

3. Arrays

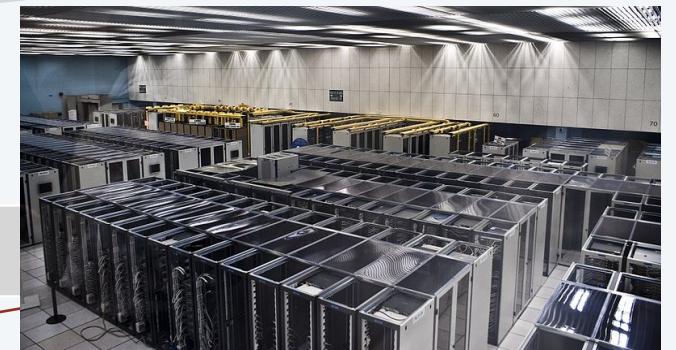
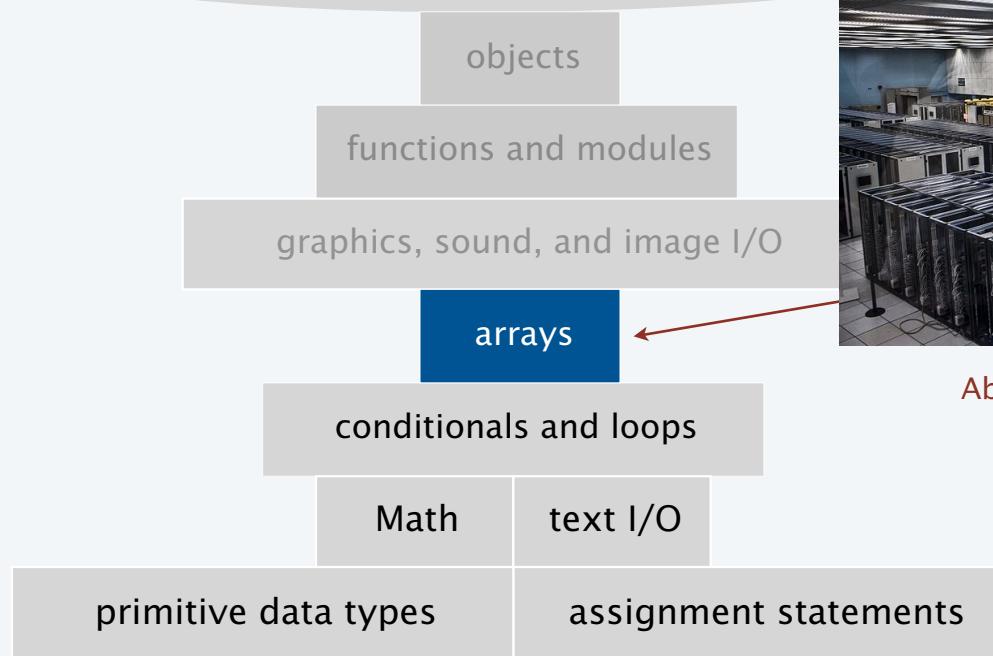


3. Arrays

- **Basic concepts**
- Typical array-processing code
- Two-dimensional arrays

Basic building blocks for programming

any program you might want to write



Ability to store and process
huge amounts of data

Your first data structure

A **data structure** is an arrangement of data that enables efficient processing by a program.

An **array** is an *indexed* sequence of values of the same type.

Examples.

- 52 playing cards in a deck.
- 100 thousand students in an online class.
- 1 billion pixels in a digital image.
- 4 billion nucleotides in a DNA strand.
- 73 billion Google queries per year.
- 86 billion neurons in the brain.
- 50 trillion cells in the human body.
- 6.02×10^{23} particles in a mole.

index	value
0	2♥
1	6♦
2	A♦
3	A♥
...	
49	3♣
50	K♣
51	4♦



Main purpose. Facilitate storage and manipulation of data.

Processing many values of the same type

10 values, without arrays

```
double a0 = 0.0;  
double a1 = 0.0;  
double a2 = 0.0;  
double a3 = 0.0;  
double a4 = 0.0;  
double a5 = 0.0;  
double a6 = 0.0;  
double a7 = 0.0;  
double a8 = 0.0;  
double a9 = 0.0;  
...  
a4 = 3.0;  
...  
a8 = 8.0;  
...  
double x = a4 + a8;
```

tedious and error-prone code

10 values, with an array

```
double[] a;  
a = new double[10];  
...  
a[4] = 3.0;  
...  
a[8] = 8.0;  
...  
double x = a[4] + a[8];
```

an easy alternative

1 million values, with an array

```
double[] a;  
a = new double[1000000];  
...  
a[234567] = 3.0;  
...  
a[876543] = 8.0;  
...  
double x = a[234567] + a[876543];
```

scales to handle huge amounts of data

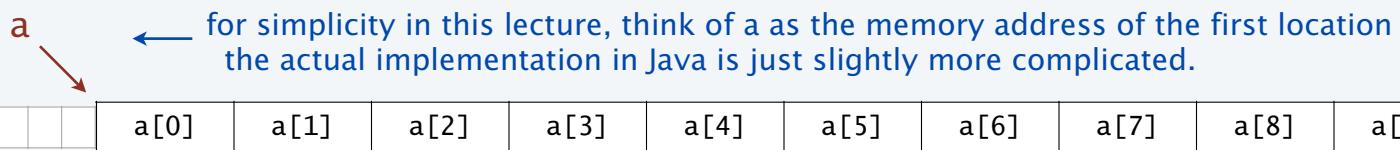
Memory representation of an array

An **array** is an indexed sequence of values of the same type.

A computer's memory is *also* an indexed sequence of memory locations.

