Midterm Review

David Sanson — 112 — 11 Feb 2020

1 First Midterm

The first midterm is on Thursday, Feb 13. Next Tuesday will be devoted to practice and review.

The midterm will contain a few very short answer questions testing your understanding of some key terms and ideas. For example,

1. Circle the main connective: $P \wedge Q \wedge R \rightarrow S$
2. Circle the antecedent: $\neg(P \rightarrow Q) \rightarrow R$

It will contain some questions that ask you to translate a symbolic sentence into English, and some questions that ask you to translate an English sentence into symbols. In both cases, a scheme of abbreviation will be supplied.

And it will contain some derivations, including some simple direct derivations, some conditional and indirect derivations, and some nested derivations.

2 Symbolization and Translation

Translate the following partially symbolic sentences all the way into symbolic sentences:

3. If P, Q
4. P if Q
5. P only if Q
6. If P then if Q then R
7. If P only if Q then R
8. It is not that if P then not Q
9. If P only if Q, Q only if P

Scheme of abbreviation: S: Sea levels will rise. T: Coastal cities will be flooded. W: Global warming continues at the current rate. X: Storms will become more extreme.

Translate the following symbolic sentences into English:

10. $\neg W \rightarrow \neg X$
11. $\neg(W \rightarrow X)$
12. $(W \rightarrow S) \rightarrow (W \rightarrow T)$
13. $S \rightarrow (X \rightarrow T)$
14. $X \rightarrow (\neg S \rightarrow W)$
15. $\neg W \rightarrow \neg(S \rightarrow W)$

Translate the following English sentences into symbolic sentences:

16. Sea levels will rise if global warming continues at the current rate.
17. Coastal cities will be flooded only if sea levels will rise.
18. Provided that global warming does not continue at the current rate, storms will not become more extreme.
19. It is not the case that if global warming does not continue at the current rate, sea levels will not rise.
20. If sea levels will rise provided that global warming continues at the current rate, then if global warming continues at the current rate, coastal cities will be flooded.
21. If coastal cities will not flood if sea levels will rise, if storms do not become more extreme coastal cities will not flood.

3 Mastering MP and MT

If you are still struggling with MP and MT, write down each rule as a pattern or template, using boxes and circles. Then write down a random conditional (or locate a random conditional on this handout), and answer two questions:

- What other sentence would I need, as an additional premise, in order to apply MP to this sentence? What other sentence would I then be allowed to infer, by MP?
- What other sentence would I need, as an additional premise, in order to apply MT to this sentence? What other sentence would I then be allowed to infer, by MP?

4 Big Picture Advice for Derivations

- This isn't algebra. There is no "cancelling out" or "simplifying". There is no "distributing negations". There are no hidden complicated moves that you are allowed to make. There are only our four simple rules (MP, MT, DNI, and DNE), and our three methods of derivation (DD, CD, and ID).
- Remember that each problem is a derivation—that is, an attempt to represent a line of reasoning, from premises to a conclusion. It is not an attempt to "break down" lines into other lines. That's a way of thinking that you are borrowing from algebra. But this isn't algebra!
- Always try to "read" the argument first, and see if you understand why it is valid. This will often give you good ideas about how the conclusion follows from the premises, and so how to show that the argument is valid. It will help keep you remember that each problem is a derivation, and help keep you from falling into the trap of seeing each problem as just a sequence of meaningless symbols. It will help keep you from falling back on patterns of thought borrowed from algebra.

5 Nitty-Gritty Advice for Derivations

- You need to know MP, MT, DNI, and DNE in order to do derivations.
- If your show line is a conditional, use CD.
- If your show line is a negation, maybe use ID. (ID is our most powerful method of derivation.)
- Never enter a show line for something you already have available.
- If you are stuck, figure out what you need to get to complete the derivation (or current subderivation), and enter a show line for that.
- If you need P, but can't get it directly, try using ID to get $\neg\neg P$.
- Remember, to complete an ID, you need *a* contradiction—any contradiction will do! Don't get fixated on trying to find one specific contradiction.
- If you have something like $\neg(P \to Q)$, try showing its unnegation, $P \to Q$. Then you will have a contradiction!

6 Easier Derivations (DD, CD, and ID)

22. $Q \vdash \neg\neg\neg\neg Q$
23. $\neg P \to \neg Q, Q \vdash P$
24. $P \to Q, Q \to R, P \vdash R$
25. $P \to Q, Q \to R, \neg R \vdash \neg P$
26. $P \to (Q \to R), P, \neg R \vdash \neg Q$
27. $Q \to P, R \to \neg P \vdash Q \to \neg R$
28. $\top \vdash Q \to Q$
29. $\top \vdash Q \vdash P \to Q$
30. $P \to Q, P \to \neg Q \vdash \neg P$

7 Harder Derivations

31. $P \to \neg(R \to S), T \to (R \to S), \neg P \to U \vdash T \to U$
32. $P \to Q, P \to \neg Q, \neg P \to \neg R \vdash \neg R$ (see (30))
33. $P, \neg P \vdash \neg\neg Z$
34. $P, \neg P \vdash Z$ (build on your solution to (33))
35. $\top \vdash P \to (\neg P \to Z)$
36. $\neg P \vdash P \to \neg\neg Q$
37. $P \vdash \neg P \to Q$
38. $P \to Q \vdash \neg(P \to Q) \to R$
39. $\neg(P \to Q) \vdash \neg Q$
40. $\neg(P \to Q) \vdash P$