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Problem 1

$$Var(U_1) = Var(U_2) = \sigma^2$$

$$x_t = U_1 \sin(2\pi\omega t) + U_2 \cos(2\pi\omega t)$$

a) since A and B are iid with zero-mean random variables $\Rightarrow$

$\Rightarrow E(x_t) = E(U_1)\sin(2\pi\omega t) + E(U_2)\cos(2\pi\omega t) = 0$ for all t (1)

(1) $\Rightarrow \gamma_h = Cov(x_t, x_{t+h}) = E\Big((U_1 \sin(2\pi\omega t) + U_2 \cos(2\pi\omega t))(U_1 \sin(2\pi\omega(t+h)) + U_2 \cos(2\pi\omega(t+h))\Big)$

$= E\big(U_1^2 \sin(2\pi\omega t)\sin(2\pi\omega(t+h)) + U_2^2 \cos(2\pi\omega(t+h))\cos(2\pi\omega t)\big) =$

U_1 and U_2 are independent

$= \sigma^2 \big(\sin(2\pi\omega t)\sin(2\pi\omega(t+h)) + \cos(2\pi\omega(t+h))\cos(2\pi\omega t)\big) =$

$Var(U_1) = Var(U_2) = E(U_1^2) \Rightarrow E(U_1^2) = E(U_2^2) = \sigma^2$

$\mu = 0$

$= \sigma^2 \cos(2\pi\omega(t+h-t)) = \boxed{\sigma^2 \cos(2\pi\omega h) = \gamma_h}$

using hint $\sin\alpha\sin\beta + \cos\alpha\cos\beta = \cos(\alpha-\beta)$

the expected value of the series does not depend on t and γ_h only depends on h, so this series is weakly stationary

b) by definition $\rho_h = \dfrac{\gamma_h}{\gamma_0} =$

$\gamma_0 = \sigma^2 \cos(2\pi\omega 0) = \sigma^2$

$$\Rightarrow \boxed{\rho_h = \frac{\sigma^2 \cos(2\pi\omega h)}{\sigma^2} = \cos(2\pi\omega h)}$$

Problem 4

a) $v_t = y_t - y_{t|t-1}$ from problem directions

$E(v_t) = E(y_t) - E(y_{t|t-1})$

$y_{t|t-1} = E(y_t | F_{t-1})$ from problem directions

$\Rightarrow E(v_t) = E(y_t) - E(E(y_t | F_{t-1}))$

$= E(y_t) - E(y_t) = 0$

using law of Iterated Expectations

$Cov(v_t\, y_j) = E(v_t\, y_j) = E(v_t\, y_j | F_{t-1}) \underset{\text{wrt } j}{=} E(y_j\, E(v_t | F_{t-1})) = 0$

$\underbrace{E(v_t | F_{t-1})}_{\text{independency}} = \underbrace{E(v_t)}_{\text{part}} = 0$

Problem 2

$$x_t = w_t w_{t-1}$$

a) $E(x_t) = E(w_t w_{t-1}) \underset{\text{independency}}{=} E(w_t) E(w_{t-1}) \underset{\text{mean of normal white noise is 0}}{=} 0$

$\gamma_{s,t} = Cov(x_s, x_t) = E[(x_s - \mu_s)(x_t - \mu_t)]$

$= E[x_s \cdot x_t] = E(w_t\, w_{t-1}\, w_s\, w_{s-1}) =$

mean is 0

$= E(w_t) E(w_{t-1}) E(w_s) E(w_{s-1}) = 0$ for $s \neq t$

independency

if $s = t \Rightarrow \gamma_{t,t} = Cov(x_t, x_t) = E(x_t x_t) = E(w_t^2 w_{t-1}^2)$