← stay tuned for many details

- Each primitive type value occupies a fixed number of locations.
- *Array values are stored in contiguous locations.*



Critical concepts

- Indices start at 0.
- Given *i*, the operation of accessing the value $a[i]$ is extremely efficient.
- The assignment $b = a$ makes the names *b* and *a* refer to the same array.

it does *not* copy the array,
as with primitive types
(stay tuned for details)

Java language support for arrays

Basic support	<i>operation</i>	<i>typical code</i>
	Declare an array	double[] a;
	Create an array of a given length	a = new double[1000];
	Refer to an array entry by index	a[i] = b[j] + c[k];
	Refer to the length of an array	a.length;

Initialization options

	<i>operation</i>	<i>typical code</i>	
	Default initialization to 0 for numeric types	a = new double[1000];	no need to use a loop like for (int i = 0; i < 1000; i++) a[i] = 0.0;
	Declare, create and initialize in one statement	double[] a = new double[1000];	BUT cost of creating an array is proportional to its length.
	Initialize to literal values	double[] x = { 0.3, 0.6, 0.1 };	

Copying an array

To copy an array, **create a new array**, then copy all the values.

```
double[] b = new double[a.length];
for (int i = 0; i < a.length; i++)
    b[i] = a[i];
```



Important note: The code **b = a** does *not* copy an array (it makes b and a refer to the same array).

```
double[] b = new double[a.length];
b = a;
```



Programming with arrays: typical examples

Access command-line args in system array

```
int stake = Integer.parseInt(args[0]);
int goal = Integer.parseInt(args[1]);
int trials = Integer.parseInt(args[2]);
```

For brevity, N is a.length and b.length in all this code.

Copy to another array

```
double[] b = new double[N];
for (int i = 0; i < N; i++)
    b[i] = a[i];
```

Create an array with N random values

```
double[] a = new double[N];
for (int i = 0; i < N; i++)
    a[i] = Math.random();
```

Print array values, one per line

```
for (int i = 0; i < N; i++)
    System.out.println(a[i]);
```

Compute the average of array values

```
double sum = 0.0;
for (int i = 0; i < N; i++)
    sum += a[i];
double average = sum / N;
```

Find the maximum of array values

```
double max = a[0];
for (int i = 1; i < N; i++)
    if (a[i] > max) max = a[i];
```

Pop quiz 1 on arrays

Q. What does the following code print?

```
public class PQarray1
{
    public static void main(String[] args)
    {
        int[] a = new int[6];
        int[] b = new int[a.length];

        b = a;
        for (int i = 1; i < b.length; i++)
            b[i] = i;

        for (int i = 0; i < a.length; i++)
            System.out.print(a[i] + " ");
        System.out.println();

        for (int i = 0; i < b.length; i++)
            System.out.print(b[i] + " ");
        System.out.println();
    }
}
```

Pop quiz 1 on arrays

Q. What does the following code print?

```
public class PQarray1
{
    public static void main(String[] args)
    {
        int[] a = new int[6];
        int[] b = new int[a.length];

        b = a; ← After this, b and a refer to the same array
        for (int i = 1; i < b.length; i++)
            b[i] = i;

        for (int i = 0; i < a.length; i++)
            System.out.print(a[i] + " ");
        System.out.println();

        for (int i = 0; i < b.length; i++)
            System.out.print(b[i] + " ");
        System.out.println();
    }
}
```

A.

```
% java PQ4_1
0 1 2 3 4 5
0 1 2 3 4 5
```

Programming with arrays: typical bugs

Array index out of bounds

```
double[] a = new double[10];  
for (int i = 1; i <= 10; i++)  
    a[i] = Math.random();
```

No a[10] (and a[0] unused)



Uninitialized array

```
double[] a;  
for (int i = 0; i < 9; i++)  
    a[i] = Math.random();
```

Never created the array



Undeclared variable

```
a = new double[10];  
for (int i = 0; i < 10; i++)  
    a[i] = Math.random();
```

What type of data does a refer to?

Image sources

http://commons.wikimedia.org/wiki/File:CERN_Server_03.jpg



3. Arrays

- Basic concepts
- **Examples of array-processing code**
- Two-dimensional arrays

Example of array use: create a deck of cards

Define three arrays

- Ranks.
- Suits.
- Full deck.

```
String[] rank = {"2", "3", "4", "5", "6", "7", "8", "9", "10", "J", "Q", "K", "A"};  
String[] suit = { "♣", "♦", "♥", "♠" };  
String[] deck = new String[52];
```



Use nested for loops to put all the cards in the deck.

```
for (int j = 0; j < 4; j++)  
    for (int i = 0; i < 13; i++)  
        deck[i + 13*j] = rank[i] + suit[j];
```

better style to use `rank.length` and `suit.length`
clearer in lecture to use 4 and 13

		j	0	1	2	3								
		suit	♣	♦	♥	♠								
i		0	1	2	3	4	5	6	7	8	9	10	11	12
	rank	2	3	4	5	6	7	8	9	10	J	Q	K	A

deck	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	...
	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣	A♣	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	...

Example of array use: create a deck of cards

```
public class Deck
{
    public static void main(String[] args)
    {
        String[] rank = {"2", "3", "4", "5", "6", "7", "8", "9", "10", "J", "Q", "K", "A" };
        String[] suit = { "♣", "♦", "♥", "♠" };

        String[] deck = new String[52];
        for (int j = 0; j < 4; j++)           no color in Unicode;
            for (int i = 0; i < 13; i++)      artistic license for lecture
                deck[i + 13*j] = rank[i] + suit[j];

        for (int i = 0; i < 52; i++)
            System.out.print(deck[i] + " ");
        System.out.println();
    }
}
```



```
% java Deck
2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣ A♣
2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦ A♦
2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥ A♥
2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠ A♠
%
```

Pop quiz 2 on arrays

Q. What happens if the order of the for loops in Deck is switched?

```
for (int j = 0; j < 4; j++)
    for (int i = 0; i < 13; i++)
        deck[i + 13*j] = rank[i] + suit[j];
```



```
for (int i = 0; i < 13; i++)
    for (int j = 0; j < 4; j++)
        deck[i + 13*j] = rank[i] + suit[j];
```

