



PHY1001 Mechanics

2022-2023 Term 2

Final Examination

May 15, 2023; Time Allowed: 3 Hours

NAME (print)

CUHKSZ ID

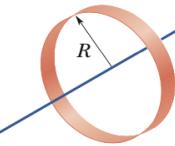
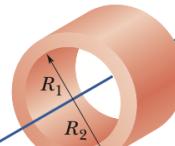
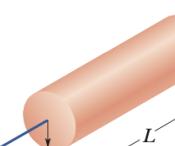
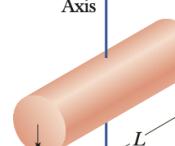
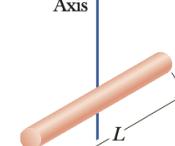
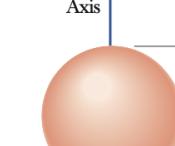
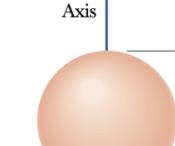
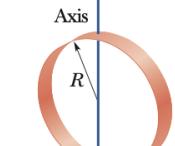
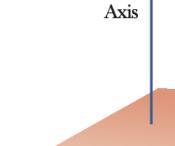
Exam Room No. and Seat No.

- **Show all your work.** Correct answers with little supporting work will not be given credit.
 - Closed Book Exam: One piece of double-sided A4 reference paper, a scientific calculator, and a paper-based dictionary are allowed.
 - Unless approved by the instructors, students who arrive more than 30 minutes late will NOT be admitted.
 - The total points are 120 points. You need to finish ALL the questions in 3 hours (180 minutes).

Summary of Basic Calculus:

$$\begin{aligned}
 \frac{d}{dx} x^n &= nx^{n-1}, \\
 \frac{d}{dx} e^{ax} &= ae^{ax}, \\
 \frac{d}{dx} \ln ax &= \frac{1}{x}, \\
 \frac{d}{dx} (uv) &= v \frac{d}{dx} u + u \frac{d}{dx} v, \\
 \int x^n dx &= \frac{x^{n+1}}{n+1} + C, \quad (n \neq -1), \\
 \int \frac{dx}{x} &= \ln x + C, \\
 \int e^{ax} dx &= \frac{1}{a} e^{ax} + C, \quad C \text{ is a constant.}
 \end{aligned}$$

Table of Moments of Inertia:

 $I = MR^2$	 $I = \frac{1}{2}M(R_1^2 + R_2^2)$	 $I = \frac{1}{2}MR^2$
 $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$	 $I = \frac{1}{12}ML^2$	 $I = \frac{2}{5}MR^2$
 $I = \frac{2}{3}MR^2$	 $I = \frac{1}{2}MR^2$	 $I = \frac{1}{12}M(a^2 + b^2)$

Bernoulli's equation

$$P + \rho gy + \frac{1}{2} \rho V^2 = \text{Constant}$$

Simple Harmonic Oscillator
Spring Mass System

$$M \frac{d^2X}{dt^2} = -kX,$$

1. A uniform solid ball with mass M and radius R is given an initial clockwise angular velocity ω_0 and zero linear velocity ($v_{com} = 0$) before it is placed on a horizontal surface with kinetic friction coefficient μ_k . (10 pts)

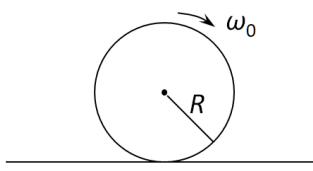


Fig. 1

* Student gets
2pts if they
write "right."

- (a) Which direction is the ball going to move towards? (Hint: Left or Right.) (2 pts)

0.5 The contact point is moving to the left relative to the surface.
0.5 Thus the friction applied to the ball is to the right of right 1pt
1pt The ball is moving toward right of right 1pt

- (b) What is the acceleration at the moment that the ball is placed on the surface? (2 pts)

Consider the friction f (kinetic).