$= E(w_t^2)\, E(w_{t-1}^2) = 1$

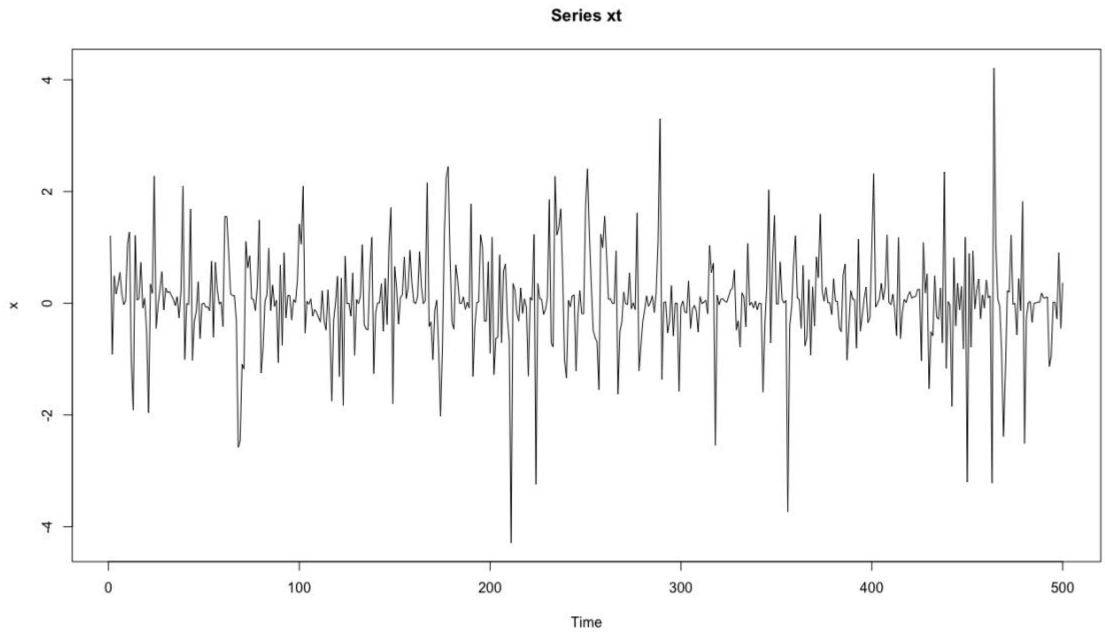
equal variance second moment

$\rho_{s,t} = \dfrac{\gamma_{s,t}}{\sqrt{\gamma_{s,s}\gamma_{t,t}}} = \dfrac{0}{\sqrt{1}} = 0$ $s \neq t$

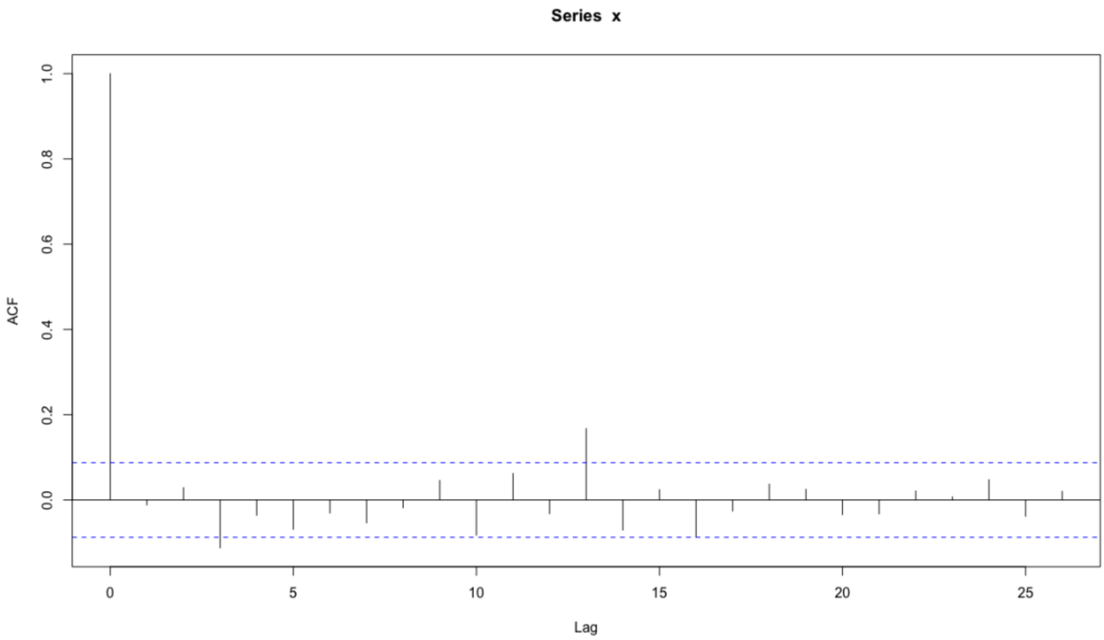
$\gamma_{t,t} = 1$ $s = t$

#Problem 2

b)



If we look at the times series, it seems to seasonal variation in number per time unit, there are peaks from time to showing this.



