Lecture 8

Poincaré

GFN1000 In Dialogue with Nature

Epistemology

Part I. Human exploration of the physical universe

Part II. Human exploration of the world of life

Part III. Our understanding of human understanding

Ancient Greek	epistēmē	Math + Philosophy
Middle Age	scientia	Theology + Math + Philosophy
Modern	wissen-schaft (German)	Science + Philosophy

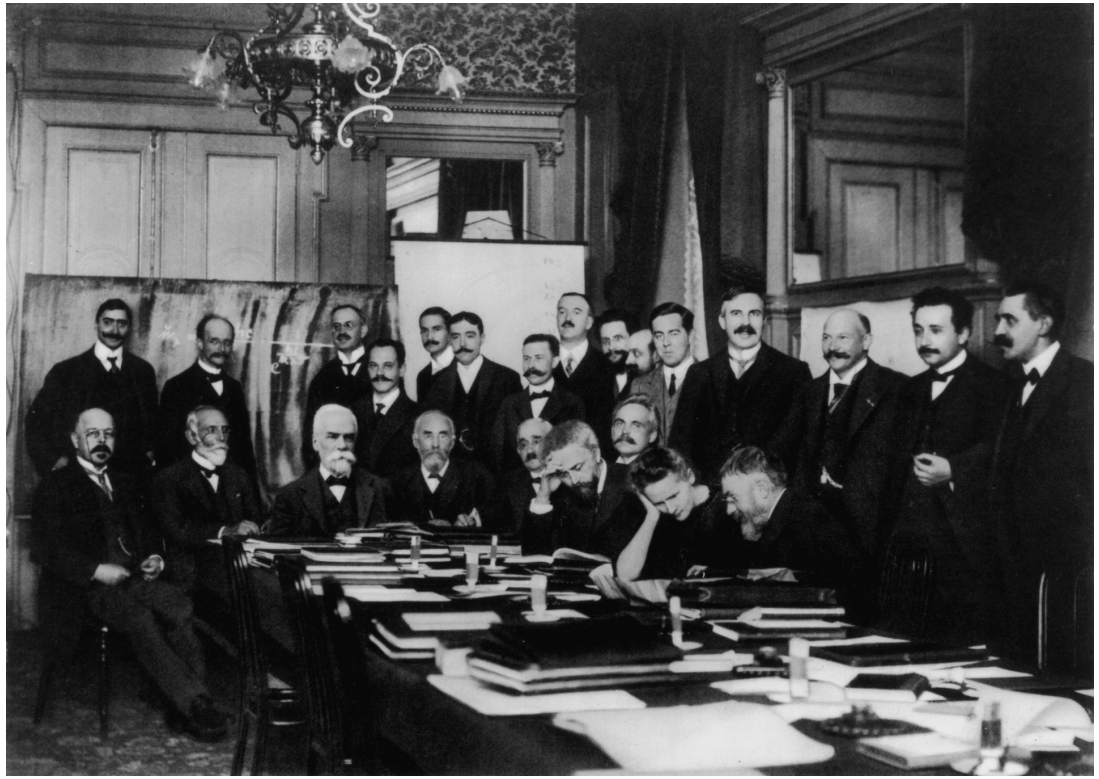
	Analytic	Synthetic
A Priori	Logical	Transcen-dental
A posteriori	Hypothe-tical	Empirical

Content

- Henri Poincaré
  - The Person
  - Poincaré Conjecture
- Science and Method
  - Reductionism
  - Scientific Discovery
  - Conventionalism
  - Intuition and Aesthetic Values

