New Rules

David Sanson — 112 — 20 Feb 2020

1 Rules for Conjunctions ("And" statements)

Our first rule captures the idea that, if "P and Q" is true, then "P" is true and "Q" is true:

Simplification (S)

$\square \wedge \bigcirc \vdash \square$
$\square \wedge \bigcirc \vdash \bigcirc$

This rule has two forms, because the order doesn't matter. The first form lets you infer the first conjunct, $\square$. The second form lets you infer the second conjunct, $\bigcirc$. Here are two instances of this rule:

1. You live in Illinois and you attend ISU. So, you live in Illinois.
2. It is raining and the streets are wet. So, the streets are wet.

Our second rule captures the idea that if "P" is true, and "Q" is true, then "P and Q" is true:

Adjuction (ADJ)

$\square, \bigcirc \vdash \square \wedge \bigcirc$

Here are two instances of this rule:

3. It is raining. I am tired. So, it is raining and I am tired.
4. You live in Illinois. You attend ISU. So, you live in Illinois and you attend ISU.

2 Rules for Biconditionals ("if and only if" statements)

The biconditional ("if and only if") is used to express the idea that a conditional relationship holds in both directions between two sentences:

- $Q \to P$: I am happy if you feed me cupcakes.
- $P \to Q$: I am happy only if you feed me cupcakes.
- $P \leftrightarrow Q$: I am happy if and only if you feed me cupcakes.

Our rules for the biconditional reflect this. The first says that, from "if P then Q" and "if Q then P", you can infer "P if and only if Q":

Conditional-Biconditional (CB)

$\square \to \bigcirc, \bigcirc \to \square \vdash \square \leftrightarrow \bigcirc$

Our second rule says that you we can also go in the other direction, and infer either conditional from a biconditional:

Biconditional-Conditional (BC)

$\square \leftrightarrow \bigcirc \vdash \square \to \bigcirc$
$\square \leftrightarrow \bigcirc \vdash \bigcirc \to \square$

Like S, BC has two forms: one lets you infer the left-to-right conditional, the other lets you infer the right-to-left conditional.

3 Rules for Disjunctions ("or" statements)

Our first rule for disjunctions captures the idea that, if "P" is true, then "P or Q" must be true. And, likwise, if "Q" is true, then "P or Q" must be true:

Addition (ADD)

$\square \vdash \square \vee \bigcirc$
$\bigcirc \vdash \square \vee \bigcirc$

5. It is raining. So either it is raining or it is snowing.
6. It is raining. So either it is raining or it is pouring.

Our second rule for disjunction is a bit trickier. Consider the following argument:

1. $S \vee W$: Either Sanders will get the nomination or one of the other candidates will.
2. $\neg S$: Sanders won't get the nomination.
3. W: So, one of the other candidates will.

This argument is valid. Imagine, for example, that you know that (1) is true, and then a fortune teller tells you that (2) is true. If the fortune teller is correct, (3) follows.

Or consider this argument:

1. $S \vee W$: Either Sanders will get the nomination or one of the other candidates will.
2. $\neg W$: None of the other candidates will.
3. S: So, Sanders will.

Again, this argument is valid.

So here is the idea: If "P or Q" is true, but it isn't P, then it must be Q. And, if "P or Q" is true, but it isn't Q, then it must be P. We call this:

Modus Tollendo Ponens (MTP)

$\square \vee \bigcirc, \neg\square \vdash \bigcirc$
$\square \vee \bigcirc, \neg\bigcirc \vdash \square$

(Modus Ponens = "the way of putting". Modus Tollens = "the way of taking". "Modus Tollendo Ponens" = "the way of putting by taking"!)

Remember, each rule **only works for one kind of connective**. You can't use MP and MT work with conditionals; ADD and MTP works with disjunctions. CB and BC work with biconditionals, and S and ADJ work with conjunctions.

4 Practice

For each of the following, what can you infer, and by which rule?

7. $P \wedge Q$
8. $P \vee Q, \neg Q$
9. $P \to Q, \neg Q$
10. $(P \wedge Q) \leftrightarrow (Q \wedge P)$
11. $(P \leftrightarrow Q) \vee (R \leftrightarrow S), \neg(P \leftrightarrow Q)$

For each of the following, what else would you need to apply MP, MT, or MTP? What would you get?

12. $P \vee Q$
13. $\neg P \vee \neg Q$
14. $P \vee Q \vee R$
15. $P \vee Q \to R$
16. $P \vee (Q \to R)$
17. $\neg(P \wedge Q) \vee (\neg P \wedge \neg Q)$