1.4 *Bistatic radar equation*

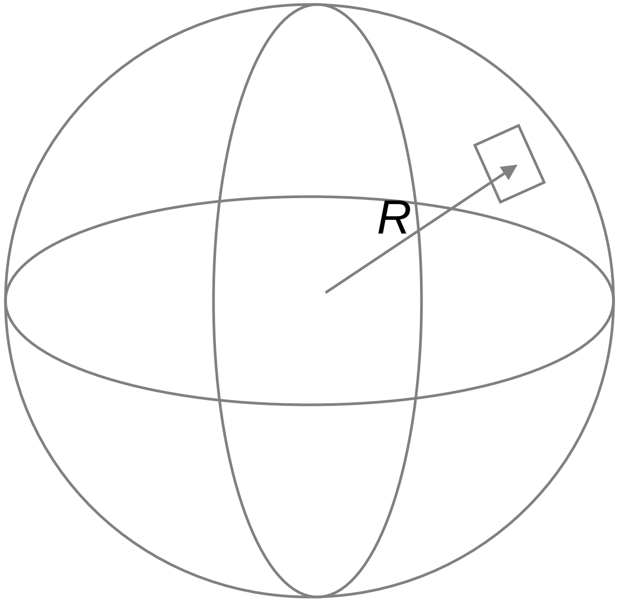
1. The monostatic radar equation (revision)
2. The bistatic radar equation
3. Ovals of Cassini

Monostatic radar equation

Consider an isotropic antenna, fed with a transmit power P_t .

The power density, in Watts/m^2 , at range R is:

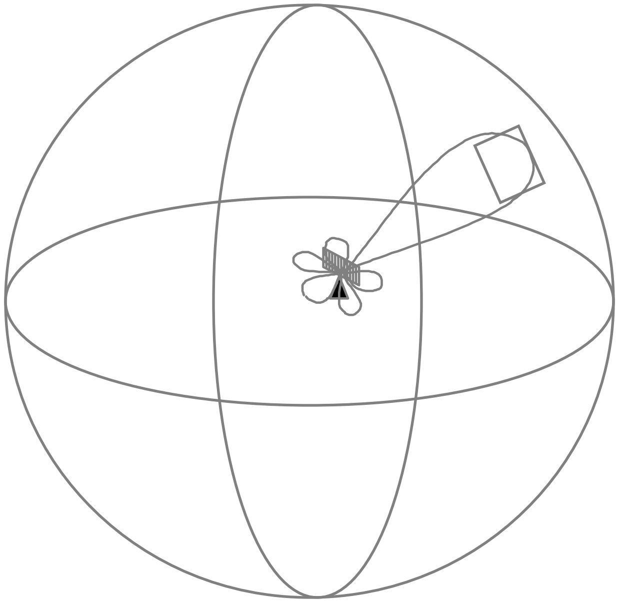
$$P_t \cdot \frac{1}{4\pi R^2}$$



Monostatic radar equation

But any practical antenna will be directive, so the power density in the direction of maximum gain is :

$$\frac{P_t G}{4\pi R^2} \quad \mathrm{W/m^2}$$



where *G* is the antenna gain.

Monostatic radar equation

Suppose this is incident on a point target. The target's scattering properties are characterised by its Radar Cross Section (RCS), which is given the symbol σ. This has the dimensions of area (m^2).

(Note that a target's RCS is not necessarily the same as its physical area).

The power intercepted by the target is therefore :

$$\frac{P_t G}{4\pi R^2} . \sigma \qquad \text{Watts}$$

Monostatic radar equation

This is scattered by the target, so the echo power density back at the receiver is :

$$\frac{P_t G}{4\pi R^2} . \sigma . \frac{1}{4\pi R^2} \quad \text{Watts} / \text{m}^2$$

Monostatic radar equation

This is intercepted by the radar antenna, whose effective area is A_e , so the received power P_r is given by :

$$P_r = \frac{P_t G}{4\pi R^2} . \sigma . \frac{1}{4\pi R^2} . A_e \qquad \text{Watts}$$