Pop quiz 2 on arrays

Q. What happens if the order of the for loops in Deck is switched?

```
for (int j = 0; j < 4; j++)
    for (int i = 0; i < 13; i++)
        deck[i + 13*j] = rank[i] + suit[j];
```



```
for (int i = 0; i < 13; i++)
    for (int j = 0; j < 4; j++)
        deck[i + 13*j] = rank[i] + suit[j];
```

A. The array is filled in a different order, but the output is the same.

j	0	1	2	3									
suit	♣	♦	♥	♠									
i	0	1	2	3	4	5	6	7	8	9	10	11	12
rank	2	3	4	5	6	7	8	9	10	J	Q	K	A

deck	0	1	2	...	12	13	14	15	...	25	26	27	28	...	38	39	40	41	...	51
	2♣	3♣	4♣	...	A♣	2♦	3♦	4♦	...	A♦	2♥	3♥	4♥	...	A♥	2♠	3♠	4♠	...	A♠

Pop quiz 3 on arrays

Q. Change Deck to put the cards in rank order in the array.

```
% java Deck  
2♣ 2♦ 2♥ 2♠ 3♣ 3♦ 3♥ 3♠ 4♣ 4♦ 4♥ 4♠ 5♣ 5♦ 5♥ 5♠ 6♣ 6♦ 6♥ 6♠ 7♣ 7♦ 7♥ 7♠ 8♣ 8♦ 8♥ 8♠ 9♣ 9♦ 9♥ 9♠  
10♣ 10♦ 10♥ 10♠ J♣ J♦ J♥ J♠ Q♣ Q♦ Q♥ Q♠ K♣ K♦ K♥ K♠ A♣ A♦ A♥ A♠  
%
```

Pop quiz 3 on arrays

Q. Change Deck to put the cards in rank order in the array.

```
% java Deck  
2♣ 2♦ 2♥ 2♠ 3♣ 3♦ 3♥ 3♠ 4♣ 4♦ 4♥ 4♠ 5♣ 5♦ 5♥ 5♠ 6♣ 6♦ 6♥ 6♠ 7♣ 7♦ 7♥ 7♠ 8♣ 8♦ 8♥ 8♠ 9♣ 9♦ 9♥ 9♠  
10♣ 10♦ 10♥ 10♠ J♣ J♦ J♥ J♠ Q♣ Q♦ Q♥ Q♠ K♣ K♦ K♥ K♠ A♣ A♦ A♥ A♠  
%
```

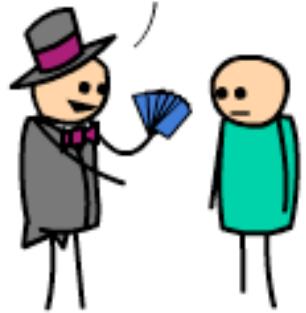
A.

```
for (int i = 0; i < 13; i++)  
    for (int j = 0; j < 4; j++)  
        deck[4*i + j] = rank[i] + suit[j];
```

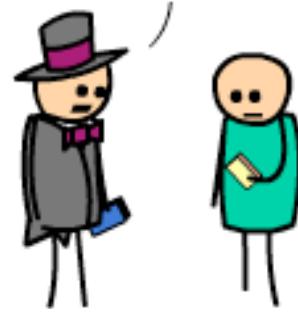
		j				
		0	1	2	3	
		suit	♣	♦	♥	♠
		i	0	1	2	3
		rank	2	3	4	5
		6	7	8	9	10
		J	Q	K	A	11
		12	...			

deck	0	1	2	3	4	5	6	7	8	9	10	11	12	...
	2♣	2♦	2♥	2♠	3♣	3♦	3♥	3♠	4♣	4♦	4♥	4♠	5♣	...

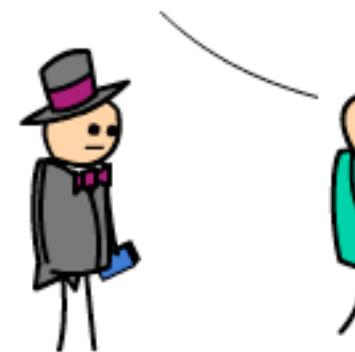
Take a card!
Any card!



That's my
credit card.



Abra kadabra.



Array application: take a card, any card

Problem: Print a random sequence of N cards.

Algorithm

Take N from the command line and do the following N times

- Calculate a random index r between 0 and 51.
- Print $\text{deck}[r]$.



Implementation: Add this code instead of printing deck in Deck.

```
for (int i = 0; i < N; i++)
{
    int r = (int) (Math.random() * 52);
    System.out.println(deck[r]);
}
```

each value between 0 and 51 equally likely

Note: Same method is effective for printing a random sequence from any data collection.

Array application: random sequence of cards

```
public class DrawCards
{
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);

        String[] rank = {"2", "3", "4", "5", "6", "7", "8", "9", "10", "J", "Q", "K", "A"};
        String[] suit = { "♣", "♦", "♥", "♠" };

        String[] deck = new String[52];
        for (int i = 0; i < 13; i++)
            for (int j = 0; j < 4; j++)
                deck[i + 13*j] = rank[i] + suit[j];

        for (int i = 0; i < N; i++)
        {
            int r = (int) (Math.random() * 52);
            System.out.print(deck[r] + " ");
        }
        System.out.println();
    }
}
```

```
% java DrawCards 10
6♥ K♦ 10♠ 8♦ 9♦ 9♥ 6♦ 10♠ 3♣ 5♦
```

```
% java DrawCards 10
2♦ A♠ 5♣ A♣ 10♣ Q♦ K♣ K♣ A♣ A♦
```

```
% java DrawCards 10
6♦ 10♦ 4♥ A♣ K♥ Q♣ K♣ 7♣ 5♦ Q♣
```

```
% java DrawCards 10
A♣ J♣ 5♥ K♥ Q♣ 5♥ 9♦ 9♣ 6♦ K♥
```

Note: Sample is *with replacement* (same card can appear multiple times).

appears twice

Array application: shuffle and deal from a deck of cards

Problem: Print N random cards from a deck.

