香港中文大學（深圳）
The Chinese University of Hong Kong, Shenzhen

Introduction to Computer Science: Programming Methodology

Lecture 9
Recursion, Stack and Queue

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Linear Recursion

- If a recursive function is designed so that each invocation of the body makes at most one new recursive call, this is known as linear recursion

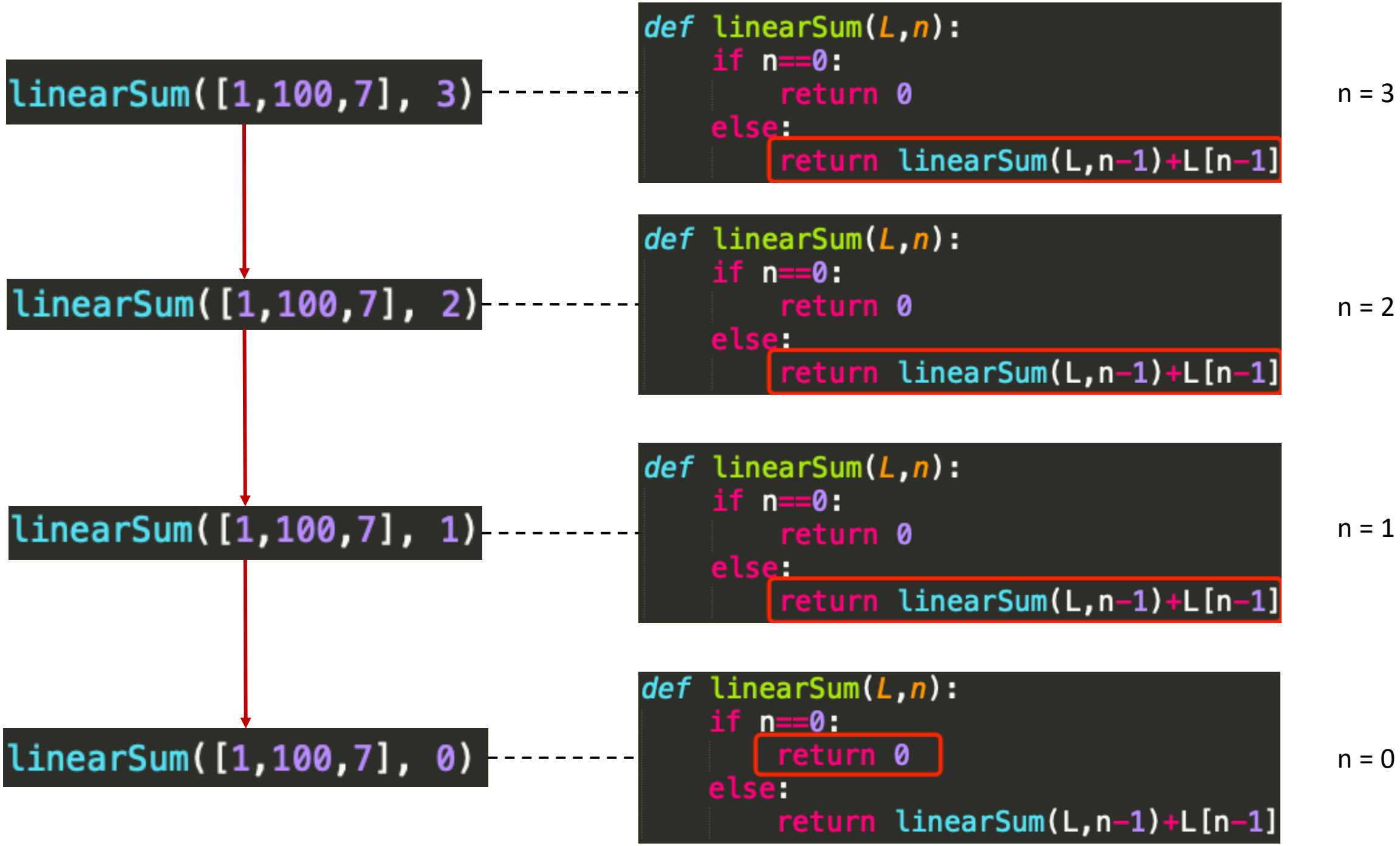
- Finding the smallest number and binary search are both linear recursive algorithms

