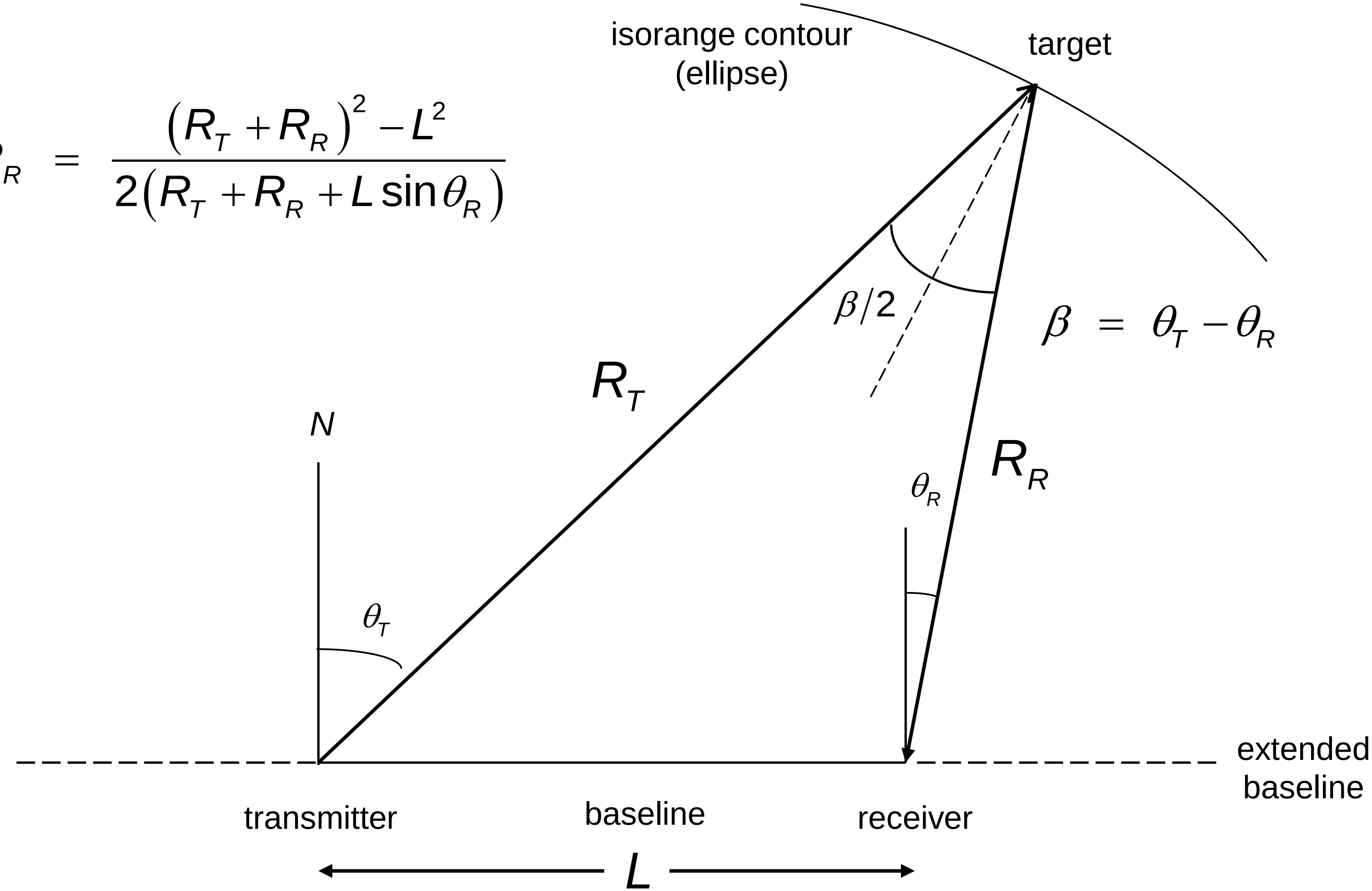
1.3 *Bistatic geometry*

1. Bistatic geometry
2. Doppler shift
3. Pulse chasing
4. The Jackson paper
5. The ambiguity function for bistatic radar

Bistatic radar geometry

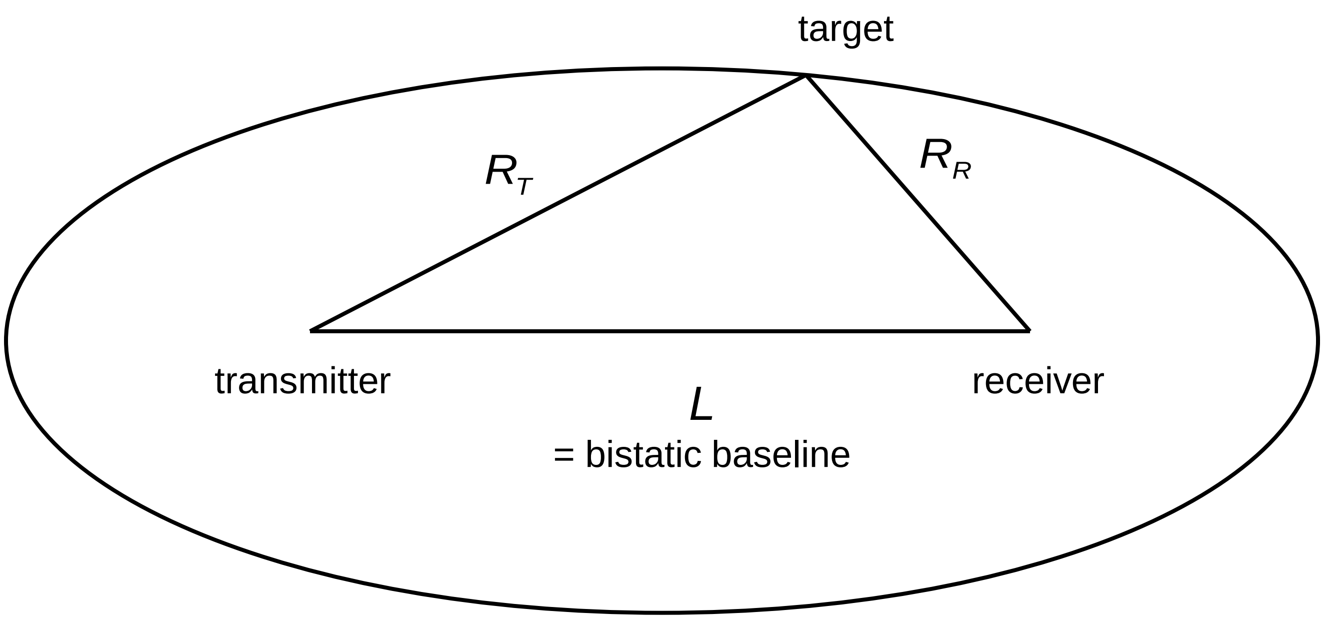
$$R_R = \frac{(R_T + R_R)^2 - L^2}{2(R_T + R_R + L\sin\theta_R)}$$



