Filter-and-Sum Beamforming

ECE 6279: Spatial Array Processing
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Lecture 8

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Where We Are in J&D

- **Lecture material drawn from:**
  - –Sec. 4.3 (all)

Temporal Filtering

- **Filter the signal collected by each sensor in time**

$$y_m(t) = \{h_m(\cdot) *_t f(\vec{x}_m, \cdot)\}(t)$$

$$= \int_{-\infty}^{\infty} h_m(\tau) f(\vec{x}_m, t-\tau) d\tau$$

- **Then do delay-and-sum beamforming on the filtered signals, using techniques from past two lectures**

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

- **Choosing $h_m(t)=\delta(t)$ gives straight delay-and-sum beamforming**

Why Filter in Time?

- **Focus on frequency range of interest**
- **Reduce noise and interference in unwanted frequency bands**
- **In digital implementations, need antialiasing filter before A/D converters**
- **If you have a statistical model for the signal and the noise, you can design a Wiener filter (see ECE6254 class and textbook)**

Temporally Filtered Signal at Sensor

The usual: try a monochromatic plane wave!

$$f(\vec{x}_m, t) = \exp\{j(\omega^0 t - \vec{k}^0 \cdot \vec{x}_m)\}$$

$$y_m(t) = \int_{-\infty}^{\infty} h_m(\tau) f(\vec{x}_m, t - \tau) d\tau$$

$$= \int_{-\infty}^{\infty} h_m(\tau) \exp\{j[\omega^0 (t - \tau) - k^0 \cdot \vec{x}_m]\} d\tau$$

$$= \exp\{j(\omega^0 t - \vec{k}^0 \cdot \vec{x}_m)\} \times$$

$$\underbrace{\int_{-\infty}^{\infty} h_m(\tau) \exp(-j\omega^0 \tau) d\tau}_{H_m(\omega^0)}$$

(Temporal Filter)-and-Sum Beamforming

$$y_m(t) = H_m(\omega^0)\exp\{j(\omega^0 t - \vec{k}^0 \cdot \vec{x}_m)\}$$

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

$$= \sum_{m=0}^{M-1} w_m H_m(\omega^0)\exp\{j(\omega^0[t - \Delta_m] - \vec{k}^0 \cdot \vec{x}_m)\}$$

$$= e^{j\omega^0 t} \underbrace{\sum_{m=0}^{M-1} w_m H_m(\omega^0)\exp\{-j(\omega^0 \Delta_m + \vec{k}^0 \cdot \vec{x}_m)\}}_{H(\vec{k}^0, \omega^0)}$$

Wavenumber-frequency response $\rightarrow H(\vec{k}^0, \omega^0)$

Identical Temporal Filters (1)

$$z(t) = \exp(j\omega^0 t) H(\vec{k}^0, \omega^0)$$

where $H(\vec{k}^0, \omega^0) =$

$$\sum_{m=0}^{M-1} w_m H_m(\omega^0) \exp\{-j(\omega^0 \Delta_m + \vec{k}^0 \cdot \vec{x}_m)\}$$

- **Notice if** $H_m(\omega) = H_0(\omega)$

then $H(\vec{k}^0, \omega^0) =$

$$H_0(\omega^0) \sum_{m=0}^{M-1} w_m \exp\{-j(\omega^0 \Delta_m + \vec{k}^0 \cdot \vec{x}_m)\}$$

Identical Temporal Filters (2)

- **If $H_m(\omega) = H_0(\omega)$ and we beamform on a planewave with slowness vector $\vec{\alpha}$**

$$H(\vec{k}^0, \omega^0) =$$

$$\Delta_m = -\vec{\alpha} \cdot \vec{x}_m$$

$$H_0(\omega^0) \sum_{m=0}^{M-1} w_m \exp\{-j(\omega^0 \Delta_m + \vec{k}^0 \cdot \vec{x}_m)\}$$

$$= H_0(\omega^0) \sum_{m=0}^{M-1} w_m \exp\{j(\omega^0 \vec{\alpha} - \vec{k}^0) \cdot \vec{x}_m\}$$

$$= H_0(\omega^0) W(\omega^0 \vec{\alpha} - \vec{k}^0)$$

Identical Temporal Filters (3)

- **If** $H_m(\omega) = H_0(\omega)$ **and we beamform on a planewave with slowness vector** $\vec{\alpha}$

$$H(\vec{k}^{\,0}, \omega^0) = H_0(\omega^0) W(\omega^0 \vec{\alpha} - \vec{k}^{\,0})$$

- **If** $H_0(\omega^0) = 1$**, i.e. no filtering, we get**

$$H(\vec{k}^{\,0}, \omega^0) = W(\omega^0 \vec{\alpha} - \vec{k}^{\,0})$$

which is the old delay-and-sum result

Quick Question and Answer

- **Question: To avoid needing *m* filters, why not just beamform first, then apply the *H*(ω) filter?**

$$H(\vec{k}^0, \omega^0) = H_0(\omega^0) W(\omega^0 \vec{\alpha} - \vec{k}^0)$$

- **Answer: filter first to avoid unwanted signals eating up the dynamic range of your A/D converters**
  - Temporally filter first
  - Then beamform

Spatial Filtering

- Suppose now we filter in space

$$y_m(t) = \{h_m(\cdot) *_{\vec{x}} f(\cdot, t)\}(\vec{x}_m)$$
$$= \int_{-\infty}^{\infty} h_m(\vec{\chi}) f(\vec{x}_m - \vec{\chi}, t) d\vec{\chi}$$

