg, 1988). Simpler but longer descriptions of the payoff distributions appear in Figures 12, 14, 15, and 16, and in <http://departments.agri.huji.ac.il/cpc2015>.

<sup>a</sup> Difference between rates in the no-FB and the with-FB trials is significant at a .05 significance level (corrected according to Hochberg, 1988).

## Appendix H

### Call for Competition Submissions and Competition Requirements

The following call was distributed over the mailing lists of leading scientific societies focused on decision research and experimental and behavioral economics in January 2015:

Ido Erev, Eyal Ert, and Ori Plonsky (henceforth “we”) invite you to participate in a new choice prediction competition. The goal of this competition is to facilitate the derivation of models that can capture the classical choice anomalies (including Allais, St. Petersburg, and Ellsberg paradoxes and loss aversion) and provide useful forecasts of decisions under risk and ambiguity (with and without feedback).

The rules of the competition are described in <http://departments.agri.huji.ac.il/cpc2015>. The submission deadline is May 17, 2015. The prize for the winners is an invitation to be a co-author of the article that summarizes the competition (the first part can be downloaded from [http://departments.agri.huji.ac.il/economics/teachers/ert\\_eyal/CPC2015.pdf](http://departments.agri.huji.ac.il/economics/teachers/ert_eyal/CPC2015.pdf)).

Here is a summary of the basic idea. We ran two experiments (replication and estimation studies, both are described in the site), and plan to run a third one (a target study) during March 2015. To participate in the competition you should e-mail us (to [eyal.ert@mail.huji.ac.il](mailto:eyal.ert@mail.huji.ac.il)) a computer program that predicts the results of the target study.

The replication study replicated 14 well-known choice anomalies. The subjects faced each of 30 problems for 25 trials, received feedback after the sixth trial, and were paid for a randomly selected choice. The estimation study examined 60 problems randomly drawn from a space of problems from which the replication problems were derived. Our analysis of these 90 problems (see <http://departments.agri.huji.ac.il/cpc2015>) shows that the classical anomalies are robust, and that the

popular descriptive models (e.g., prospect theory) cannot capture all the phenomena with one set of parameters. We present one model (a baseline model) that can capture all the results, and challenge you to propose a better model. The models will be compared based on their ability to predict the results of the new target experiment. You are encouraged to use the results of the replication and estimation studies to calibrate your model. The winner will be the acceptable model (see criteria details in the site) that provides the most accurate predictions (lowest mean squared deviation between the predicted choice rates and the choice rates observed in the target study).

In addition to the call, the competition’s web site included, among other things, the raw data from the replication and calibration studies, a summary of this data, examples of acceptable submissions (specifically, using BEAST as an example), and the submission rules and requirements. These rules stated that each submission must include its written verbal description and its implementation (coded using either SAS, Matlab, or R). Three requirements were imposed on the submitted models. First, the model was required to have replicated the 14 qualitative phenomena described in Table 1 in the main text (the exact replication criteria were detailed on the competition’s web site). Second, the verbal description was required to be no more than 1,500 words long in addition to up to 300 words of footnotes. Third, the verbal description was required to be clear. The clarity of the model’s description was evaluated by asking skilled behavioral modelers to reproduce the model and its output (using a programming language of their choice) based only on the verbal description.

(Appendices continue)