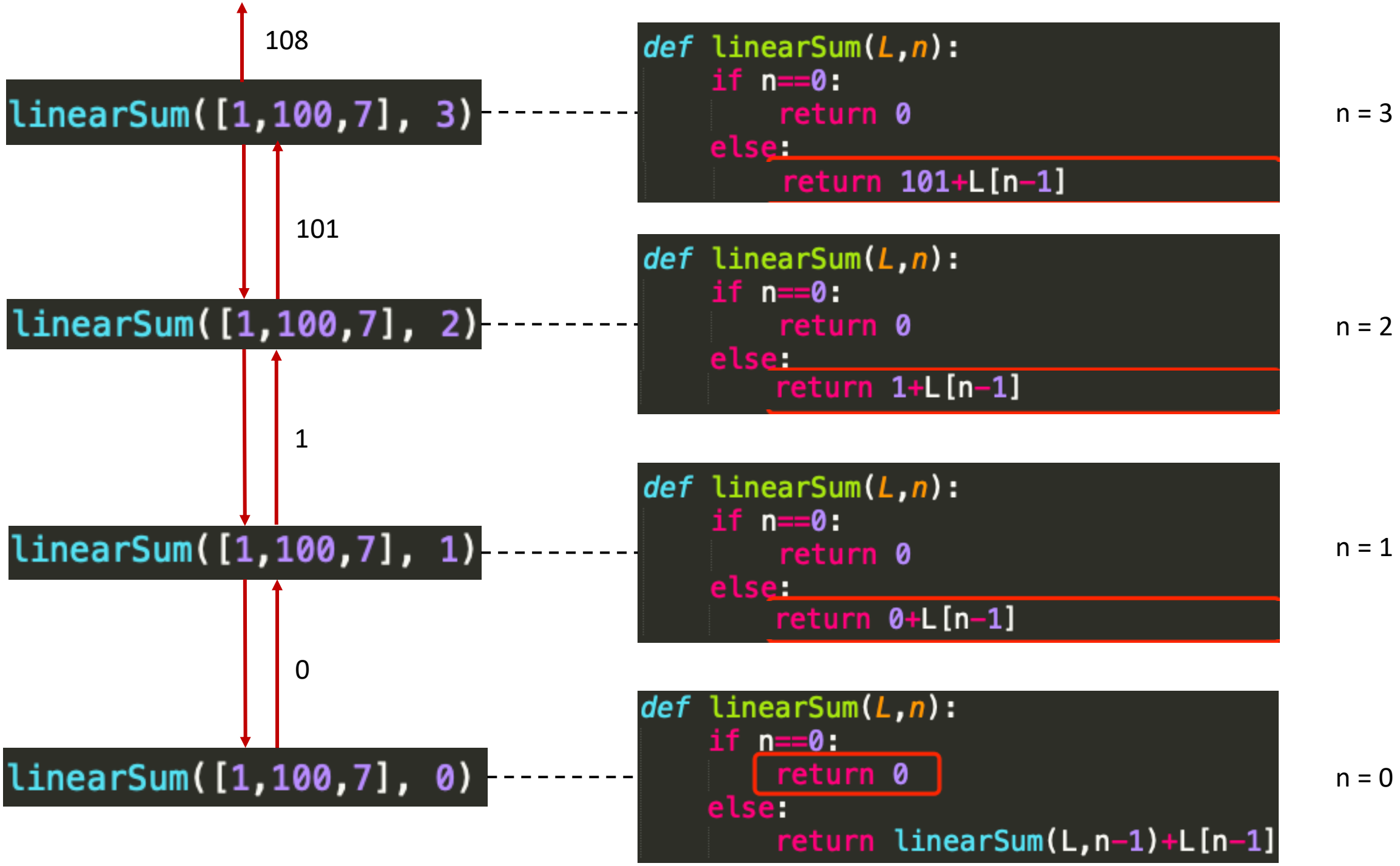
Practice: Sum of a list

- Given a list of numbers, write a program to calculate the sum of this list using recursion

