Nested Derivations

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1 First Midterm

The first midterm is on Thursday, Feb 13. We have one more thing to cover before then: indirect derivations. That's what Chapter 6 is on, so that's what you will be working on for next time. Next Tuesday will be devoted to practice and review.

The midterm will contain a few very short answer questions testing your understanding of some key terms and ideas. For example,

1. Circle the main connective: $P \wedge Q \wedge R \rightarrow S$
2. Circle the antecedent: $\neg(P \rightarrow Q) \rightarrow R$

It will contain some questions that ask you to translate a symbolic sentence into English, and some questions that ask you to translate an English sentence into symbols. In both cases, a scheme of abbreviation will be supplied.

And it will contain some derivations, including some simple direct derivations, some conditional and indirect derivations, and some nested derivations.

2 Self-Assessment Quiz

Do you know the basic rules yet?

3. What else would you need to apply MP to $(P \rightarrow Q) \rightarrow \neg R$? What would that allow you to infer?
4. What else would you need to apply MT to $(P \rightarrow Q) \rightarrow \neg R$? What would that allow you to infer?

The following derivation contains a mistake. What is it?

5. $R \rightarrow Q, Q \rightarrow P \vdash P \rightarrow R$

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1. Show: P->R
2.   R      :AS
3.   R->Q   :PR
4.   Q      :MP 2,3
5.   Q->P   :PR
6.   P      :MP 4,5
7. :CD 6
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Construct derivations for:

6. $\neg P \rightarrow \neg Q, Q \vdash P$
7. $Q \rightarrow \neg R, S \rightarrow R \vdash S \rightarrow \neg Q$

3 Nested Derivations

Except in unusual circumstances, you can't derive a conditional *directly*. That's why we have CD, our method for deriving conditionals. Sometimes in the middle of a derivation, you need to derive a conditional. So, sometimes, in the middle of a derivation, you need to use CD. The result is a *nested derivation*—a derivation with another derivation inside it.

8. $P \rightarrow (Q \rightarrow R), R \rightarrow S \vdash P \rightarrow (Q \rightarrow S)$

First, translate this into English, letting P stand for 'You are happy', Q for 'You know it', R stand for 'You clap your hands', and S stand for 'You stomp your feet.' Then we will work through the derivation together.

3.1 Available and Unavailable Lines

You can only apply rules to **available lines**. But which lines are available, and which are not?

- Show lines are not available *until* they have been *shown*. Once they have been shown, they become available.
- Once a subderivation is complete, the lines inside that subderivation are no longer available.

In the original version of the system we are using, this was emphasized with a visual system of *boxing* and *cancelling*.

3.2 Strategy: Using vs. Getting

Using Conditionals We **use** conditionals by applying MP or MT to them.
Getting Conditionals We **get** conditionals by using CD.

Don't confuse these two tasks. If you already have a conditional on an available line, use it by applying MP or MT. If you need to get a conditional, enter a show line and use CD.

3.3 Strategy: When to Enter Show Lines?

You are always allowed to enter a show line for anything you want. But doing so is often a bad idea. How do you know when to enter a show line, and for what?

- You always start a derivation by entering a show line for the conclusion. That's because the conclusion is the thing you need to **get** in order to complete the problem.
- If you are in the middle of a derivation, and you can't find anything useful to do with what you have, think about what you need to **get** to complete the problem. For example, if you are trying to complete a CD, you need to **get** the consequent of your show line. Is that consequent a conditional? If so, enter a show line for it, and try to get it by CD.

3.4 Exercises

As you do these derivations, try to move back and forth between the abstract symbolic patterns, on the one hand, and the line of reasoning, on the other. This isn't like algebra, where you learned a mechanical process for simplifying and solving equations. Constructing a derivation requires *thinking about what the symbols mean*, and thinking of each step as a step in a line of reasoning. It is a creative process, not a mechanical process.

1. $P \to (Q \to R) \vdash P \to (\neg R \to \neg Q)$
2. $P \to Q \vdash P \to (R \to Q)$
3. $R \to \neg(P \to Q), Q \vdash \neg R$
4. $P \to Q, R \to \neg Q \vdash R \to \neg P$
5. $P \to \neg(Q \to R) \vdash (Q \to R) \to \neg P$