PHY1001: Mechanics (Week 10)

1 Fluids

1.1 Density and pressure

Density ρ is mass per unit volume.

$$\rho = \frac{m}{V} \tag{1}$$

The specific gravity of a material is the ratio of its density to the density of water at 4.0 °C , 1000kg/m^3. It is a pure number without units. For example, the specific gravity of gold is roughly 19. In fact, specific gravity is a poor term, since it has nothing to do with gravity. The name "relative density" would be much better since it represents its true physical meaning.

1.2 Pressure in a fluid at rest

Pressure (P) is normal force (the force perpendicular to the area) per unit area.

$$P = \frac{dF_\perp}{dA} \tag{2}$$

If the pressure is the same at all points of a finite plane surface with area A, then $p = F_\perp/A$. The SI unit of pressure is the pascal, where

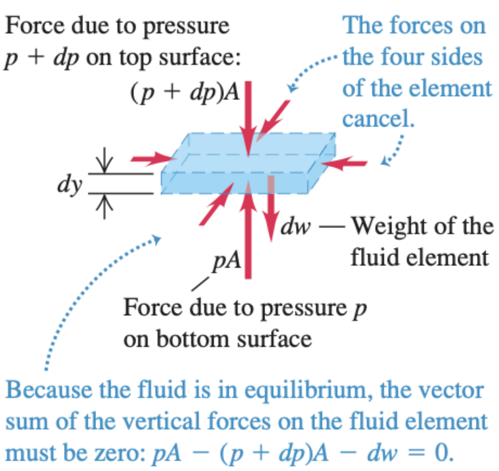
$$1\,pascal = 1\,\mathrm{Pa} = 1\,\mathrm{N/m^2} \tag{3}$$

In meteorology, we use the unit bar, equal to 10^5 Pa.

Atmospheric pressure p_a is the pressure of the earth's atmosphere, the pressure at the bottom of this sea of air in which we live. This pressure varies with weather changes and with elevation. Normal atmospheric pressure at sea level (an average value) is 1 atm, defined to be exactly 101, 325 Pa. To four significant figures,

$$1\,atm = 1.013 \times 10^5\,Pa = 1.013\,bar \tag{4}$$

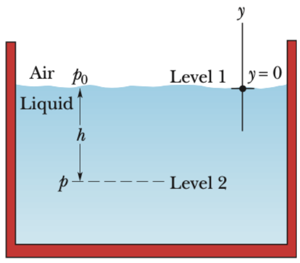
Estimate: How much pressure force is on the desk?



Pressure and Depth By taking into account the pressure gradient, we can find that the pressure in a fluid at rest below the surface is increase by an amount of ρgh due to gravity.

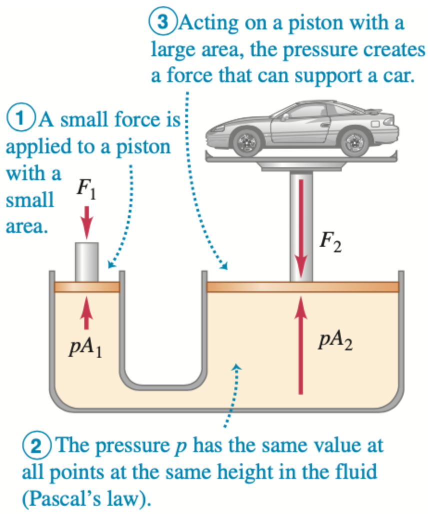
$$\frac{dp}{dy} = -\rho g \quad \Rightarrow \quad p = p_0 + \rho gh, \quad \text{(pressure at depth h)} \tag{5}$$

where ρ is the density of the fluid and h is the depth. p is said to be the total pressure, or absolute pressure, at level 2. The pressure p at level 2 consists of two contributions: (1) p_0, the pressure due to the atmosphere, which bears down on the liquid, and (2) ρgh, the pressure due to the liquid above level 2. In general, the difference between an absolute pressure and an atmospheric pressure is called the gauge pressure (because we use a gauge to measure this pressure difference ρgh).



1.3 Pascal's law

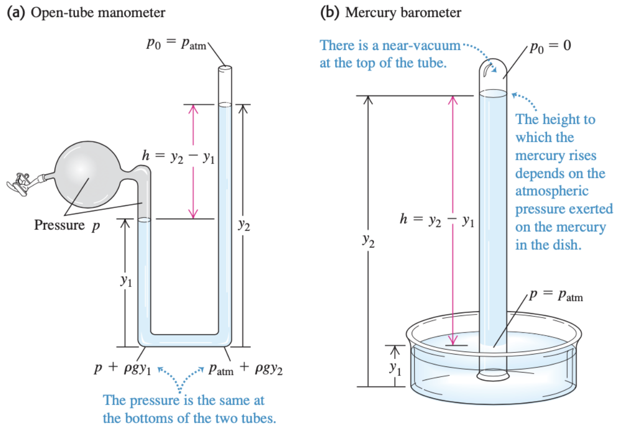
Pascal's law states that pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid. Loosely speaking, Pascal's law means the pressure is everywhere and in every direction in an enclosed fluid.