$$f = \mu_k N \quad \xrightarrow{\text{0.5 pt}}$$

$$N = Mg \quad \xrightarrow{\text{0.5 pt}}$$

$$\Rightarrow a = \frac{f}{M} = \mu_k g \quad \xrightarrow{\text{1 pt}}$$

- (c) Show that the corresponding angular acceleration of the ball is $\alpha = 5\mu_k g / (2R)$? (2 pts)

choose C.O.M as the axis

$$\text{with } f = \mu_k Mg \Rightarrow fR = Id \quad \xrightarrow{\text{1 pt}}$$

$$I = \frac{2}{5}MR^2 \quad \xrightarrow{\text{1 pt}}$$

$$\Rightarrow \alpha = \frac{5\mu_k g}{2R}$$

- (d) How long does it take for the ball to roll without slipping? (Hint: The rolling without slipping happens when $\omega R + v = 0$) (4 pts)

$$\left\{ \begin{array}{l} \omega = -\omega_0 + \alpha t \\ v = at \end{array} \right.$$

1pt

1pt

$$\Rightarrow \omega R + v = 0 \quad \xrightarrow{\text{1 pt}}$$

$$\Rightarrow -\omega_0 R + \frac{5}{2}\mu_k g t + \mu_k g t = 0$$

$$\Rightarrow t = \frac{2\omega_0 R}{7\mu_k g} \quad \xrightarrow{\text{1 pt}}$$

2. A uniform solid cylinder of mass M is supported on a ramp that rises at angle θ above the horizontal by a wire that is wrapped around its rim and pulls on it tangentially parallel to the ramp as shown in the figure below. The static friction coefficient is $\mu_s = 0.5$. The cylinder is at equilibrium state. (10 pts)

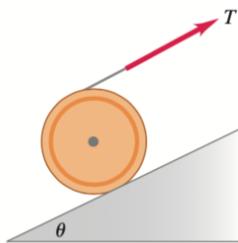


Fig. 2

- (a) Show that there must be friction on the surface for the cylinder to stay at equilibrium state. (2 pts)

1 pt equilibrium state \Rightarrow zero torque and force.

choose COM as axis $fR - TR = 0$

1 pt thus there must be friction f Other correct reasoning can get full 2 pts as well!

- (b) From the torque equation, show that the friction f is equal to the tension T ($f = T$). (2 pts)

choose COM as axis

$$fR - TR = 0$$

$$\Rightarrow f = T.$$

there are other methods

- (c) Find the frictional force (in terms of Mg and θ) to keep the cylinder at rest. (2 pts)

From $\sum F_i = 0$ along the ramp.

together with $f = T$.

$$\Rightarrow f + T - Mg \sin\theta = 0$$

$$\Rightarrow f = \frac{1}{2}Mg \sin\theta$$

other methods ok

- (d) Find the normal force N from the ramp to keep the cylinder at rest. (2 pts)

1 pt \perp the ramp

$$N - Mg \cos\theta = 0 \rightarrow$$

$$\Rightarrow N = Mg \cos\theta \rightarrow$$

1 pt

- (e) Find the maximum angle θ to keep the cylinder at rest. (2 pts)

$$\text{1 pt} f = \frac{1}{2}Mg \sin\theta \leq \mu_s N = \mu_s Mg \cos\theta$$

$$\boxed{+go \leq 2\mu_s = 1.}$$

1 pt \rightarrow This is $\theta_{\max} = 45^\circ$ or $\frac{\pi}{4}$.

3. As shown in the figure below, a point particle of mass m_1 is a distance d from one end of a uniform thin rod with length L and mass M . (10 pts)

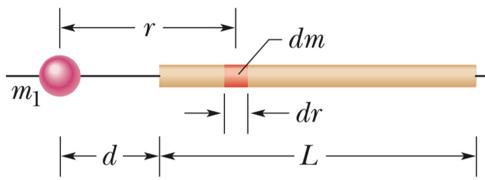


Fig. 3

- (a) What is the magnitude of the gravitational force on the particle from the rod? (Hint: As shown above, use $\int \frac{dr}{r^2} = -\frac{1}{r}$ to show that magnitude is $\frac{GMm_1}{d(d+L)}$). (5 pts)

Only compute
the amplitude

$$dF = \frac{Gm_1 dm}{r^2} = \frac{Gm_1 M}{L} \frac{dr}{r^2} \quad \text{~} \rightarrow \text{2pt}$$

one or

two steps

can be
skipped !!