But A_e is related to the antenna gain by $G = \frac{4\pi A_e}{\lambda^2}$

so

$$P_r = \frac{P_t G}{4\pi R^2} . \sigma . \frac{1}{4\pi R^2} . \frac{G\lambda^2}{4\pi}$$

$$= \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

Monostatic radar equation

The receiver noise power P_n is given by :

$$P_n = kT_0BF$$

where : k = Boltzmann's constant = 1.38×10^{-23} W/K/Hz
T_0 = noise reference temperature = 290 K
B = receiver bandwidth (Hz)
F = receiver noise figure

(For a simple pulsed radar, B would be matched to the pulse length τ such that $B \approx 1/\tau$).

So the signal-to-noise ratio is given by :

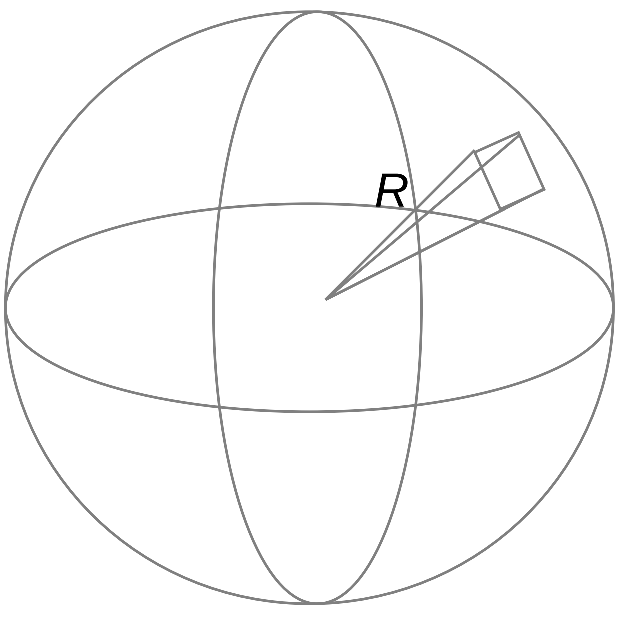
$$SNR = \frac{P_r}{P_n} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 B F}$$

(note that $kT_0 = 4 \times 10^{-21}$ W/H = –204 dBW/Hz = – 174 dBm/Hz)

Antenna gain and effective area

Consider a rectangular antenna aperture of dimensions a x b. The beamwidths in the respective planes are λ/a and λ/b (radians) which subtend a rectangular area $R\lambda/a \times R\lambda/b$ on the surface of a sphere of radius R.

The gain of the antenna is the ratio of the power density in the main beam to that which would be obtained from an isotropic antenna, i.e. the ratio of the total surface area of the sphere to the area $R\lambda/a \times R\lambda/b$:



$$G = \frac{4\pi R^2}{R^2\lambda^2 / ab}$$

$$= \frac{4\pi A_e}{\lambda^2}$$

where $A_e = a \times b$ is the effective area of the antenna

Receiver noise - revision

The total noise power is the sum of the noise from the environment plus noise added by the receiver :

$$P_n \; = \; kT_0B \; + \; kT_sB$$

where : k = Boltzmann's constant = 1.38×10^{-23} W/K/Hz
T_0 = noise reference temperature = 290 K
B = receiver bandwidth (Hz)
T_s = receiver noise temperature

Noise Figure F is defined as :

$$F = \frac{SNR|_{input}}{SNR|_{output}} = \frac{P_s/(kT_0B)}{P_s/(kT_0B + kT_sB)} = 1 + \frac{T_s}{T_0}$$

So

$$P_n = kT_0B\left(1 + \frac{T_s}{T_0}\right) = kT_0BF$$

Monostatic radar equation

We should also include the effect of losses :

propagation losses (two-way)
'plumbing' losses
beamshape losses
:
:

These can be characterised in terms of a factor L (≥ 1), so :

$$\boxed{\frac{P_r}{P_n} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L \, k T_0 B F}}$$

Monostatic radar equation

This can also be rearranged to show the maximum detection range for a given target. If the minimum signal-to-noise ratio required for detection is S/N_{min}, then :

$$S/N_{\min} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R_{\max}^4 L\, kT_0 BF}$$

or :

$$R_{\max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 L\, kT_0 BF\, S/N_{\min}} \right]^{1/4}$$

Monostatic radar equation – in dB

All of the quantities in the radar equation can be expressed in dB, and added or subtracted rather than multiplied or divided.