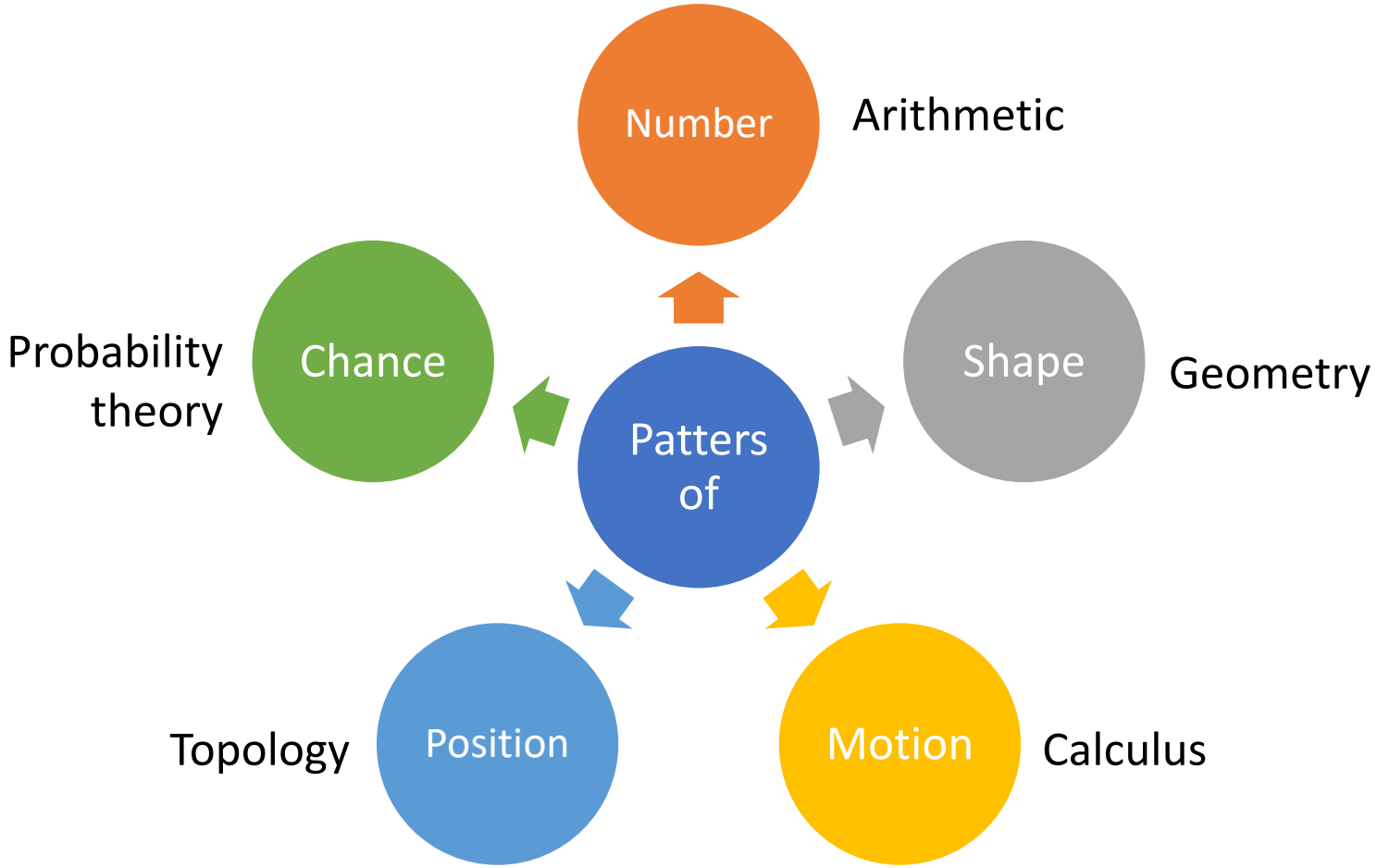
Henri Poincaré (1854-1912)

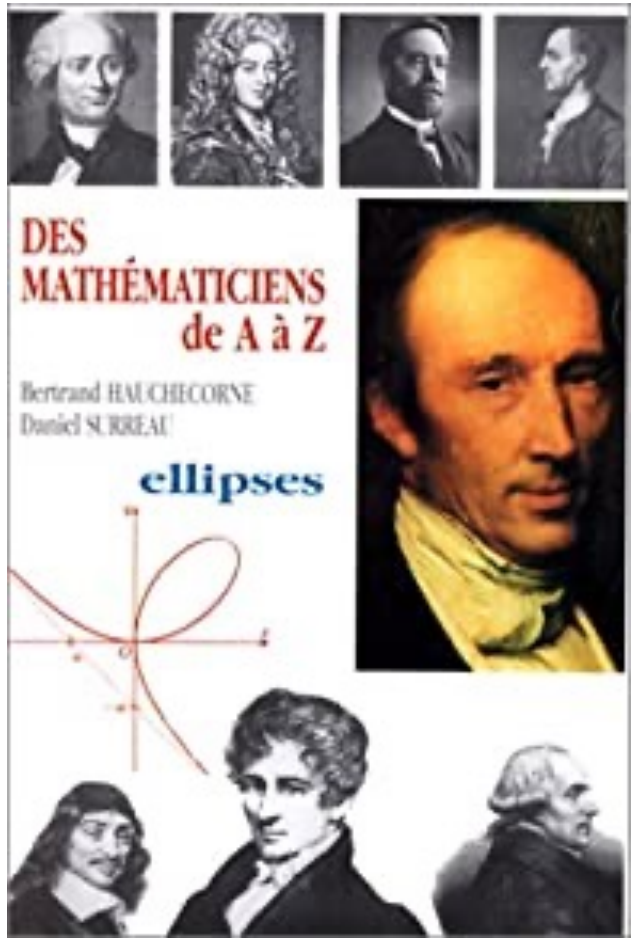
- Polymath: French mathematician, theoretical physicist, and philosopher of science
  - Einstein viewed Poincaré as being one of the pioneers of relativity.
- The only member elected to every one of the five sections of the French Academy of Sciences
  - Geometry, mechanics, physics, geography, and navigation sections