Algorithm: Shuffle the deck, then deal.

- Consider each card index i from 0 to 51.
 - Calculate a random index r between i and 51.
 - Exchange $\text{deck}[i]$ with $\text{deck}[r]$
- Print the first N cards in the deck.



Implementation

```
for (int i = 0; i < 52; i++)
{
    int r = i + (int) (Math.random() * (52-i));
    String t = deck[r];
    deck[r] = deck[i];
    deck[i] = t;
}
for (int i = 0; i < N; i++)
    System.out.print(deck[i]);
System.out.println();
```

each value
between i and 51
equally likely

Array application: shuffle a deck of 10 cards (trace)

```
for (int i = 0; i < 10; i++)  
{  
    int r = i + (int) (Math.random() * (10-i));  
    String t = deck[r];  
    deck[r] = deck[i];  
    deck[i] = t;  
}
```

Q. Why does this method work?

- Uses only exchanges, so the deck after the shuffle has the same cards as before.
- $N-i$ equally likely values for `deck[i]`.
- Therefore $N \times (N-1) \times (N-2) \dots \times 2 \times 1 = N!$ equally likely values for `deck[]`.

Initial order is immaterial.

i	r	deck									
		0	1	2	3	4	5	6	7	8	9
		2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣
0	7	9♣	3♣	4♣	5♣	6♣	7♣	8♣	2♣	10♣	J♣
1	3	9♣	5♣	4♣	3♣	6♣	7♣	8♣	2♣	10♣	J♣
2	9	9♣	5♣	J♣	3♣	6♣	7♣	8♣	2♣	10♣	4♣
3	9	9♣	5♣	J♣	4♣	6♣	7♣	8♣	2♣	10♣	3♣
4	6	9♣	5♣	J♣	4♣	8♣	7♣	6♣	2♣	10♣	3♣
5	9	9♣	5♣	J♣	4♣	8♣	3♣	6♣	2♣	10♣	7♣
6	8	9♣	5♣	J♣	4♣	8♣	3♣	10♣	2♣	6♣	7♣
7	9	9♣	5♣	J♣	4♣	8♣	3♣	10♣	7♣	6♣	2♣
8	8	9♣	5♣	J♣	4♣	8♣	3♣	10♣	7♣	6♣	2♣
9	9	9♣	5♣	J♣	4♣	8♣	3♣	10♣	7♣	6♣	2♣

Note: Same method is effective for randomly rearranging any type of data.

Array application: shuffle and deal from a deck of cards

```
public class DealCards
{
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);

        String[] rank = {"2", "3", "4", "5", "6", "7", "8", "9", "10", "J", "Q", "K", "A" };
        String[] suit = { "♣", "♦", "♥", "♠" };

        String[] deck = new String[52];
        for (int i = 0; i < 13; i++)
            for (int j = 0; j < 4; j++)
                deck[i + 13*j] = rank[i] + suit[j];

        for (int i = 0; i < 52; i++)
        {
            int r = i + (int) (Math.random() * (52-i));
            String t = deck[r];
            deck[r] = deck[i];
            deck[i] = t;
        }

        for (int i = 0; i < N; i++)
            System.out.print(deck[i]);
        System.out.println();
    }
}
```



% java DealCards 5
9♣ Q♥ 6♥ 4♦ 2♠

random poker hand

% java DealCards 13
3♣ 4♥ 10♦ 6♥ 6♦ 2♠ 9♣ 8♠ A♠ 3♥ 9♣ 5♠ Q♥

random bridge hand

Coupon collector

Coupon collector problem

- M different types of coupons.
 - Collector acquires random coupons, one at a time, each type equally likely.
- Q. What is the expected number of coupons needed to acquire a full collection?

Example: Collect all ranks in a random sequence of cards ($M =$

Sequence

9♣	5♠	8♥	10♦	2♠	A♠	10♥	Q♦	3♠	9♥	5♦	9♣	7♦	2♦	8♣	6♣	Q♥	K♣	10♥	A♦	4♦	J♥
----	----	----	-----	----	----	-----	----	----	----	----	----	----	----	----	----	----	----	-----	----	----	----

Collection

2	3	4	5	6	7	8	9	10	J	Q	K	A								
2♠	3♠	4♦	5♠	6♣	7♦	8♥	9♣	10♦	J♥	Q♦	K♣	A♠								
2♦			5♦			8♣	9♥	10♥		Q♥		A♦								
						9♣	10♥													

22 cards needed
to complete
collection

Array application: coupon collector

Coupon collector simulation

- Generate random `int` values between 0 and $M-1$.
- Count number used to generate each value at least once.

Key to the implementation

- Create a boolean array of length M . (Initially all `false` by default.)
- When r generated, check the r th value in the array.
 - If `true`, ignore it (not new).
 - If `false`, count it as new distinct value (and set r th entry to `true`)

```
public class Coupon
{
    public static void main(String[] args)
    {
        int M = Integer.parseInt(args[0]);
        int cards = 0;      // number of cards collected
        int distinct = 0;   // number of distinct cards

        boolean[] found = new boolean[M];
        while (distinct < M)
        {
            int r = (int) (Math.random() * M);
            cards++;
            if (!found[r])
            {
                distinct++;
                found[r] = true;
            }
        }

        System.out.println(cards);
    }
}
```

```
% java Coupon 13  
46
```

```
% java Coupon 13  
22
```

```
% java Coupon 13  
54
```

```
% java Coupon 13  
27
```

