Conditional Derivations

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1 Self-Assessment Quiz

This page of the handout is a self-assessment quiz. Complete the quiz, then we will talk about it. The point is to help you get a better sense of whether or not there are important things that you should know, but do not yet know.

1. Symbolize: if P, Q.
2. Symbolize: P if Q.
3. Symbolize: P only if Q.
4. Write down an instance of MP that contains some negations.
5. Write down an instance of MT that contains more than two negations.
6. Write down an instance of DNI that involves a conditional.
7. Write down an instance of DNE that involves a conditional.
8. Write down an instance of MP that involves a conditional that has a conditional as its antecedent and a conditional as its consequent.
9. Write down an instance of MT that involves a conditional that has a conditional as its antecedent and a conditional as its consequent.

Each of the following derivations contains a mistake. Identify the mistake:

10. $R \to P, P \vdash R$

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1. Show: R
2.   R->P    :PR
3.   P       :PR
4.   R       :MP 2,3
5. :DD 4
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11. $\neg P \to Q, P \vdash \neg Q$

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1. Show: ~Q
2.   ~P->Q   :PR
3.   P       :PR
4.   ~~P     :DNI 3
5.   ~Q      :MT 2,4
6. :DD 5
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12. $P \to (P \to Q), P \vdash Q$

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1. Show: Q
2.   P->(P->Q)  :PR
3.   P          :PR
4.   Q          :MP 2,3
5. :DD 4
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2 Hypothetical Reasoning

13. You just bet $100 on the Niners to win the Superbowl. Your rent is due by February 5th, and if you lose this bet, you won't be able to make rent. This won't be the first time (you've let this gambling habit get out of control). So if that happens, you'll get evicted. And your partner is just using you for your apartment, so if that happens, they will leave you. Your Mom has always said you'd be much happier if you weren't in this toxic relationship, and, at least in this case, she's absolutely right.

    Complete the following informal (no need for symbols) conditional derivation:

    To be shown: If the Niners lose, then I will be happier than I am now.

    > Assume that the Niners lose.
    > I bet on the Niners to win (PR).
    > If Niners lose, I lose the bet (PR)
    > So, I lose the bet (by MP)
    > ⋮
    > So, I will be happier than I am now.

    QED!

14. A King declares that all and only those who tell a lie in his city will be executed. Mullā Nasr al-Dīn goes to the city, and, at the gates, when the guards ask him his business, he says, "I am here to be executed." What happens to the King if he executes Mullā Nasr al-Dīn? What happens to the King if he doesn't?

    Once you've figured out the answer, try to present it in the form of two informal conditional proofs. That is, something like this:

    To be shown: If the King executes Mullā Nasr al-Dīn, then _____

    > Assume the King executes Mullā Nasr al-Dīn.
    > ⋮

    QED!

    To be shown: If the King does not execute Mullā Nasr al-Dīn, then _____

    > Assume the King does not execute Mullā Nasr al-Dīn.
    > ⋮

    QED!

3 Derivations

As you do these derivations, try to move back and forth between the abstract symbolic patterns, on the one hand, and the line of reasoning, on the other. This isn't like algebra, where you learned a mechanical process for simplifying and solving equations. Constructing a derivation requires *thinking about what the symbols mean*, and thinking of each step as a step in a line of reasoning. It is a creative process, not a mechanical process.

15. $P \to (Q \to R) \vdash P \to R$
16. $\top \vdash P \to (Q \to P)$
17. $\neg(\neg Q \to P) \to Q \vdash \neg Q \to P$
18. $\neg(P \to Q) \to Q \vdash \neg Q \to \neg P$
19. $(P \to Q) \to R, Q \to \neg R \vdash (P \to Q) \to \neg Q$