Solvay Congress 1911

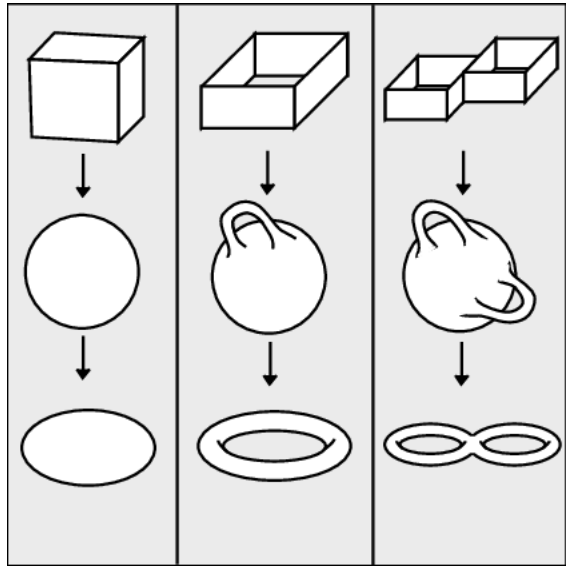
The Last Universalist

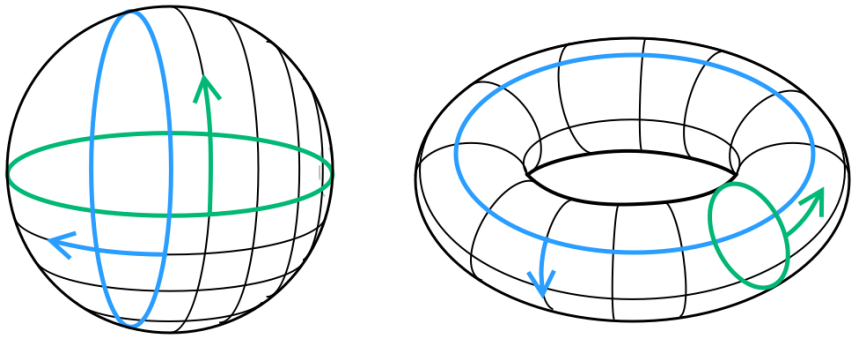




Poincaré Conjecture

- Topology is interested in the properties that remain unchanged under continuous deformations.
- Every simply connected, closed 3-manifold is **homeomorphic** (topologically equivalent) to the 3-sphere.
- Grigori Perelman solved the problem in 2003 but declined the Fields Medal.

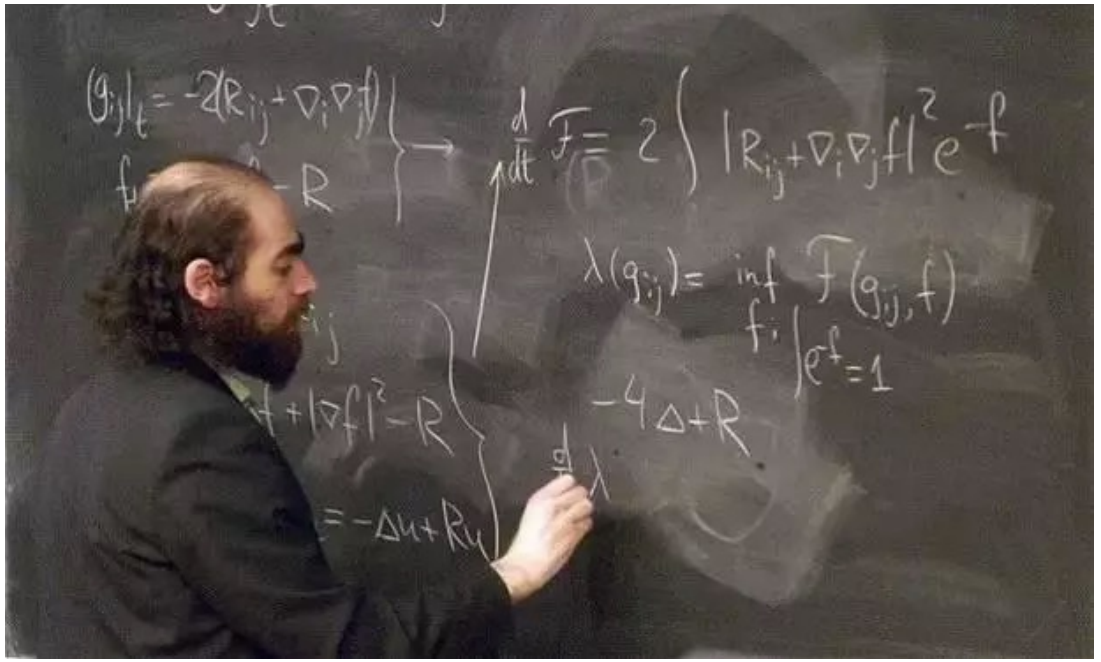




The Millennium Problems

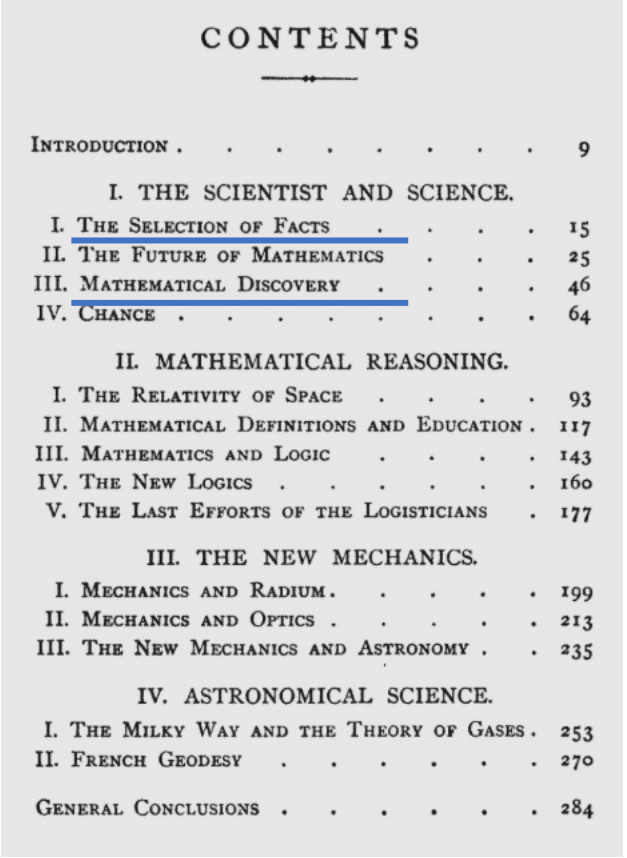
Millennium Prize Problems (US$1,000,000)

