Finite Difference Time Domain

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December 20, 2021

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Introduction

To learn and do three-dimensional electromagnetic simulation using the finite-difference time-domain (FDTD) method.

Type of material:

1. Free space
2. Complex dielectric material
3. Frequency-dependent material

Some choice that have been made:

1. The use of Normalised Units Maxwell's equations have been normalized by substituting

$$\widetilde{E} = \sqrt{\frac{\varepsilon_0}{\mu_0}} E$$

this is a system similar to Gaussian units.
The reason for using it here is the simplicity in the formulation. The E and H fields have the same order of magnitude. This has an advantage in formulating the PML.

2. Maxwell's Equations with the Flux Density Time-domain Maxwell's equations from which the FDTD formulation is developed

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0} \nabla \times H$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0} \nabla \times E$$

straight forward formulation

$$\frac{\partial D}{\partial t} = \nabla \times H$$

$$D = \varepsilon_0 \varepsilon_r^* E$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0} \nabla \times E$$

formulation using the flux density in this formulation, it is assumed that the materials being simulated are non-magnetic, that is, $H = (1/\mu_0)B$

Pulse propagating in free space in one dimension

Time-dependent Maxwell's curl equations for free space:

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0} \nabla \times H$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0} \nabla \times E$$

simple one-dimensional case:

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0} \frac{\partial H_y}{\partial z}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0} \frac{\partial E_x}{\partial z}$$

- The formulation of Equations assume that the E and H fields are interleaved in both space and time.
- The new value of E_x is calculated from the previous value of E_x and the most resent values of H_y. This is the fundamental paradigm of the FDTD method.

governing equations,

$$\widetilde{E}_x^{n+1/2}(k) = \widetilde{E}_x^{n-1/2}(k) - \frac{\Delta t}{\sqrt{\varepsilon_0 \mu_0} \cdot \Delta x}\left[H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)\right]$$

$$H_y^{n+1}\left(k+\frac{1}{2}\right) = H_y^n\left(k+\frac{1}{2}\right) - \frac{\Delta t}{\sqrt{\varepsilon_0 \mu_0} \cdot \Delta x}\left[\widetilde{E}_x^{n+1/2}(k+1) - \widetilde{E}_x^{n+1/2}(k)\right]$$

Once the cell size Δx is choosen, then the time step Δt is determined by

$$\Delta t = \frac{\Delta x}{2 \cdot c_0},$$

where c_0 is the speed of light in free space. Therefore, remembering that $\varepsilon_0 \mu_0 = 1/(c_0)^2$,

$$\frac{\Delta t}{\sqrt{\varepsilon_0 \mu_0} \cdot \Delta x} = \frac{\Delta x}{2 \cdot c_0} \cdot \frac{1}{\sqrt{\varepsilon_0 \mu_0} \cdot \Delta x} = \frac{1}{2}$$

$$\widetilde{E}_x^{n+1/2}(k) = \widetilde{E}_x^{n-1/2}(k) - \frac{1}{2}\left[H_y^n\left(k + \frac{1}{2}\right) - H_y^n\left(k - \frac{1}{2}\right)\right]$$

$$H_y^{n+1}\left(k + \frac{1}{2}\right) = H_y^n\left(k + \frac{1}{2}\right) - \frac{1}{2}\left[\widetilde{E}_x^{n+1/2}(k+1) - \widetilde{E}_x^{n+1/2}(k)\right]$$

Simulation in free space