1. Compute $\displaystyle\int_0^1 e^{x^2}\, dx$ using the trapezoidal rule and trapezoidal rule with end corrections using the first and third derivatives. Perform this by subdividing $[0,1]$ into $N \in \{2, 5, 10, 20, 50, 100, 200, 500, 1000\}$ panels and plot the decay of the absolute error using the **three methods**. The value of the integral accurate upto 16 digits is 1.4626517459071816.

2. Use the Euler-Macluarin to obtain
$$\log(n!) = \log\left(C\left(\frac{n}{e}\right)^n \sqrt{n}\right) + \mathcal{O}(1/n)$$
where $C$ is some constant.

3. We will now determine $C$ in the above question as follows.
   - Use integration by parts to obtain an expression for $I_k = \displaystyle\int_0^{\pi/2} sin^k(x)\, dx$ (It might be easier to look at the even and odd cases separately)
   - Prove that $I_k$ is a monotone decreasing sequence.
   - Show that
   $$\lim_{m\to\infty} \frac{I_{2m-1}}{I_{2m+1}} = 1$$
   - Conclude that
   $$\lim_{m\to\infty} \frac{I_{2m}}{I_{2m+1}} = 1$$
   - Hence, infer that the central binomial coefficient is asymptotically given by
   $$\binom{2m}{m} \sim \frac{4^m}{\sqrt{m\pi}}$$
   where $f(m) \sim g(m) \implies \lim_{m\to\infty} \dfrac{f(m)}{g(m)} = 1$
   - Conclude that $C$ in the above question is $\sqrt{2\pi}$
   - Hence, obtain the Stirling formula:
   $$n! \sim \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$
   - Obtain the relative error in $n!$ using the Stirling formula for $n \in \{20, 50\}$
   - Obtain a better estimate for $n!$, which is accurate upto $\mathcal{O}(1/n^3)$