EE269
Signal Processing for Machine Learning

Lecture 20

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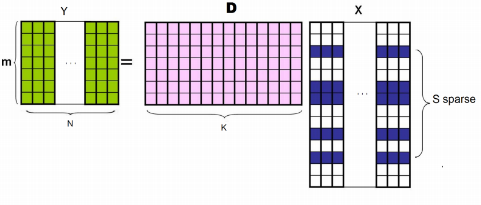
Dictionary Learning

$$\min_{x_1,\ldots x_n,D} \sum_{i=1}^{n} \|Dx_i - y_i\|_2^2 \qquad \text{s.t. } ||x_i||_0 \leq s$$



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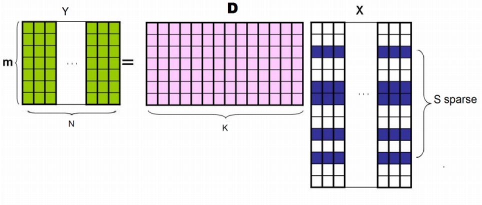
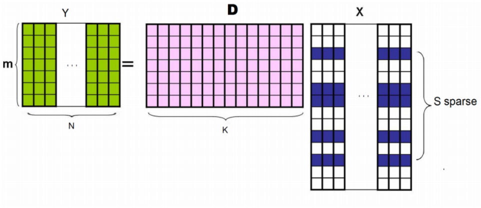


- Matrix factorization interpretation:
  $DX \approx Y$ where X has sparse columns

Dictionary Learning : ℓ_1 relaxation

$$\min_{x_1,\ldots x_n,D} \sum_{i=1}^{n} \|Dx_i - y_i\|_2^2 + \lambda||x_i||_1$$



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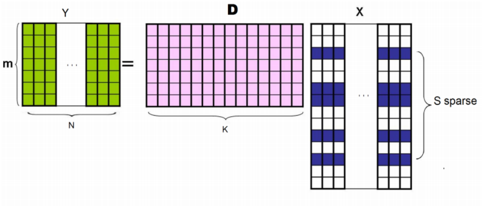


- heuristic algorithm

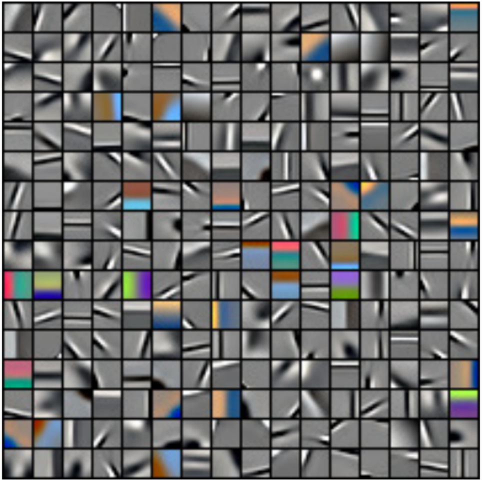
  minimize over x_i : ℓ_1 regularized Least Squares problem

  gradient descent step over D:

  $D \leftarrow D - \mu \sum_i (Dx_i - y_i)x_i^T$

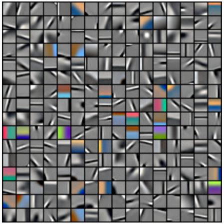
Learned Dictionaries from Images

$$\min_{x_1,\ldots x_n, D} \sum_{i=1}^{n} \|Dx_i - y_i\|_2^2 + \lambda ||x_i||_1$$

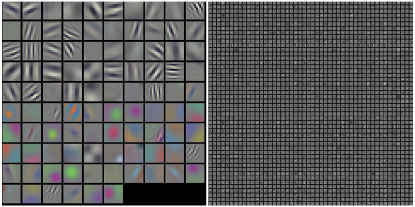


Dictionary Learning vs Deep Learning

- Dictionary learning



- Alexnet: Convolutional net for image classification



Dictionary Learning Applications

[Mairal, Sapiro, and Elad, 2008d]

Dictionary Learning Applications

Inpainting, [Mairal, Elad, and Sapiro, 2008b]

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Deep Dictionary Learning

- The dictionary can be structured, e.g., hierarchical, convolutional etc

$$\min_{x_1,\ldots x_n, D_1, D_2, \ldots, D_L} \sum_{i=1}^{n} \|D_1 D_2 \ldots D_L x_i - y_i\|_2^2 + \lambda ||x_i||_1$$

Clustering

$$\min \sum_{j=1}^{k} \sum_{x_i \in C_j} \|x_i - c_j\|^2 \; = \; \|X - CB^T\|_F^2$$

