Wildfire Spreading Model using a Parallel Implementation of Cellular Automata

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***Abstract*—Wildfires may have devastating social and economic impact in our societies, destroying thousands of forest hectares that also dramatically damage the ecosystems. There is a pressing need to better understand, framed in disaster response decision making, and control wildfires in different geographical areas. There are continuous and discrete mathematical models that aim to explain and model wildfire's dynamics, but is necessary to integrate a wildfire model into a larger container that enables first responders to simulate and rehearse responses to wildfire propagation dynamics. This work presents a Cellular Automata-based model incorporated into a simulator built for the Chilean forest protection agency (CONAF), which is implemented as a service for a video game-based simulator. Results include very encouraging evaluation from the Chilean forest agency, both for the dynamics shown by the model but also for the realistic experience thanks to the implementation of the simulator.**

***Index Terms*—Wildfire, Cellular Automata, Parallel Programming.**

I. Introduction

Wildfires are one of the most harmful phenomena in Chile burning thousands of forest hectares every year destroying significant flora and fauna, and affecting the air, water cycles, and the ecosystem. Wildfires start by varied reasons, ranging from reckless human behavior to extreme weather and environmental conditions. In Chile, around 50000 hectares are burnt yearly, and in certain occasions up to 100000 hectares have been consumed by fire [1].

Wildfire spreading dynamics has gathered large attention from the scientific community. In general, wildfires are modelled using continuous or discrete models, or a combination of both [2], [3]. Many authors use differential equations for continuous modelling [4], [5], [6], [7], but discrete modelling has also gained significant interest from the scientific community; specially the use Cellular Automata [8], [9], [10], [11], to model wildfire dynamics.

This research effort is part of a research project titled "Serious video games to improve decision making for disaster response systems" [12] funded by Chile's national science foundation. The research presented in this article uses discrete modeling because of the possibility to easily incorporate lower level details about geographically dependent features, such as flora, water areas, and location-based responses or fire combat, and fine granularity to incorporate dynamic interplay among temperature, humidity, and wind characteristics. While there is significant consensus on modelling core components of wildfire propagation models, such as terrain characteristics, fuel type, meteorological conditions, statistical past patterns, the dynamic interaction between temperature, humidity, pressure, and wind characteristics are usually an elusive component in most modelling efforts.

The goal for this research initiative is to build a suitable model to be incorporated into a video-game-based simulator to allow assessment of decision making during wildfire combat involving different agencies which are part of the response system in Chile; the National Forest Agency (CONAF) and the National Emergencies Agency. For this goal, instead of developing complex continuous or discrete models that may forecast and reproduce wildfire dynamics in a realistic manner based only on mathematical models, this work proposes the use of a hybrid approach, which provides mathematical models for core capability of wildfire propagation dynamics, but also incorporates the expert role who may qualitatively alter the model dynamics in real-time to generate realistic cases for further use in the video-game-based simulator. The expert executes actions in the wildfire propagation dynamics through parameters such as wind speed and direction, temperature, and humidity as well as through changes in transition functions in the cellular automata.

II. Formalization

The problem presented in this paper is related to the complexity of incorporating a hybrid approach in the development of the simulator, which allows qualitatively modeling the wildfire and also adds the dynamic component in which the expert role makes changes in real time.

Due to the objective of the model is to be as realistic as possible, the model must handle a high resolution of the terrain to be modeled, involving the management of high dimension matrices consecutively.

Within the motivations of the development of this simulator there is the possibility of training the technical and professional personnel involved in the combat of this type of disaster. It is clear that proposing a realistic experience allows a positive impact on the decision-making processes and complements in an important way the classical techniques of teaching and training in this area.

III. Solution

For the development of the simulator, the fire model was constructed using environmental factors, forest fuel, topography and the cellular automata which interact and evolve in discrete time steps. Then, to include the dynamic component, it was necessary decomposing the computations in order to map tasks to threads. The elements involves in the development of the simulator are described in detail below.