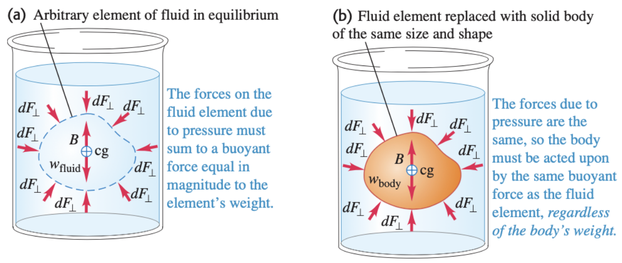
With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance. A piston with small cross-sectional area A_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure $p = F_1/A_1$ is transmitted through the connecting pipe to a larger piston of area A_2. The applied pressure is the same in both cylinders, so that $F_2 = (F_1/A_1)A_2$. E.g., Car lifts, etc.

1.4 Measuring Pressure

The Open-Tube Manometer ($p = p_a + \rho g h$) and the Mercury Barometer ($p = \rho g h$, it reads the atmospheric pressure p_a directly from the height of the mercury column).



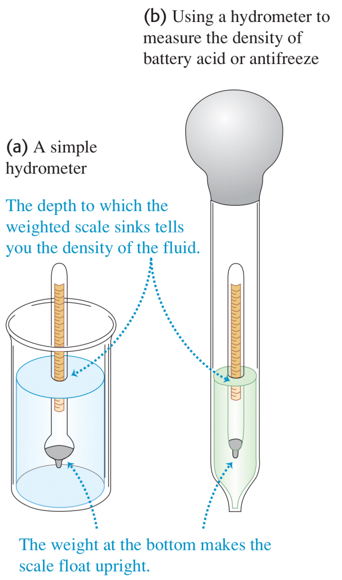
1.5 Buoyancy



Archimedes's principle states that when a body is immersed in a fluid, the fluid exerts an upward buoyant force on the body equal to the weight of the fluid that the body displaces. The magnitude of the buoyancy is

$$B = \rho g V, \tag{6}$$

where V is the volume that the body occupies in the fluid and ρ is the density of the fluid. Suggested reading: the interesting story of "Eureka!".



Fluid in Motion: Previously, we have discussed the static fluid when it is at rest. Now let us introduce the fluid dynamics and discuss the fluid in motion (flow).

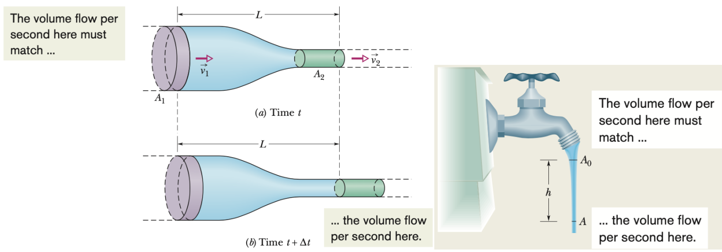
The motion of real fluids is very complicated and not yet fully understood. Instead, we shall discuss the motion of an ideal fluid, which is simpler to handle mathematically and yet provides useful results. Here are four assumptions that we make about our ideal fluid; they all are concerned with flow:

1. Steady (or laminar) flow: The path of an individual particle in a moving fluid is called a flow line. If the overall flow pattern does not change with time, the flow is called steady flow. In turbulent flow (irregular and chaotic flow), there is no steady-state pattern; the flow pattern changes continuously. (defines velocity)
2. Incompressible flow: fluid density has a constant, uniform value. (volume conservation)
3. Nonviscous flow: the ideal fluid has no internal friction (called viscosity).
4. Irrotational flow. Only consider linear motion.

1.6 Continuity

Mass is conserved as the fluid flows. $dm = \rho dV =$ const. For incompressible fluid, how much going in must equal how much goes out in a pipe

$$\Rightarrow \quad A_1 v_1 = A_2 v_2. \tag{7}$$



A water stream narrows as it falls: the above figure shows how the stream of water emerging from a faucet "necks down" as it falls. This change in the horizontal cross-sectional area is characteristic of any laminar (non-turbulant) falling stream because the gravitational force increases the speed of the stream. Continuity and energy conservation give

$$v^2 = v_0^2 + 2gh = \frac{A^2}{A_0^2} v^2 + 2gh, \tag{8}$$

$$v_0 A_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}} A_0, \quad \text{volume flow rate.} \tag{9}$$

1.7 Bernoulli's equation

Fluid flow satisfies the Bernoulli's equation

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2, \tag{10}$$

which can be derived from the work-energy theorem ($dW = dK + dU$). The work down by the pressure force equals the change of kinetic energy and potential energy for a small flow element.