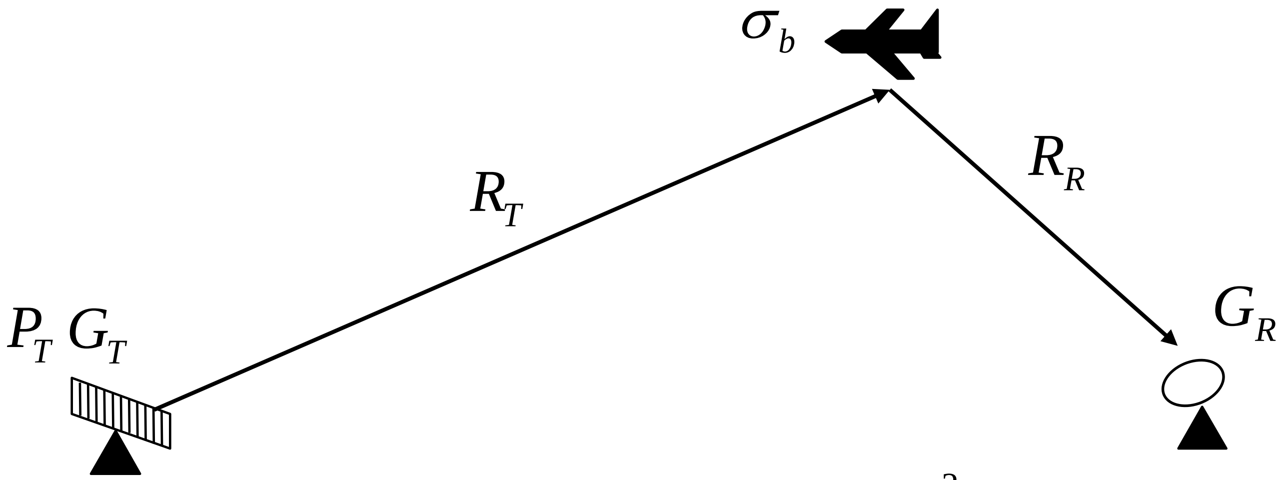
Thus :

$$SNR = P_t + 2G + 20\log_{10}\lambda + \sigma - L - 30\log_{10}(4\pi)$$

$$- 40\log_{10} R - 10\log_{10}(kT_0B) - F$$

Bistatic radar equation

This is derived in the same way as the monostatic radar equation :



$$\frac{P_R}{P_N} = \frac{P_T G_T}{4\pi R_T^2} . \sigma_b . \frac{1}{4\pi R_R^2} . \frac{G_R \lambda^2}{4\pi} . \frac{1}{L} . \frac{1}{k T_0 B F}$$

$$= \frac{P_T G_T G_R \lambda^2 \sigma_b}{(4\pi)^3 R_T^2 R_R^2 \ L \ k T_0 B F}$$

The dynamic range of signals to be handled is reduced, because of the defined minimum range.

Bistatic radar equation

To be complete, we should also include pattern propagation factors, to take account of multipath, diffraction, etc:

$$\frac{P_R}{P_N} = \frac{P_T G_T G_R F_T^2 F_R^2 \lambda^2 \sigma_b}{(4\pi)^3 R_T^2 R_R^2 L k T_0 B F}$$

