PHY1001: Mechanics (Week 2)

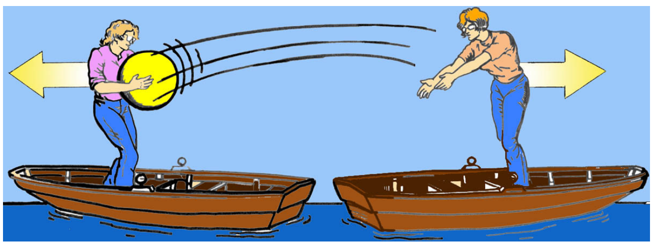
We are going to discuss the Newton's law of Motion, including the concept and significance of forces, the relationship between force and acceleration, as well as forces between two bodies. Furthermore, we apply these laws and solve problems involving objects in motion and at rest.

1 Force and Interaction

In everyday language, a force is a push or a pull. There are many types of forces, such as contact force, normal force, friction force, tension force and gravitational force, etc. Forces have directions and they can be measured in the unit of Newton (N=kg $\cdot$ m/s^2) with the dimension of ML/T^2. The superposition of forces can be written as the sum vectors.

Some particular forces:

- Gravitational force (weight) on the Earth $F = mg$ with $g = 9.8 m/s^2$ and it is directed down toward the ground.
- A normal force is the force on a body from a surface against which the body presses. The normal force is always perpendicular to the surface.
- A frictional force f is the force on a body when the body slides or attempts to slide along a surface. The force is always parallel to the surface and directed so as to oppose the sliding. (It is always against the relative motion.) On a frictionless surface, the frictional force is negligible.
- String tension: When a cord is under tension, each end of the cord pulls on a body. The pull is directed along the cord, away from the point of attachment to the body. For a massless cord (a cord with negligible mass), the pulls at both ends of the cord have the same magnitude. Note that there is only pull $T > 0$ in the cord. When it is taut, the tension force is not zero. When the rope is slack, $T = 0$. (In comparison, a elastic spring can provide both pull and push forces.)
- A contact force is any force that requires contact to occur. A non-contact force is a force which acts on an object without coming physically in contact with it. (All four known fundamental interactions are non-contact forces.)



In modern physics language, particles usually do not directly interact with each others, instead they exchange mediating particles between each other as illustrated in the above cartoon. A better definition of forces are interactions between particles or between a body and its environment. Traditionally, physicists categorize all interactions observed in nature in terms of four basic interactions that occur between elementary particles.

1. The gravitational interaction—the long-range interaction between particles due to their mass. It is believed by some that the gravitational interaction involves the exchange of hypothetical particles called gravitons.

2. The electromagnetic interaction—the long-range interaction between electrically charged particles involving the exchange of photons.

   Macroscopic forces, such as contact forces including the normal force, friction, and fluid resistance, are the results of electromagnetic interactions between atoms of the interacting bodies, if we try to view these forces microscopically.

3. The weak interaction—the extremely short-range interaction between sub-nuclear particles involving the exchange or production of W and Z bosons.

   The electromagnetic and weak interactions are now viewed as a single unified interaction called the electroweak interaction.

4. The strong interaction—the long-range interaction between hadrons, which themselves consist of quarks, that binds protons and neutrons together to form the atomic nuclei. It involves the exchange of mesons between hadrons, or gluons between quarks.

We experience first two of the above interactions in our everyday life, while we can not directly observe the other two without using experimental apparatuses. The distinction of these four forces come from the fact they exchange different type of force mediating particles.

2 Newton's Laws and Dynamics

We have learned about the kinematics, which is the description of the motion of moving bodies without considering the forces that caused the motion. Now we are studying the subject of dynamics, which is the relationship of motion to the forces that causes it.

Newton's law of motion, published in Newton's book “Mathematical Principles of Natural Philosophy" in 1687, can be summarized into three statements.

