Selection Probability of Data Channels in Bluetooth Low Energy

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***Abstract*—Bluetooth Low Energy (BLE) is a ubiquitous technology allowing applications in healthcare, wellness and household sensors to have wireless functionality. Even though a frequency hopping mechanism is employed by BLE to combat fading and interference in the 2.4 GHz Industrial, Scientific and Medical (ISM) band, a noticeable probability of collision is expected to arise when the number of communicating BLE devices in the same vicinity increases.**

In this paper, an analytical model to find the selection probability of each of the 37 BLE data channels is presented. The results, which are confirmed by brute-force simulation, are then used to find the probability of collision between collocated BLE pairs and their maximum achievable aggregate throughput.

I. Introduction

Bluetooth Low Energy (BLE) offers a low-cost, low-power, and low-complexity solution for connecting wireless sensors, smartphones, and household devices. BLE can be found in commercially available products such as Fitbit activity and wellness trackers [1], Qualcomm 2netTM Hub wireless gateway [2], iBeacon [3] and others.

The unlicensed 2.4 GHz Industrial Scientific Medical (ISM) band, which is already congested with other technologies, is divided into 40 channels of 2 MHz bandwidth. Three channels are dedicated as advertising channels and carefully positioned to avoid interference from 802.11 networks. The remaining 37 channels are used to exchange data using a frequency hopping mechanism, where each channel is labelled by a channel index that is then selected for next-hop by a dedicated selection algorithm .

In addition to its popular usage as preferred communication protocol in medical devices, BLE is identified as a key enabling technology for the Internet of Things [4], therefore a large number of devices communicating over BLE is expected to exist within close proximity. In this study, we report an analytical model to find the selection probability of each of the 37 BLE data channels and use it to derive the probability of collision between multiple concurrent BLE connections.

The remainder of this paper is organized as follows: Section II provides an overview of literature focusing on BLE. Section III describes BLE data channel selection algorithm. The probability of selecting a given BLE data channel is derived in Section IV. Collision probability and maximum aggregate throughput for concurrent BLE connections is found in Section V. Section VI concludes the paper.

II. Background

BLE was introduced as a part of Bluetooth Specification Version 4.0 [5]. Research found in literature is focused mainly on creating applications for the new technology with an emphasis on healthcare related scenarios. Electrocardiogram (ECG) monitoring systems based on BLE were proposed in [6]–[9]. Glucose [10], blood pressure [11] and other physiological signals monitoring and reporting systems were presented as well [12]. Suggested applications were expanded to include personal security systems [13], motion sensing [14], indoor localization [15], [16] and even investigating BLE as a candidate technology for Vehicular Ad-Hoc Networks (VANET) [17].

Because of the low-energy nature of BLE, several studies were conducted to quantify the energy consumption of devices running BLE. Siekkinen et al. compared BLE energy consumption with that of ZigBee in [18] and concluded that BLE offers superior energy performance. Liu et al. focused on energy consumption of BLE device discovery process in [19] while the work of Kindt et al. provided an extensive analytical model for the energy performance of BLE [20]. Current consumption, latency and other parameters were studied by Gomez et al. in [21] and an analytical model supported by simulation that determines the maximum achievable throughput of BLE to be 236.7 kbps was published in [22]. Hardware supporting BLE was proposed in [23]–[25]. Coexistence of BLE with interfering technologies in the ISM band is addressed in [26], [27].

There is a clear lack of studies addressing the performance of newly presented BLE data channel selection algorithm, which comes in contrast to the numerous studies focusing on Classic Bluetooth frequency hopping algorithm [28], [29]. The results presented in this paper have been confirmed by brute-force supercomputer simulation in previous work of the authors [30]. A detailed investigation of BLE's data channel selection algorithm was reported and a supercomputer was used to execute a loop iterating on all possible inputs of the data channel selection algorithm to observe the data channel selection probability on the output.

