PHY1001 Mechanics

2022-2023 Term 2

Final Examination

May 15, 2023;   Time Allowed: 3 Hours

NAME (print)
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CUHKSZ ID
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Exam Room No. and Seat No.
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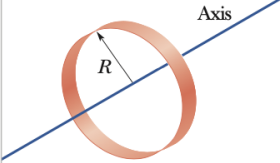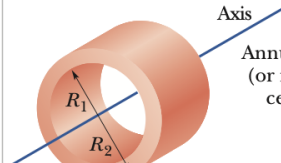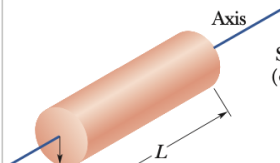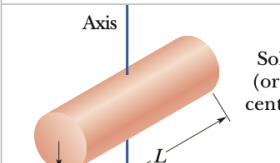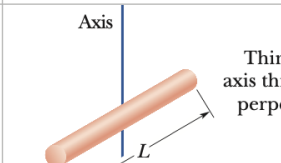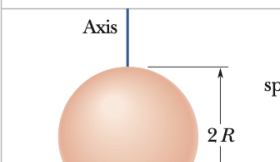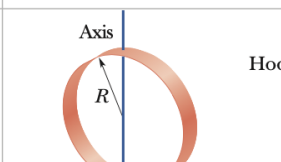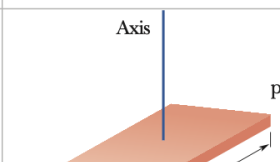
- **Show all your work**. Correct answers with little supporting work will not be given credit.

- Closed Book Exam: One piece of double-sided A4 reference paper, a scientific calculator, and a paper-based dictionary are allowed.

- Unless approved by the instructors, students who arrive more than 30 minutes late will NOT be admitted.

- The total points are 120 points. You need to finish ALL the questions in 3 hours (180 minutes).

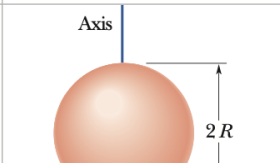| #1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	TOTAL
10	10	10	10	10	10	10	10	10	10	10	10	120

Summary of Basic Calculus:

$$\frac{d}{dx}x^n = nx^{n-1},$$

$$\frac{d}{dx}e^{ax} = ae^{ax},$$

$$\frac{d}{dx}\ln ax = \frac{1}{x},$$

$$\frac{d}{dx}(uv) = v\frac{d}{dx}u + u\frac{d}{dx}v,$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \ (n \neq -1),$$

$$\int \frac{dx}{x} = \ln x + C,$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C, \quad C \text{ is a constant.}$$

Table of Moments of Inertia:

Axis · Hoop about central axis $I = MR^2$ *(a)*	Axis · Annular cylinder (or ring) about central axis $I = \frac{1}{2}M(R_1^2 + R_2^2)$ *(b)*	Axis · Solid cylinder (or disk) about central axis $I = \frac{1}{2}MR^2$ *(c)*
Axis · Solid cylinder (or disk) about central diameter $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ *(d)*	Axis · Thin rod about axis through center perpendicular to length $I = \frac{1}{12}ML^2$ *(e)*	Axis · Solid sphere about any diameter $I = \frac{2}{5}MR^2$ *(f)*
Axis · Thin spherical shell about any diameter $I = \frac{2}{3}MR^2$ *(g)*	Axis · Hoop about any diameter $I = \frac{1}{2}MR^2$ *(h)*	Axis · Slab about perpendicular axis through center $I = \frac{1}{12}M(a^2 + b^2)$ *(i)*

Bernoulli's Equation

$$P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant.}$$

Spring Mass System; Simple Harmonic Oscillator

$$m\frac{d^2x}{dt^2} = -kx.$$

1. A uniform solid ball with mass M and radius R is given an initial <u>clockwise</u> angular velocity ω_0 and zero linear velocity ($v_{com} = 0$) before it is placed on a horizontal surface with kinetic friction coefficient μ_k. **(10 pts)**
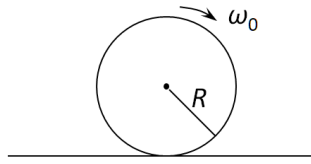
Fig. 1

    (a) <u>Which direction</u> is the ball going to move towards? (Hint: Left or Right.) **(2 pts)**

    (b) What is the <u>acceleration</u> at the moment that the ball is placed on the surface? **(2 pts)**

    (c) Show that the corresponding <u>angular acceleration</u> of the ball is $\alpha = 5\mu_k g/(2R)$? **(2 pts)**