From the ACF test, we can see that the autocorrelation at lag 1 is just touching the significance bounds.

c)
Box.test(x,lag=1,type='Ljung')
m=1
data: x
X-squared = 0.067359, df = 1, p-value = 0.7952

Box.test(x,lag=2,type='Ljung')
m=2
data: x
X-squared = 0.48274, df = 2, p-value = 0.7856

Box.test(x,lag=3,type='Ljung')
m=3
data: x
X-squared = 6.8458, df = 3, p-value = 0.07698

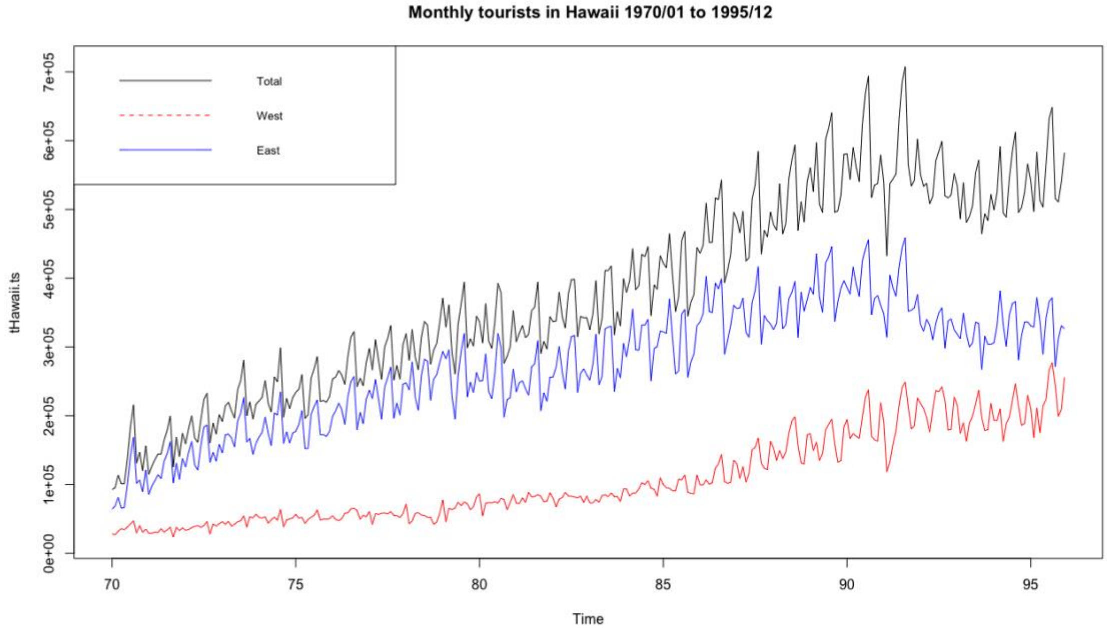
Box.test(x,lag=4,type='Ljung')
m=4
data: x
X-squared = 7.4957, df = 4, p-value = 0.1119

Box.test(x,lag=5,type='Ljung')
m= 5
data: x
X-squared = 9.8958, df = 5, p-value = 0.07824

Box.test(x,lag=6,type='Ljung')
m=6
data: x
X-squared = 10.373, df = 6, p-value = 0.1098

Problem 3

a)