# Appendix I

## The Choice Problems and the Main Results in the Test Set

Prob.	Option A			Option B			Lottery		Corr	Amb	B-rate						
	H	pH	L	H	pH	L	Num	Shape			No-FB	With-FB					All
											B1	B2	B3	B4	B5		
91	7	1	7	16	.1	10	1	—	0	0	.92	.96	.99	.99	1.0	.99	
92	8	.8	−37	102	.2	−29	1	—	0	0	.39	.28	.35	.31	.32	.32	
93	5	1	5	103	.1	−9	4	L-skew	0	1	.56	.48	.44	.33	.30	.39 <sup>a</sup>	
94	7	1	7	6	.75	1	1	—	0	0	.10	.04	.02	.03	.02	.03	
95	−3	.05	−9	42	.4	−24	6	L-skew	1	0	.72	.60	.57	.57	.57	.58 <sup>a</sup>	
96	35	.5	−47	−10	.75	−15	1	—	0	0	.30	.34	.37	.34	.31	.34	
97	10	1	10	45	.2	−5	1	—	0	0	.22	.14	.27	.24	.21	.22	
98	94	.5	−40	36	.75	−21	7	Symm	0	0	.51	.45	.43	.44	.45	.44	
99	22	1	22	44	.4	15	5	Symm	0	0	.65	.74	.75	.71	.71	.73	
100	18	.6	−29	−1	1	−1	1	—	0	0	.54	.46	.50	.54	.53	.51	
101	28	1	28	73	.05	27	3	Symm	0	0	.83	.83	.79	.77	.76	.79	
102	11	1	11	25	.5	−3	3	Symm	0	1	.39	.51	.59	.62	.59	.58 <sup>a</sup>	
103	27	.8	−4	77	.1	22	6	R-skew	−1	0	.83	.82	.73	.78	.77	.78	
104	−6	1	−6	3	.99	−27	1	—	0	0	.85	.95	.98	.97	.98	.97 <sup>a</sup>	
105	30	1	30	90	.01	36	1	—	0	0	.90	.97	1.0	1.0	1.0	.99	
106	2	1	2	34	.05	−5	5	Symm	0	0	.20	.14	.15	.15	.12	.14	
107	25	1	25	65	.25	9	5	Symm	0	0	.39	.4	.40	.37	.32	.37	
108	16	1	16	91	.2	−11	1	—	0	0	.27	.17	.23	.19	.18	.19	
109	11	1	11	26	.5	−9	1	—	0	0	.33	.41	.47	.38	.40	.42	
110	12	1	12	29	.8	−35	2	L-skew	0	0	.56	.72	.71	.70	.77	.73 <sup>a</sup>	
111	28	1	28	47	.6	−13	1	—	0	0	.25	.41	.41	.40	.39	.40 <sup>a</sup>	
112	−7	1	−7	28	.2	−18	7	Symm	0	0	.51	.36	.31	.29	.26	.31 <sup>a</sup>	
113	9	.95	0	37	.25	−3	6	R-skew	0	0	.35	.37	.37	.38	.37	.37	
114	72	.01	−2	112	.25	−33	1	—	−1	0	.44	.45	.40	.42	.32	.40	
115	50	.4	5	20	.8	−17	7	Symm	0	0	.17	.19	.23	.20	.20	.21	
116	2	1	2	45	.05	3	5	Symm	0	0	.95	.99	1.0	1.0	1.0	1.0	
117	−6	1	−6	7	.5	−30	1	—	0	0	.33	.39	.36	.35	.34	.36	
118	26	1	26	46	.5	10	6	L-skew	0	0	.47	.56	.62	.59	.57	.59	
119	19	.4	12	100	.25	−12	2	R-skew	0	0	.33	.32	.34	.34	.35	.34	
120	−9	.95	−26	−1	.1	−11	1	—	0	0	.57	.41	.43	.46	.42	.43	
121	−8	1	−8	21	.01	0	3	Symm	0	0	.79	.95	.99	1.0	.99	.98 <sup>a</sup>	
122	68	.05	−14	−11	.9	−36	1	—	0	0	.36	.39	.42	.46	.40	.42	
123	28	.75	−13	57	.1	16	1	—	0	0	.74	.64	.66	.70	.63	.66	
124	15	.95	7	42	.01	19	1	—	0	0	.85	.96	.97	.97	.98	.97 <sup>a</sup>	
125	28	1	28	41	.4	12	1	—	0	0	.29	.36	.33	.33	.36	.35	
126	−8	1	−8	80	.2	−18	7	Symm	0	0	.53	.52	.51	.49	.52	.51	
127	4	1	4	29	.6	−40	1	—	0	1	.26	.33	.44	.46	.44	.42 <sup>a</sup>	
128	−3	1	−3	32	.4	−16	1	—	0	0	.64	.57	.56	.57	.56	.57	
129	−2	1	−2	−2	1	−2	9	Symm	0	0	.46	.53	.51	.48	.53	.51	
130	72	.4	−41	16	.01	1	1	—	0	0	.61	.60	.56	.54	.54	.56	
131	18	1	18	45	.01	11	1	—	0	0	.19	.09	.08	.08	.06	.08 <sup>a</sup>	
132	11	1	11	20	.99	4	7	Symm	0	0	.81	.94	.97	.97	.98	.97 <sup>a</sup>	
133	3	1	3	8	.99	−17	9	Symm	0	0	.71	.91	.92	.92	.94	.92 <sup>a</sup>	
134	27	.05	24	31	.5	10	3	Symm	0	0	.34	.34	.38	.38	.37	.37	
135	6	1	6	8	.5	−1	1	—	0	0	.25	.32	.31	.29	.29	.30	
136	4	1	4	25	.01	−5	1	—	0	0	.16	.07	.07	.05	.05	.06	
137	3	1	3	4	.4	3	5	Symm	0	1	.73	.86	.90	.90	.90	.89 <sup>a</sup>	
138	23	1	23	21	.8	16	1	—	0	0	.13	.07	.01	.02	.02	.03 <sup>a</sup>	
139	14	1	14	35	.6	−9	7	Symm	0	0	.48	.67	.70	.64	.65	.67 <sup>a</sup>	
140	−2	1	−2	9	.25	8	1	—	0	0	.91	.98	.98	.99	.98	.98	
141	28	.8	−26	22	.75	2	1	—	0	0	.77	.70	.62	.60	.62	.64	
142	23	1	23	29	.8	−8	1	—	0	0	.30	.44	.43	.51	.54	.48 <sup>a</sup>	