Jackson, M.C., ‘The geometry of bistatic radar systems’; *IEE Proc*., Vol.133, Pt.F, No.7, pp604-612, December 1986.

Bistatic radar geometry

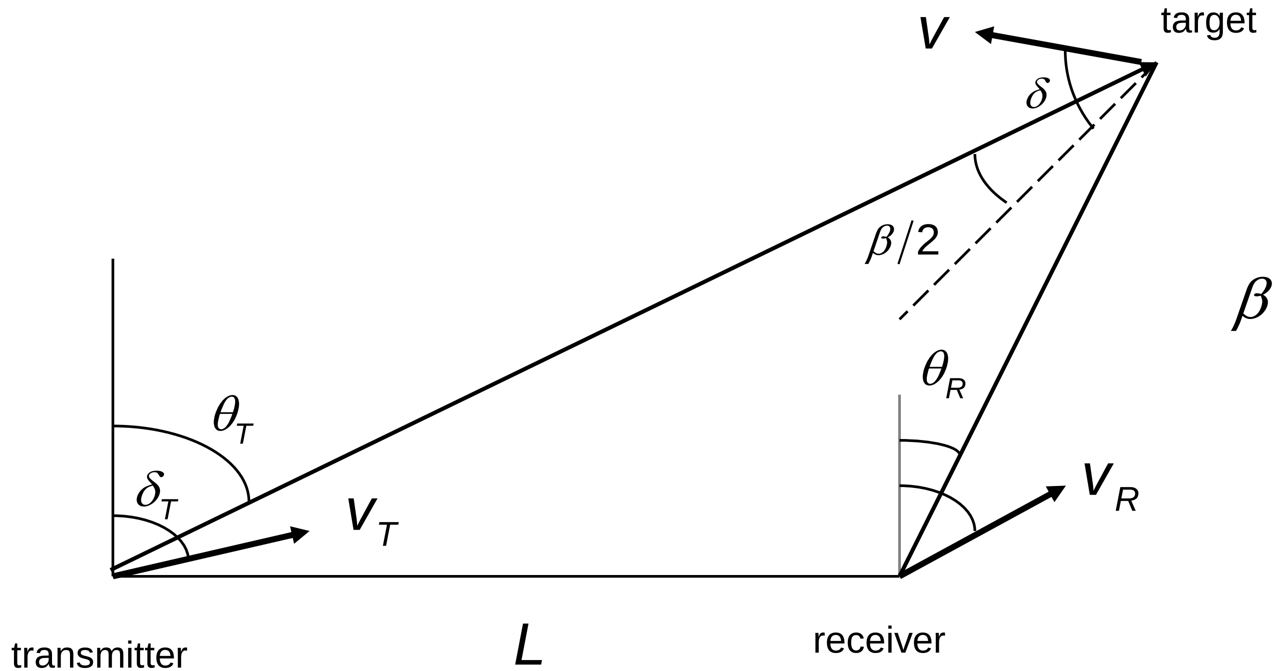
Contours of constant bistatic range are ellipses, with the transmitter and receiver as the two foci



$$R_T + R_R = \text{const}$$

Targets lying on the transmitter-receiver baseline have zero bistatic range.

