Derivations

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1 Inference Rules

The textbook introduces four rules: MP, MT, DNI, and DNE. Each of the following is an instance of one of these rules. Which one is it?

1. If you are made of wood, then you are flammable. You are not flammable. Therefore, you are not made of wood.
2. If you weigh the same as a duck, then you float. You weigh the same as a duck. Therefore, you float.
3. You aren't not flammable. So you are flammable.
4. $Q \to R, Q \vdash R$
5. $\neg Q \vdash \neg\neg\neg Q$
6. $\neg\neg(P \to Q) \vdash P \to Q$
7. $S \to T, \neg T \vdash \neg S$
8. $\neg S \to \neg T, \neg S \vdash \neg T$
9. $\neg\neg(P \to Q) \to R, \neg\neg(P \to Q) \vdash R$
10. $(P \to Q) \to R, \neg R \vdash \neg(P \to Q)$
11. If you float if you are made of wood, then if you are witch you are flammable. You float if you are made of wood. So, if you are a witch you are flammable.

Some of the following are instances of MP or MT, and some of them are instances of the fallacies, Affirming the Consequent or Denying the Antecedent. Which are which?

11. If you are a duck, you float. You float. So you are a duck.
12. If you are a duck, you float. You are a duck. So you float.
13. If you are a duck, you float. You are not a duck. So you don't float.
14. If you are a duck, you float. You don't float. So you aren't a duck.
15. $P \to Q, Q \vdash P$
16. $Q \to P, Q \vdash P$
17. $Q \to P, \neg Q \vdash \neg P$
18. $Q \to P, \neg P \vdash \neg Q$
19. $P \to Q, \neg P \vdash \neg Q$
20. $R \to P, \neg P \vdash \neg R$

2 Direct Derivations

1. $\neg\neg P, P \to Q \vdash \neg\neg Q$
2. $P, \neg\neg P \to \neg\neg Q \vdash Q$
3. $Q, \neg P \to \neg Q \vdash P$
4. $P \to Q, \neg Q, \neg P \to R \vdash R$
5. $\neg P, Q \to P, R \to Q \vdash \neg R$
6. $\neg(P \to Q) \to R, \neg R, P \vdash Q$
7. $(P \to Q) \to (Q \to R), Q \to (P \to Q), Q \vdash R$
8. $\neg(P \to Q) \to (Q \to R), P \to \neg(Q \to R), P \vdash Q$

3 Arguments

Symbolize (come up with your own scheme of abbreviation). Derive. Read through the derivation in English.

1. If you are a witch, you float. If you float, you weight the same as a duck. If you weigh the same as a duck, then you are flammable. You are not flammable. Therefore, you are not a witch.
2. If you are a witch if you float, then if you are a witch you we the same as a duck. It is not the case that if you are a witch you weigh the same as a duck. So, it is not the case that if you float, you are a witch.
3. You are a witch only if you float. You float only if you weigh the same as a duck. You don't weigh the same as a duck. So, you are not a witch.
4. Provided that you are a witch only if you float, you are not a witch. You are a witch. So it is not the case that if you are a witch, you float.