Array application: coupon collector (trace for M = 6)

```

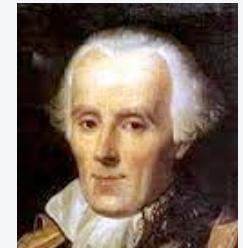
boolean[] found = new boolean[M];
while (distinct < M)
{
    int r = (int) (Math.random() * M);
    cards++;
    if (!found[r])
    {
        distinct++;
        found[r] = true;
    }
}
    
```

r	found							distinct	cards
	0	1	2	3	4	5			
	F	F	F	F	F	F	0	0	0
2	F	F	T	F	F	F	1	1	1
0	T	F	T	F	F	F	2	2	2
4	T	F	T	F	T	F	3	3	3
0	T	F	T	F	T	F	3	4	4
1	T	T	T	F	T	F	4	5	5
2	T	T	T	F	T	F	4	6	6
5	T	T	T	F	T	T	5	7	7
0	T	T	T	F	T	T	5	8	8
1	T	T	T	F	T	T	5	9	9
3	T	T	T	T	T	T	6	10	10

Simulation, randomness, and analysis (revisited)

Coupon collector problem

- M different types of coupons.
 - Collector acquires random coupons, one at a time, each type equally likely.
- Q. What is the expected number of coupons needed to acquire a full collection?



Pierre-Simon Laplace
1749-1827

A. (known via mathematical analysis for centuries) About $M \ln M + .57721M$.

<i>type</i>	M	<i>expected wait</i>
playing card suits	4	8
playing card ranks	13	41
baseball cards	1200	9201
Magic™ cards	12534	125508

```
% java Coupon 4  
11  
% java Coupon 13  
38  
% java Coupon 1200  
8789  
% java Coupon 12534  
125671
```

Remarks

- Computer simulation can help validate mathematical analysis.
- Computer simulation can also validate software behavior.

Example: Is `Math.random()` simulating randomness? ←

Simulation, randomness, and analysis (revisited)

Once simulation is debugged, experimental evidence is easy to obtain.

Gambler's ruin simulation, previous lecture

```
public class Gambler
{
    public static void main(String[] args)
    {
        int stake  = Integer.parseInt(args[0]);
        int goal   = Integer.parseInt(args[1]);
        int trials = Integer.parseInt(args[2]);

        int wins   = 0;
        for (int i = 0; i < trials; i++)
        {
            int t = stake;
            while (t > 0 && t < goal)
            {
                if (Math.random() < 0.5) t++;
                else                      t--;
            }
            if (t == goal) wins++;
        }
        System.out.println(wins + " wins of " + trials);
    }
}
```

Analogous code for coupon collector, this lecture

```
public class CouponCollector
{
    public static void main(String[] args)
    {
        int M = Integer.parseInt(args[0]);
        int trials = Integer.parseInt(args[1]);
        int cards = 0;
        boolean[] found;

        for (int i = 0; i < trials; i++)
        {
            int distinct = 0;
            found = new boolean[M];
            while (distinct < M)
            {
                int r = (int) (Math.random() * M);
                cards++;
                if (!found[r])
                {
                    distinct++;
                    found[r] = true;
                }
            }
        }
        System.out.println(cards/trials);
    }
}
```

Simulation, randomness, and analysis (revisited)

Coupon collector problem

- M different types of coupons.
 - Collector acquires random coupons, one at a time, each type equally likely.
- Q. What is the expected number of coupons needed to acquire a full collection?

Predicted by mathematical analysis

type	M	$M \ln M + .57721M$
playing card suits	4	8
playing card ranks	13	41
playing cards	52	236
baseball cards	1200	9201
magic cards	12534	125508



Observed by computer simulation

```
% java CouponCollector 4 1000000  
8  
% java CouponCollector 13 1000000  
41  
% java CouponCollector 52 100000  
236  
% java CouponCollector 1200 10000  
9176  
% java CouponCollector 12534 1000  
125920
```



Hypothesis. Centuries-old analysis is correct and `Math.random()` simulates randomness.

Image sources

http://www.vis.gr.jp/~nazoya/cgi-bin/catalog/img/CARDSBIC809_red.jpg

http://www.alegriphotos.com/Shuffling_cards_in_casino-photo-deae1081e5ebc6631d6871f8b320b808.html

<http://iveypoker.com/wp-content/uploads/2013/09/Dealing.jpg>

[http://upload.wikimedia.org/wikipedia/commons/b/bf/Pierre-Simon,_marquis_de_Laplace_\(1745-1827\)_-_Guérin.jpg](http://upload.wikimedia.org/wikipedia/commons/b/bf/Pierre-Simon,_marquis_de_Laplace_(1745-1827)_-_Guérin.jpg)



3. Arrays

- Basic concepts
- Examples of array-processing code
- **Two-dimensional arrays**

Two-dimensional arrays

A **two-dimensional array** is a *doubly-indexed* sequence of values of the same type.

Examples

- Matrices in math calculations.
- [Grades for students in an online class.](#)
- Outcomes of scientific experiments.
- Transactions for bank customers.
- [Pixels in a digital image.](#)
- Geographic data
- ...

	grade						
	0	1	2	3	4	5	...
0	A	A	C	B	A	C	
1	B	B	B	B	A	A	
2	C	D	D	B	C	A	
3	A	A	A	A	A	A	
4	C	C	B	C	B	B	
5	A	A	A	B	A	A	
...							