(Appendices continue)

Appendix I (continued)

Prob.	Option A			Option B			Lottery		Corr	Amb	B-rate						
	H	pH	L	H	pH	L	Num	Shape			No-FB	With-FB					
											B1	B2	B3	B4	B5	All	
143	67	.5	−39	93	.25	−15	1	—	0	0	.53	.58	.54	.60	.63	.59	
144	16	.8	12	15	1	15	9	Symm	0	0	.42	.50	.44	.42	.42	.45	
145	17	.5	−27	3	.75	−35	7	Symm	0	0	.42	.43	.42	.34	.34	<b>.38</b>	
146	45	.2	3	75	.05	13	5	Symm	0	0	<b>.79</b>	.82	.84	.87	.84	<b>.84</b>	
147	29	1	29	36	.1	32	7	Symm	0	0	<b>.88</b>	.96	.99	.98	.97	<b>.98<sup>a</sup></b>	
148	65	.01	1	12	.01	3	1	—	−1	1	<b>.73</b>	.81	.82	.84	.85	<b>.83</b>	
149	12	1	12	31	.1	12	3	Symm	0	0	<b>.86</b>	.90	.92	.91	.94	<b>.92</b>	
150	16	1	16	24	.05	12	3	L-skew	0	0	<b>.35</b>	.25	.22	.19	.17	<b>.21<sup>a</sup></b>	

*Note.* B-rates are mean choice rates for Option B, presented according to blocks of five trials each or according to availability of feedback: no-FB (no feedback) or with-FB (with feedback). The rightmost column shows the mean B-rate across all four with-FB blocks. Values in bold (in the no-FB and all-with-FB columns) are significantly different from .5 at .05 significance level (corrected for multiple testing according to the procedure in Hochberg, 1988).

<sup>a</sup> Difference between rates in the no-FB and the with-FB trials is significant at a .05 significance level (corrected according to Hochberg, 1988).

## Appendix J

### The Winning Model's Additions to BEAST

The full verbal description submitted by the winning team, as well as the model's code, are presented on the competition's web site. Below, we only explain how Cohen's BEAST differs from the baseline BEAST.

Cohen and Cohen (hereinafter, CC) observed that the BEAST model predictions deviate systematically from the actual choices reported, in two different cases: (a) Option B includes more than two outcomes (i.e.,  $LotNum > 1$ ); (b) the payoff distribution of option B is unknown (ambiguous problems, i.e.,  $Amb = 1$ ).