$$\begin{aligned} F &= \frac{Gm_1 M}{L} \int_d^{L+d} \frac{dr}{r^2} \\ &= \frac{Gm_1 M}{L} \left(-\frac{1}{r} \right) \Big|_d^{L+d} \\ &= \frac{Gm_1 M}{d(L+d)} \end{aligned} \quad \text{~} \rightarrow \text{2pt} \quad \text{~} \rightarrow \text{1pt}$$

- (b) Find the gravitational potential energy U between the particle and the rod. (Take the potential energy to be zero when the rod and particle are infinitely far apart.) (5 pts)

$$dU = -\frac{Gm_1 dm}{r} \quad \text{~} \rightarrow \text{1pt}$$

$$= -\frac{Gm_1 M}{L} \frac{dr}{r} \quad \text{~} \rightarrow \text{1pt}$$

$$U = -\frac{Gm_1 M}{L} \int_d^{L+d} \frac{dr}{r} \quad \text{~} \rightarrow \text{2pt}$$

$$= -\frac{Gm_1 M}{L} \ln \frac{L+d}{d} \quad \text{~} \rightarrow \text{1pt}$$

4. A rock with irregular shape is suspended by a light string. When the rock is hanging in the air (suppose the air buoyancy is negligible), the tension in the string is $T_1 = 20.0 \text{ N}$. When the rock is totally immersed in water (mass density of water is $\rho_w = 1.0 \times 10^3 \text{ kg/m}^3$), the tension is $T_2 = 10.2 \text{ N}$. (10 pts)

(a) What is the volume of the rock?

(3 pts)

$$(1\text{pt}) \quad T_1 = W$$

$$(1\text{pt}) \quad T_2 = W - f_w g V$$

$$0.5\text{pt} \quad V = \frac{(T_1 - T_2)}{g \rho_w}$$

$$0.5\text{pt} \quad = 1.0 \times 10^{-3} \text{ m}^3$$

(b) What is the average mass density of the rock?

(4 pts)

$$(2\text{pts}) \quad f_R g V = W = T_1$$

$$(1\text{pt}) \quad f_R = \frac{T_1}{g V}$$

$$(1\text{pt}) \quad = 2.0 \times 10^3 \text{ kg/m}^3$$

$$\text{or } 2.04 \times 10^3 \text{ kg/m}^3$$

(c) When the rock is totally immersed in an unknown liquid, the tension is $T_3 = 7.9 \text{ N}$. What is the density of the unknown liquid? (3 pts)

$$(1\text{pt}) \quad T_3 + f_0 g V - W = 0$$

$$(1\text{pt}) \quad f_0 = \frac{W - T_3}{g V}$$

$$(1\text{pt}) \quad = 1.2 \times 10^3 \text{ kg/m}^3$$

$$\text{or } 1.23 \times 10^3 \text{ kg/m}^3$$

5. In the figure below, the fresh water behind a reservoir dam has depth $D = 12 \text{ m}$. A horizontal pipe passes through the dam at depth $d = 6.0 \text{ m}$. The inner and outer parts of the pipe have cross-section areas of $S_1 = 18.8 \text{ cm}^2$ and $S_2 = 12.5 \text{ cm}^2$. A plug secures the pipe opening. (10 pts)

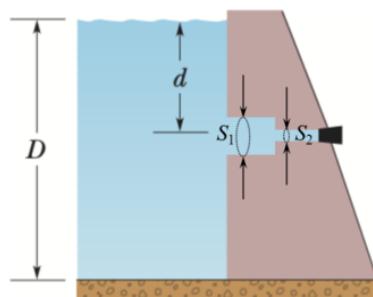


Fig. 4

- (a) Find the values of gauge pressure in the horizontal pipe? (2 pts)

(Hint: The two diameters of the horizontal pipe are much smaller than d and D . The gauge pressure of water at the cross-section of S_1 approximately equals to that at the cross-section of S_2 .) (The gauge pressure is defined as the difference between an absolute pressure (P) and the atmospheric pressure (P_{atm})).

1 pt gauge pressure $\Delta P = \rho g d$

1 pt $= 5.9 \times 10^4 \text{ Pa}$

or $5.88 \times 10^4 \text{ Pa}$.