- P versus NP.
- Hodge conjecture.
- Riemann hypothesis.
- Yang–Mills existence and mass gap.
- Navier–Stokes existence and smoothness.
- Birch and Swinnerton-Dyer Conjecture.
- ~~Poincaré conjecture.~~



Poincaré's Work on Philosophy of Science

- Poincaré is known for his critique of logicism and formalism. He argued for conventionalism.
  - *Science and Hypothesis* (1902)
  - *The Value of Science* (1905)
  - *Science and Method* (1908)
  - *The Foundations of Science* (1913)
- *Science and Method* rethinks and redefines the boundary between **reason** and **instinct**, the boundary between **utility** and **beauty**, and the boundary between **conscious** and **unconscious**.



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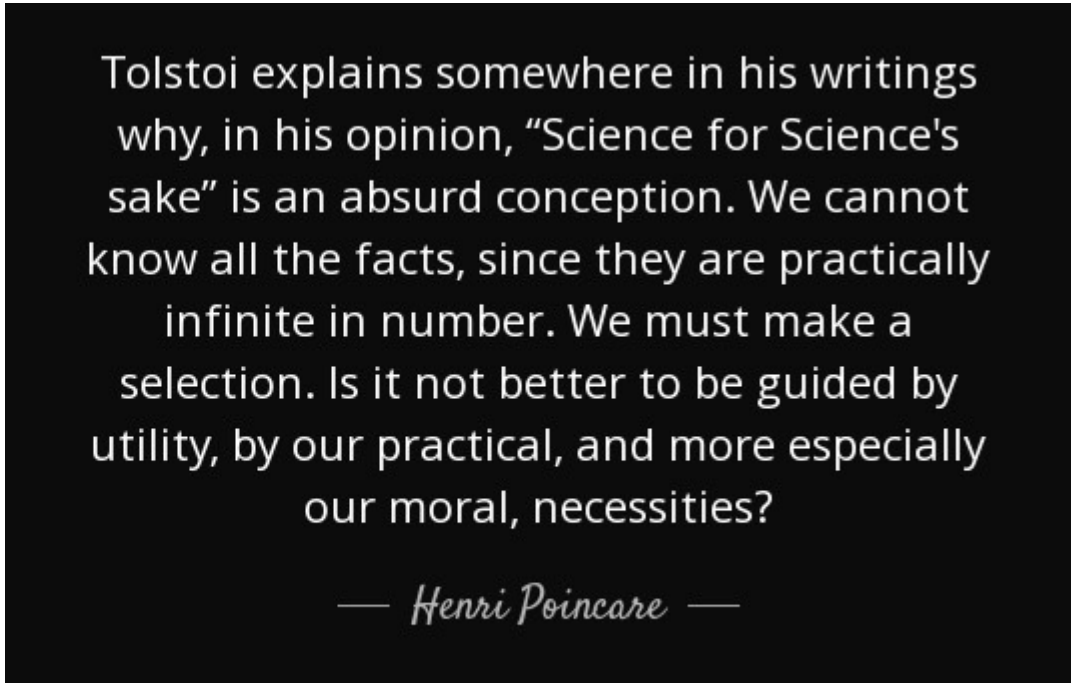
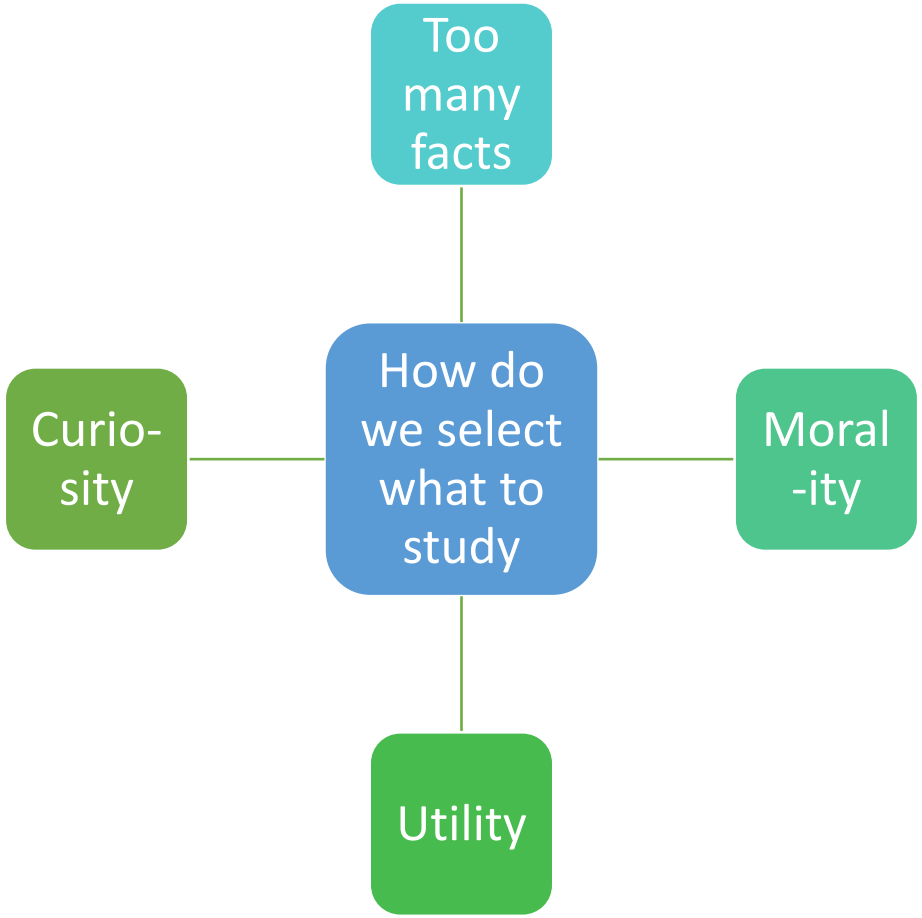
Science for the Sake of Science?



