Truth Tables

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1 Characteristic Truth Tables

All of our connectives are **truth-functional**: if you know the truth value of each part, you can calculate the truth value of the whole. You already know how to do this in many cases:

1. T/F: Normal is in Illinois and Chicago is in Wisconsin.
2. T/F: Normal is in Illinois and Chicago is in Illinois.
3. T/F: Normal is in Wisconsin and Chicago is in Wisconsin.
4. T/F: Chicago is in Wisconsin or Chicago is in Illinois.
5. T/F: Chicago is in Wisconsin or Chicago is in Indiana.

We can represent the truth-function for each connective using a **characteristic truth table**. Each column of the table represents a sentence. Each row of the truth table represents a possible assignment of truth values:

$\Box$	$\neg\Box$
T	
F	

$\Box$	$\bigcirc$	$\Box \wedge \bigcirc$
T	T	
T	F	
F	T	
F	F	

$\Box$	$\bigcirc$	$\Box \vee \bigcirc$
T	T	
T	F	
F	T	
F	F	

$\Box$	$\bigcirc$	$\Box \rightarrow \bigcirc$
T	T	
T	F	
F	T	
F	F	

$\Box$	$\bigcirc$	$\Box \leftrightarrow \bigcirc$
T	T	
T	F	
F	T	
F	F	

Try to fill these truth characteristic truth tables out, relying on your understanding of the meaning of each connective. If you are not sure, put a "?". (Use pencil so you can correct your mistakes.) Remember:

- $\vee$ expresses *inclusive* disjunction: e.g., "If you can roll your tongue, then either your mother can roll her tongue *or* your father can roll his tongue."
- $\leftrightarrow$ expresses *agreement*: e.g., "I will go the party if and only if you go to the party."
- $- >$ expresses the *material* conditional.

1.1 The Material Conditional

'If. . . then. . . ' in English is not truth-functional. To see this, answer the following True/False questions, using your own judgment, given your knowledge of the world:

6. T/F: If Pritzker is Governor, then a Republican is Governor.
7. T/F: If Normal is a city in Illinois, then Normal is bigger than Chicago.
8. T/F: If Pritzker is Governor, then a Democrat is Governor.
9. T/F: If Normal is a city in Illinois, then a Democrat is Governor.
10. T/F: If Obama is Governor, then a Democrat is Governor.
11. T/F: If Rauner is Governor, then a Democrat is Governor.
12. T/F If Rauner is Governor, then a Republican is Governor.
13. T/F: If Obama is Governor, then a Republican is Governor.

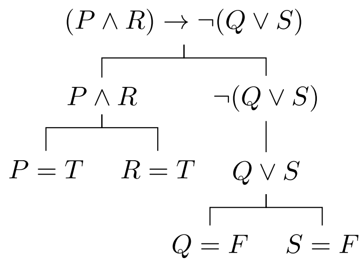
We need *our* '$\rightarrow$' to be truth-functional, even if English's 'if. . . then. . . ' is not truth-functional. So we take it to express what is called the **material conditional**—the closest truth-function you can get to 'if. . . then. . . ' in English. The key to understanding the truth-table for the material conditional: *it is only false when it absolutely has to be*, that is, when *the antecedent is true* but *the consequent is false*. In all other cases, it is true. So (6) and (7) are both false. The rest are all true.

1.2 Calculating Truth

We can calculate the truth value of a complex sentence given an assignment of truth values to the atomic sentences it contains. For example,

P	Q	R	S	$(P \wedge R) \to \neg(Q \vee S)$
T	F	T	F	?

First, we parse the sentence, and indicate the truth values assigned to the atomic sentences:



Now, we calculate the truth of the whole by working our way up the tree, using the characteristic truth tables.

Assume P and R are true, and Q and S are false. Calculate the truth of each of the following:

14. $\neg(\mathrm{P} \wedge \mathrm{Q})$
15. $\neg(\mathrm{Q} \vee \mathrm{S})$
16. $\mathrm{P} \to \mathrm{Q} \vee \mathrm{R}$
17. $\neg(\mathrm{Q} \wedge \mathrm{R}) \to \mathrm{S}$
18. $\neg\mathrm{P} \to (\neg\mathrm{Q} \leftrightarrow \neg\mathrm{R})$
19. $\neg(\mathrm{P} \vee \mathrm{Q} \leftrightarrow \mathrm{Q} \wedge \mathrm{S})$