CC's model adds an additional criterion that decides the direction of the deviations from the actual choices made by participants. This criterion is the difference between the lowest possible outcome of option A ( $L_A$ ) and the expected value of the lottery option B ( $H_B$ ; i.e. the criterion is  $|L_A - H_B|$ ). To correct for this deviation, CC added a new parameter, *Diffbias*, dependent on the two cases and criterion above.

CC employ these two cases after BEAST has produced its prediction of B choice rates for each of the problems. Then, for each problem, CC's model checks the following: If Option B in the current problem has more than two possible outcomes (i.e.  $Lot-$

$Num > 1$ ), the *Diffbias* parameter is added to each of BEAST's block predictions in the following manner: when  $|L_A - H_B| > 16$ , ( $-Diffbias$ ) is subtracted from the predicted choice rate. When  $|L_A - H_B| \leq 16$ , ( $+Diffbias$ ) is added. Similarly, if the payoff distribution of Option B is unknown ( $Amb = 1$ ), then when  $|L_A - H_B| > 20$ , ( $-Diffbias$ ) is added to BEAST's final prediction for a given problem (across all five blocks), and when  $|L_A - H_B| \leq 20$ , ( $+Diffbias$ ) is added.

If a problem has more than two possible outcomes and is also an ambiguous problem, *Diffbias* is added and/or subtracted (depending on the level of the criterion) both times. Thus, the process of adding or subtracting the parameter is serial and independent. For example, in Problem 69,  $LotNum > 1$  and  $Amb = 1$ . Because for this problem  $|L_A - H_B| = |31 - 11| = 20$ , the first rule subtracts ( $-Diffbias$ ) and then the second rule adds ( $+Diffbias$ ).

The parameter *Diffbias* is a property of the agent, and is assumed to be drawn from a uniform distribution between 0 and *Diffbias*:  $Diffbias_i \sim Uni(0, Diffbias)$ . Best fit of the Calibration set was obtained with  $Diffbias = 0.07$ .

(Appendices continue)