- First Law. An object at rest stays at rest unless acted on by an external force. An object in motion continues to travel with constant speed in a straight line unless acted on by an external force. It is also known as the law of inertia. Inertia can be viewed as the resistance to any change in an object's velocity. Or the tendency of a body to keep moving once it is set in motion.
- Second Law. The acceleration of an object is directly proportional to the net force acting on it, and the reciprocal of the mass of the object is the constant of proportionality. Thus, $\vec{F}_{\text{net}} = m\,\vec{a}$, where $\vec{F}_{\text{net}} = \sum \vec{F}$.
- Third Law. When two bodies interact, the force $\vec{F}_{BA}$ exerted by object B on object A is equal in magnitude and opposite in direction to the force $\vec{F}_{AB}$ exerted by object A on object B. Thus, $\vec{F}_{BA} = -\vec{F}_{AB}$.

These laws can not be proved, therefore they are considered as fundamental principles. These three laws are valid only in the inertial frames of reference!

Superposition of forces principle: Tested by experiments, any number of forces applied at a point on a body have the same effect as a single force equal to the vector sum of the forces. In addition, a single force can be decomposed into different components.

Range of validity: Newtonian mechanics does not apply to all situations. For fast-moving objects (close to c), we must replace Newtonian mechanics with Einstein's special theory of relativity. For small particles, we must use quantum mechanics instead. Newtonian mechanics is the special case of these two more comprehensive theories.

2.1 Newton's First Law of Motion

- First Law. An object acted on by no net force moves with constant velocity and zero acceleration.
- It is also known as the law of inertia.
- Inertial frames of reference: the frame in which Newton's first law is valid. If no forces act on an object, any reference frame for which the acceleration of the object remains zero is an inertial reference frame. Inertial frame should be a frame with no acceleration with respect to other inertial frames. There are many inertial frames, since any frame B moving with a constant velocity with respect to an inertial frame A is also an inertial frame of reference. The velocity in frame B is $\vec{v}_{p/B} = \vec{v}_{p/A} + \vec{v}_{A/B} = \vec{v}_{p/A} - \vec{v}_{B/A}$ which is a constant as long as $\vec{v}_{p/A}$ and $\vec{v}_{B/A}$ are constants. The earth is approximately an inertial frame, due to the small acceleration associated with its rotations.

Inertia is the resistance to any change in an object's velocity. Or the tendency of a body to keep moving once it is set in motion as shown above.

2.2 Newton's Second Law of Motion

$$\vec{F} = m\vec{a}, \quad \text{vector equation} \tag{1}$$

$$F_x = ma_x, \quad F_x = ma_x, \quad F_x = ma_x, \quad \text{components.} \tag{2}$$

What is mass?
$m = |F|/a$ tells us that the ratio $|F|/a$ is a constant for a given body. We call this ratio the inertial mass, or simply the mass. Mass is a quantitative measure of inertia. The greater the mass, the more a body "resists" being accelerated (change of motion).

Relativity
In Einstein's special relativity, there is mass-energy equivalence (relation) $m = E/c^2$. ("Does the inertia of a body depend upon its energy-content?") In general relativity, the equivalence principle is the equivalence of gravitational and initial mass.

- **Heads up**: first application of vectors in physics.
- $F = 0$ gives constant velocity.
- For the same force, the ratio of the masses is the the inverse of the ratio of the accelerations $\frac{m_1}{m_2} = \frac{a_2}{a_1}$. It is harder to accelerate a heavy object than a light object.
- Mass is not the same as weight, although we treat them the same in our daily life. Weight is a force $W = mg$ with $g = 9.8\text{N/kg} = 9.8\text{m/s}^2$.
- Non-inertial frame: Imagine we drop two objects A and B simultaneously (at the same time) and let them free fall. Choosing the object A as the reference frame, we find that object B is at rest relative to object A. However, $F_B = mg$ is not zero, which means that Newton's first and second laws are no longer valid. This is due to the reason that object A is accelerating and it is not an inertial frame.

