Linear Regression :

For the data set given below fit a Linear regression line :

x	1	2	3	4
y	1.5	2	2.5	3

. By assuming cost function 'Mean Squared error' and by assuming $(w, w_0) = (1, 1)$, Apply Gradient Descent Alg to compute updated weights and new costs? Assume the learning rate to be 0.1.

Step ① Regression Line finding

x	y	x^2	xy
1	1.5	1	1.5
2	2	4	4
3	2.5	9	7.5
4	3	16	12
10	9	30	25

$\Sigma x = 10, \quad \Sigma y = 9, \quad \Sigma x^2 = 30$

$\Sigma xy = 25, \quad N = 4$

$$m = \frac{N\Sigma xy - \Sigma x \Sigma y}{N\Sigma x^2 - (\Sigma x)^2} = \frac{4(25) - (10)(9)}{4(30) - 100}$$

$$= \frac{10}{20} = 0.5$$

$$c = \frac{\Sigma y - m\Sigma x}{N} = \frac{9 - 0.5(10)}{4}$$

$$= 1$$

$\therefore$ $y = mx + c \Rightarrow y = 0.5x + 1$

Step (2) Linear Regression with Gradient Descent :

$y = w * two \Rightarrow y$

Step (2) Linear Regression with Gradient Descent :

$y = wx + w_0 \Rightarrow y = 0.5x + 1$

Cost function $E = \frac{1}{N} \sum_{i=0}^{N} (y_i - \hat{y}_i)^2$

$= \frac{1}{N} \sum_{i=0}^{N} [y_i - (mx_i + c)]^2$

$D_m = -\frac{2}{N} \sum_{i=0}^{N} x_i (y_i - \hat{y}_i)$, $D_c = -\frac{2}{N} \sum_{i=0}^{N} (y_i - \hat{y}_i)$

$\boxed{w = w - \alpha D_m}$, $\boxed{w_0 = w_0 - \alpha D_c}$

$\alpha = 0.1$ (given)

$N = 4$

Iteration (1) Since $w, w_0 = 1$, $\boxed{\hat{y} = x + 1}$ $\because \hat{y} = wx + w_0$

$(w, w_0) = (1, 1)$

x	y	$\hat{y}$	$y - \hat{y}$	$x_i(y_i - \hat{y})$
1	1.5	2	−0.5	−0.5
2	2	3	−1	−2
3	2.5	4	−1.5	−4.5
4	3	5	−2	−8
			−5	−15

$$D_m = -\frac{2}{4}(-15) = 7.5$$

$$D_c = -\frac{2}{4}(-5) = 2.5$$

$$w = w - \alpha D_m = 1 - 0.1(7.5)$$

$$= 0.25$$

$$w_0 = c - \alpha D_c = 1 - 0.1(2.5)$$

$$= 0.75$$

$\therefore y = wx + w_0 \Longrightarrow y = 0.25x + 0.75$ //

Iteration (2) $\hat{y} = 0.25x + 0.75$

$w = 0.25$

$w_0 = .75$

$N = 4$

x	y	$\hat{y}$	$y - \hat{y}$	$x_i(y_i - \hat{y})$
1	1.5	1	0.5	0.5
2	2	1.25	0.75	1.5
3	2.5	1.5	1	3
4	3	1.75	1.25	7

$D_m = -\frac{2}{4}[12] = -6$ $\alpha = 0.1$

$D_c = -\frac{2}{4}[4] = -2$

$w = w - \alpha D_m = 0.25 - 0.1(-6)$

$= 0.85$

$w_0 = w_0 - \alpha D_c = 0.75$

$- 0.1(-2)$

$= 0.95$

$y = 0.85x + 0.95$