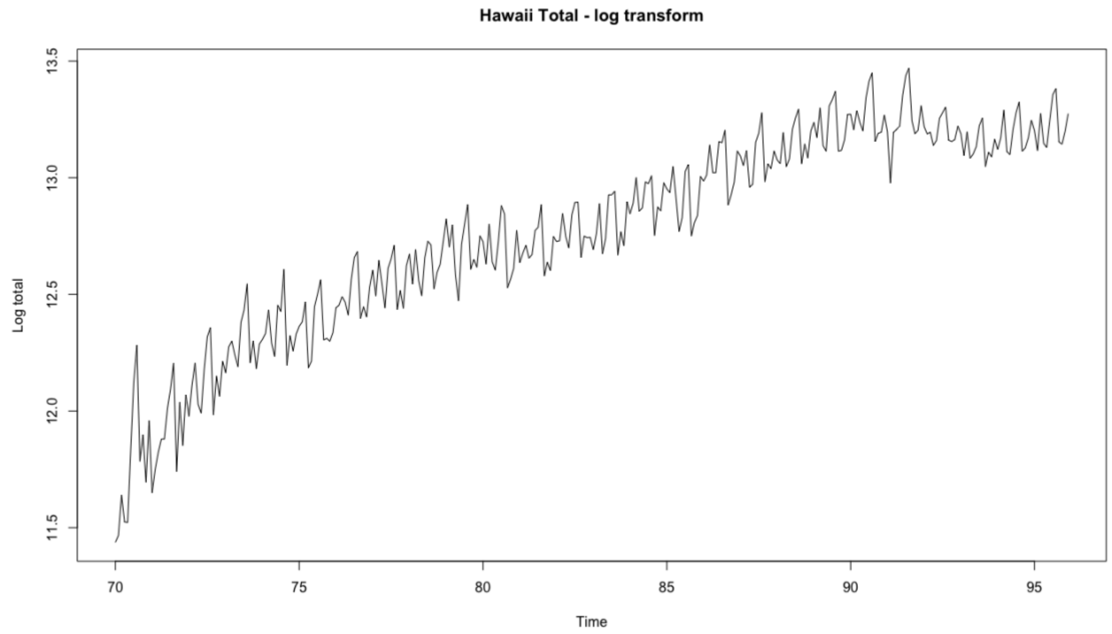


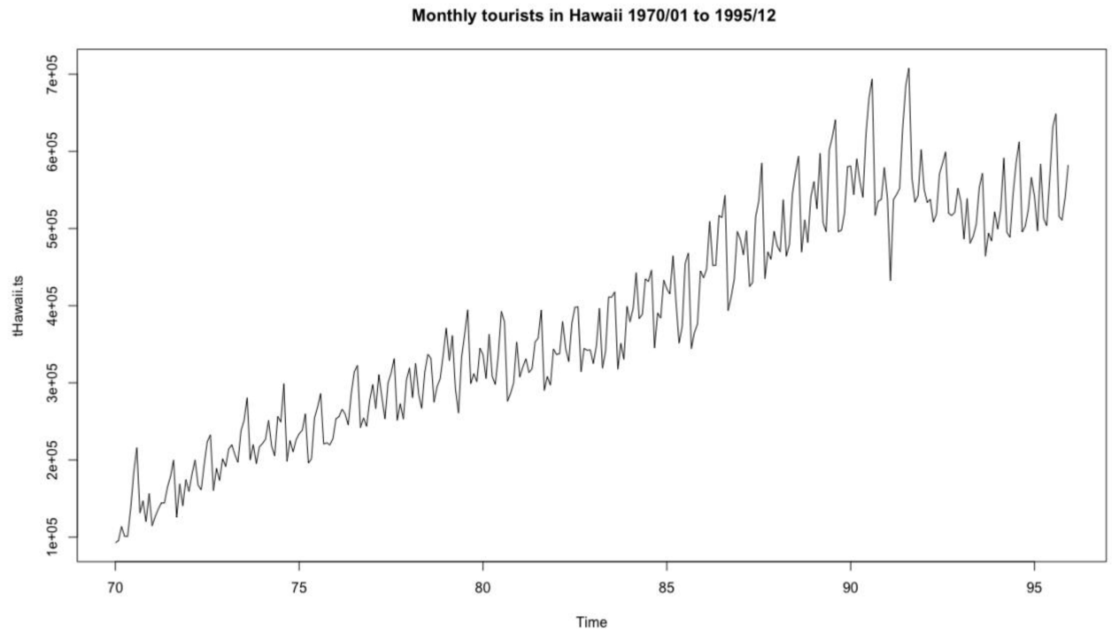
The Total and East the plots seems to follow an upward linear trend until the year 92 aproximately, where a shift down happens, a more a drastical one for the East plot. Then the Total returns to the upward trend,but the East remains in a horizontal trend. In the West plot, we can see that it keeps an upward trend the whole time, what seems to be a quadratic trend.

Also the three present a seasonal pattern, higher on the summer months and lower on the winter ones.