    (d) <u>How long</u> does it take for the ball to <u>roll without slipping</u>? (Hint: The rolling without slipping happens when $\omega R + v = 0$) **(4 pts)**

2. A uniform solid cylinder of mass M is supported on a ramp that rises at angle θ above the horizontal by a wire that is wrapped around its rim and pulls on it tangentially parallel to the ramp as shown in the figure below. The static friction coefficient is $\mu_s = 0.5$. The cylinder is at equilibrium state. **(10 pts)**
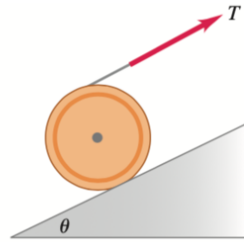
Fig. 2

(a) <u>Show</u> that there must be <u>friction on the surface</u> for the cylinder to stay at equilibrium state. **(2 pts)**

(b) From the torque equation, <u>show</u> that the friction f is equal to the tension T ($f = T$). **(2 pts)**

(c) <u>Find</u> the <u>frictional force</u> (in terms of Mg and θ) to keep the cylinder at rest. **(2 pts)**

(d) <u>Find</u> the <u>normal force</u> N from the ramp to keep the cylinder at rest. **(2 pts)**

(e) <u>Find</u> the <u>maximum angle</u> θ to keep the cylinder at rest. **(2 pts)**

3. As shown in the figure below, a point particle of mass m_1 is a distance d from one end of a uniform thin rod with length L and mass M. **(10 pts)**

Fig. 3
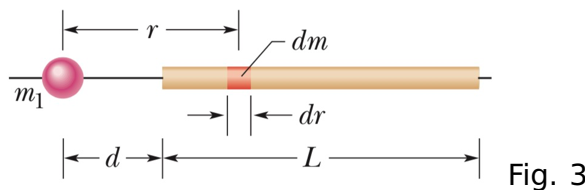
(a) What is the <u>magnitude of the gravitational force</u> on the particle from the rod? (Hint: As shown above, use $\int \dfrac{dr}{r^2} = -\dfrac{1}{r}$ to show that magnitude is $\dfrac{GMm_1}{d(d+L)}$.) **(5 pts)**

(b) Find the <u>gravitational potential energy</u> U between the particle and the rod. (Take the potential energy to be zero when the rod and particle are infinitely far apart.) **(5 pts)**

4. A rock with irregular shape is suspended by a light string. When the rock is hanging in the air (suppose the air buoyancy is negligible), the tension in the string is $T_1 = 20.0$ N. When the rock is totally immersed in water (mass density of water is $\rho_w = 1.0 \times 10^3$ kg/m^3), the tension is $T_2 = 10.2$ N. **(10 pts)**

   (a) What is the volume of the rock? **(3 pts)**

   (b) What is the average mass density of the rock? **(4 pts)**

   (c) When the rock is totally immersed in an unknown liquid, the tension is $T_3 = 7.9$ N. What is the density of the unknown liquid? **(3 pts)**

5. In the figure below, the fresh water behind a reservoir dam has depth $D = 12$ m. A horizontal pipe passes through the dam at depth $d = 6.0$ m. The inner and outer parts of the pipe have cross-section areas of $S_1 = 18.8 \text{cm}^2$ and $S_2 = 12.5 \text{cm}^2$. A plug secures the pipe opening. **(10 pts)**
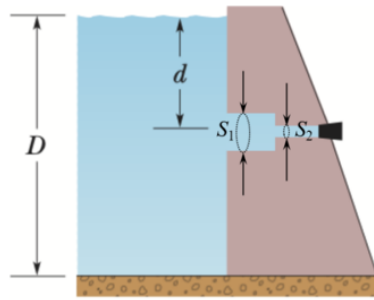

Fig. 4

(a) Find the values of <u>gauge pressure</u> in the horizontal pipe? **(2 pts)**
(Hint: The two diameters of the horizontal pipe are much smaller than d and D. The gauge pressure of water at the cross-section of S_1 approximately equals to that at the cross-section of S_2.) (The gauge pressure is defined as the difference between an absolute pressure (P) and the atmospheric pressure (P_{atm}).)