A. Weather

The weather elements are closely related with the fire propagation. The wind guides the fire's direction, brings oxygen, and increases the speed of fire propagation, dried fuel and transports sparks. Temperature evaporates the humidity from fuels, increasing the chances of to burn. If the relative humidity is low, the fuel is drier than the reverse case, it is therefore more flammable. There is not vertical movement of air masses when the atmospheric is quiet, on the contrary with atmospheric instability there are strong wind current stimulating the fire spreading.

Since we work in a discrete world, the environmental factors were discretized using the typical weather values in Chile. We can summarize the relations between the fire and environmental factors as: the temperature and wind have a direct and the humidity and pressure have an inversely effect to the fire spread speed.

B. Forest Fuel

The forest fuel are all the vegetation elements, woody or herbaceous, alive or dead that may be flammable. These elements are classified by: type, location, size, amount, humidity, compression, continuity, and chemical content. A simple scalar is used in the model to moderate the impact of this component of the probability for each cell of the cellular automata to start and stop burning.

C. Topography

The topography factor influences over the two above factors, the fuel and weather, modifying or altering them. The fire dynamics may be affected by: (a) the slope; it favors the rise of convection currents of warm air preheating fuels above, accelerating the wildfire; (b) altitude: at higher altitudes, the temperature is cooler; there is greater precipitation and snow reducing the amount of oxygen in the air; (c) exposure: affects the behavior of wind and hot sun; (d) landform: affects wind patterns and temperature in a given locality. Further versions of this work will include more elaborate versions of sub models that will embrace the articulation of the topography factors.

In addition, according to CONAF, there are 3 models for wildfire progression: (a) circular: in this case, the terrain is flat, there is no wind presence and the fuel is uniform; (b) oval: In this case, the land has a smooth slope, there is soft wind and the fuel is heterogeneous; and (c) extended: The terrain is sloped, exist strong wind and the fuel is heterogeneous. Most of the scenarios involved in this work were flat, the type of wildfire progression was considered circular, in absence of wind and other factors. Further work will include the impact of this element for scenarios that exhibit these three types of wildfire progression.

D. Cellular Automata

John Von Neumann introduced the Cellular Automata (CA) in 1966, this appears as an alternative option to models that use partial differential equations (PDE) and satisfactory results have demonstrated in physics models [13]. CA have been used to model complex systems [14] and applied to some physics problems where local iterations are involved [15], [16]. The discrete nature of CA allows plenty of opportunity to include elements at local level that have the potential to achieve a suitable emerging solution for wildfire dynamics modeling. The main characteristics of the CA used in this work are as follows:

1) Spatial dimensions: The number of spatial dimensions gives the sense of simulation space. In this model, the simulation has two spatial dimensions, that is, the CA has cells in (i, j) in a grid of size $M \times N$.

2) Neighborhood: This CA model use the Moore Neighborhood [17] that is a squared region used to define a set of cells around a cell (i_0, j_0). The Moore neighborhood of range r is defined by

$$N^M_{(i_0,j_0)} = \{(i,j): \ |i - i_0| \leq r, \ |j - j_0| \leq r\}. \qquad (1)$$

The range used in this work was $r = 1$.

3) Cell states: To represent the fire spreading phenomena applied to the CA theory is necessary to define the cell states. In this work we define: nonflammable = 0, flammable = 1, burning = 2, burnt = 3 and extinguished = 4.

4) Transition Rule: The transition rule computes the future value of the respective cells. For this model the cell's next state is computed as:

$$S^{t+1}_{i,j} = F(S^t_{i,j}, S^t_{i,j+1}, S^t_{i+1,j+1}, S^t_{i+1,j}, S^t_{i+1,j-1}, S^t_{i,j-1}, S^t_{i,j-1}, S^t_{i-1,j-1}, S^t_{i-1,j}, S^t_{i-1,j+1}, T_{i,j}, H_{i,j}, Ws_{i,j}, Wd_{i,j}, F_{ab})$$

where T is the temperature, H the humidity, Ws the speed and Wd direction of wind, F_{ab} the probability of change from state a to b. All these variables are matrices.