where F_T is the pattern propagation factor for the transmitter-to-target path

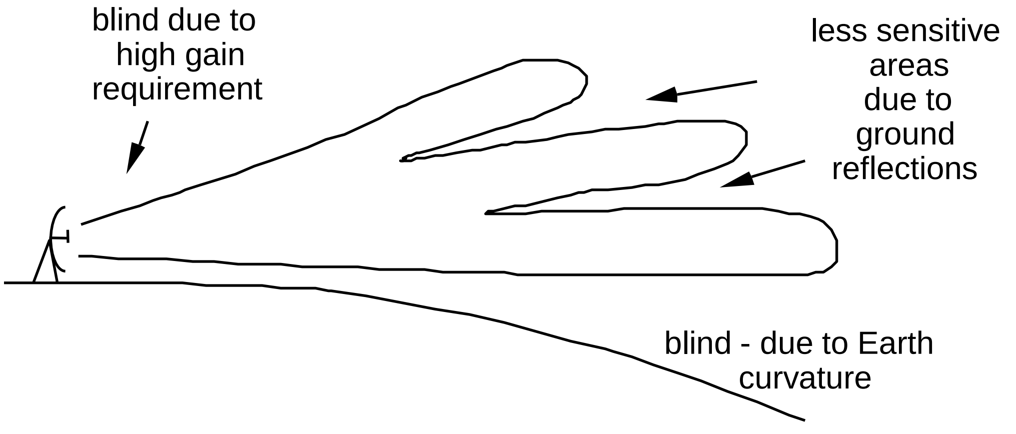
and F_R is the pattern propagation factor for the target-to-receiver path

Pattern propagation factor

Loss is not the only propagation effect that the radar signal may experience. In practice there will be several other factors :

- Earth curvature
- variation of atmospheric refractive index with height
- ground reflections
- anomalous propagation

Their combined effect is known as the ‘pattern propagation factor’.



Bistatic radar equation

$$\frac{P_R}{P_N} = \frac{P_T G_T G_R \lambda^2 \sigma_b}{(4\pi)^3 R_T^2 R_R^2 L k T_0 B F}$$

$\dfrac{1}{R_T R_R}$, and hence the signal-to-noise ratio, has a minimum value for $R_\mathrm{T} = R_\mathrm{R}$

(which can be verified by differentiation in the usual way).

The signal-to-noise ratio is highest for targets close to the transmitter or close to the receiver.

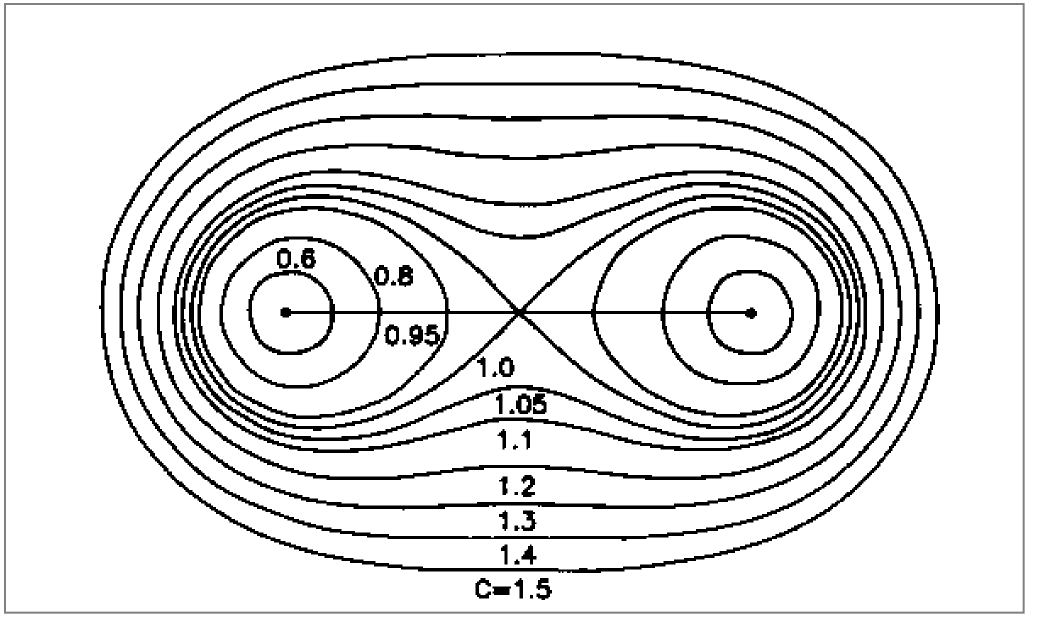
Ovals of Cassini

We can see from the bistatic radar equation

$$\frac{P_R}{P_N} = \frac{P_T G_T G_R \lambda^2 \sigma_b}{(4\pi)^3 R_T^2 R_R^2 L k T_0 B F}$$