Solution:

```
def linearSum(L, n):
    if n==0:
        return 0
    else:
        return linearSum(L, n-1)+L[n-1]

def main():
    L = [1, 2, 3, 4, 5, 9, 100, 46, 7]
    print('The sum is:', linearSum(L, len(L)))
```


linearSum([1,100,7], 3)
linearSum([1,100,7], 2)
linearSum([1,100,7], 1)
linearSum([1,100,7], 0)
def linearSum(L,n):
if n==0:
return 0
else:
return linearSum(L,n-1)+L[n-1]
n = 3
n = 2
n = 1
n = 0


108
linearSum([1,100,7], 3)
101
linearSum([1,100,7], 2)
1
linearSum([1,100,7], 1)
0
linearSum([1,100,7], 0)
def linearSum(L,n):
if n==0:
return 0
else:
return 101+L[n-1]
n = 3
def linearSum(L,n):
if n==0:
return 0
else:
return 1+L[n-1]
n = 2
def linearSum(L,n):
if n==0:
return 0
else:
return 0+L[n-1]
n = 1
def linearSum(L,n):
if n==0:
return 0
else:
return linearSum(L,n-1)+L[n-1]
n = 0

Practice: Power function

- Write a program to calculate the power function $f(x, n) = x^n$ using Recursion. The time complexity of the program should be O(logn)

x*x*x* … *x

(n-1) is O(n)

A better recursive definition of power function

$$power(x,n) = \begin{cases} 1 & \text{if } n = 0 \\ x \cdot \left(power\left(x, \lfloor \frac{n}{2} \rfloor\right)\right)^2 & \text{if } n > 0 \text{ is odd} \\ \left(power\left(x, \lfloor \frac{n}{2} \rfloor\right)\right)^2 & \text{if } n > 0 \text{ is even} \end{cases}$$

Solution:

```
def myPower(x,n):
    if n==0:
        return 1
    else:
        partial = myPower(x,n//2)
        result = partial * partial
        if n%2==1:
            result = result * x
        return result
```