Formally, we propose the follow transition rule:

$$S^{t+1}_{i,j} = \begin{cases} 2, & \text{if } S^t_{i,j} = 1 \text{ and } f_{12} \geq F_{12} \\ 2, & \text{if } S^t_{i,j} = 4 \text{ and } f_{42} \leq F_{42} \\ 3, & \text{if } S^t_{i,j} = 2 \text{ and } f_{23} \leq F_{23} \\ 4, & \text{if } S^t_{i,j} = 3 \text{ and } f_{24} \leq F_{24} \end{cases}, \qquad (2)$$

where the functions f are determined by the following equations:

- Function f_{12}: represents the probability that a cell begin to burn and depend of:

- The environmental factors E a matrix with values:

$$E_{i,j} = \frac{a\ C_{i,j}\ T_{i,j}\ Ws_{i,j}}{H_{i,j}\ P_{i,j}}, \tag{3}$$

where a is a factor that depends of wind direction. This factor is computed by

$$a = \begin{cases} 1 & \text{if cell in the same direction} \\ & \text{of } Wd_{i,j} \text{ is burning.} \\ 2/3 & \text{if contiguous cells in the same} \\ & \text{direction of } Wd_{i,j} \text{ are burning.} \\ 1/3 & \text{otherwise.} \end{cases}, \tag{4}$$

and $C_{i,j}$ is terrain fuel composition with values $0 \leq C_{i,j} \leq 1$.

- The burning states of neighborhood $p(N_b)$ defined by

$$p(N_b)_{i,j} = \frac{N_b}{8}, \tag{5}$$

where N_b is the number of burning state neighboring cells.

Finally, f_{12} is computed by

$$f_{12} = \alpha\ E_{i,j} + \beta\ p(N_b)_{i,j}. \tag{6}$$

α and β ($\alpha + \beta = 1$) is used to weight the effect of weather condition and neighborhood respectively. F_{12} is a random value between 0 and 1.

- Function f_{23}: represents the probability that a cell ends up burning and is defined as random value between 0 and 1.
- Function f_{24}: represents the probability that a cell is extinguished and is defined as random value between 0 and 1.
- Function f_{42}: represents the probability that an extinct cell burns again and is defined as random value between 0 and 1.

F_{23}, F_{24}, F_{42} are threshold parameters defined through try and error with feedback from experts observing visual outcome representation of wildfire dynamics.

E. Multithreading

Multithreading is a type of execution model that allows multiple threads to exist within the context of a process such that they execute independently but share their process resources. A thread maintains a list of information relevant to its execution including the priority schedule, exception handlers, a set of CPU registers, and stack state in the address space of its hosting process.

In this work we used this technique for operating over independents blocks of CA to process faster the computation of future's state of the cells inside each matrix block. The variable $N_{threads}$ defines the number of threads used to improve the results in the algorithm computation.

F. Implementation

The development of the model needs to perform a tessellation of the area to be simulated, where each cell represents the state of a square portion of the terrain. Because different components are integrated in the modeling, it is proposed to work with a world discretized by layers (DW - Figure 1), where each layer contains the information of the components described before, i.e. environmental conditions, fuel, etc.