student ID



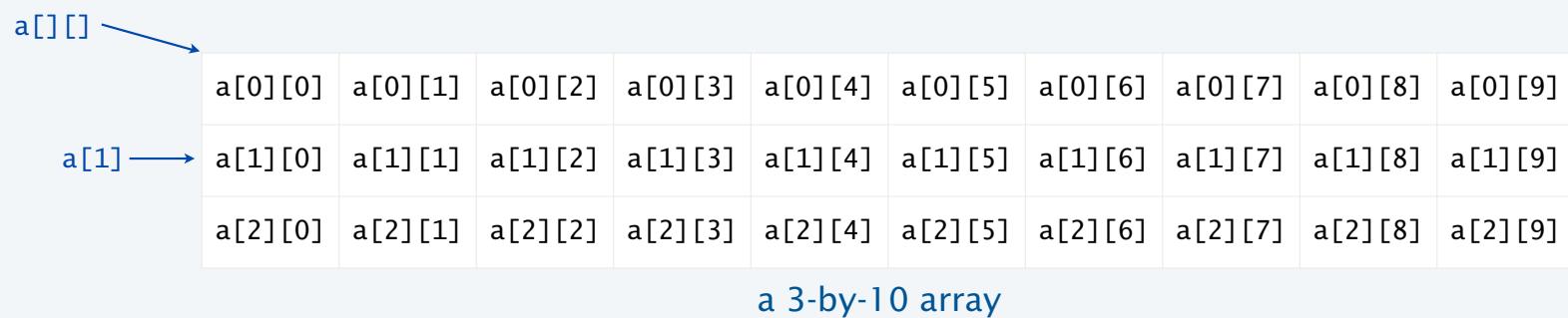
y-coordinate

x-coordinate

Main purpose. Facilitate storage and manipulation of data.

Java language support for two-dimensional arrays (basic support)

<i>operation</i>	<i>typical code</i>
Declare a two-dimensional array	<code>double[][] a;</code>
Create a two-dimensional array of a given length	<code>a = new double[1000][1000];</code>
Refer to an array entry by index	<code>a[i][j] = b[i][j] * c[j][k];</code>
Refer to the number of rows	<code>a.length;</code>
Refer to the number of columns	<code>a[i].length;</code> ← can be different for each row
Refer to row <i>i</i>	<code>a[i]</code> ← no way to refer to column <i>j</i>



Java language support for two-dimensional arrays (initialization)

<i>operation</i>	<i>typical code</i>	
Default initialization to 0 for numeric types	<code>a = new double[1000][1000];</code>	no need to use nested loops like <code>for (int i = 0; i < 1000; i++) for (int j = 0; j < 1000; j++) a[i][j] = 0.0;</code>
Declare, create and initialize in a single statement	<code>double[][] a = new double[1000][1000];</code>	BUT cost of creating an array is proportional to its size.
Initialize to literal values	<pre>double[][] p = { { .92, .02, .02, .02, .02 }, { .02, .92, .32, .32, .32 }, { .02, .02, .02, .92, .02 }, { .92, .02, .02, .02, .02 }, { .47, .02, .47, .02, .02 }, };</pre>	

Application of arrays: vector and matrix calculations

Mathematical abstraction: vector
Java implementation: 1D array

Mathematical abstraction: matrix
Java implementation: 2D array

Vector addition

```
double[] c = new double[N];
for (int i = 0; i < N; i++)
    c[i] = a[i] + b[i];
```

$$\begin{array}{ccc} .30 & .60 & .10 \end{array} + \begin{array}{ccc} .50 & .10 & .40 \end{array} = \begin{array}{ccc} .80 & .70 & .50 \end{array}$$

Matrix addition

```
double[][] c = new double[N][N];
for (int i = 0; i < N; i++)
    for (int j = 0; j < N; j++)
        c[i][j] = a[i][j] + b[i][j];
```

$$\begin{array}{ccc|ccc|ccc} .70 & .20 & .10 & .80 & .30 & .50 & 1.5 \\ .30 & .60 & .10 & .10 & .40 & .10 & .40 \\ .50 & .10 & .40 & .10 & .30 & .40 & .60 \end{array} = \begin{array}{ccc} .20 & .10 & .00 \end{array}$$

Application of arrays: vector and matrix calculations

Mathematical abstraction: vector
Java implementation: 1D array

Mathematical abstraction: matrix
Java implementation: 2D array

Vector dot product

```
double sum = 0.0;  
for (int i = 0; i < N; i++)  
    sum += a[i]*b[i];
```

$$\begin{array}{|c|c|c|} \hline .30 & .60 & .10 \\ \hline \end{array} \quad \cdot \quad \begin{array}{|c|c|c|} \hline .50 & .10 & .40 \\ \hline \end{array} \quad = \quad \begin{array}{|c|} \hline .25 \\ \hline \end{array}$$

i	x[i]	y[i]	x[i]*y[i]	sum
0	0.3	0.5	0.15	0.15
1	0.6	0.1	0.06	0.21
2	0.1	0.4	0.04	0.25

end-of-loop trace

Matrix multiplication

```
double[][] c = new double[N][N];  
for (int i = 0; i < N; i++)  
    for (int j = 0; j < N; j++)  
        for (int k = 0; k < N; k++)  
            c[i][j] += a[i][k] * b[k][j];
```

$$\begin{array}{|c|c|c|} \hline .70 & .20 & .10 \\ \hline \end{array} \quad * \quad \begin{array}{|c|c|c|} \hline .80 & .30 & .50 \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|c|} \hline .59 & .32 & .41 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline .30 & .60 & .10 \\ \hline \end{array} \quad * \quad \begin{array}{|c|c|c|} \hline .10 & .40 & .10 \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|c|} \hline .31 & .36 & .25 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline .50 & .10 & .40 \\ \hline \end{array} \quad * \quad \begin{array}{|c|c|c|} \hline .10 & .30 & .40 \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|c|} \hline .45 & .31 & .42 \\ \hline \end{array}$$

Pop quiz 4 on arrays

Q. How many multiplications to multiply two N -by- N matrices?

```
double[][] c = new double[N][N];
for (int i = 0; i < N; i++)
    for (int j = 0; j < N; j++)
        for (int k = 0; k < N; k++)
            c[i][j] += a[i][k] * b[k][j];
```

1. N
2. N^2
3. N^3
4. N^4

Pop quiz 4 on arrays

Q. How many multiplications to multiply two N -by- N matrices?

```
double[][] c = new double[N][N];  
for (int i = 0; i < N; i++)  
    for (int j = 0; j < N; j++)  
        for (int k = 0; k < N; k++)  
            c[i][j] += a[i][k] * b[k][j];
```

1. N
2. N^2
3. N^3  Nested for loops: $N \times N \times N$
4. N^4

Self-avoiding random walks

A dog walks around at random in a city, never revisiting any intersection.