## Appendix K

## Descriptions of Models not Statistically Inferior to the Winner

Model ID	Rank	Prediction MSD	Problems better	Short description
DB49	2	.0093	28	BEAST with the additional assumptions that (a) in trivial problems the error term is small but positive, (b) $pBias$ does not change after more than 10 trials with feedback, (c) the <i>Sign</i> tool is used twice as often as the other biased tools, and (d) the estimates of the model's parameters are slightly different (smaller $K$ and $\psi$ , larger $\sigma$ and $\beta$ ).
EK8	3	.0093	28	In the absence of feedback, uses BEAST. When feedback is available, uses IBL with initial expectations ("prepopulated instances") based on BEAST.
DS24	4	.0096	27	BEAST with increased attractiveness for the option with the higher maximal gain in the gain domain, or the lower minimal loss in the loss domain. Additionally, slightly delayed effect for feedback, increased aversion for symmetric mixed gambles compared to certain outcomes, and some noise in the first two blocks when predictions are extreme.
NS50	5	.0096	27	BEAST, with faster convergence towards using the <i>unbiased</i> tool. Additionally, replacement of the <i>uniform</i> tool with a pessimistic heuristic likely to give more weight to minimal outcomes, and changes to relative probabilities of using the biased tools.
MK51	6	.0096	24	BEAST with changes to relative probabilities of using biased tools (usually less use of the <i>uniform</i> tool)
BEAST	NA	.0098	24	See main text
DA45	7	.0098	26	BEAST with increased pessimism for ambiguous problems.
BS53	8	.0103	30	BEAST with a chance of using mental draw ( <i>luck-level</i> ) for the <i>unbiased</i> tool even after obtaining feedback. Additionally, sensitivity to sequences of outcomes in ambiguous problems and small but positive error in trivial problems.
MS20	9	.0106	25	In the absence of feedback, uses BEAST. When feedback is available, uses a weighted average of BEAST and win-stay-lose-shift strategy.
OY42	11	.0107	18	In the absence of feedback, uses BEAST. When feedback is available, uses either BEAST or a strategy selecting the option providing higher payoff in a few recent trials (equally likely). Additionally, uses diminishing sensitivity for outcomes in all cases.
SH23	12	.0115	22	BEAST, with changes to sampling tools: (a) the <i>unbiased</i> tool replaced with draw from distribution function with probabilities transformed as in CPT; (b) with feedback, the <i>uniform</i> tool draws from actual observed outcomes (as in BEAST's <i>unbiased</i> ); and (c) with feedback, <i>contingent pessimism</i> sometimes becomes optimism (maximin). Additionally, $K = 1$ and the probability of using the new "unbiased" tool converges slower.
GN27	13	.0124	26	A support vector regression with 24–28 features per block per problem including: most of the problem's defining parameters; the differences between the EVs assuming unbiased distributions, assuming outcomes are equally likely, assuming the outcomes are sign-transformed, and assuming the distributions are transformed as in CPT; the probability of one option generating better outcome than the other; <i>SignMax</i> and <i>RationMin</i> (as in BEAST); the prediction of a stochastic version of CPT; the difference between the options' variances; the difference between the options' entropies; the sign of the majority of the possible outcomes; and the model's predictions for the previous blocks. In ambiguous problems, where relevant, uses BEAST mechanisms to get features' values. Additionally, the prediction is weighted with that of stochastic CPT when the problem is considered particularly difficult or with perfect maximization when it is particularly easy (trivial).
WH9	14	.0127	28	A weighted average of two models. One first classifies the problem into one of seven classes and then provides predictions based on a log-linear model. A problem's class depends on the number of outcomes in each option and on their sign. In each class, the log-linear model uses one or more of the following as explanatory variables: the EV; the probability of the option generating better outcome than the other; the value, sign, and probability of the maximal outcome; the sign of the minimal outcome; the expected rank of an obtained outcome relative to all possible outcomes; the sum of cues in which the option is better than the other (cues are EV, expected rank, maximal and minimal outcomes, possible regret, and probability of maximum). The second model is a stochastic version of CPT where probabilities in ambiguous problems are considered equally likely, very small probabilities are neglected, probabilities are slightly skewed towards uniform, and updating of probabilities to be perceived as more extreme with feedback. Additionally, dominant options are automatically chosen.

*Note.* Only submitted models for which a bootstrap analysis of the test set problems suggests they are not statistically inferior (according to their MSD) to the competition's winner are listed. Rank refers to the official ranking in the competition, based on the prediction MSD. *Problems better* refers to the number of problems in the test set (of 60 possible) in which the model provides a better prediction than the competition's winner.

(Appendices continue)



## Appendix L

### Analysis of BEAST's Features Using Random Forest

We have identified 13 features that capture the essence of the baseline model BEAST (Plonsky, Erev, Hazan, & Tennenholtz, in press). The first two features,  $dBEV_0$  and  $dBEV_{FB}$ , capture the differences between the two options' BEVs (best estimate of expected values) as implied by BEAST. Two features are required because BEAST assumes an ambiguous option's BEV changes with feedback. Specifically,

$$dBEV_0 = \begin{cases} EV_B - EV_A, & Amb = 0 \\ (Min_B + UEV_B + EV_A)/3 - EV_A, & Amb = 1 \end{cases}$$

$$dBEV_{FB} = \begin{cases} EV_B - EV_A, & Amb = 0 \\ \frac{1}{2}[(Min_B + UEV_B + EV_A)/3 + EV_B] - EV_A, & Amb = 1 \end{cases}$$

Two additional features capture the logic of sampling using the *unbiased* tool. It should be noted that the average outcome drawn using this tool is already captured by the first two features, but using this tool also implies sensitivity to the probability that one option will generate a higher outcome than the other (i.e., probability of immediate regret in choosing the alternative). Again, two features are required because BEAST assumes different abstractions of this tool before and with feedback. The first feature,  $pBetter_0$ , equals the difference between the probability that Option B generates the better outcome when sampling according to the *luck-level* procedure and the probability that Option A generates the better outcome with this procedure. The second feature,  $pBetter_{FB}$ , equals the same difference, except that instead of using *luck-level*, the draw is made from the observed outcomes (thus, it is sensitive to the correlation between the outcomes).