- (b) Find the magnitude of the frictional force between plug and pipe wall. (2 pts)

1 pt $f - \Delta P S_2 = 0$

1 pt $f = \Delta P S_2 = 74 \text{ N}$ (or 73.5 N)

- (c) When the plug is removed, what is the water volume that exits the pipe in 1.0 hour? (3 pts)

use Bernoulli's equation

Or Work-Energy theorem
 $\rho g d S_2 v dt = \frac{1}{2} (\rho S_2 v dt) v^2 \Rightarrow P_0 + \frac{1}{2} \rho V^2 = P_0 + \rho g d \Rightarrow$

1 pt
1 pt

$$V = \sqrt{2gd}$$

$$\text{Volume} = VS_2 t = 49 \text{ m}^3 \text{ (or } 50 \text{ m}^3\text{)}$$

1 pt

- (d) What is the speed of water flowing through the cross-section of S_1 ? (3 pts)

2 pts $VS_2 = V_1 S_1$ Continuity equation

1 pt $\Rightarrow V_1 = \frac{VS_2}{S_1} = 7.2 \text{ m/s}$

6. As shown in Fig. 5, two stars with masses M_1 and M_2 orbit around their center of mass (COM) at the same angular velocity ω . Both orbits are circular. Let R_1 be the distance between star M_1 and COM, and R_2 be the distance between star M_2 and COM.

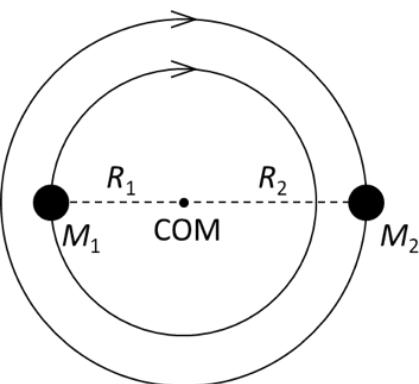


Fig. 5

- (a) Find the gravitational force (magnitude and direction) of one star on the other. **(2 pts)**

$$F = \frac{GM_1M_2}{(R_1+R_2)^2}$$

(ipt) F points from each star to CDM

attractive force is OK.

- (b) Show that $M_1 R_1 = M_2 R_2$.

(b) Show that $M_1 R_1 = M_2 R_2$.

Ans

$$F = M_1 \frac{V_1^2}{R_1} = M_2 \frac{V_2^2}{R_2}$$

$$\Rightarrow M_1 R_1 = M_2 R_2$$

w is the same for M_1 and M_2

- (c) Show that $(R_1 + R_2)^3 \omega^2 = G(M_1 + M_2)$.

$$(1 \text{ pt}) \quad \left\{ \frac{GM_1 M_2}{(R_1 + R_2)^2} = M_1 R_1 \omega^2 \Rightarrow \frac{GM_2}{(R_1 + R_2)^2} = R_1 \omega^2 \right.$$

$$\text{1 pt} \quad \left\{ \frac{(R_1+R_2)^2}{GM_1 M_2} = M_2 R_2 \omega^2 \Rightarrow \frac{GM_1}{(R_1+R_2)^2} = R_2 \omega^2 \right.$$

$$\textcircled{1} \text{pt} \Rightarrow G(M_1 + M_2) = (R_1 + R_2)^3 \omega^2$$

- (d) Find angular momentum of star M_1 about COM, including the magnitude and direction.

(Hint: is the direction pointing in or out of the exam paper)

(2 pts)

(Hint: is the direction pointing in or out of the exam paper) (2 pts)

there are two possible answers!

Another answer

$$L_1 = M_1 V_1 R_1$$

$$= M_1 \omega R_1^2$$

$$= M_1 R_1^2 \sqrt{\frac{G(M_1 + m)}{(R_1 + R_2)^3}}$$

OK

use $M_1 = \frac{M_2 R_2}{R_1}$

1 pt

1 pt

According to the right hand rule, angular momentum points inside

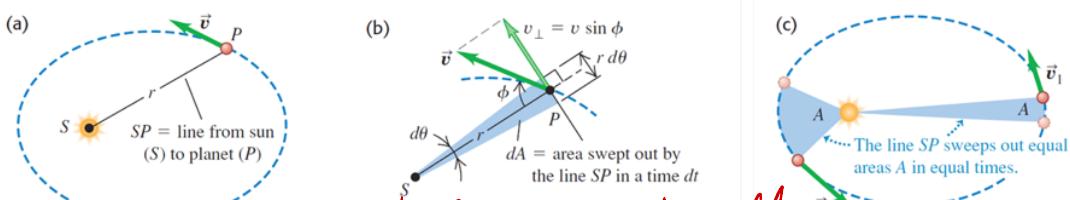
- (e) How much energy would be required to separate the two stars to infinity?