- For Tolstoy, for art to be art "the spectators or auditors are infected by the feelings the author has felt" and "a means of **union** among men."
- "We have the terrible probability to consider, that while fearful sacrifices of the labor and lives of men, and of **morality** itself, are being made to art, that same art may be not only useless but even harmful."

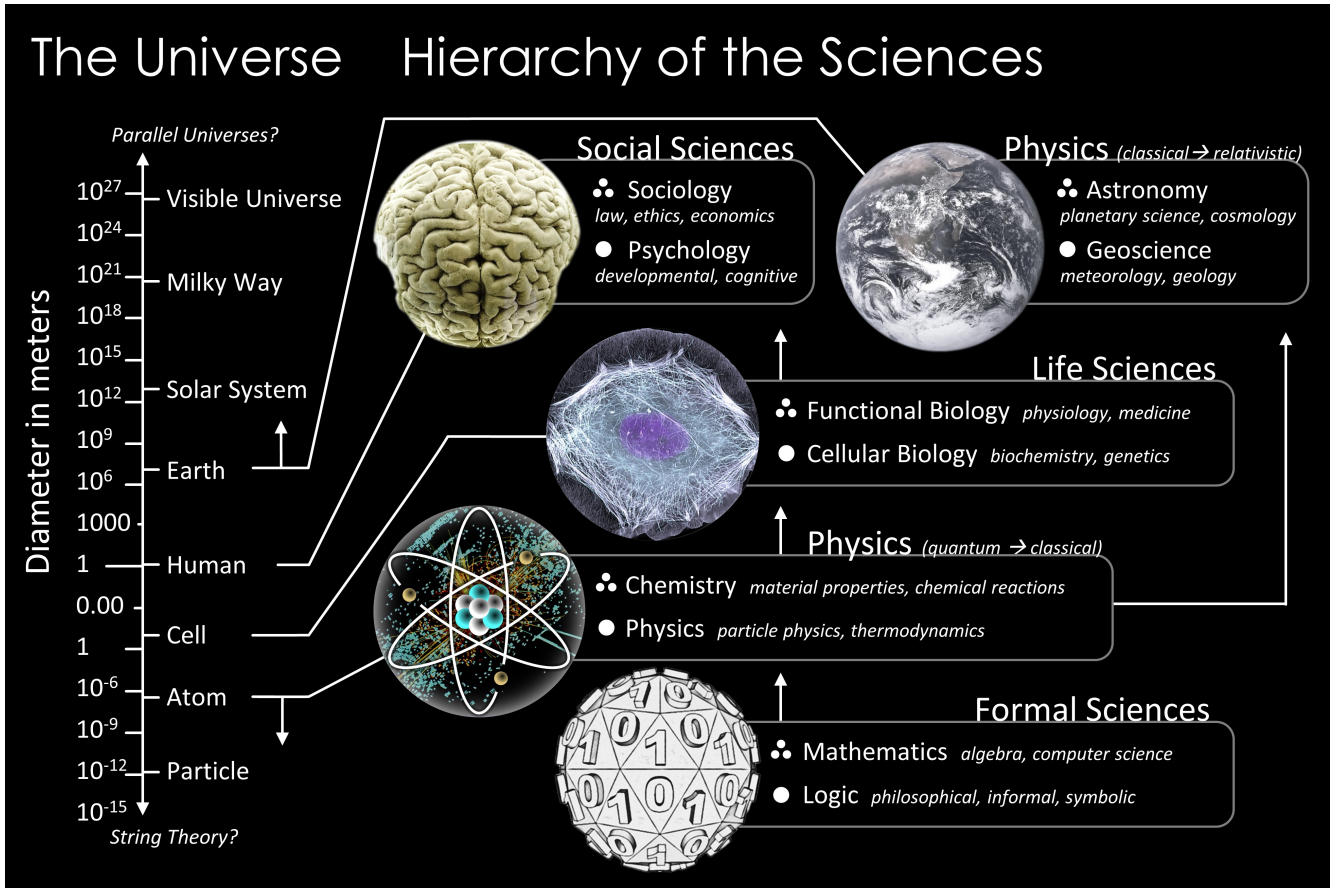
How Do We Select What to Study?



- There is an **infinite** number of facts; it is impossible for us to know them all.
- All facts are **not equal**. Some facts are more important for science than others.
- Poincaré rejects the idea that **utility** should be the **standard** of this selection, because utility is only concerned with the immediate task and is unlikely to guide us to facts with wider application.