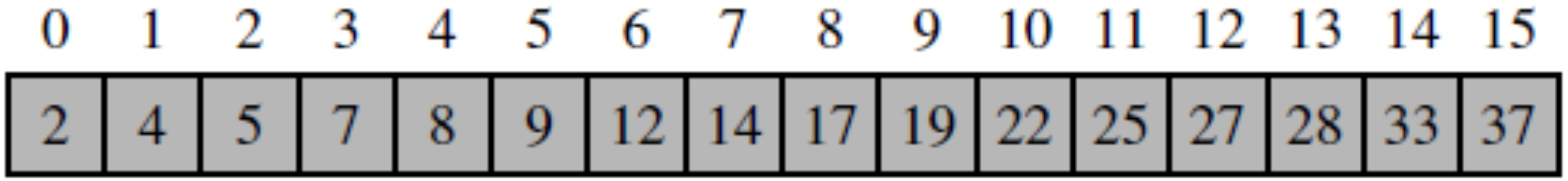
Multiple re

- When a functi ·sive calls, we say that it
- Drawing the E ɹrsion program

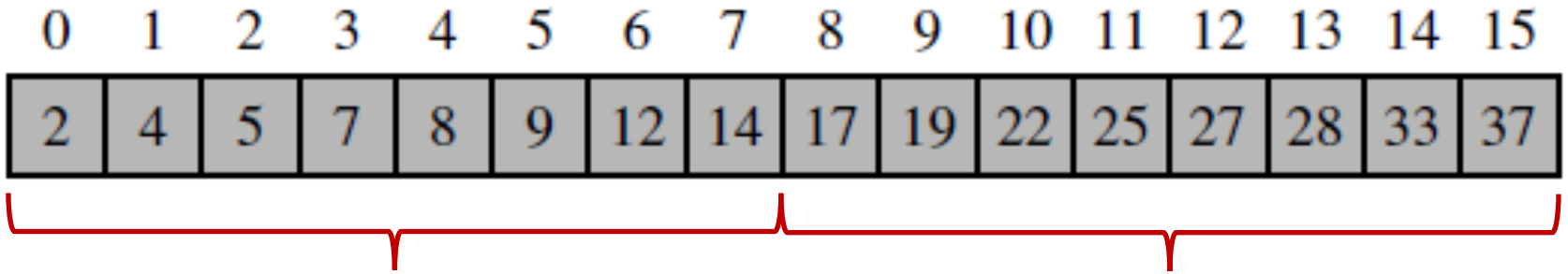
```
def draw_line(tickLen, tickLabel=''):
    line = '-'*tickLen
    if tickLabel:
        line+=' '+tickLabel
    print(line)

def draw_interval(centerLen):
    if centerLen>0:
        draw_interval(centerLen-1)
        draw_line(centerLen)
        draw_interval(centerLen-1)

def draw_ruler(numInch, majorLen):
    draw_line(majorLen, '0')

    for j in range(1, 1+numInch):
        draw_interval(majorLen-1)
        draw_line(majorLen, str(j))
```

Practice: Binary sum

- Write a function binarySum() to calculate the sum of a list of numbers. Inside binarySum() two recursive calls should be made

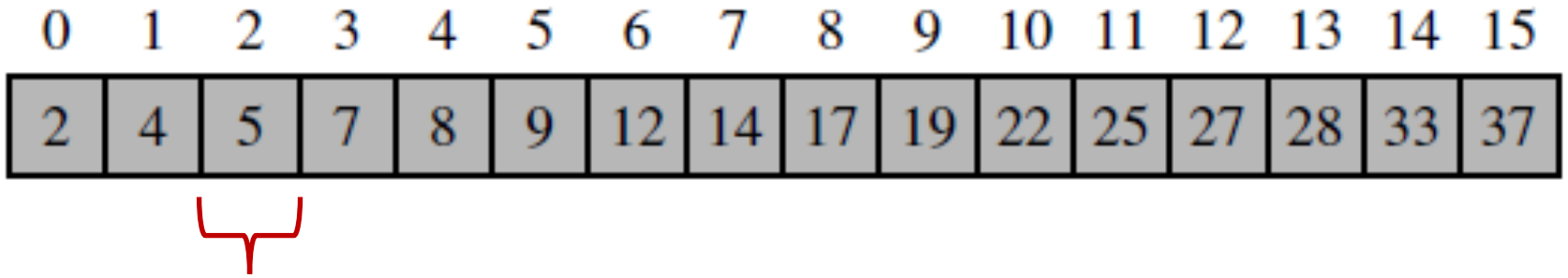


Practice: Binary sum




```
binarySum(L,start,mid)+binarySum(L,mid,stop)
```

Practice: Binary sum




```
elif start==stop-1:
    return L[start]
```

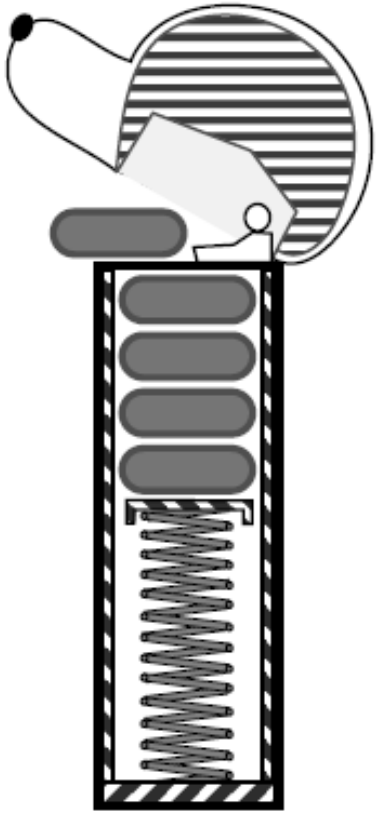
Solution:

```
def binarySum(L, start, stop):
    if start>=stop:
        return 0
    elif start==stop - 1:
        return L[start]
    else:
        mid = (start+stop)//2
        return binarySum(L,start,mid)+binarySum(L,mid,stop)

def main():
    L = [1,2,3,4,5,6,7]
    print(binarySum(L,0,len(L)))
```