2 Truth Tables and Tautologies

A **truth table** is a table that represents **all** the logical possibilities for a given sentence, and the resulting truth value of the sentence in each possible situation. For example,

P	$P \vee \neg P$
T	
F	

P	Q	$P \vee \neg Q$
T	T	
T	F	
F	T	
F	F	

P	Q	R	$P \wedge Q \to R$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

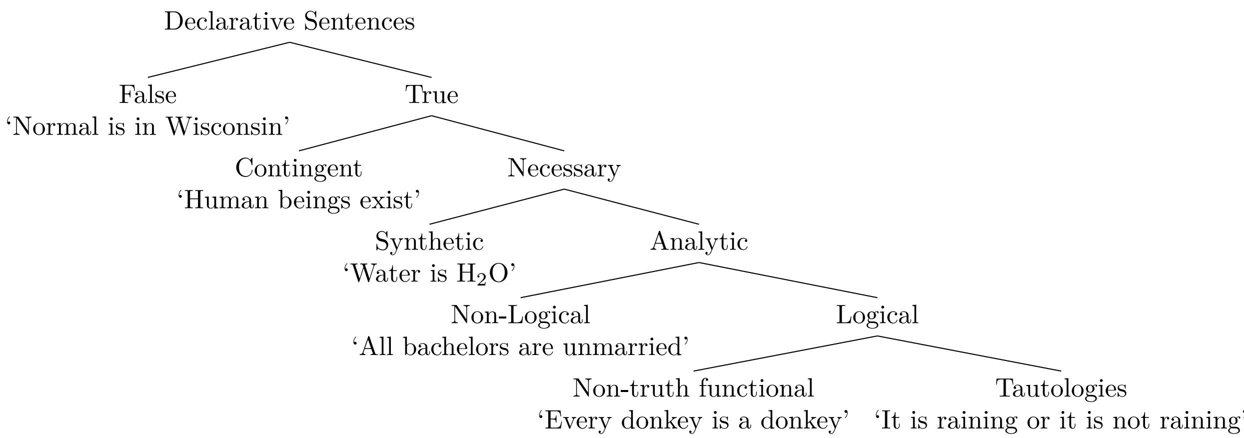
The number of logical possibilities for a given sentence, and so the number of rows of its truth table, depends on how many sentence letters the sentence contains: for a sentence that contains n sentence letters, there will be 2^n possibilities, and so 2^n rows. When you construct a truth table, you should always write down the possibilities using the same pattern illustrated in these examples.

We "complete" the truth table by calculating the truth value of the sentence on each row. Note that the point is to *show* the calculation, not just the result. So you need to follow the process described in the book.

2.1 Tautologies

Some sentences are true, and some are false. Among the true sentences, some are **contingent** ("There are cows"), and some are **necessary** (e.g., '$2 + 2 = 4$', 'Donkeys are mammals'). Among the necessary truths, some are **analytic**—true in virtue of meaning and logic (e.g., 'All bachelors are unmarried')—and some are not ('Water is H_2O'). Among analytic truths, some are **logical truths**—true in virtue of logic alone ('Every donkey is a donkey'). Among logical truths, some are true in virtue of truth-functional logic alone. We call these **tautologies**.

of their logical form (e.g., 'Whatever will be, will be'). We call these **logical truths**. Among logical truths, some are true in virtue the logical form we can represent using truth-functional sentential connectives (e.g., 'If it rains, it rains'). We call these **tautologies**.



We can use a truth table to figure out whether or not a sentence is a tautology:

> A sentence is a **tautology** if and only it is assigned T on every row of its truth table.

So if there is at least one row on which the sentence is assigned F, then the sentence is not a tautology. We call such a row a "Counterexample".

Construct truth tables to determine whether or to the following sentences are tautologies. If the sentence is not a tautology, circle a row that contains a counterexample.

20. $\neg P \to \neg P$
21. $P \to \neg P$
22. $\neg(P \wedge \neg P)$
23. $Q \to \neg P \wedge Q$
24. $Q \to \neg P \vee Q$
25. $\neg(P \wedge Q) \leftrightarrow \neg P \vee \neg Q$
26. $\neg(P \to Q) \vee \neg(Q \to P)$
27. $\neg P \vee R \leftrightarrow P \vee \neg R$
28. $P \wedge (P \to Q) \to Q$
29. $Q \wedge (P \to Q) \to P$
30. $\neg Q \wedge (P \to Q) \to \neg P$
31. $\neg P \wedge (P \to Q) \to \neg Q$