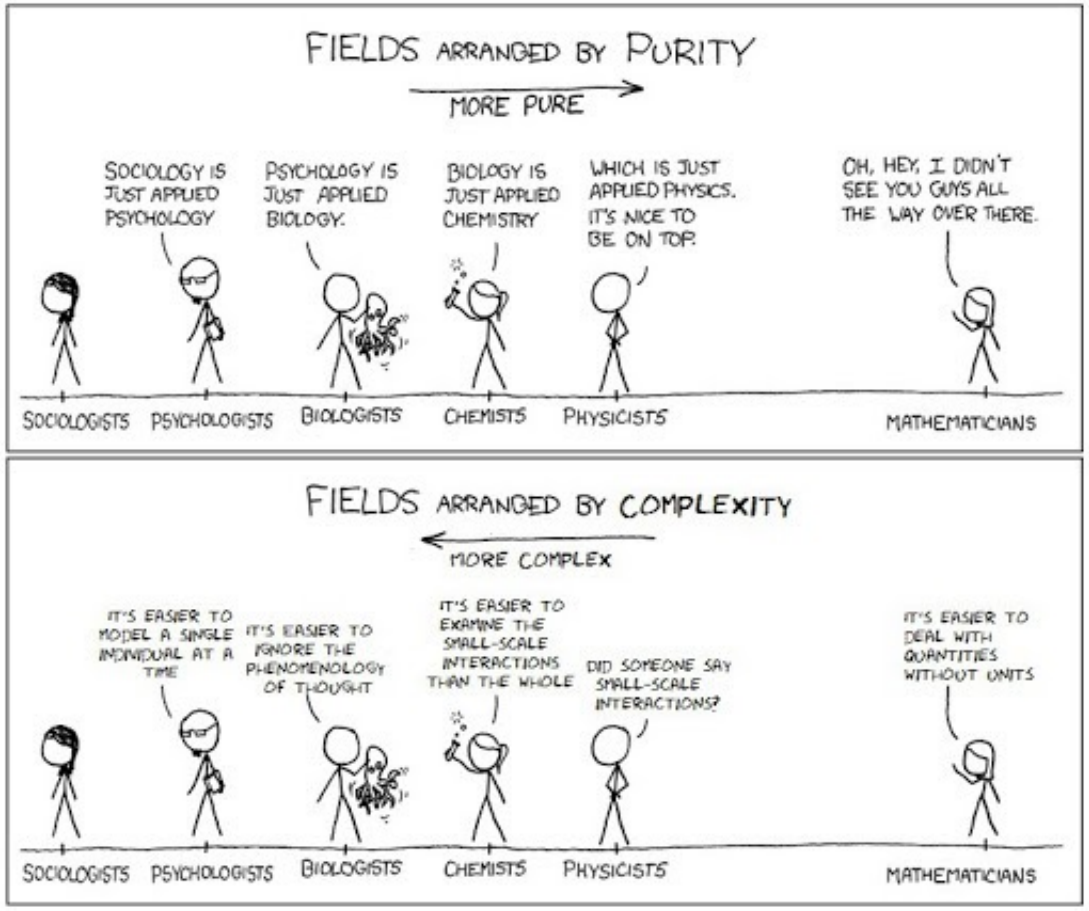
Generality and Simplicity

- The more **general** a law is, the greater its value.
  - A chance of **recurring**
  - Used several times
- Simplicity: Select facts that can be **simplified**, because they have a better chance of **wider application**.

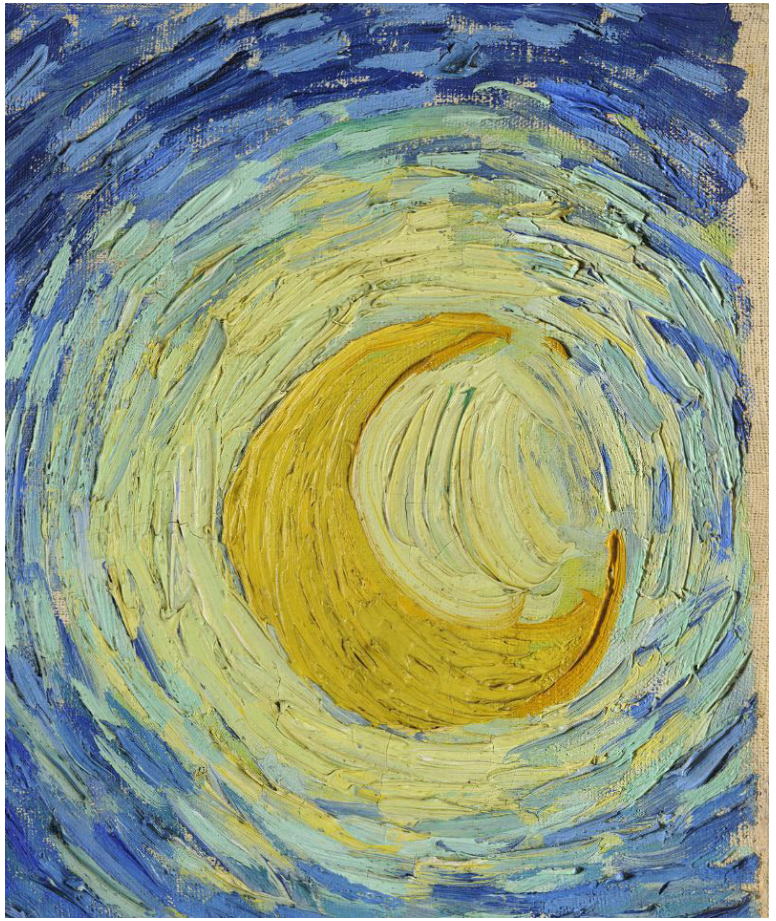


Reductionism vs. Holism

- Reductionism: Entities are **reducible** to **simpler** or **more basic** entities. Complex phenomena should be explained by the **simplest** underlying principles possible.
- Holism: The whole is more than the sum of the parts, contrasted with reductionism or atomism.