Stack

- A stack is a collection of objects that are inserted and removed according to the last-in, first-out (LIFO) principle

- A user may insert objects into a stack at any time, but may only access or remove the most recently inserted object that remains (at the so-called “top” of the stack)



Example: Web Browser

- Internet Web browsers store the addresses of recently visited sites in a stack. Each time a user visits a new site, that site's address is "pushed" onto the stack of addresses. The browser then allows the user to "pop" back to previously visited sites using the "back" button.

Example: Text editor

- Text editors usually provide an “undo” mechanism that cancels recent editing operations and reverts to former states of a document. This undo operation can be accomplished by keeping text changes in a stack.

The stack class

- Generally, a stack may contain the following methods:

S.push(e):	Add element e to the top of stack S.
S.pop( ):	Remove and return the top element from the stack S; an error occurs if the stack is empty.
S.top( ):	Return a reference to the top element of stack S, without removing it; an error occurs if the stack is empty.
S.is_empty( ):	Return True if stack S does not contain any elements.
len(S):	Return the number of elements in stack S; in Python, we implement this with the special method __len__.

The Code of Stack Class

```
class ListStack:

    def __init__(self):
        self.__data = list()

    def __len__(self):
        return len(self.__data)

    def is_empty(self):
        return len(self.__data) == 0

    def push(self,e):
        self.__data.append(e)

    def top(self):
        if self.is_empty():
            print('The stack is empty.')
        else:
            return self.__data[self.__len__()-1]

    def pop(self):
        if self.is_empty():
            print('The stack is empty.')
        else:
            return self.__data.pop()
```

The code to use stack class

```
def main():
    s = ListStack()
    print('The stack is empty? ',s.is_empty())
    s.push(100)
    s.push(200)
    s.push(300)
    print(s.top())
    print(s.pop())
    print(s.top())
```

Practice: Reverse a list using stack

- Write a program to reverse the order of a list of numbers using the stack class

Solution:

```
from stack import ListStack

def reverse_data(oldList):
    s = ListStack()
    newList = list()

    for i in oldList:
        s.push(i)

    while (not s.is_empty()):
        mid = s.pop()
        newList.append(mid)

    return newList

def main():
    oldList = [1,2,3,4,5]
    newList = reverse_data(oldList)
    print(newList)
```

Practice: Brackets match checking

- In correct arithmetic expressions, the opening brackets must match the corresponding closing brackets. Write a program to check whether all the opening brackets have matched closing brackets.

Solution:

```
from stack import ListStack

def is_matched(expr):
    lefty = '([{'
    righty = ')]}'

    s = ListStack()

    for c in expr:

        if c in lefty:
            s.push(c)
        elif c in righty:
            if s.is_empty():
                return False
            if righty.index(c)!=lefty.index(s.pop()):
                return False
    return s.is_empty()

def main():
    expr = '1+2*(3+4)-[5-6]'
    print(is_matched(expr))
    expr = '(((()}]'
    print(is_matched(expr))
```

Practice: Matching Tags in HTML Language

- HTML is the standard format for hyperlinked documents on the Internet

- In an HTML document, portions of text are delimited by HTML tags. A simple opening HTML tag has the form “<name>” and the corresponding closing tag has the form “</name>”

HTML Tags

- Commonly used HTML tags that are used in this example include

  - body: document body
  - h1: section header
  - center: center justify
  - p: paragraph
  - ol: numbered (ordered) list
  - li: list item

An example of HTML document

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine.  The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```

(a)