$C = [c_1, ..., c_k]$ is the centroid matrix

B denotes the clustering assignment

$B_{ij} = 1$ if i-th observation is assigned to the j-th cluster center c_j

Matching Pursuit

Matching pursuit

$$\min_{\alpha \in \mathbb{R}^m} \frac{1}{2} \| x - D\alpha \|_2^2 \quad \text{s. t.} \quad \| \alpha \|_0 \leq L$$

1. Initialization: $\alpha = 0$, residual $r = x$
2. while $||\alpha||_0 < L$
3. Select the element with maximum correlation with the residual

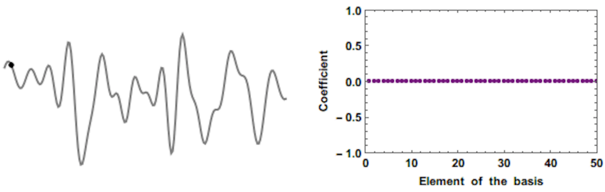
$$\hat{i} = \arg\max_{i=1,\dots,m} | d_i^T r |$$

4. Update the coefficients and residual

$$\alpha_{\hat{i}} = \alpha_{\hat{i}} + d_{\hat{i}}^T r$$
$$r = r - (d_{\hat{i}}^T r) d_{\hat{i}}$$

5. End while

Matching Pursuit: Example



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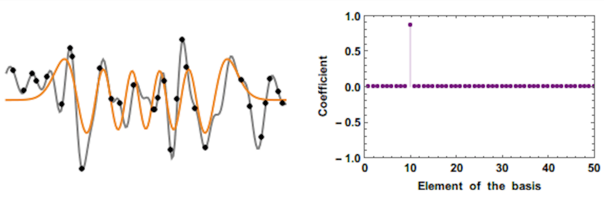
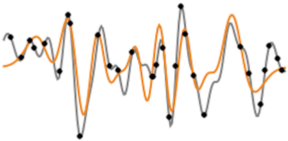
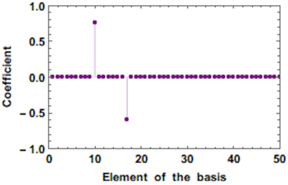


Matching Pursuit: Example





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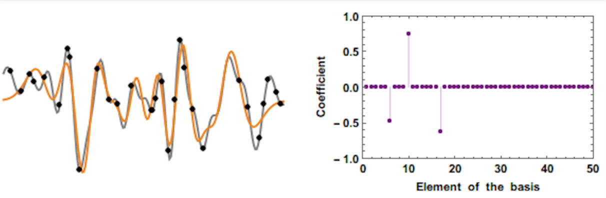

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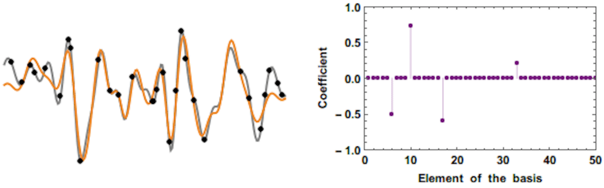

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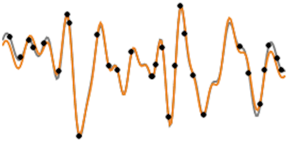

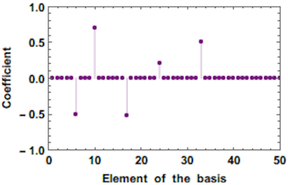

Further applications

- Mechanical monitoring

  Engines, elevators, buildings, power plants etc.
- Medical monitoring

  Pulmonary (lung) sounds, cardiac sounds, speech etc.
- Security surveillance

  Interesting event discovery, accident detection etc.

Other SP/ML methods

- Independent Component Analysis
- Recurrent Neural Nets
- Generative Models and Generative Adversarial Networks
- Manifold learning methods (e.g., Laplacian Eigenmaps, Isomap)
- Tensor Decompositions (e.g. Tensor Principal Component Analysis)
- Latent and Mixture Models (Hidden Markov Models, Low rank models)
- Bayesian Methods
- State Space Models and tracking (e.g., Kalman filter)

Other Courses

- EE270 - Large Scale Matrix Computation, Optimization and Learning
- EE364a - Convex Optimization I
- EE364b - Convex Optimization II
- EE 264 - Digital Signal Processing
- EE373A - Adaptive Signal Processing
- CS 229 - Machine Learning
- CS 230 - Deep Learning