As supposed, the total plot is higher than both West and East, but as times goes, especially since year 88 aproximately, the West plot begins to grow more while the East begans to go down.

b)

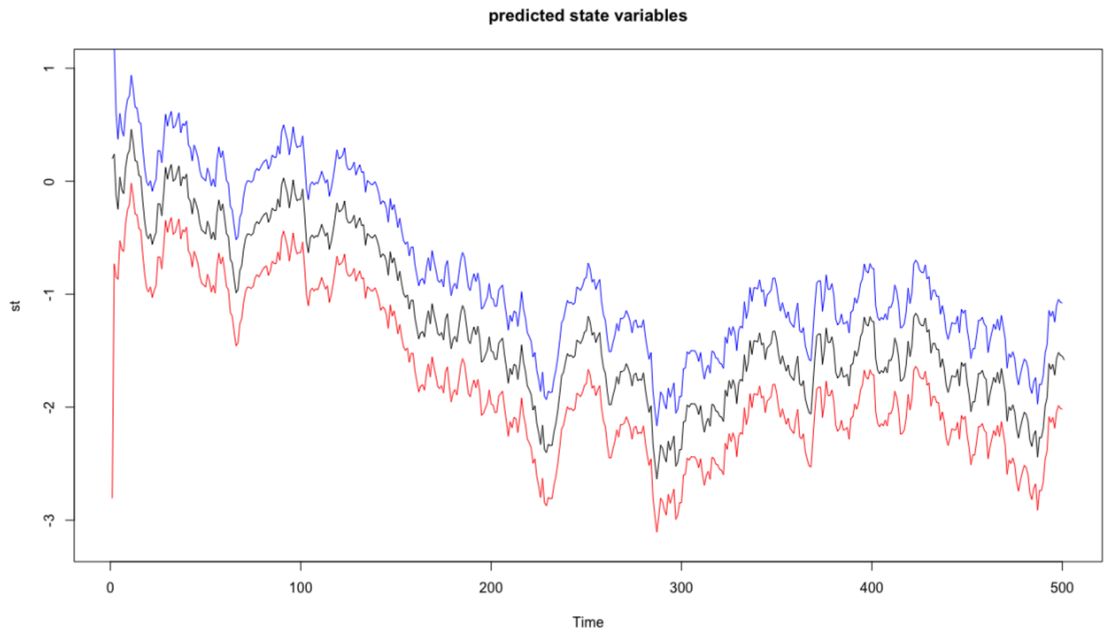




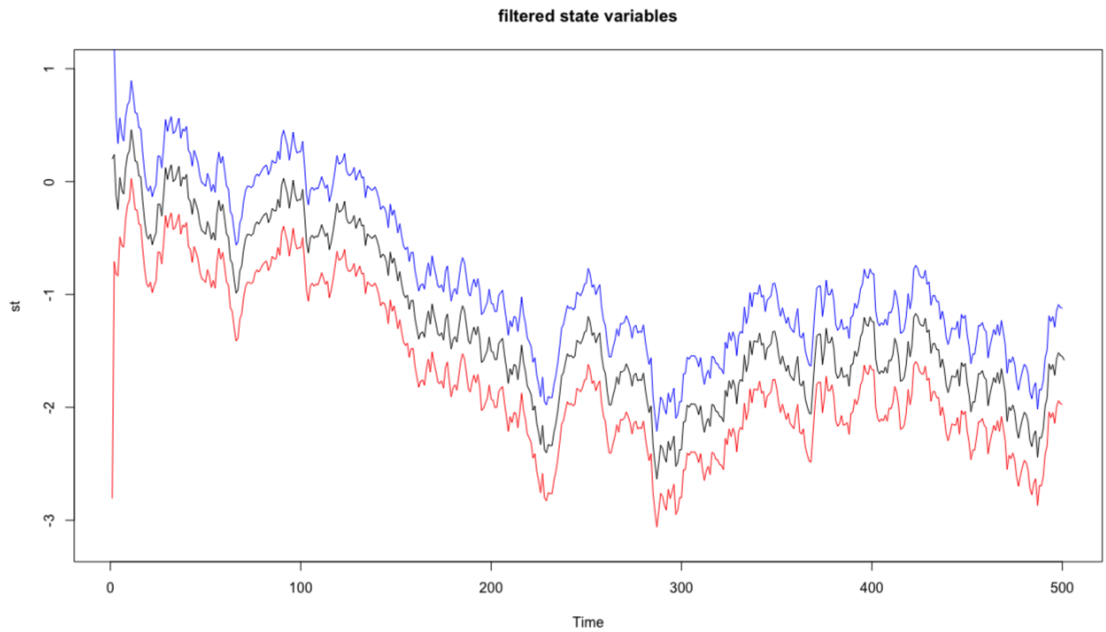
As we can see when comparing both graphs, the trend from the log transformation is practically the same as the original time series, so nothing really changes when performing the log transformation.