III. BLE Data Channel Selection Algorithm

A BLE connection is divided into numerous consecutive connection events; for each event a data channel index is selected through the data channel index selection algorithm. A BLE device can be in one of the following states: standby, advertising, scanning, initiating or connection. A device in scanning state listens to another device in advertising state,

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and then shifts to initiating state to initialize a connection with the advertiser. A CONNECTION_REQ message is then communicated from initiator to advertiser that contains necessary connection parameters, among which are those necessary for the data channel selection algorithm: *hopIncrement (hI)* and *Channel Map (ChM)*.

hI is a random value between 5 and 16 used to govern hopping sequence for consecutive connection events. Using *hI* is intended to simplify the original data channel selection algorithm used in Bluetooth BR/EDR, which required additional inputs (e.g., device clock and BD_ADDR).

ChM is a sequence of 40 bits with three most significant bits reserved for advertising channels that is always set to 0. The first 37 bits correspond to data channels. Bits in *ChM* are labelled bit0 to bit39. A data channel x is classified into either used (represented by setting the corresponding bitX to 1) or unused (represented by setting the corresponding bitX to 0). Used channels form the set of channels S_1 from which the algorithm will select an output. Unused channels form the set of channels S_0 that are avoided by the algorithm. Channel classification is provided by the host managing the BLE controller and can be achieved by either collecting information about the channel conditions or passive band scanning.

Requirement I: The standard [5] requires *ChM* to contain at least two used channels; otherwise a connection cannot be possibly established.

The algorithm selects a data channel index for use during the next connection event by determining the intermediary variable $unmappedChannel$:

$$unmappedChannel = (hI + lastUnmappedChannel) \ mod \ 37 \quad (1)$$

where $lastUnmappedChannel$ is set to 0 for the first connection event and then sequentially assigned the previous value of $unmappedChannel$. We will denote $unmappedChannel$ as uCh.

If uCh is a used channel (i.e., the corresponding bit in *ChM* is set to 1) then $data\ channel\ index = uCh$.
If uCh is an unused channel, the algorithm will remap it to a used channel by creating a remapping table that contains used channel indices in ascending order and employing $remappingIndex = uCh \ mod \ numUsedCh$, where $numUsedCh$ is the count of 1s in a given ChM, as a pointer to select a used channel from among those in the table.

IV. Selection Probability

A data channel index x can be selected through one of the following events:

- E_1: Selecting $uCh = x$, and then finding that x is a used channel according to *ChM*.
- E_2: Selecting another index $uCh = y$, and then finding that y is unused according to *ChM* before remapping y to x through the second stage of the index selection algorithm.

We will denote the event of remapping y to x as R_{yx}.

$$Pr[chIdx = x] = Pr[E_1] + Pr[E_2] \quad (2)$$

$$Pr[E_1] = Pr[uCh = x]Pr[x \in S_1] \quad (3)$$

$$Pr[E_2] = \sum_{y \neq x} Pr[uCh = y]Pr[y \in S_0]Pr[R_{yx}] \quad (4)$$

Assumption I: *ChM* possible combinations occur with equal probability, i.e., *ChM* values that satisfy the BLE specification condition of having at least two bits set to 1 are uniformly distributed with probability:

$$Pr[ChM] = \frac{1}{\sum_{i=2}^{37} \binom{37}{i}} \quad (5)$$

We will denote the set of *ChM* values that satisfy *Assumption I* as S_{ChI}
The probability of any given bitX in a channel map entry in S_{ChI} to be equal to 1 (i.e., the data channel that bitX represents $x \in S_1$) can be found as the ratio of all combinations that have bitX set to 1 over all possible combinations:

$$p = Pr[bitX = 1] = Pr[x \in S_1] = \frac{\sum_{i=1}^{36} \binom{36}{i}}{\sum_{i=2}^{37} \binom{37}{i}} \quad (6)$$

It can be shown using simulation that uCh is uniformly distributed with probability $\frac{1}{37}$ for a sufficiently large number of connection events.

$$Pr[uCh = x] = \frac{1}{37} \quad (7)$$

Substituting (6),(7) in (3), we get:

$$Pr[E_1] = \frac{1}{37}p \quad (8)$$

Moving to investigate $Pr[E_2]$, we discuss $Pr[numUsedCh = u]$ and $Pr[R_{yx}]$.