How much energy would be required to separate the two stars to infinity?

$$\text{1pt} \quad \Delta E = E_2 - (K_r + U) = 0 - \frac{1}{2}M_1V_1^2 - \frac{1}{2}M_2V_2^2 + \frac{G M_1 M_2}{(R_1 + R_2)} = \frac{G M_1 M_2}{2(R_1 + R_2)}$$

(1pt) $W = \Delta E = \frac{GM_1M_2}{2(R_1+R_2)}$ OK
Correct answer gets 2 extra

7. Demonstration of Kepler's second law (A planet sweeps out equal areas of the ellipse over equal time intervals.) for a planet that orbits the Sun in an elliptical path. (10 pts)



magnitude with direction labelled above is ok. Fig. 6

(a) Write down the gravitational force \vec{F}_g exerted to the planet by the Sun. (2 pts)

2pt

$$\vec{F}_g = -\frac{GM_S M}{r^2} \hat{r}$$

or

$$= -\frac{GM_S m}{r^2} \hat{r}$$

any similar result is OK

(b) Show that the angular momentum of the planet is conserved. (3 pts)

$$\frac{d\vec{L}}{dt} = \vec{T} \rightsquigarrow 1pt$$

$$\vec{T} = (\vec{r} \times \vec{F}) = 0 \rightsquigarrow 1pt$$

$$\Rightarrow \frac{d\vec{L}}{dt} = 0 \rightsquigarrow 1pt$$

(c) Show that the area swept out by the line SP per unit time $dA/dt = L/(2m)$. (3 pts)

1pt

$$dA = \frac{1}{2} r (v dt) \sin \phi$$

1pt

$$= \frac{1}{2} |\vec{r} \times \vec{v}| dt$$

$$\vec{T} = \vec{r} \times m\vec{v} \quad \text{and} \quad L = |\vec{r} \times m\vec{v}|$$

$$\Rightarrow \frac{dA}{dt} = \frac{L}{2m} \quad 1pt$$

(d) Demonstrate the Kepler's second law is true, namely, dA/dt is a constant. (2 pts)

First method: L angular momentum is conserved from point (b). $L = \text{constant}$, thus $\frac{dA}{dt} = \text{constant}$

2pt

Second method $\frac{d}{dt} \left(\frac{dA}{dt} \right) = \frac{1}{2m} \frac{d}{dt} (L) = 0 \rightsquigarrow 1pt$

$$\Rightarrow \frac{dA}{dt} = \text{constant.} \rightsquigarrow 1pt$$

8. The Tale of Two Springs.

(10 pts)

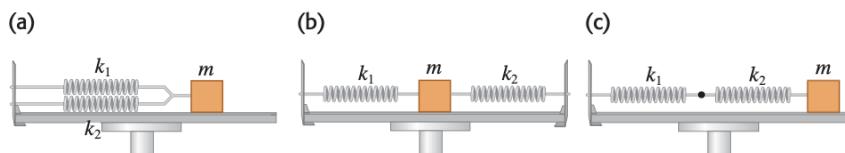


Fig. 7

Two springs with the same unstretched length but different force constants k_1 and k_2 are attached to a block with mass m on a level, frictionless surface.

- (a) Use Newton's 2nd law, write down the equation of motion for the oscillator in case (a) as shown in the above Fig. 7 (a). Find the corresponding angular frequency ω_a . (4 pts)

Suppose the displacement x .

$$\text{2pt} \quad m \frac{d^2x}{dt^2} = -k_1 x - k_2 x = -(k_1 + k_2)x$$

or $ma = -\frac{-}{k_1 + k_2}$

$$\text{2pt} \quad \Rightarrow \quad \omega_a = \sqrt{\frac{k_1 + k_2}{m}}$$

- (b) Write down the equation of motion for the oscillator in case (b) as shown in the above Fig. 7 (b) and find the corresponding angular frequency ω_b . (3 pts)

$$\text{1pt} \quad m \frac{d^2x}{dt^2} = -(k_1 + k_2)x$$

or $ma = -\frac{-}{k_1 + k_2}$

$$\text{2pt} \quad \Rightarrow \quad \omega_b = \sqrt{\frac{k_1 + k_2}{m}}$$

- (c) Write down the equation of motion for the oscillator in case (c) as shown in the above Fig. 7 (c) and find the corresponding angular frequency ω_c . (3 pts)