Problem 4

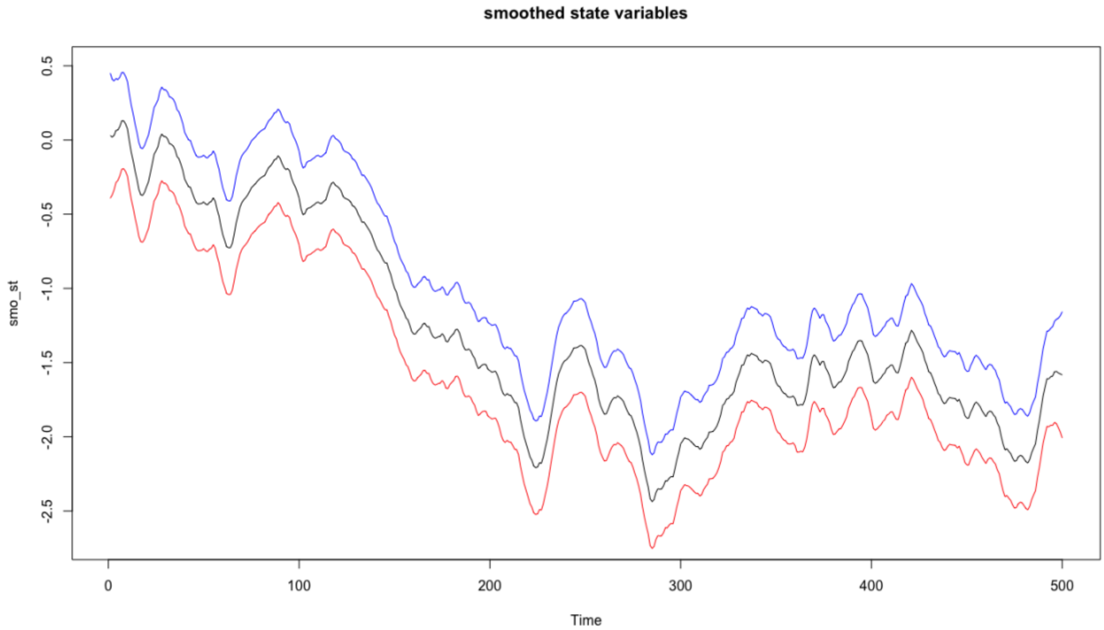
b)


predicted state variables
st
1
0
-1
-2
-3
0
100
200
300
400
500
Time

c)


filtered state variables
st
1
0
-1
-2
-3
0
100
200
300
400
500
Time

d)


smoothed state variables
smo_st
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0
100
200
300
400
500
Time