$numUsedCh$ takes values in $[2, 36]$ following *Requirement I*, given that when $numUsedCh = 37$ the second stage of the algorithm would not have been reached. The probability of $numUsedCh$ equal to u, as examined on S_{ChI}, can be found as the ratio of all combinations that have exactly u bits set to 1 over all possible combinations:

$$Pr[numUsedCh = u] = \frac{\binom{37}{u}}{\sum_{i=2}^{36} \binom{37}{i}} \quad (9)$$

Remapping index, denoted $r : r = y \ mod \ numUsedCh$, will be used as a pointer to select one channel index from among the remapping table entries. Remapping table, denoted T, contains used channel indices of a given *ChM* in ascending order.
Let $T^u_{x,r}$ be the occurrence of a remapping table T constructed from a *ChM* with u bits set to 1, adequate for selecting x as output using r as a remapping index.

$$Pr[R_{yx}] = \sum_{u=2}^{36} Pr[numUsedCh = u]Pr[T^u_{x,r}] \quad (10)$$

We divide any *ChM* bit sequence to three parts: leading bits (LB), bitX, and trailing bits (TB). To yield a given output x,

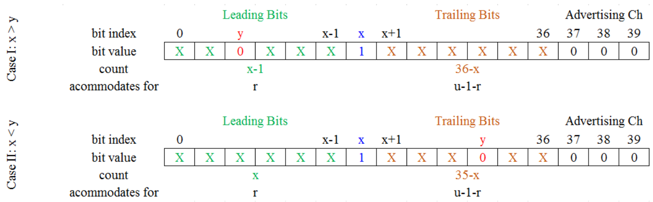

Fig. 1. *ChM* segmentation

a combination of both r and T must occur with respect to the following conditions:

- C_1: `LB` must have exactly r ones (e.g., if $r = 3$ and $x = 7$, the leading bits from `bit0` to `bit6` must contain exactly 3 ones so that the fourth entry in remapping table would be 7 and be chosen by r).
- C_2: `TB` must have exactly the remaining $u - 1 - r$ ones.

In addition to C_1 and C_2, the relative position of `bitX` and `bitY` in *ChM* and the value of u form restrictions on the values of x, r and u, for x to be chosen on output. Therefore, we identify two cases, as illustrated in Fig.1.

Case I: bitY $\in LB \Rightarrow x > y$
The following must be maintained to accommodate an adequate number of 1's:

$$Count\ of\ LB = x - 1 \geq r \tag{11}$$

$$Count\ of\ TB = 36 - x \geq u - 1 - r \tag{12}$$

The solution for these two inequalities is the following:

$$\begin{cases} x = r + 1 & \text{if } u = 36 \\ r + 1 \leq x \leq r - u + 37 & \text{if } u < 36 \end{cases} \tag{13}$$

We define the following indicator function to model restriction (13):

$$I_1(x, u, r) = \begin{cases} 1 & \text{if (13) } is\ true \\ 0 & \text{if (13) } is\ false \end{cases} \tag{14}$$

In S_{ChI}, there exists $\binom{x-1}{r}$ combinations that satisfy C_1 and $\binom{36-x}{u-1-r}$ combinations that satisfy C_2, where $\binom{a}{b}$ is the binomial coefficient. Grouping C_1,C_2, and (14), we arrive at the following for *Case I*:

$$Pr[T^u_{x,r}]_{x>y} = \frac{\binom{x-1}{r}\binom{36-x}{u-1-r}}{\binom{36}{u}} I_1(x, u, r) \tag{15}$$

Case II: bitY $\in TB \Rightarrow x < y$

$$Count\ of\ LB = x \geq r \tag{16}$$

$$Count\ of\ TB = 35 - x \geq u - 1 - r \tag{17}$$

The solution for these two inequalities is the following:

$$\begin{cases} x = r & \text{if } u = 36 \\ r \leq x \leq r - u + 36 & \text{if } u < 36 \end{cases} \tag{18}$$