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

(b)

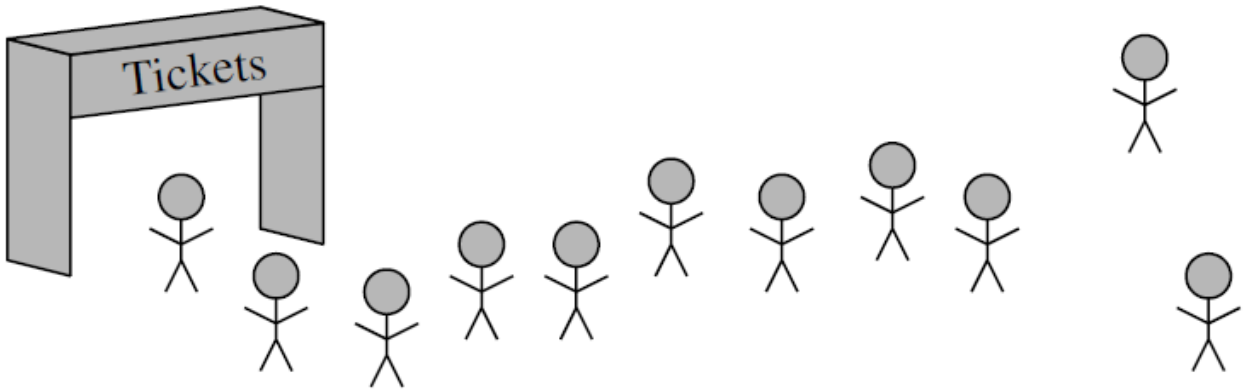
Solution:

```
from stack import ListStack

def is_matched_html(raw):
    s = ListStack()
    j = raw.find('<')

    while j!=-1:
        k = raw.find('>', j+1)
        if k==-1:
            return False
        tag = raw[j+1:k]

        if not tag.startswith('/'):
            s.push(tag)
        else:
            if s.is_empty():
                return False
            if tag[1:]!=s.pop():
                return False
        j = raw.find('<', k+1)

    return s.is_empty()

def main():
    fhand = open('sampleHTML.txt','r')
    raw = fhand.read()
    print(raw)
    print(is_matched_html(raw))
```