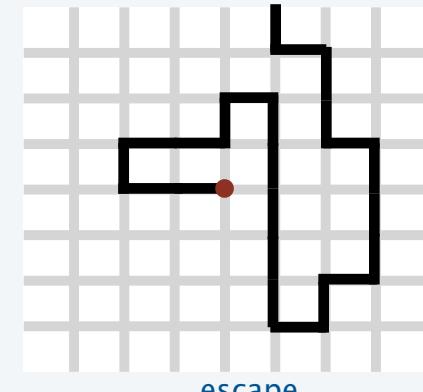
Q. Does the dog escape?

Model: a random process in an N -by- N lattice

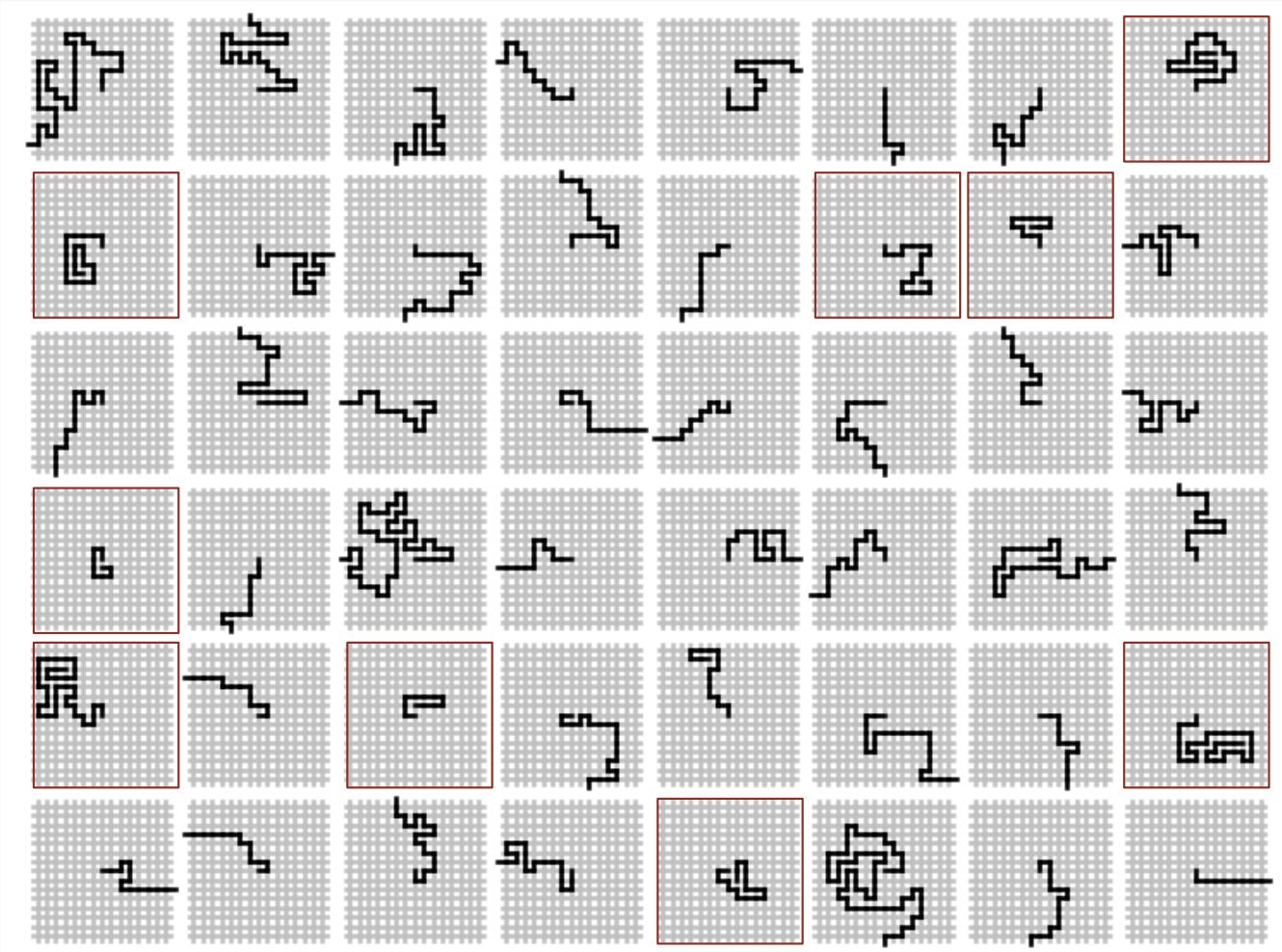
- Start in the middle.
- Move to a random neighboring intersection but *do not revisit any intersection*.
- Outcome 1 (escape): reach edge of lattice.
- Outcome 2 (dead end): no unvisited neighbors.

Q. What are the chances of reaching a dead end?

Approach: Use Monte Carlo simulation, recording visited positions in an N -by- N array.