2.3 Newton's Third Law of Motion

If body A exerts a force on body B (also called an action), the body B exerts a force on body A (a reaction). These two forces have the same magnitude but are opposite in direction. These two forces act on different bodies. The action and reaction are two opposite forces, which often are referred as an action-reaction pair.

3 Applying Newton's Laws

Free body diagram
A free body diagram is a diagram showing the chosen

body by itself, "free" of its surrounding, with vectors drawn to show the magnitudes and directions of all the external forces applied to the body by other bodies interacting with it.

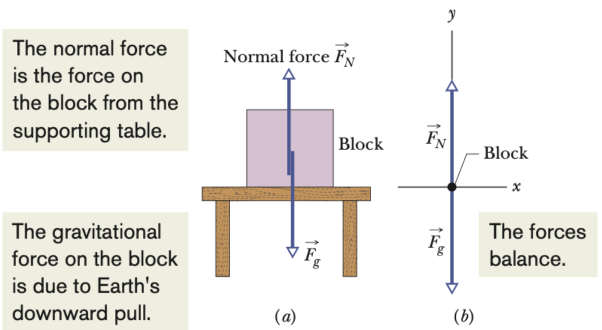
First, in a coordinate system, we represent the body as a dot, each force on the body is a vector arrow with its tail on the body.

Second, a collection of two or more bodies is called a system, and any force on the bodies inside the system caused by external agents outside the system is called an external force. If the bodies are rigidly connected, then we can treat the system as one composite body, and the net force F_{net} on it is the vector sum of all external forces.

Now let us study some simple examples.

Box on a horizontal surface

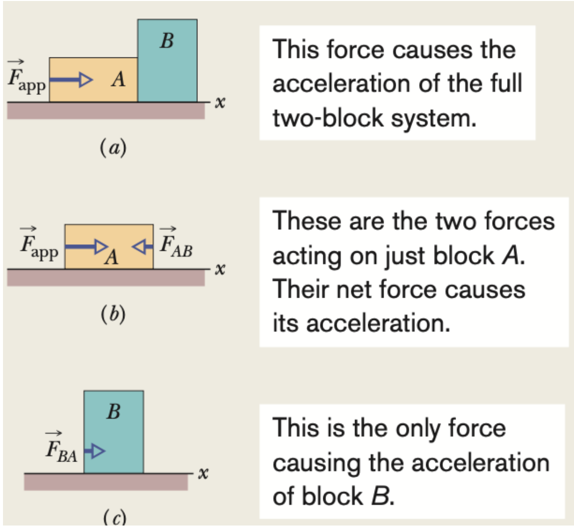
A block resting on a table experiences a normal force F_B perpendicular to the tabletop.



$$F_N - mg = ma_y \quad \Rightarrow \quad F_N = ma_y + mg. \tag{3}$$

- If the table is at rest or moving at constant velocity, it is an inertial frame itself. So $a_y = 0$. Thus $F_N = mg$.
- If the table is in an accelerating elevator, then a_y is the same as the acceleration of the elevator. It could be positive if the elevator is accelerating upward, and negative if the elevator is accelerating downward.
- Consistency check: When $a_y > 0$, the normal force F_N needs to not only overcome the gravitational force but also provide the upward acceleration. Therefore, $F_N > mg$ in this case. Similarly, when $a_y < 0$, we can understand $F_N < mg$. (Caution: We have already included the sign for g but a_y can be positive or negative here.)
- It is also interesting to note that the table and the block will no longer be in contact when $a_y \leq -g$. In this extreme case, the table (elevator) accelerates at a_y faster than g, the block leaves the surface of the tabletop and starts to float in the elevator, thus $F_N = 0$. This is known as the apparent weightlessness. It is important to note that we measure (define) the apparent weight through F_N by imaging that there is a scale between the block and tabletop.