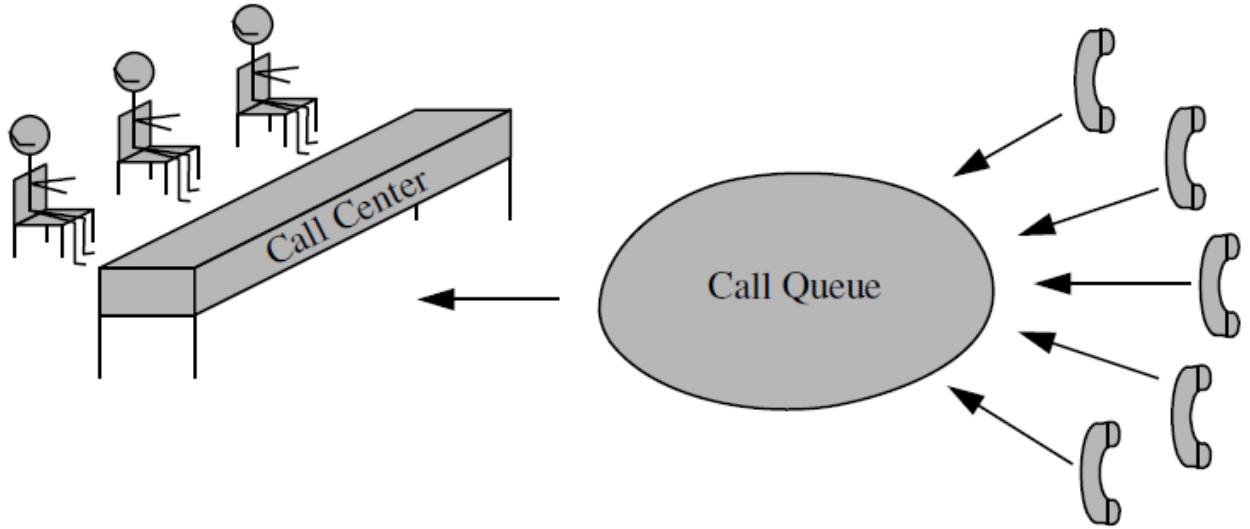
Queue

- Queue is another fundamental data structure
- A queue is a collection of objects that are inserted and removed according to the first-in, first-out (FIFO) principle
- Elements can be inserted at any time, but only the element that has been in the queue the longest can be next removed

Applications of Queue



(a)



(b)

The queue class

- The queue class may contain the following methods:

Q.enqueue(e):	Add element e to the back of queue Q.
Q.dequeue( ):	Remove and return the first element from queue Q; an error occurs if the queue is empty.
Q.first( ):	Return a reference to the element at the front of queue Q, without removing it; an error occurs if the queue is empty.
Q.is_empty( ):	Return True if queue Q does not contain any elements.
len(Q):	Return the number of elements in queue Q; in Python, we implement this with the special method __len__.

The code of queue class

```
class ListQueue:
    default_capacity = 5

    def __init__(self):
        self.__data = [None]*ListQueue.default_capacity
        self.__size = 0
        self.__front = 0
        self.__end = 0

    def __len__(self):
        return self.__size

    def is_empty(self):
        return self.__size ==0

    def first(self):
        if self.is_empty():
            print('Queue is empty.')
        else:
            return self.__data[self.__front]
```

```
def dequeue(self):

    if self.is_empty():
        print('Queue is empty.')
        return None

    answer = self.__data[self.__front]
    self.__data[self.__front] = None
    self.__front = (self.__front+1) \
                   % ListQueue.default_capacity
    self.__size -=1
    return answer

def enqueue(self,e):
    if self.__size == ListQueue.default_capacity:
        print('The queue is full.')
        return None

    self.__data[self.__end] = e
    self.__end = (self.__end+1) \
                 % ListQueue.default_capacity
    self.__size += 1

def outputQ(self):
    print(self.__data)
```

Practice: Simulating a web service

- An online video website handles service requests in the following way:
  1) It maintains a service queue which stores all the unprocessed service requests.
  2) When a new service request arrives, it will be saved at the end of the service queue.
  3) The server of the website will process each service request on a “first-come-first-serve” basis.

- Write a program to simulate this process. The processing time of each service request should be randomly generated.

Solution

```
from ListQueue import ListQueue
from random import random
from math import floor

class WebService():
    default_capacity = 5
    def __init__(self):
        self.nameQ = ListQueue()
        self.timeQ = ListQueue()

    def taskArrive(self,taskName,taskTime):
        if self.nameQ.__len__() < WebService.default_capacity:
            self.nameQ.enqueue(taskName)
            self.timeQ.enqueue(taskTime)
            print('A new task 《'+taskName+'》 has arrived and is waiting for processing...')
        else:
            print('The service queue of our website is full, the new task is dropped.')

    def taskProcess(self):
        if (self.nameQ.is_empty() == False):
            taskName = self.nameQ.dequeue()
            taskTime = self.timeQ.dequeue()
            print('Task 《'+taskName+'》 has been processed, it costs '+str(taskTime)+' seconds.')
```

Solution

```
def main():
    ws = WebService()
    taskNameList = ['Dark knight','X-man','Kungfu','Shaolin Soccer','Matrix','Walking in the clouds'\
                    ,'Casino Royale','Bourne Supremacy','Inception','The Shawshank Redemption']

    print('Simulation starts...')
    print('-------------------------------------')
    for i in range(1,31):
        rNum = random()
        if rNum<=0.6:
            taskIndex = floor(random()*10)
            taskTime  = floor(random()*1000)/100
            ws.taskArrive(taskNameList[taskIndex],taskTime)
        else:
            ws.taskProcess()
    print('-------------------------------------')
    print('Simulation finished.')
```