Self-avoiding random walks



Application of 2D arrays: self-avoiding random walks

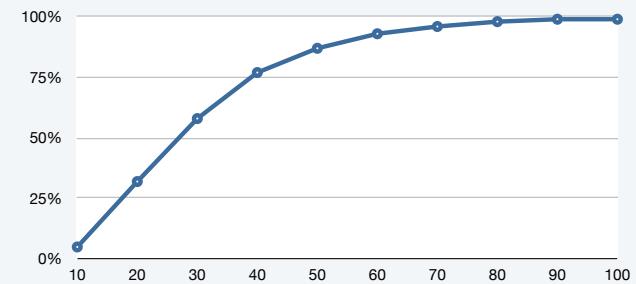
```
public class SelfAvoidingWalker
{
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        int trials = Integer.parseInt(args[1]);
        int deadEnds = 0;
        for (int t = 0; t < trials; t++)
        {
            boolean[][] a = new boolean[N][N];
            int x = N/2, y = N/2;

            while (x > 0 && x < N-1 && y > 0 && y < N-1)
            {
                if (a[x-1][y] && a[x+1][y] && a[x][y-1] && a[x][y+1])
                { deadEnds++; break; }

                a[x][y] = true;
                double r = Math.random();
                if (r < 0.25) { if (!a[x+1][y]) x++; }
                else if (r < 0.50) { if (!a[x-1][y]) x--; }
                else if (r < 0.75) { if (!a[x][y+1]) y++; }
                else if (r < 1.00) { if (!a[x][y-1]) y--; }
            }

            System.out.println(100*deadEnds/trials + "% dead ends");
        }
    }
}
```

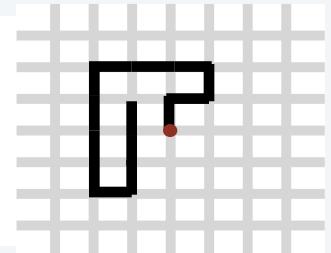
```
% java SelfAvoidingWalker 10 100000
5% dead ends
% java SelfAvoidingWalker 20 100000
32% dead ends
% java SelfAvoidingWalker 30 100000
58% dead ends
% java SelfAvoidingWalker 40 100000
77% dead ends
% java SelfAvoidingWalker 50 100000
87% dead ends
% java SelfAvoidingWalker 60 100000
93% dead ends
% java SelfAvoidingWalker 70 100000
96% dead ends
% java SelfAvoidingWalker 80 100000
98% dead ends
% java SelfAvoidingWalker 90 100000
99% dead ends
% java SelfAvoidingWalker 100 100000
99% dead ends
```



Simulation, randomness, and analysis (revisited again)

Self-avoiding walk in an N -by- N lattice

- Start in the middle.
- Move to a random neighboring intersection (do not revisit any intersection).



Applications

- Model the behavior of solvents and polymers.
- Model the physics of magnetic materials.
- (many other physical phenomena)



Paul Flory
1910-1985
Nobel Prize 1974

Q. What is the probability of reaching a dead end?

A. Nobody knows (despite decades of study). ←

Mathematicians and
physics researchers
cannot solve the problem.

A. 99% for $N > 100$ (clear from simulations). ← YOU can!

Computational models play
an essential role in modern
scientific research.

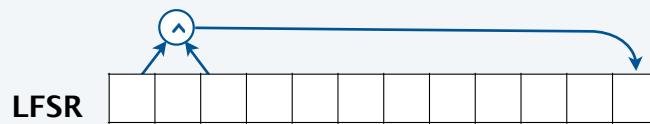
Remark: Computer simulation is often the *only* effective way to study a scientific phenomenon.

Your first data structure

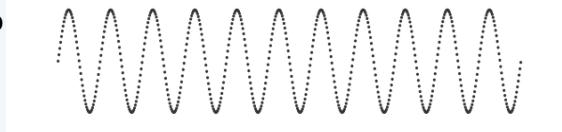
Arrays: A basic building block in programming

- They enable storage of large amounts of data (values all of the same type).
- With an index, a program can instantly access a given value.
- Efficiency derives from low-level computer hardware organization (stay tuned).

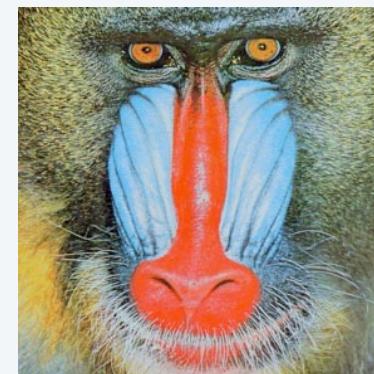
Some applications in this course:



digital audio



digital images



N-body simulation

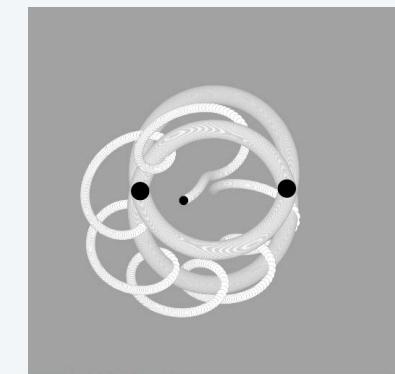
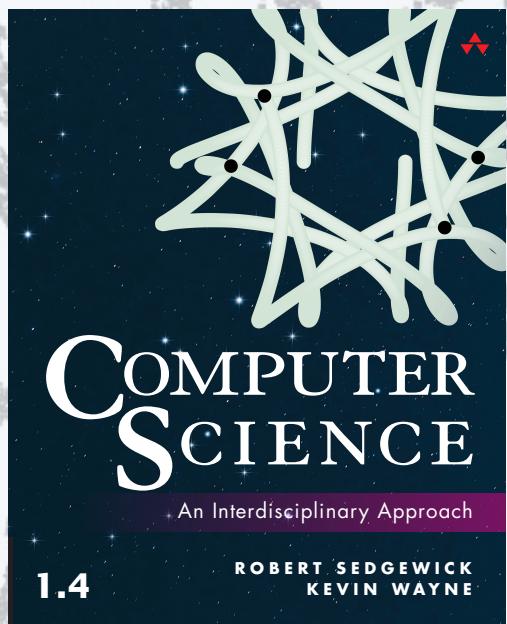


Image sources

http://en.wikipedia.org/wiki/Airedale_Terrier#mediaviewer/File:Airedale_Terrier.jpg

http://www.nobelprize.org/nobel_prizes/chemistry/laureates/1974/flory_postcard.jpg



3. Arrays