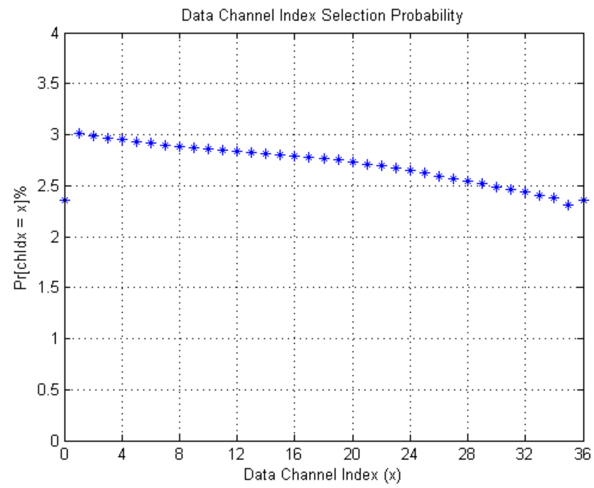

Fig. 2. Probability of selecting a channel index in BLE.

TABLE I. List of Variables

Variable	Description
x	Output of channel index selection algorithm. $x \in [0, 36]$
y	Unused index, selected by the algorithm first stage and remapped to x in the second stage. $y \neq x, y \in [0, 36]$
u	Number of used channels in a given channel map. $u \in [2, 36]$
r	Remapping index. $r \in [0, u]$
p	The probability of any given `bitX` in a channel map in S_{ChI} to be equal to 1. Defined in (6)

We define the following indicator function to model restriction (18):

$$I_2(x, u, r) = \begin{cases} 1 & \text{if (18) } is\ true \\ 0 & \text{if (18) } is\ false \end{cases} \tag{19}$$

In S_{ChI}, there exists $\binom{x}{r}$ combinations that satisfy C_1 and $\binom{35-x}{u-1-r}$ combinations that satisfy C_2. Grouping C_1,C_2, and (19), we arrive at the following for *Case II*:

$$Pr[T^u_{x,r}]_{x<y} = \frac{\binom{x}{r}\binom{35-x}{u-1-r}}{\binom{36}{u}} I_2(x, u, r) \tag{20}$$

Substituting in (10) and then in (4), we find:

$$Pr[E_2] = \frac{1}{37}(1-p)\Big(\sum_{y:y<x}\sum_{u=2}^{36} \frac{\binom{37}{u}}{\sum_{i=2}^{36}\binom{37}{i}} \frac{\binom{x-1}{r}\binom{36-x}{u-1-r}}{\binom{36}{u}} I_1(x,u,r) + \sum_{y:y>x}\sum_{u=2}^{36} \frac{\binom{37}{u}}{\sum_{i=2}^{36}\binom{37}{i}} \frac{\binom{x}{r}\binom{35-x}{u-1-r}}{\binom{36}{u}} I_2(x,u,r)\Big) \tag{21}$$

Ultimately, the probability of selecting a channel index x is:

$$f(x) = Pr[chIdx = x] = \frac{1}{37}p + \frac{1}{37}(1-p) \Big(\sum_{y:y<x}\sum_{u=2}^{36} \frac{\binom{37}{u}}{\sum_{i=2}^{36}\binom{37}{i}} \frac{\binom{x-1}{r}\binom{36-x}{u-1-r}}{\binom{36}{u}} I_1(x,u,r) +$$

$$\sum_{y:y>x} \sum_{u=2}^{36} \frac{\binom{37}{u}}{\sum_{i=2}^{36}\binom{37}{i}} \frac{\binom{x}{r}\binom{35-x}{u-1-r}}{\binom{36}{u}} I_2(x,u,r)) \quad (22)$$

Equation (22) is illustrated in Fig. 2. This result has been confirmed by brute-force simulation of the data channel selection algorithm by iterating on all possible inputs, and then calculating the selection probability [30]. Table I provides a summary of the employed variables in (22).