Appendix

R Code

```
#Problem 2
#b)
wNoise = rnorm(500,0,1)                # 500 N(0,1) variates
x <- array(dim=500)
x[1] <- wNoise[1]
for(i in 2:500)
  x[i] = wNoise[i]*wNoise[i-1]

plot.ts(x, main="Series xt")
acf(x)

#If we look at the times series, it seems to seasonal variation in number per time unit,
there are peaks from time to showing this.

#From the ACF test, we can see that the autocorrelation at lag 1 is just touching the
significance bounds

#c)
Box.test(x,lag=1,type='Ljung')
Box.test(x,lag=2,type='Ljung')
Box.test(x,lag=3,type='Ljung')
Box.test(x,lag=4,type='Ljung')
Box.test(x,lag=5,type='Ljung')
Box.test(x,lag=6,type='Ljung')


#Problem 3

hawaii <- read.table('hawaii-new.dat', col.names = c('year_month', 'total', 'west',
'east'))
hawaii[1,]
tHawaii=hawaii[,2] ## total
wHawaii=hawaii[,3] ## west
eHawaii=hawaii[,4] ## east

tHawaii.ts=ts(tHawaii,frequency=12,start=c(70,1))
wHawaii.ts=ts(wHawaii,frequency=12,start=c(70,1))
eHawaii.ts=ts(eHawaii,frequency=12,start=c(70,1))

#a)
plot.ts(tHawaii.ts, ylim=c(20000,710000), main=" Monthly tourists in Hawaii 1970/01
to 1995/12")
lines(wHawaii.ts, col = 'red')
```

```
lines(eHawaii.ts, col = 'blue')
legend("topleft" , legend=c("Total", "West", "East"), col=c("black", "red", "blue"),
lty=1:2, cex=0.8)

# The Total and East the plots seems to follow an upward linear trend until the year 92
aproximately,
# where a shift down happens, a more a drastical one for the East plot. Then the Total
returns to the upward trend,
# but the East remains in a horizontal trend. In the West plot, we can see that it keeps
an upward trend the
# whole time, what seems to be a quadratic trend.
# Also the three present a seasonal pattern, higher on the summer months and lower
on the winter ones.
# As supposed, the total plot is higher than both West and East, but as times goes,
especially since year
# 88 aproximately, the West plot begins to grow more while the East begans to go
down.

#b)
log.tHawaii=log(tHawaii.ts) ## log transform
plot.ts(log.tHawaii,ylab="Log total",main="Hawaii Total - log transform")
plot.ts(tHawaii.ts, main=" Monthly tourists in Hawaii 1970/01 to 1995/12")

#The trend from the log transformation is practically the same as the original time
series, so nothing really
# changes when performing the log transformation.

#c)
# tot <- c(1:312)
# tHawaii.lm1=lm(log.tHawaii~tot)
# tHawaii.lm2=lm(log.tHawaii~tot+I(tot^2))
# tHawaii.lm3=lm(log.tHawaii~tot+I(tot^2)+I(tot^3))
# tHawaii.lm4=lm(log.tHawaii~tot+I(tot^2)+I(tot^3)+I(tot^4))
# tHawaii.lm5=lm(log.tHawaii~tot+I(tot^2)+I(tot^3)+I(tot^4)+I(tot^5))
#
# summary(tHawaii.lm1)
# summary(tHawaii.lm2)
# summary(tHawaii.lm3)
# summary(tHawaii.lm4)
# summary(tHawaii.lm5)



#Problem 4
yt <- scan("lt.txt")
#b
#initialize variables from problem description
```

```
sigma <- array(500)
sigma[1] <-  2.26
st <- array(500)
st[1] <- 0.2
kt <- array(500)
vt <- array(500)
Vt <- array(500)

for(i in 1:500){
  vt[i]=yt[i]-st[i]
  Vt[i]=sigma[i]+0.25
  kt[i]=sigma[i]/Vt[i]
  st[i+1]=st[i]+kt[i]*vt[i]
  sigma[i+1]=(1-kt[i])*sigma[i]+0.01
}

upper <- array(500)
lower <- array(500)
for(i in 1:500){
  upper[i]=st[i]+2*sqrt(sigma[i])
  lower[i]=st[i]-2*sqrt(sigma[i])
}

plot.ts(st,ylim=c(-3.2,1.0),main="predicted state variables")
lines(upper,col='blue')
lines(lower,col='red')
#c
flt_sigma <- array(500)
flt_sigma[1] <- 2.26

for(i in 1:500){
  flt_sigma[i+1]=(1-kt[i])*sigma[i]
}

flt_upper <- array(500)
flt_lower <- array(500)
for(i in 1:500){
  flt_upper[i]=st[i]+2*sqrt(flt_sigma[i])
  flt_lower[i]=st[i]-2*sqrt(flt_sigma[i])
}
plot.ts(st,ylim=c(-3.2,1.0),main="filtered state variables")
lines(flt_upper,col='blue')
lines(flt_lower,col='red')

#d
smo_lt <- array(500)
smo_qt <- array(501)
```

```
smo_mt <- array(501)
smo_qt[501] <- 0
smo_mt[501] <- 0
smo_st <- array(500)
smo_sigma <- array(500)
for(i in 1:500){
  smo_lt[i]=1-kt[i]
}

for(i in 500:1){
  smo_qt[i]=vt[i]/Vt[i]+smo_lt[i]*smo_qt[i+1]
  smo_mt[i]=1/Vt[i]+smo_lt[i]*smo_lt[i]*smo_mt[i+1]
  smo_st[i]=st[i]+sigma[i]*smo_qt[i]
  smo_sigma[i]=sigma[i]-sigma[i]*sigma[i]*smo_mt[i]
}

smo_upper <- array(500)
smo_lower <- array(500)

for(i in 1:500){
  smo_upper[i]=smo_st[i]+2*sqrt(smo_sigma[i])
  smo_lower[i]=smo_st[i]-2*sqrt(smo_sigma[i])
}

plot.ts(smo_st,ylim=c(-2.7,0.5),main="smoothed state variables")
lines(smo_upper,type='l',col='blue')
lines(smo_lower,type='l',col='red')
```