Suppose a total displacement $x = x_1 + x_2$

$$\text{1pt} \quad k_1 x_1 = k_2 x_2$$

$$\Rightarrow x_1 = \frac{k_2}{k_1 + k_2} x ; \quad x_2 = \frac{k_1}{k_1 + k_2} x$$

$$\text{1pt} \quad \text{EOM} \Rightarrow m \frac{d^2x}{dt^2} = -k_1 x_1 = -\frac{k_1 k_2}{k_1 + k_2} x$$

or $ma = -\frac{-}{k_1 k_2}$

$$\text{1pt} \quad \Rightarrow \quad \omega_c = \sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}$$

9. A certain transverse wave on a rope is described by

$$y_1(x, t) = (6.50 \times 10^{-3} \text{ m}) \cos 2\pi \left(\frac{x}{0.280 \text{ m}} - \frac{t}{0.0360 \text{ s}} \right)$$

(a) Which direction does the wave propagate (travel)? (1 pts)

Ipt $\rightarrow +x$ direction

(b) Determine the wave's amplitude. (1 pts)

Ipt $A = 6.50 \times 10^{-3} \text{ m}$

(c) Find the wavelength λ . (1 pts)

Ipt $\lambda = 0.280 \text{ m}$

(d) What is the period T ? (1 pts)

Ipt $T = 0.0360 \text{ s}$

(e) What is its speed of propagation (the wave traveling speed)? (1 pts)

Ipt $V = \frac{\lambda}{T} = 7.78 \text{ m/s}$

(f) The mass per unit length of the rope is $\mu = 0.0500 \text{ kg/m}$. Find the tension F_T . (1 pts)

Ipt $\leftarrow V = \sqrt{\frac{F_T}{\mu}} \Rightarrow F_T = V^2 \mu = 3.02 \text{ N}$ (0.5pt) (0.5pt)

(g) Find the average power $P_{av} = \frac{1}{2} \sqrt{\mu F_T} \omega^2 A^2$ of this wave. (1 pts)

Ipt $\leftarrow P_{av} = \frac{1}{2} \sqrt{\mu F_T} \cdot \omega^2 A^2$
 $= \frac{1}{2} \sqrt{3.02 \times 0.05} \times \left(\frac{2\pi}{T} \right)^2 \cdot A^2 = 0.250 \text{ (W)}$

(h) The reflection of the above wave can be described by

$$y_2(x, t) = -(6.50 \times 10^{-3} \text{ m}) \cos 2\pi \left(\frac{x}{0.280 \text{ m}} + \frac{t}{0.0360 \text{ s}} \right).$$

The superposition of y_1 and y_2 gives rise to a standing wave. Find the form of the standing wave $y(x, t) = y_1(x, t) + y_2(x, t)$ and the location of nodes. (3 pts)

Ipt $\leftarrow y = A \cos(kx - \omega t) - A \cos(kx + \omega t)$
 $= 2A \sin kx \sin \omega t$

Ipt $\leftarrow = 2 \times (6.50 \times 10^{-3} \text{ m}) \sin \frac{2\pi x}{0.280 \text{ m}} \sin \frac{2\pi t}{0.0360 \text{ s}}$

locations of nodes $kx = n\pi = \frac{2\pi}{\lambda} x \rightarrow$ **Ipt**
 $\Rightarrow x = \frac{1}{2} n\lambda = (0.140)n \text{ (m)}$

10. Doppler Effects: $f' = \frac{v \pm v_d}{v \pm v_s} f$

(10 pts)

A firetruck's siren emits a sinusoidal wave with frequency $f = 170$ Hz. The speed of sound is 340 m/s and the air is still.

(a) Find the wavelength of the sound wave if the firetruck is at rest.