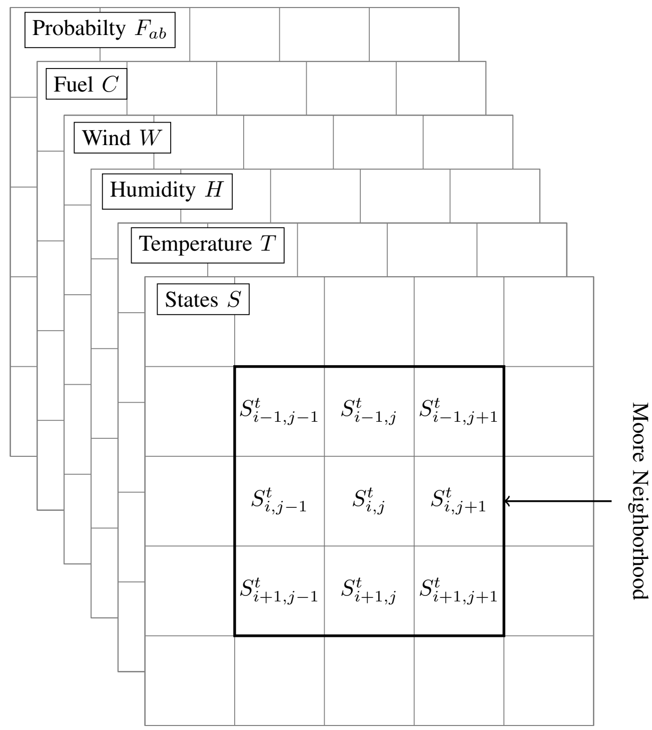

Fig. 1. Discrete world (DW)

The main portion of the code (Algorithm 1) processes the discrete world states S^t for each time.

Algorithm 1 Main Algorithm

$S^0 \leftarrow$ Initialize cell's states.
$DW \leftarrow$ Initialize discrete world.
for $t = 0$ to T_{max} **do**
    $S^{t+1} \leftarrow spreading(S^t, DW)$
end for

For the computation of one step of time we need to call the function *Spreading* where we made the matrix partition for *thread* implementation (Algorithm 2).

Finally, we use Algorithm 3 to apply the transition function. In *SubSpreading* we compute the independent pieces of matrix using a *Thread* for each matrix block defined in *Spreading*. A simple sketch summarizing the last algorithm is shown in Figure 2.

IV. Evaluation

In this section we present some simulations to compare the performance of our proposal model. We assume the use of

Algorithm 2 Spreading Algorithm

procedure SPREADING(S^t, DW)
  $N \leftarrow$ number of rows in DW.
  for $i = 0$ to $N_{threads} - 1$ **do**
    $delta \leftarrow N/N_{threads}$
    $start \leftarrow i \cdot delta$
    if $i = N_{threads} - 1$ **then**
      $end \leftarrow$ number of rows in DW.
    else
      $end \leftarrow (i+1) \cdot delta$
    end if
    $S_{start:end,\cdot} \leftarrow subSpreading(start, end, S^t, DW)$
  end for
  for $i = 0$ to $N_{threads} - 1$ **do**
    Thread's join.
  end for
  return S
end procedure

Algorithm 3 Sub-spreading Algorithm

procedure SUBSPREADING($start, end, S^t, DW$)
  $N \leftarrow$ number of columns in DW.
  for $i = start$ to end **do**
    for $j = 0$ to N **do**
      Compute $S^t_{i,j}$ using equation (2) with S^t and DW.
    end for
  end for
  return $S^t_{start:end,\cdot}$
end procedure

a square grid of size $N \times N$ and, defining the maximum number of discrete times T_{max} we estimate a computational complexity of $O(T_{max}\ N^2)$.

It is important to mention that in the case we choose $T_{max} << N$ the complexity is about $O(N^2)$, but it may change a lot if $T_{max} \sim N$ increasing the complexity to $O(N^3)$. This is the main motivation of to include the use of threads in the computing of the model.

The experiments were made using a fixed $T_{max} = 50$, for 1 to 4 number of threads, repeating the simulations 10 times per N. The average of these repetitions are summarized in Table I.

TABLE I
SUMMARY OF THE AVERAGE TIMES IN SECONDS.

Threads	$N = 100$	$N = 500$	$N = 1000$	$N = 1500$
1	0.198	8.023	32.861	72.464
2	0.135	5.147	19.695	44.407
3	0.163	4.825	18.852	41.626
4	0.220	4.650	18.055	41.458

To show a better representation of results and compare to the theoretical complexity estimated, we present the Figure 3.