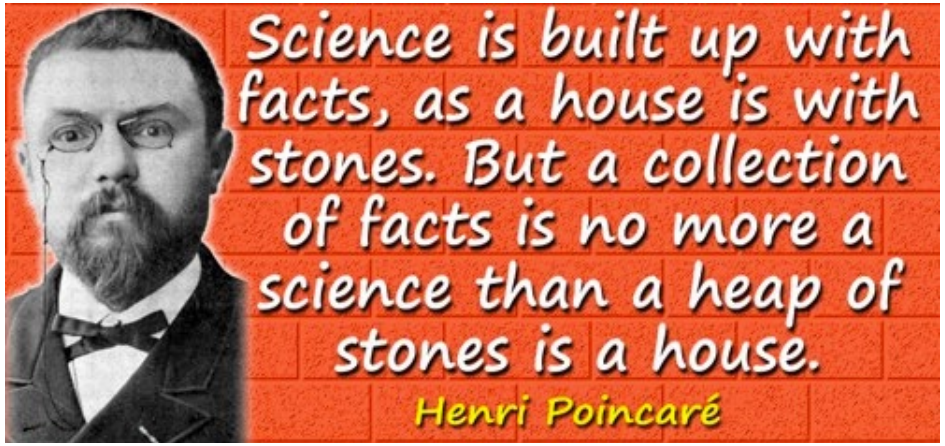


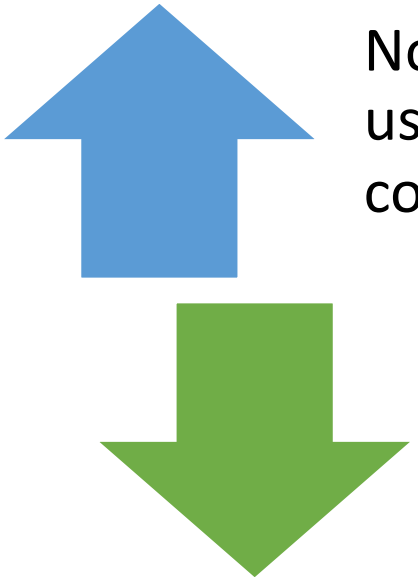
Limits of Reductionism



Mathematical Discovery

- For Poincare, mathematical discovery [invention] is about the selection of the small minority of combinations that are useful.
- The **useful combinations** are those conducive to the discovery of laws relating to facts that were previously believed to be unrelated.
- By **intuition** (unconscious) rather than by **rationalization**.



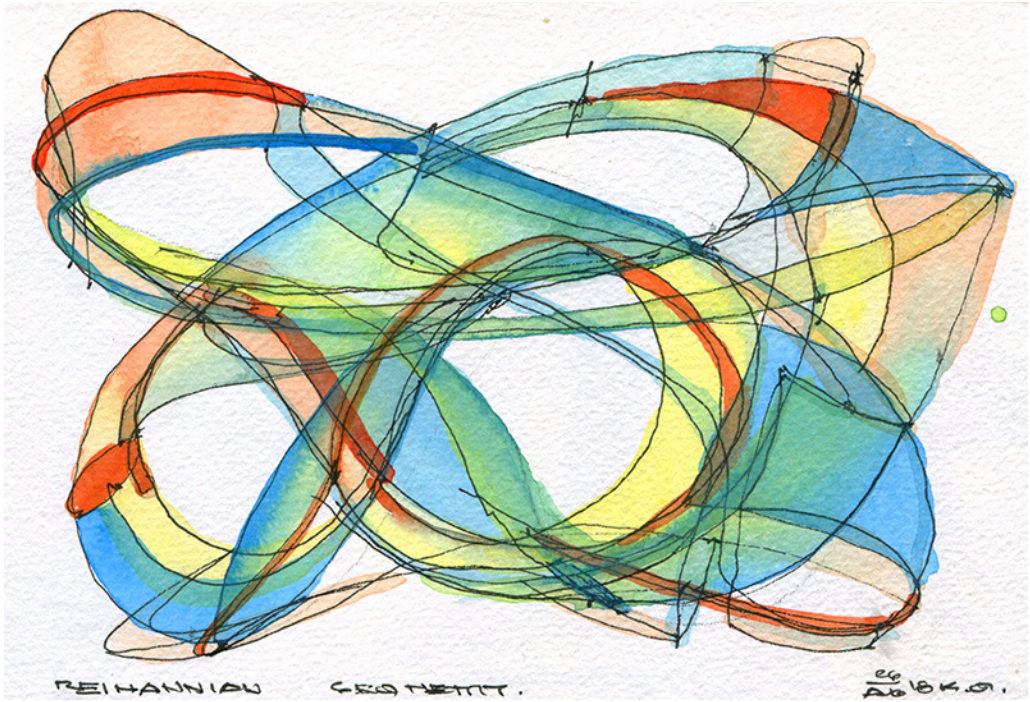


Not making new useless combinations

Selecting useful constructions that lead to mathematical law

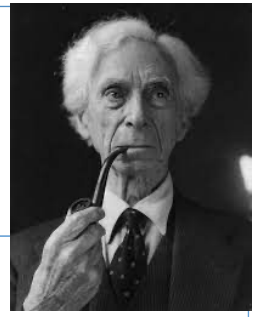
Conventionalism

- Neither is geometry *a priori* nor *a posteriori* based on empirical facts. It is a **convention**.
- Conventions are chosen by scientists based on **simplicity** and **efficiency**, but they are not necessarily true.
- The use of Euclidean geometry and Newtonian mechanics would not be overturned experimentally.