that contours of constant detection range are defined by $R_T R_R$ = constant = c.

These are *Ovals of Cassini*



Ovals of Cassini

- Definition:

  Locus of the vertex of a (bistatic) triangle when the product of the sides adjacent to the vertex (R_T, R_R) is constant and the length of the opposite side (L) is fixed.

- Caveat:

  Only valid under free space conditions and requiring constant:

  Antenna gains

  Pattern propagation factors

  Radar cross section

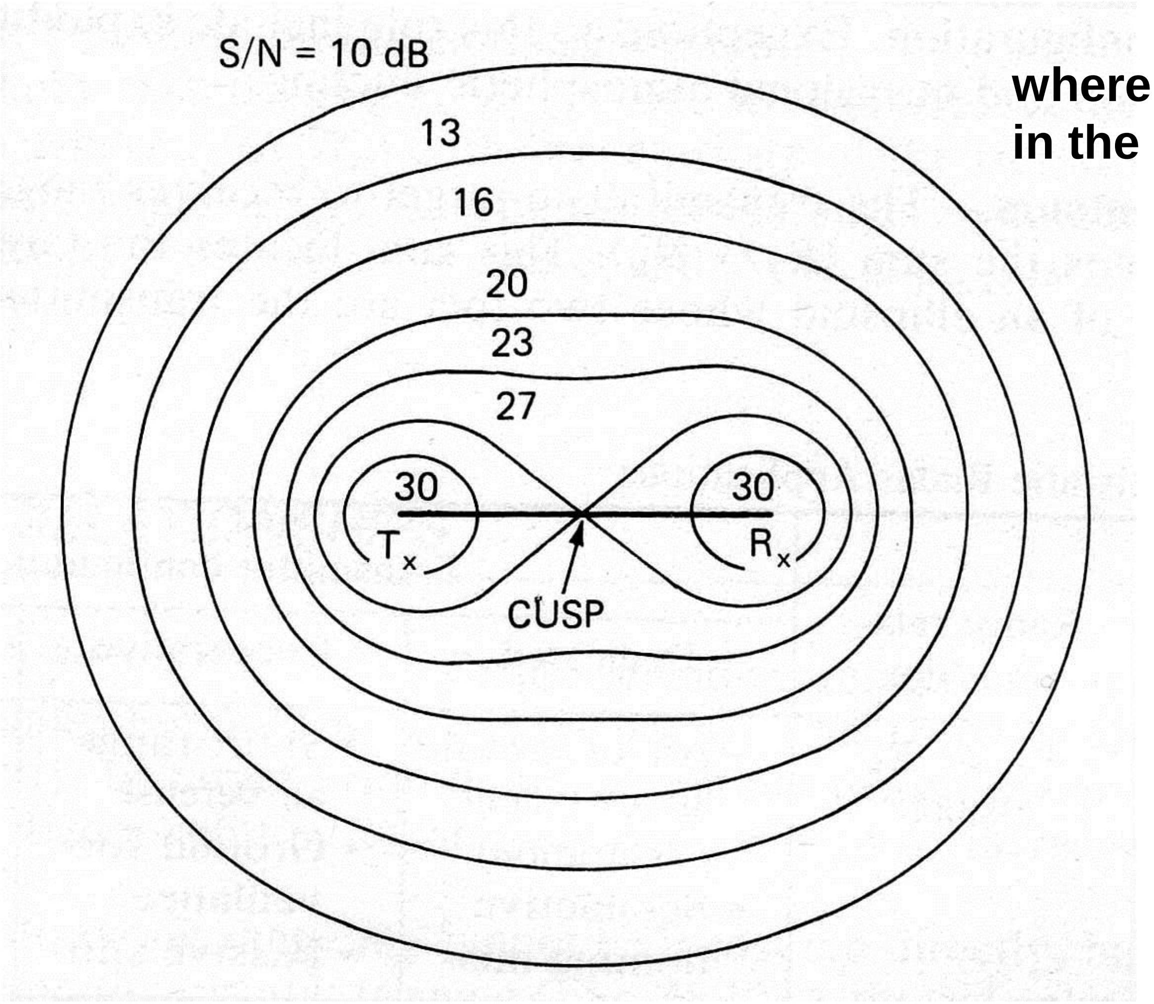
  System noise temperature

- Typically plotted as a function of S/N, for a fixed baseline L:

$$R_T R_R = K / (S/N)^{1/2}$$

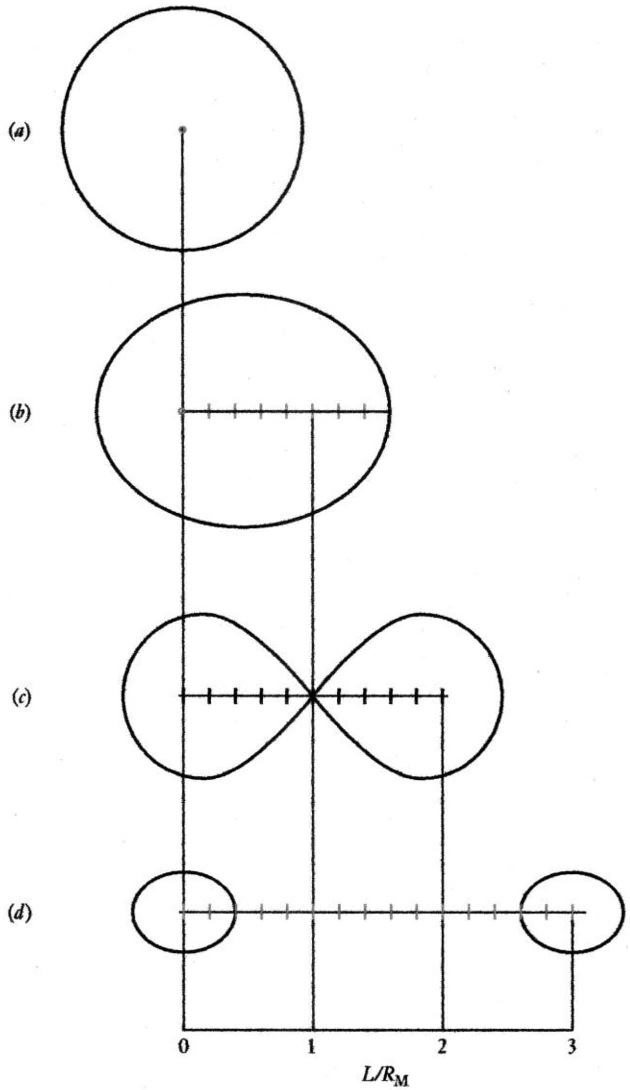
Ovals of Cassini

WHEN PLOTTED AS CONTOURS OF CONSTANT *S*/*N*



where *S*/*N* replaces *E*/*N*$_o$ in the range equation

Now for the (leminscate) twist:



OVALS PLOTTED AS A FUNCTION OF THE BASELINE L, NORMALIZED TO THE EQUIVALENT MONOSTATIC RANGE R_M: (L/R_M)

KEY:

Tx = 0

Rx = 1→ 3…

Tgt detected on oval

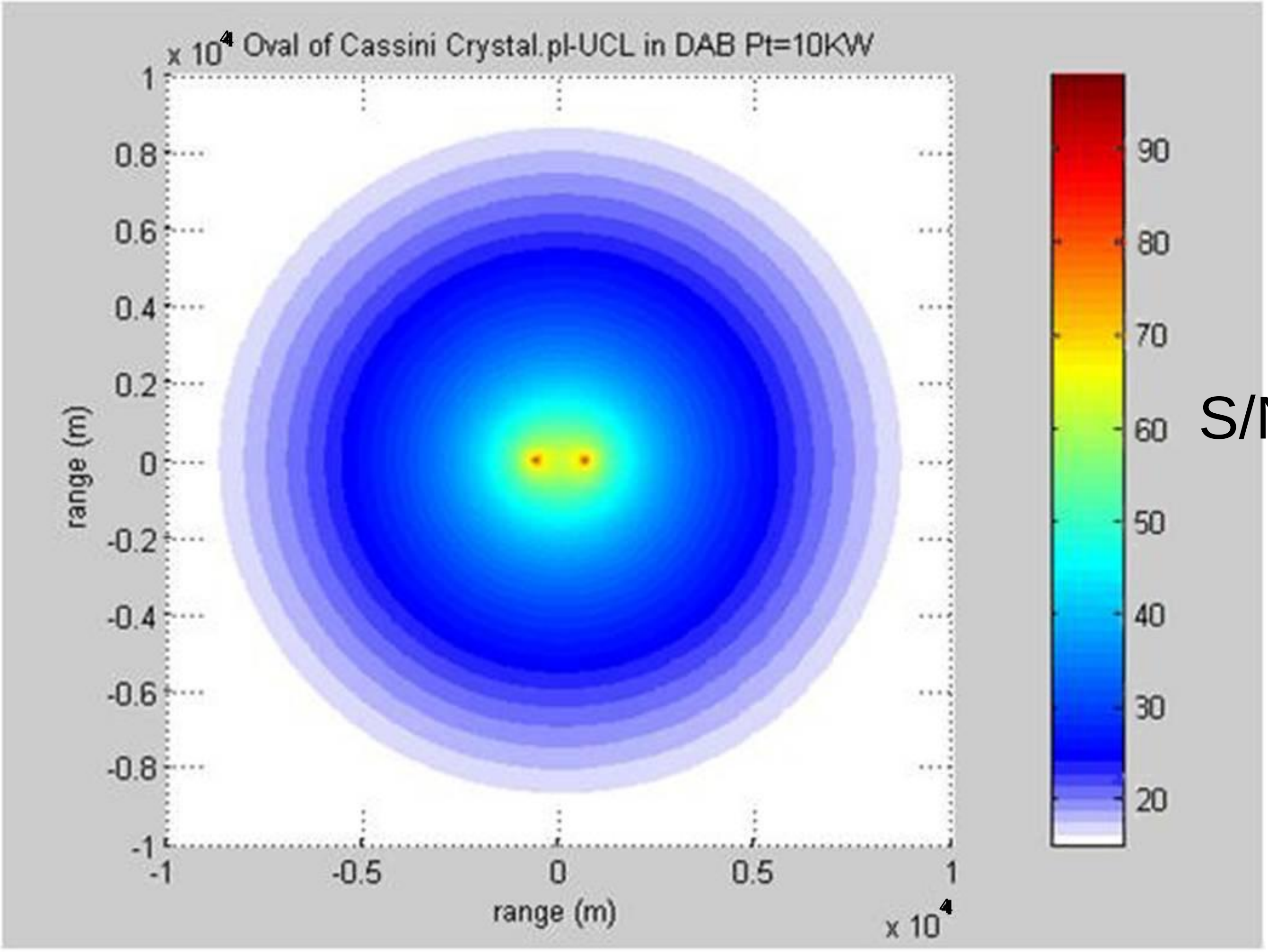
Case (a) = Monostatic (and equiv. monostatic) case

Case (d): $R_{R(av)} \sim R_M^2 / L$ = radius of circle with area equal to one oval:

EXAMPLE OF S/N_o OVAL PREDICTION for:

10 kW DAB Tx at Crystal Palace, Rx at UCL

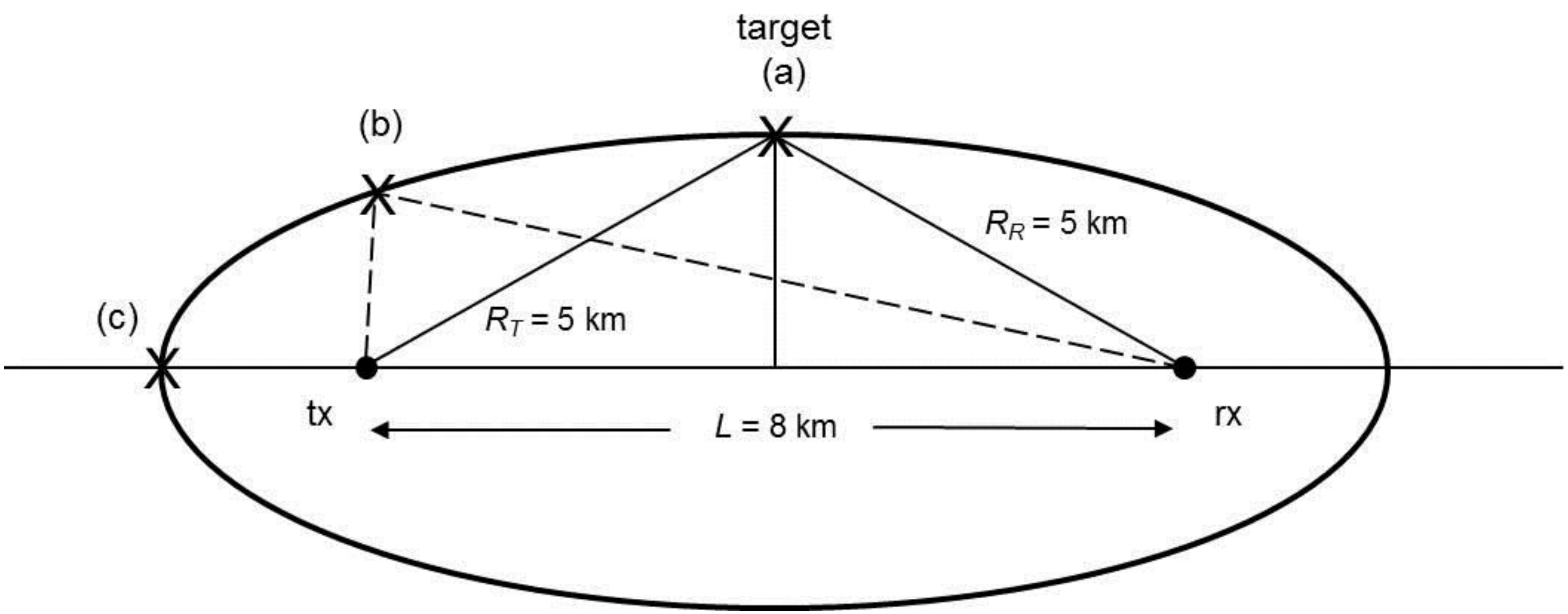
L ~ 2 km $F_n L_T L_R$ = 30 dB R_R ~ 9 km for S/N_o = 15 dB



S/N

Test your understanding

A target lies equidistant between transmitter and receiver such that $R_T + R_R = 10$ km. The baseline $L = 8$ km. Assuming that the transmit and receive antennas are omnidirectional, and that all other parameters remain unchanged, what is the echo signal power, relative to that when the target is at (a), when it is instead at (b) or at (c) – both points lying on the constant range sum ellipse ?



Summary

- The bistatic radar equation is derived in much the same way as the monostatic radar equation
- Many of the properties of bistatic radar are a function of the bistatic geometry.
- Contours of constant bistatic range are ellipses, with the transmitter and receiver as the two focal points
- Contours of constant target echo power, under the assumption of omnidirectional transmit and receive antenna patterns, are Ovals of Cassini
- The target echo power is minimum when the target is equidistant from transmitter and receiver, and greatest when the target is either close to the transmitter or close to the receiver