DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5720 Num. Methods | **Session 09** | **10 April 2018**
Time: 2:00 pm - 5:00 pm | **[Total: 10 points]**

Goal of this session:

1. ODE and PDE solvers.
2. Please upload your plots (.pdf file) on moodle and submit this lab sheet by **Monday 16 April 2018, 5:00 pm**.

1 Problems:

1. By setting up the following ODE solvers:
   (a) Euler
   (b) Runge-Kutta second order
   (c) Runge-Kutta fourth order

   solve the following differential equation that governs the decay of radioactive particles.

   $$\frac{dN(t)}{dt} = -N(t) \tag{1}$$

   using $N(0) = 2.0$ for time t in the range $[0, 1]$. For each solver write out the relative error as a function of N, the number of points in the solver and extract the power law dependence of the relative error on N. The analytic solution is:

   $$N(t) = N(0)\exp(-t) \tag{2}$$

   Please write your results below:

2. In this exercise, you are going to solve the Diffusion Equation seen in class in 1-D. This is given by:

   $$\frac{\partial^2 T(x,t)}{\partial x^2} = D\frac{\partial T(x,t)}{\partial t} \tag{3}$$

   where $T(x,t)$ is the temperature gradient and $D = C\rho/\kappa$. C is the specific heat, ρ is the density of the material and κ is the thermal conductivity of the material. We can re-scale this equation by defining $x = \alpha\widehat{x}$ such that $\alpha^2 D = 1$. Then Eq. 3 becomes:

   $$\frac{\partial^2 T(\widehat{x},\widehat{t})}{\partial \widehat{x}^2} = \frac{\partial T(\widehat{x},\widehat{t})}{\partial \widehat{t}} \tag{4}$$

Consider a 1D rod of length $L = 1$ and let its ends be dipped in a sink which is at $T = 0$. Let us assume that the rest of the rod except for the end points are at a constant temperature $T_0 = 100$. Note these are just numbers since the equation is now dimensionless. Your task is to find the solution $T(x,t)$, where x and t are still dimensionless but the hat has been dropped for notational simplicity, following the steps outlined below:

(a) Using the Implicit scheme solve for $T(x,t)$. Use the appropriate solver from GSL.

(b) Use the 3d plotter file and insert the data file that you have and make a 3D plot using the following command for the Implicit scheme:

```
gnuplot 3d_plot.plt
```