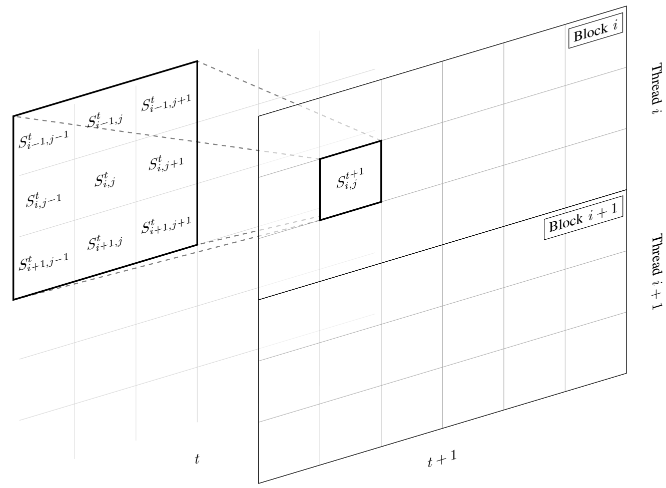

Fig. 2. *SubSpreading* computing a matrix block.

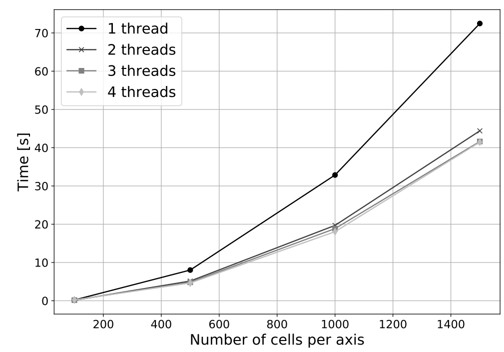

Fig. 3. Threads' performance comparison

We note that the curves approximate the estimated complexity. In addition we can appreciate the impact of the parallel implementation in the computation time of the simulations. Computing the Speedup S as

$$S = \frac{Time_{sequential}}{Time_{parallel}}, \tag{7}$$

we resume the *Speedup* results in Table II.

TABLE II
SPEEDUPS RESULTS.

Threads	$N = 100$	$N = 500$	$N = 1000$	$N = 1500$
2	1.47	1.56	1.67	1.63
3	1.21	1.66	1.74	1.74
4	0.90	1.73	1.82	1.75

For a qualitative evaluation Figure 4 shows a piece of map simulated for $N = 1500$, $T = 30°\text{C}$, $H = 50\%$, $W_s = 40$ km/hr, $W_d = 90°$, $P = 50$ hPa, $F_{23} = F_{24} = F_{42} = 0.1$.

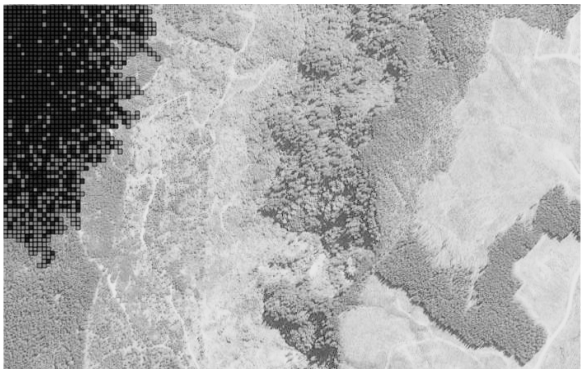

Fig. 4. Simulation qualitative result.

V. Validation/Discussion

According to [18] fire models can be grouped into three types: risk models, propagation and effect. The first ones are associated to quantify the probability and potential effects before possible episodes of fire. Depending on the variables used, different jobs can be found, for example the estimation of the probability of occurrence of a fire depending on the location and the day of the year [19], [20], fire hazard and climate indices [21], [22], among others [23], [24], [25]. The second group tries model the movement of fire and exist from physical approaches [4] to models of regression to estimate the propagation rate [26]. Generally this type of models assumes that the fuel (place where the fire occurs) can be tiled by a regular mesh where each cell has associated a probability of burning and that depends on the conditions of the neighboring cells. There are different tools associated with this type of models and can be reviewed in [27], [28], [29], [30], [31]. Finally, the effect models are important to study the administration and management of fuels, for example the control of tree mortality or analysis of ecosystems, among others [32], [33], [34].