- Usually we don't do this deliberately - occurs due to the nature of the given sensors
- Let's study the filter-and-sum beamformer with spatial filtering:

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

Spatially Filtered Signal at Sensor

$$f(\vec{x}_m, t) = \exp\{j(\omega^0 t - \vec{k}^0 \cdot \vec{x}_m)\}$$

$$y_m(t) = \int_{-\infty}^{\infty} h_m(\vec{\chi}) f(\vec{x}_m - \vec{\chi}, t) d\vec{\chi}$$

$$= \int_{-\infty}^{\infty} h_m(\vec{\chi}) \exp\{j(\omega^0 t - \vec{k}^0 \cdot [\vec{x}_m - \vec{\chi}])\} d\vec{\chi}$$

$$= e^{j(\omega^0 t - \vec{k}^0 \cdot \vec{x}_m)} \underbrace{\int_{-\infty}^{\infty} h_m(\vec{\chi}) \exp(j\vec{k}^0 \cdot \vec{\chi}) d\vec{\chi}}_{H_m(\vec{k}^0)}$$

(Spatial Filter)-and-Sum Beamforming

$$y_m(t) = H_m(\vec{k}^0)\exp\{j(\omega^0 t - \vec{k}^0 \cdot \vec{x}_m)\}$$

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

$$= \sum_{m=0}^{M-1} w_m H_m(\vec{k}^0)\exp\{j(\omega^0[t - \Delta_m] - \vec{k}^0 \cdot \vec{x}_m)\}$$

$$= e^{j\omega^0 t} \underbrace{\sum_{m=0}^{M-1} w_m H_m(\vec{k}^0)\exp\{-j(\omega^0 \Delta_m + \vec{k}^0 \cdot \vec{x}_m)\}}_{H(\vec{k}^0,\omega^0)}$$

Wavenumber-frequency response $\rightarrow H(\vec{k}^0,\omega^0)$

Identical Spatial Filters (1)

$$z(t) = \exp(j\omega^0 t) H(\vec{k}^0, \omega^0)$$

where $H(\vec{k}^0, \omega^0) =$

$$\sum_{m=0}^{M-1} w_m H_m(\vec{k}^0) \exp\{-j(\omega^0 \Delta_m + \vec{k}^0 \cdot \vec{x}_m)\}$$

- **Notice if** $H_m(\vec{k}) = H_0(\vec{k})$

  then $H(\vec{k}^0, \omega^0) =$

$$H_0(\vec{k}^0) \sum_{m=0}^{M-1} w_m \exp\{-j(\omega^0 \Delta_m + \vec{k}^0 \cdot \vec{x}_m)\}$$

Identical Spatial Filters (2)

- **If** $H_m(\vec{k}) = H_0(\vec{k})$ **and we beamform on a planewave with slowness vector** $\vec{\alpha}$

$$H(\vec{k}^0, \omega^0) =$$

$$\Delta_m = -\vec{\alpha}\cdot\vec{x}_m$$

$$H_0(\vec{k}^0)\sum_{m=0}^{M-1} w_m \exp\{-j(\omega^0\Delta_m + \vec{k}^0\cdot\vec{x}_m)\}$$

$$= H_0(\vec{k}^0)\sum_{m=0}^{M-1} w_m \exp\{j(\omega^0\vec{\alpha} - \vec{k}^0)\cdot\vec{x}_m\}$$

$$= H_0(\vec{k}^0)W(\omega^0\vec{\alpha} - \vec{k}^0)$$

Identical Spatial Filters (3)

- **If** $H_m(\vec{k}) = H_0(\vec{k})$ **and we beamform on a planewave with slowness vector** $\vec{\alpha}$

$$H(\vec{k}^0, \omega^0) = H_0(\vec{k}^0) W(\omega^0 \vec{\alpha} - \vec{k}^0)$$

- **If** $H_0(\vec{k}^0) = 1$**, i.e. no filtering, we get**

$$H(\vec{k}^0, \omega^0) = W(\omega^0 \vec{\alpha} - \vec{k}^0)$$

which is the old delay-and-sum result

Spatiotemporal Filtering

- **Might filter in both space and time!**

$$y_m(t) = \{h_m * f\}(\vec{x}_m, t)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_m(\vec{\chi}, \tau) f(\vec{x}_m - \vec{\chi}, t - \tau) \, d\vec{\chi} \, d\tau$$

- **By now we can guess the-filter and-sum beamformer output will look like**

$$z(t) = \exp(j\omega^0 t) H(\vec{k}^0, \omega^0)$$

where $H(\vec{k}^0, \omega^0) =$

$$\sum_{m=0}^{M-1} w_m H_m(\vec{k}^0, \omega^0) \exp\{-j(\omega^0 \Delta_m + \vec{k}^0 \cdot \vec{x}_m)\}$$

Idential Spatiotemporal Filters

- If $H_m(\vec{k},\omega) = H_0(\vec{k},\omega)$ and we beamform on a planewave with slowness vector $\vec{\alpha}$

$$H(\vec{k}^0,\omega^0) = H_0(\vec{k}^0,\omega^0) W(\omega^0\vec{\alpha} - \vec{k}^0)$$

Waveno-freq response of individual sensor

Waveno-freq response of beamformed array