The next two features capture the use of the *uniform* sampling tool. The first feature,  $pBetter_{Uni}$ , equals the difference between the probability B generates the better outcome when sampling using the *uniform* tool and the probability that A generates the better outcome when sampling using this tool. The second feature,  $dUniEV$ , captures the average outcomes drawn using this tool. It equals the difference between the options' EVs after their distributions are transformed to uniform distributions.

Three other features capture sampling using the *sign* tool.  $dSignEV$  equals the difference between the options' EVs after they are transformed according to the *sign* tool. The two other features,  $pBetter_{Sign0}$  and  $pBetter_{SignFB}$ , capture the differences between the probabilities of drawing a better outcome in B and those of drawing a better outcome in A, before and with feedback (similarly to the  $pBetter_0$  and  $pBetter_{FB}$ ).

The next three features capture the *contingent pessimism* tool. The first,  $dMins$ , equals the difference between the options' minimal outcomes. The other two features capture the conditions under which BEAST assumes this tool is not used. Specifically,  $RatioMin$  is computed as in Equation (2) in the main text, whereas  $SignMax$  equals the sign of the maximal possible outcome in that problem. The final feature captures the sensitivity of BEAST's error term to the presence of dominant options. The feature  $Dom$  equals 1 if Option B dominates Option A, -1 if A dominates B, and 0 otherwise.

In addition to these 13 features, a *block* feature (equals 1, 2, . . . 5) was supplied as input to the Random Forest algorithm. This was done to allow the algorithm to differentiate between the five different blocks generated by each of the 90 problems of the training set, and to allow it to produce different predictions for the different blocks in the test set. Implementing the algorithm with all these features implies an MSD score of 0.0098 on the test set, virtually identical to the MSD of BEAST.

Next, we ran the algorithm 13 additional times, each time removing one of the psychological features described above. The results suggest that all 13 algorithms have similar predictive power on the test set. Their MSDs ranged between 0.0093 and 0.0113. Notice, however, that many of the features are very highly correlated (e.g., the correlation between  $dBEV_0$  and  $dBEV_{FB}$  is 0.93) and therefore removal of one of the features may not have greatly affected the algorithm's performance because another feature compensated for its absence.

To obtain a clearer picture of the importance of BEAST's mechanisms, we re-evaluated the algorithm's performance after removing sets of features, each capturing a different mechanism of BEAST. Specifically, running the algorithm without either  $dBEV_0$  or  $dBEV_{FB}$ —that is without assuming sensitivity to the best estimates of the expected values—implies an MSD score of 0.0168, a 72% decrease in the predictive performance relative to the baseline that includes this mechanism. Running the algorithm without either  $pBetter_0$  or  $pBetter_{FB}$ —that is without assuming a tendency to minimize immediate regret—implies an MSD score of 0.0156, a 59% decrease in the algorithm's relative predictive performance. These results highlight the importance of including both mechanisms for a model to provide useful predictions for the current data.

In contrast, removal of each of the three sets of features that capture the use of each of the biased sampling tools does not lead to a great decrease in predictive performance. Specifically, removal of the two *uniform* features leads to MSD of 0.0104; removal of the three *sign* features leads to MSD of 0.0109; and removal of the three *contingent pessimism* features leads to MSD of 0.0105. However, this does not mean that the use of biased sampling tools is unwarranted. Indeed, removal of all eight features that capture the use of these biased tools implies an MSD score of 0.0162, a 66% decrease in the model's relative predictive performance. Therefore, while it may be possible to provide useful predictions with different abstractions of biased sampling tools (that correspond to certain behavioral tendencies), BEAST correctly captures the importance of incorporating such biases. One advantage of BEAST over the abstractions implied by the various Random Forest algorithms is its relative interpretability.

Finally, running the algorithm without the *Dom* feature leads, surprisingly, to slightly better predictive power: It implies MSD of 0.0094. It seems that BEAST's assumption of a completely different error mechanism for "trivial" games is too strong.

Received January 18, 2016

Revision received December 15, 2016

Accepted December 19, 2016 ■