As mentioned in the introduction, modeling with differential equations is one of the most used approaches. The main problem with wildfire models based on this approach is the computational cost associated with the resolution of the componentes in equations. For example, the reaction-convection-diffusion equation [35] is a mathematical model that models temperature and involves the effect of a flow and the reaction rate of chemicals interpreted as fuel. One of the most used numerical methods to solve diffusion equations is the Crank-Nicolson method, which is known to have computational cost problems when solving the problem in two dimensions, given the size of the system of equations that is required to solve for each iteration in time. In addition, the flow or wind, which is represented by a vector field, must satisfy the Navier-Stokes equations, a problem that does not have a general solution [36], so it can usually only be approximated with a high computational cost. The dynamic implementation is a problem for differential equations since the increment of complexity unlike the model proposed by us for example in the modification of environmental conditions that only implies to set values in the matrices of the discrete world.

The wildfire dynamics generated with the prototypes developed are consistent with what CONAF has observed in past devastating wildfires occurring in Chile. Nevertheless, the effort so far has focused in creating a framework to include the core of modelling found in related work and mainly the expert's knowledge and expertise through manipulation of quantitative and qualitative variables affecting the dynamics.

Dynamic interplay among temperature, pressure, and humidity is a key element to be included in models for wildfire in Chilean context. While this has been approach from a qualitative decision making perspective in this version of the model, more elaborate local level rules for the interplay among these variables will be explore in a future version of the wildfire dynamics model.

VI. Conclusion

This article describes a qualitative approach to calibrate and generate a suitable model for realistic wildfire dynamics. This modelling effort is based on discrete simulation using a Cellular Automata to simulate fire spreading, which is easy to manipulate at both quantitative low level and qualitative high level. As expected, the system's dynamics emerging from cell's switching rules involving environmental, topographic, and fuel components, as well as experts' suggested features, such as smoke column characteristics, facilitates the use of an expert-based try and error approach to achieve suitable and realistic wildfire dynamics.

The incorporation of parallel techniques allows the model to compute enough states in discrete times to show a qualitatively realistic result for the specialists requirements. The decrease in computing time allows us to conclude that the use of multithreads is a good strategy to apply to this problem, given the characteristics of the discrete world and the independence of states between the times t and $t+1$.

On the other hand, the incorporation of experts into the development, calibration, and evaluation of the wildfire dynamics model being built became a key component in achieving models that resemble the dynamics of devastating wildfires occurring in the locations modeled in this work. The provision of all the parameters governing the dynamics of the wildfire in the prototypes built allowed the experts to use their expertise and knowledge to manipulate those parameters to gradually achieve suitable and realistic outcomes.

Finally, positive qualitative evaluation has been obtained from wildfire combat experts, from the Chilean forest fire agency in terms of functionality, usability, performance, and overall quality of the prototypes built upon the wildfire model designed in this research effort. These experts also highlighted that the introduction of the expert to be able to change at will the conditions and ongoing wildfire dynamics is of extreme value to provide not only suitable and realistic simulations, but also for increasing the learning experience on professionals who would be using the simulator, from which the model presented in this article is part of.

VII. Future Work

Future work will seek to deliver more solid components, or finer granularity in some cases, for the core characteristics of the wildfire dynamics, such as the topographic and fuel components.

Another possibility of improvement is related to the use of a parallel architecture of the pipeline type, taking advantage of the refresh rate in the interface of the simulator. It is evident that in addition to the efforts in the speed of computation we must include work in the optimization of communication and memory usage.

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