(b) <u>Find</u> the <u>magnitude of the frictional force</u> between plug and pipe wall. **(2 pts)**

(c) When the plug is removed, what is the <u>water volume</u> that exits the pipe in 1.0 hour? **(3 pts)**

(d) What is the speed of water flowing through the cross-section of S_1? **(3 pts)**

6. As shown in Fig. 5, two stars with masses M_1 and M_2 orbit around their center of mass (COM) at the same angular velocity ω. Both orbits are circular. Let R_1 be the distance between star M_1 and COM, and R_2 be the distance between star M_2 and COM.
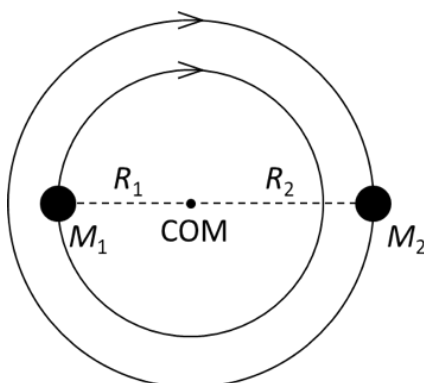
Fig. 5

(a) Find the gravitational force (<u>magnitude and direction</u>) of one star on the other. **(2 pts)**

(b) Show that $M_1 R_1 = M_2 R_2$. **(2 pts)**

(c) Show that $(R_1 + R_2)^3 \omega^2 = G(M_1 + M_2)$. **(2 pts)**

(d) Find angular momentum of star M_1 about COM, including the <u>magnitude and direction</u>. (Hint: is the direction pointing in or out of the exam paper?) **(2 pts)**

(e) <u>How much energy</u> would be required to separate the two stars to infinity? **(2 pts)**

7. Demonstration of Kepler's second law (A planet sweeps out equal areas of the ellipse over equal time intervals.) for a planet that orbits the Sun in an elliptical path. **(10 pts)**

Fig. 6

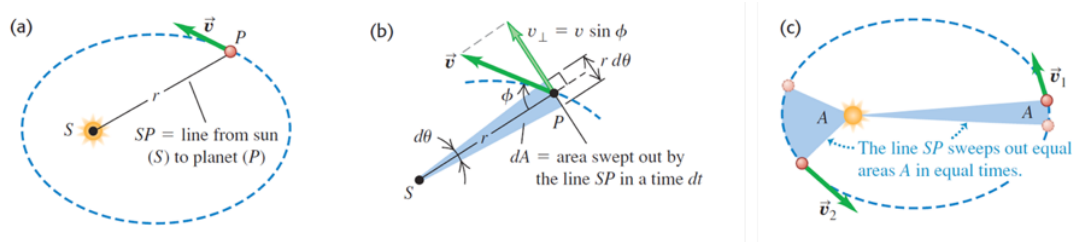(a) <u>Write down</u> the gravitational force $\vec{F}_g$ exerted to the planet by the Sun. **(2 pts)**

(b) <u>Show</u> that the <u>angular momentum</u> of the planet is <u>conserved</u>. **(3 pts)**

(c) <u>Show</u> that the area swept out by the line SP per unit time $dA/dt = L/(2m)$. **(3 pts)**

(d) <u>Demonstrate</u> the Kepler's second law is true, namely, dA/dt is a <u>constant</u>. **(2 pts)**
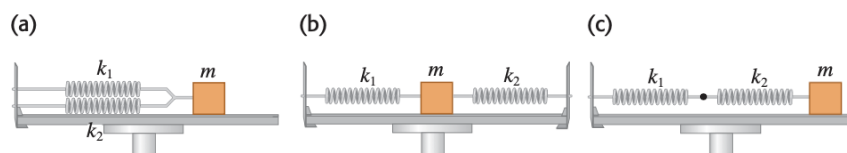
8. The Tale of Two Springs. **(10 pts)**

Fig. 7

Two springs with the same unstretched length but different force constants k_1 and k_2 are attached to a block with mass m on a level, frictionless surface.

(a) Use Newton's 2nd law, <u>write down</u> the equation of motion for the oscillator in case (a) as shown in the above Fig. 7 (a). <u>Find</u> the corresponding angular frequency ω_a. **(4 pts)**

(b) <u>Write down</u> the equation of motion for the oscillator in case (b) as shown in the above Fig. 7 (b) and <u>find</u> the corresponding angular frequency ω_b. **(3 pts)**

(c) <u>Write down</u> the equation of motion for the oscillator in case (c) as shown in the above Fig. 7 (c) and <u>find</u> the corresponding angular frequency ω_c. **(3 pts)**

9. A certain transverse wave on a rope is described by

$$y_1(x, t) = (6.50 \times 10^{-3}\,\text{m}) \cos 2\pi \left(\frac{x}{0.280\,\text{m}} - \frac{t}{0.0360\,s} \right)$$