This model is limited by *Assumption I*, where interference in a wireless environment causes *ChM* to occur with equal probability. Future work would include deriving *ChM* distribution in actual environments either through spectrum surveys or simulation.

V. Probability of Collision and Aggregate Throughput

The probability of selecting a given data channel by a system running BLE has been quantified in the previous section, we can use that information to derive the probability of collision between concurrent BLE connections.

We assume n collocated BLE pairs communicating simultaneously and that selecting the same channel index by any two pairs would result in total loss of packets for both pairs. The probability of successful transmission for n collocated BLE connections can be expressed as:

$$P_S(n) = \sum_{x=0}^{36}(f(x)(1-f(x))^{n-1}) \quad (23)$$

Therefore, the probability of collision is:

$$P_{col}(n) = 1 - \sum_{x=0}^{36}(f(x)(1-f(x))^{n-1}) \quad (24)$$

Fig. 3 provides a plot for $P_{col}(n)$. We notice that the probability of collision increases with the number of concurrent BLE communicating pairs.

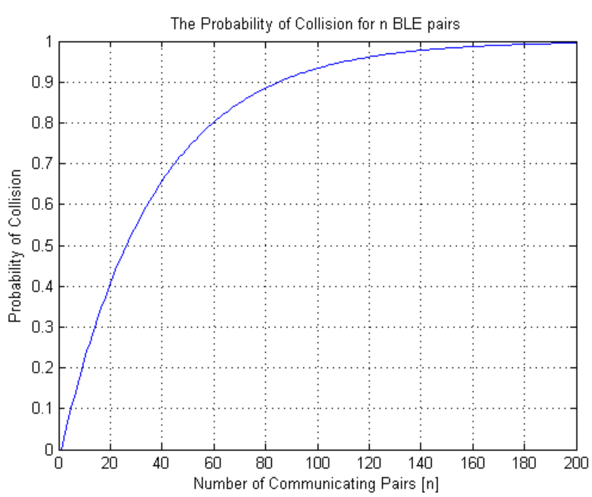

Fig. 3. Probability of collision for n simultaneous BLE connections.

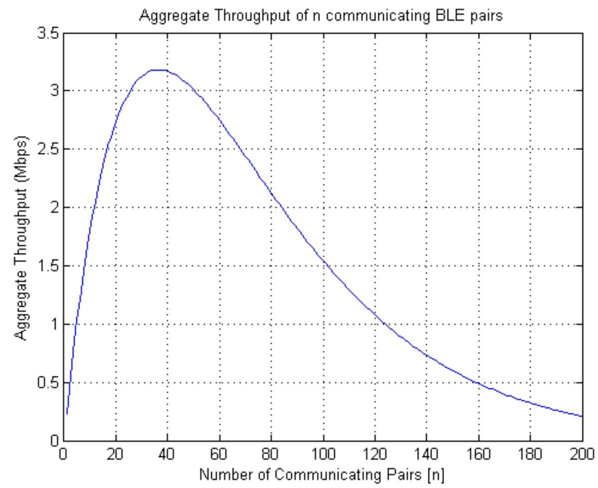

Fig. 4. Maximum aggregate throughput for n simultaneous BLE connections.

The aggregate throughput for n simultaneously communicating BLE pairs is:

$$S(n) = nS_0P_S(n) \quad (25)$$

where S_0 is the maximum achievable BLE throughput on a single connection. S_0 is found to be equal to 236.7 kbps in [22].

Maximum aggregate throughput can then be determined as $S_{maximum} = 3.18\ Mbps$ at $n \approx 36$. Fig. 4 illustrates the behavior of aggregate throughput of n communicating BLE pairs. We notice that the aggregate throughput increases until it reaches $S_{maximum}$ and then starts to decrease as the number of concurrent communicating pairs increases.

VI. Conclusion

The probability of selecting a data channel index in BLE has been derived, and then used to find the probability of collision between multiple BLE connections and the maximum achievable aggregate throughput. Results suggest that the maximum achievable aggregate throughput for n simultaneously communicating BLE pairs is 3.18 $Mbps$. Findings conform with those obtained through brute-force supercomputer simulation.

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