Schools of Philosophy of Mathematics

Logicism



- Russell
- Purely formal systems based on axioms and arbitrarily chosen symbols
- Math can be reduced to **logic**.

Intuitionism

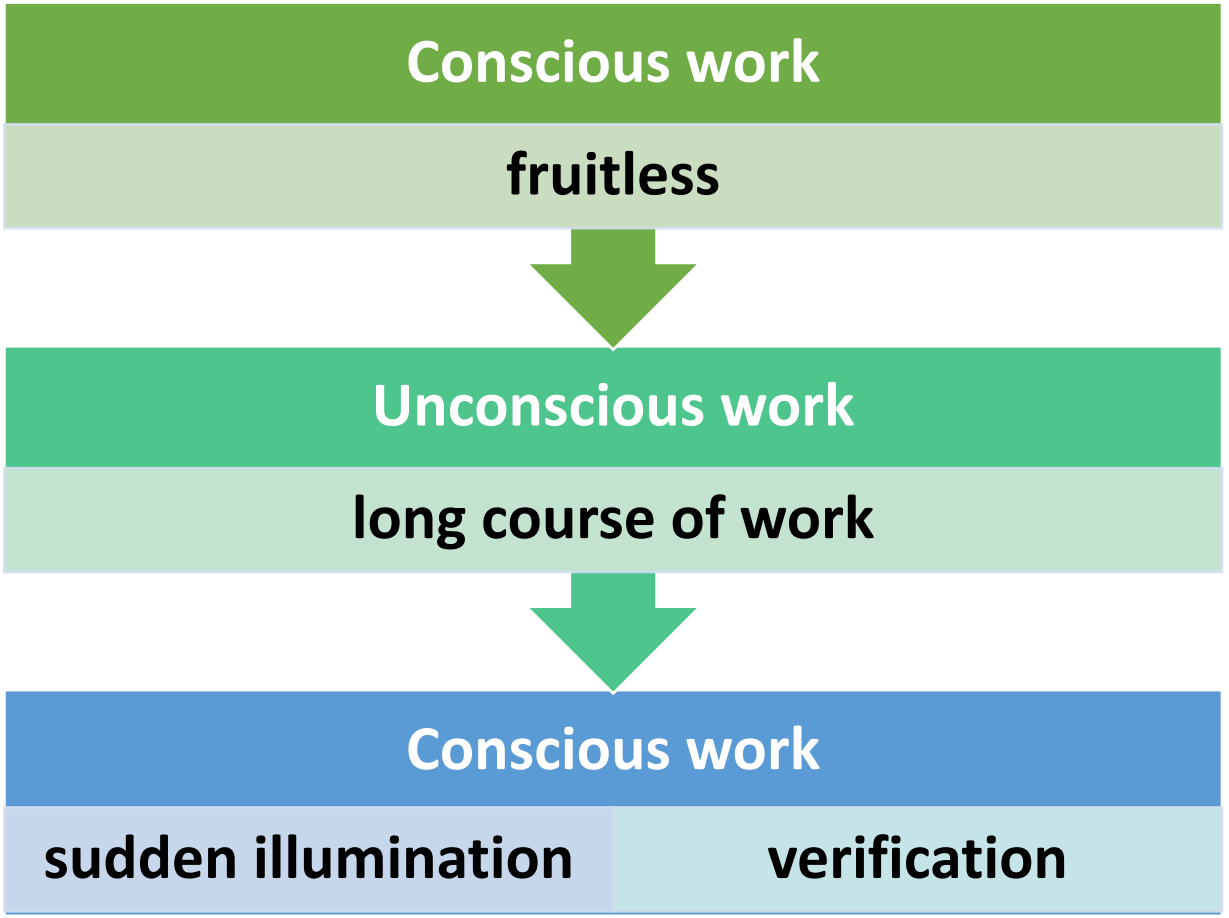
- Poincare
- No pre-existence of mathematical reality
- Math is the creation of human minds for us to invent.
- Essentially an activity of **construction**.

Formalism

- Hilbert
- Prior to experience
- Manipulation of **symbols** and rules
- Just a **game**, nothing else.
- Internally **consistent**

Intuition and Subliminal Ego

- Selection of fruitful combination through subliminal ego
- "Sudden inspirations after voluntary efforts"
- "Ideas rose in clouds; I felt them collide until pairs interlocked, so to speak, making a stable combination."



Aesthetic Value

- Scientists do not study nature for its **use**, but for its **aesthetic pleasure**.
  - "The useful combinations are precisely the most beautiful" because they "take in the whole without neglecting the detail."
- Not **sensual beauty**, but **intellectual beauty**, arising from "the sense of the **harmony** of the world".