(a) Which direction does the wave propagate (travel)?   **(1 pts)**

(b) Determine the wave's amplitude.   **(1 pts)**

(c) Find the wavelength λ.   **(1 pts)**

(d) What is the period T?   **(1 pts)**

(e) What is its speed of propagation (the wave traveling speed)?   **(1 pts)**

(f) The mass per unit length of the rope is $\mu = 0.0500 kg/m$. Find the tension F_T.   **(1 pts)**

(g) Find the average power $P_{av} = \dfrac{1}{2}\sqrt{\mu F_T}\omega^2 A^2$ of this wave.   **(1 pts)**

(h) The reflection of the above wave can be described by

$$y_2(x, t) = -(6.50 \times 10^{-3}\,\text{m}) \cos 2\pi \left(\frac{x}{0.280\,\text{m}} + \frac{t}{0.0360\,s} \right).$$

The superposition of y_1 and y_2 gives rise to a standing wave. Find the form of the standing wave $y(x, t) = y_1(x, t) + y_2(x, t)$ and the location of nodes.   **(3 pts)**

10. Doppler Effects: $f' = \dfrac{v \pm v_d}{v \pm v_s} f$. **(10 pts)**

A firetruck's siren emits a sinusoidal wave with frequency $f = 170$ Hz. The speed of sound is 340 m/s and the air is still.

(a) Find the wavelength of the sound wave if the firetruck is at rest. **(2 pts)**

(b) What frequency will the stationary listener hear if the firetruck is moving toward him with a speed $v_s = 40$ m/s ? **(2 pts)**

(c) What frequency will the stationary listener hear if the firetruck is moving away from him with a speed $v_s = 60$ m/s ? **(2 pts)**

(d) Now the firetruck is at rest and the listener is moving toward the firetruck with a speed $v_d = 50$ m/s, what frequency will the moving listener hear? **(2 pts)**

(e) Now the firetruck is at rest and the listener is moving away from the firetruck with a speed $v_d = 30$ m/s, what frequency will the moving listener hear? **(2 pts)**

11. A uniform disk with Mass M and radius R mounted on ceiling. Two blocks with mass m_1 and m_2 ($m_2 > m_1$) hang from a massless cord that is wrapped around the rim of the disk as shown in the figure below. **(10 pts)**

Fig. 8
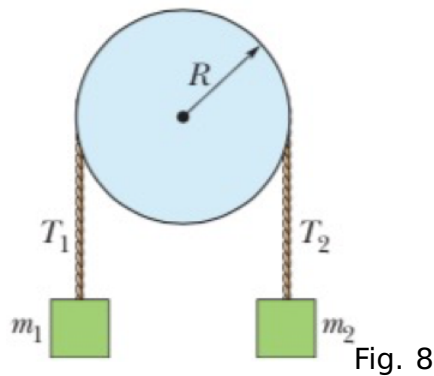
(a) Let us first set mass of the disk $M = 0$, <u>find</u> the tensions T_1 and T_2 as indicated in the above graph. **(2 pts)**

(b) Now assume the disk mass M is nonzero and there is no slipping between the cord and the disk (the disk is rotating as the blocks move), <u>find</u> the tensions T_1 and T_2 as indicated in the above graph. **(4 pts)**

(c) When block m_2 drops a height of h, what is the <u>angular speed</u> of the disk? **(4 pts)**

12. Oscillation in general. **(10 pts)**

(a) For a simple harmonic oscillator, the net force on the body with mass m is given by $F_x = -kx$. What is the potential energy function for this simple harmonic oscillator if we take $U = 0$ at $x = 0$? **(1 pts)**

(b) Use energy conservation, show that the velocity v_x as a function of x is

$$v_x = \frac{dx}{dt} = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}.$$

where A is the amplitude of the oscillator. **(1 pts)**

(c) Separate the variable by writing all factors containing x on one side and all factors containing t on the other side so that each side can be integrated, and use the identity $\int_0^1 \frac{du}{\sqrt{1-u^2}} = \frac{\pi}{2}$ to find the period T of the simple harmonic oscillator. (Hint: One-quarter $(1/4)$ of T is the time for the body to move from $x = 0$ to $x = A$.) **(2 pts)**

(d) For a second oscillator, the net force on the body with mass m is given by $F_x = -cx^3$. What is the underlined potential energy function for this oscillator if we take $U = 0$ at $x = 0$? **(2 pts)**

(e) Follow part (d), compute the underlined period T_2 for the second oscillator. (Hint: You may find the identity $\int_0^1 \dfrac{du}{\sqrt{1-u^4}} = 1.31$ useful.) **(3 pts)**

(f) According to the result you obtained in part (e), does the underlined period T_2 depend on the amplitude A of the motion? Is this oscillation underlined simple harmonic? **(1 pts)**