(2 pts)

$$\lambda = V_T = V \frac{1}{f}$$

$$= \frac{340}{170} \text{ m} = 2.00 \text{ m}$$

→ (1pt)

→ (1pt)

(b) What frequency will the stationary listener hear if the firetruck is moving toward him with a speed $v_s = 40$ m/s ?

(2 pts)

$$f_b = \frac{V}{V - V_s} f$$

$$= \frac{340}{300} \cdot 170 = 193 \text{ Hz}$$

→ (1pt)

→ (1pt)

close to 190 is fine.

(c) What frequency will the stationary listener hear if the firetruck is moving away from him with a speed $v_s = 60$ m/s ?

(2 pts)

$$f_c = \frac{V}{V + V_s} f$$

$$= \frac{340 \times 170}{400} = 145 \text{ Hz}$$

→ (1pt)

→ (1pt)

close to 145 is OK.

(d) Now the firetruck is at rest and the listener is moving toward the firetruck with a speed $v_d = 50$ m/s, what frequency will the moving listener hear?

(2 pts)

$$f_d = \frac{V + V_d}{V} f$$

$$= \frac{390}{340} \times 170 = 195 \text{ Hz}$$

→ (1pt)

→ (1pt)

(e) Now the firetruck is at rest and the listener is moving away from the firetruck with a speed $v_d = 30$ m/s, what frequency will the moving listener hear?

(2 pts)

$$f_e = \frac{V - V_d}{V} f$$

$$= \frac{340 - 30}{340} \times 170 = 155 \text{ Hz}$$

→ (1pt)

→ (1pt)

11. A uniform disk with Mass M and radius R mounted on ceiling. Two blocks with mass m_1 and m_2 ($m_2 > m_1$) hang from a massless cord that is wrapped around the rim of the disk as shown in the figure below. (10 pts)

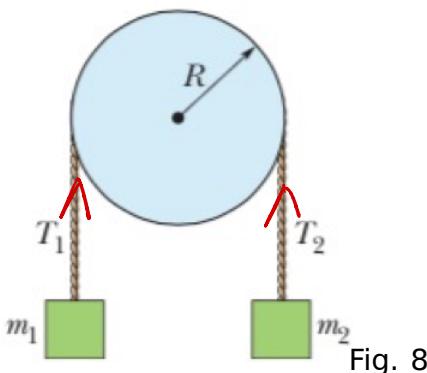


Fig. 8

- (a) Let us first set mass of the disk $M = 0$, find the tensions T_1 and T_2 as indicated in the above graph.

$$\begin{aligned} \text{(0.5)} \quad T_1 - m_1 g &= m_1 a & T_1 = T_2 & \text{(0.5)} \\ \text{(0.5)} \quad M_2 g - T_2 &= m_2 a & \Rightarrow \left\{ \begin{array}{l} a = \frac{m_2 - m_1}{m_1 + m_2} g \\ T_1 = T_2 = \frac{2m_1 m_2}{m_1 + m_2} g \end{array} \right. \end{aligned}$$

- (b) Now assume the disk mass M is nonzero and there is no slipping between the cord and the disk (the disk is rotating as the blocks move), find the tensions T_1 and T_2 as indicated in the above graph. (4 pts)

$$T_1 - m_1 g = m_1 a \quad m_2 g - T_2 = m_2 a$$

$$\text{(1pt)} \quad -(T_2 - T_1)R = \frac{1}{2}MR^2\alpha$$

$$\text{(1pt)} \quad \text{(1pt)} \quad R\alpha + a = 0 \quad \Rightarrow T_1 = \frac{(2m_2 + \frac{1}{2}M)m_1 g}{M_1 + M_2 + \frac{1}{2}M} ; T_2 = \frac{(2m_1 + \frac{1}{2}M)m_2 g}{M_1 + M_2 + \frac{1}{2}M} \quad \text{(1pt)}$$

- (c) When block m_2 drops a height of h , what is the angular speed of the disk? (4 pts)