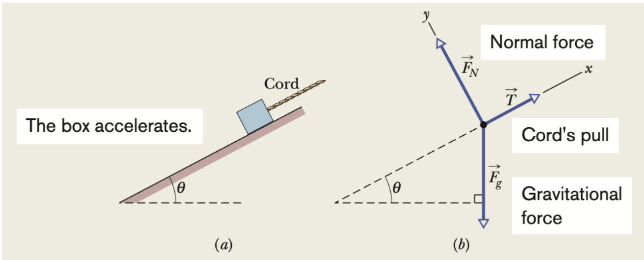
Two boxes moving together: (A pair of action and reaction forces)



There could be three equations: object A, object B and $A + B$. Only two of them are independent.

Homework problem

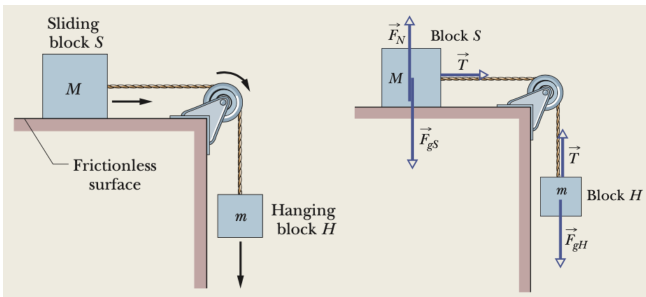
Box on a tilted slope



Note: Newton's second law now becomes a vector equation, which be projected onto x and y axes as follows

$$\text{y direction}: \quad F_N - mg\cos\theta = ma_y = 0 \tag{4}$$
$$\text{x direction}: \quad T - mg\sin\theta = ma_x \tag{5}$$

Ropes and pulleys

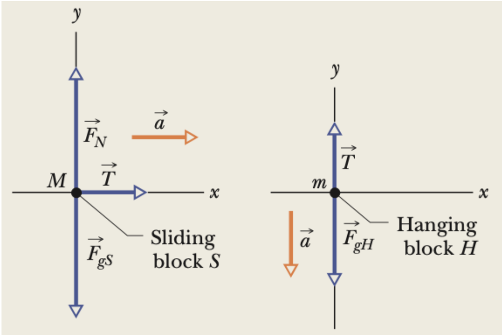


Note: One thing you should note: We assume that the cord does not stretch, therefore the blocks move together and their accelerations have the same magnitude a.

Rope: Usually taut but not stretched, pass the force from one object to anther. Pulley: Its only function is to change the cord's orientation.

Now apply Newton's second law to these two blocks

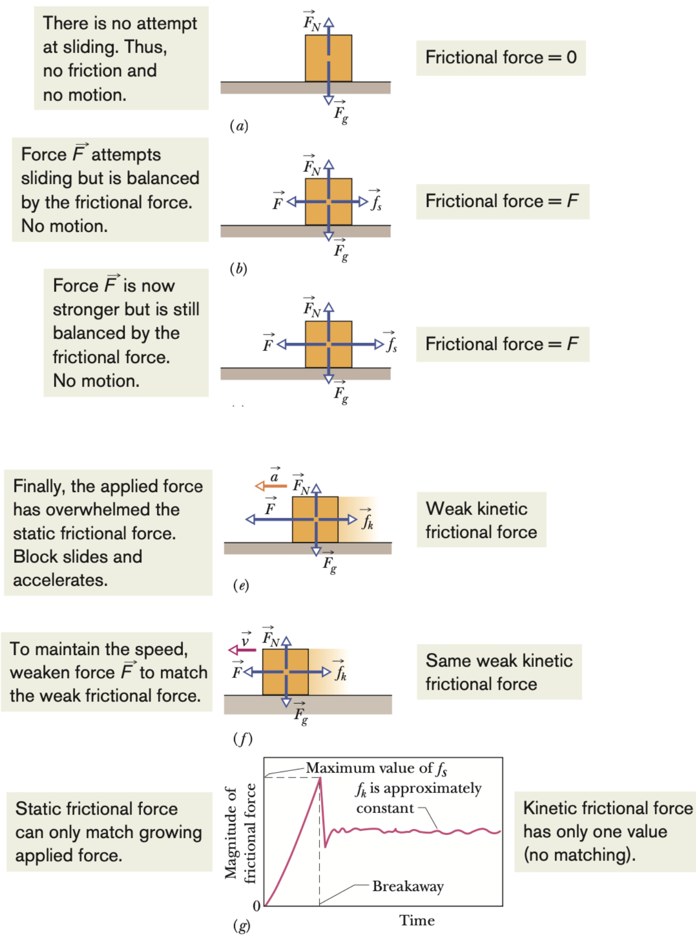
$$\text{x direction, block S}: \quad T = Ma \tag{6}$$
$$\text{y direction, block H}: \quad mg - T = ma \tag{7}$$
$$\text{Eliminate } T :\Rightarrow \quad a = \frac{mg}{m+M}. \tag{8}$$



Friction

Properties of friction: It is always against relative motion between two surfaces.

- Direction: along the surface, against the attempted motion or actual motion.
- Magnitude: For static case, the friction can vary, and there is a maximum value given by $f_{s,max} = \mu_s F_N$ where μ_s is the coefficient of static friction. For the sliding case, the magnitude of the frictional force is given by the value $f_k = \mu_k F_N$ where μ_k is the coefficient of kinetic friction. Usually, $\mu_s > \mu_k$. (Sometimes, for simplicity, we do not distinguish them, and use roughly the same value for them.)



Drags and terminal speed

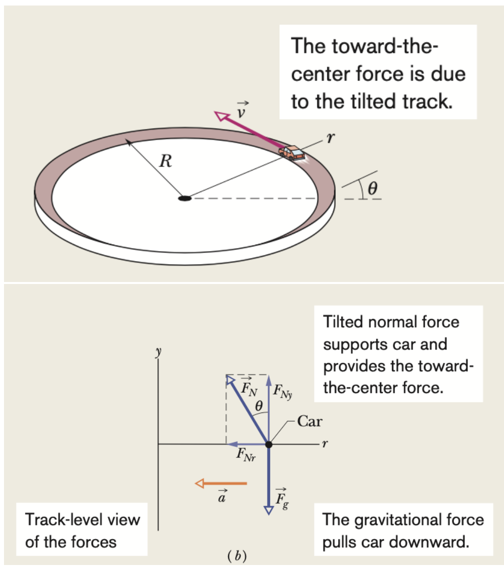
Sky divers in a horizontal "spread eagle" maximize air drag. To increase the terminal speed, one can reorient his body head-down so as to minimize A and maximize his downward speed.

The drag force is a special friction between a moving object and a fluid (liquid or gas).

- For small objects moving at low speeds, the magnitude of the fluid resistance $f = kv$ is approximately proportional to the metal ball's speed, where k is a proportionality constant that depends on the shape and size of the body and the properties of the fluid.
- For the case in which the relative motion is fast enough, the magnitude of the drag force, f is related to the relative speed v by an experimentally determined drag coefficient C according to
$$f = D = \frac{1}{2}C\rho A v^2,$$
where ρ is the air density and A is the effective cross-sectional area. C (dimensionless) is around $0.4 \sim 1$.
- When divers jump out of the plane, $ma = mg - C\rho A v^2/2$, the speed increases, and the drag force also increases, until finally it is equal in magnitude to the weight. At this time, $mg = C\rho A v^2/2$, the acceleration becomes zero, and there is no further increase in speed. The terminal speed is then given by $v_t = \sqrt{(2mg)/(C\rho A)}$.

Circular motion

Rounding a banked curve: On highway or racetrack, the curved road is banked so that cars can safely make the turn even without friction.



Car is in equilibrium along the Vertical Axis
$$F_N \cos\theta - mg = 0 \quad \Rightarrow \quad F_N = mg/\cos\theta. \tag{9}$$
Radial direction: there should be centripetal acceleration to maintain the circular motion in the horizontal plane
$$F_N \sin\theta = ma_{rad} = m\frac{v^2}{R} \quad \Rightarrow \quad F_N = m\frac{v^2}{R}/\sin\theta. \tag{10}$$