Two methods

$$\textcircled{1} \quad \text{use } a = \frac{(m_2 - m_1)g}{M_1 + M_2 + \frac{1}{2}M} \quad \text{(2pt)} \quad \text{(1pt)}$$

$$V = \sqrt{2ah} \quad \text{(1pt)} \quad \omega = \frac{V}{R} = \frac{1}{R} \sqrt{\frac{2(m_2 - m_1)gh}{M_1 + M_2 + \frac{1}{2}M}} \quad \text{(1pt)}$$

$$\textcircled{2} \quad \text{energy conservation} \quad (m_2 - m_1)gh = \frac{1}{2}m_1 V^2 + \frac{1}{2}m_2 V^2 + \frac{1}{2}\frac{1}{2}MR^2\omega^2$$

$$\text{(2pt)} \quad \text{(1pt)} \quad R\omega = V \quad \Rightarrow W = \frac{1}{R} \sqrt{\frac{2(m_2 - m_1)gh}{M_1 + M_2 + \frac{1}{2}M}} \quad \text{(1pt)}$$

- (a) For a simple harmonic oscillator, the net force on the body with mass m is given by $F_x = -kx$. What is the potential energy function for this simple harmonic oscillator if we take $U = 0$ at $x = 0$? (1 pts)

use work-energy theorem

$$U(x) = - \int_0^x dx F_x = \underline{\underline{\frac{1}{2} k x^2}}. \quad \text{--- } \textcircled{1pt}$$

- (b) Use energy conservation, show that the velocity v_x as a function of x is

$$v_x = \frac{dx}{dt} = \sqrt{\frac{k}{m}(A^2 - x^2)}.$$

where A is the amplitude of the oscillator.

(1 pts)

$$\begin{aligned} \frac{1}{2} k A^2 + 0 &= \frac{1}{2} m v_x^2 + \frac{1}{2} k x^2 \rightsquigarrow \textcircled{1pt} \\ \text{potential} &\quad \text{kinetic} \\ \Rightarrow v_x &= \frac{dx}{dt} = \sqrt{\frac{k}{m}(A^2 - x^2)} \end{aligned}$$

- (c) Separate the variable by writing all factors containing x on one side and all factors containing t on the other side so that each side can be integrated, and use the identity $\int_0^1 \frac{du}{\sqrt{1-u^2}} = \frac{\pi}{2}$ to find the period T of the simple harmonic oscillator. (Hint: One-quarter (1/4) of T is the time for the body to move from $x = 0$ to $x = A$.) (2 pts)

Separate variables

$$\int_0^A \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^{T/4} \sqrt{\frac{k}{m}} dt \rightsquigarrow \textcircled{1pt}$$

$$\text{let } u = x/A$$

$$\Rightarrow \int_0^1 \frac{du}{\sqrt{1-u^2}} = \frac{\pi}{2} = \sqrt{\frac{k}{m}} \frac{T}{4}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{m}{k}} \rightsquigarrow \textcircled{1pt}$$

- (d) For a second oscillator, the net force on the body with mass m is given by $F_x = -cx^3$. What is the potential energy function for this oscillator if we take $U = 0$ at $x = 0$? (2 pts)

$$U(x) = - \int_0^x F_x dx \rightsquigarrow (1 \text{ pt})$$

$$= -\frac{1}{4}Cx^4 \rightsquigarrow (1 \text{ pt})$$

- (e) Follow part (d), compute the period T_2 for the second oscillator. (Hint: You may find the identity $\int_0^1 \frac{du}{\sqrt{1-u^4}} = 1.31$ useful.) (3 pts)

use energy conservation

$$(1 \text{ pt}) \rightarrow \frac{1}{4}CA^4 = \frac{1}{4}Cx^4 + \frac{1}{2}mV_x^2$$

$$\Rightarrow V_x = \frac{dx}{dt} = \sqrt{\frac{C}{2m}(A^4 - x^4)}$$

$$(1 \text{ pt}) \Rightarrow \int_0^{T_2/4} \sqrt{\frac{C}{2m}} dt = \int_0^A \frac{dx}{\sqrt{A^4 - x^4}} = \frac{1}{A} \int_0^1 \frac{du}{\sqrt{1-u^4}}$$

$$(1 \text{ pt}) \Rightarrow T_2 = 4 \sqrt{\frac{2m}{C}} \cdot 1.31 \frac{1}{A} = \frac{7.41}{A} \sqrt{\frac{m}{C}}$$

- (f) According to the result you obtained in part (e), does the period T_2 depend on the amplitude A of the motion? Is this oscillation simple harmonic? (1 pts)

Yes, T_2 depends on A . (0.5pt)

No, it is not simple harmonic. (0.5pt)