Bistatic radar Doppler



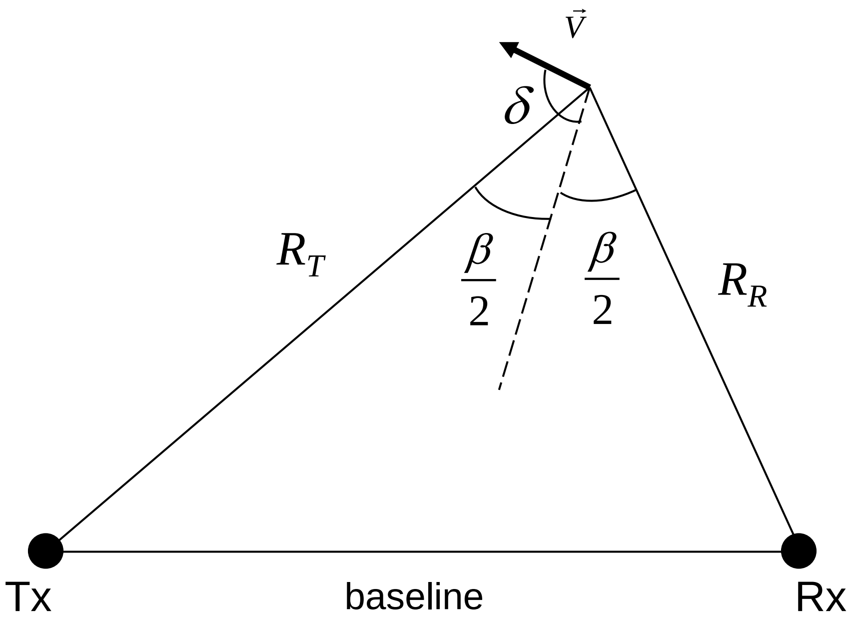
$$\beta = \theta_T - \theta_R$$

$$f_D = \frac{1}{\lambda}\left(\frac{dR_T}{dt} + \frac{dR_R}{dt}\right)$$

Jackson, M.C., 'The geometry of bistatic radar systems'; *IEE Proc.*, Vol.133, Pt.F, No.7, pp604-612, December 1986.

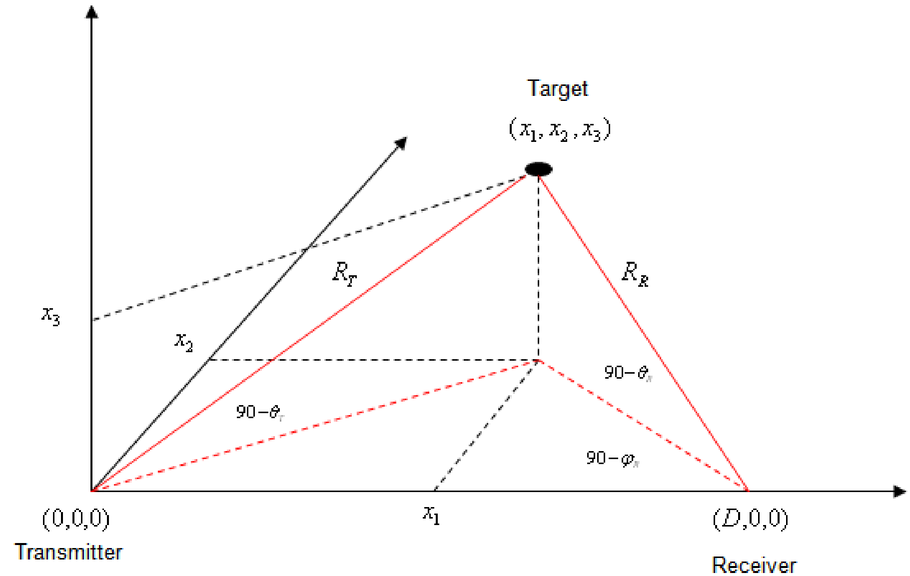
Bistatic Doppler

Given a target with a velocity $\vec{V}$, the bistatic Doppler shift depends on the geometry and the transmitted frequency only.



$$f_D = \frac{1}{\lambda}\left(\frac{dR_T}{dt} + \frac{dR_R}{dt}\right) = \frac{1}{\lambda}\left|V\right|\cos\left(\frac{\beta}{2}\right)\cos\delta$$

Target tracking



The Doppler shift induced by a moving target can be expressed as a function of its position and velocity.

$$f_D = \frac{1}{\lambda}\left(\frac{dR_T}{dt} + \frac{dR_R}{dt}\right)$$

$$f_D = \frac{1}{\lambda}\left(\frac{x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3}{\sqrt{x_1^2 + x_2^2 + x_3^2}} + \frac{(x_1 - D)\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3}{\sqrt{(x_1 - D)^2 + x_2^2 + x_3^2}}\right)$$

Bistatic radar Doppler

For $V_T = V_R = 0$; $V \neq 0$ $$f_D = \left(\frac{2V}{\lambda}\right)\cos\delta\cos(\beta/2)$$

special cases :

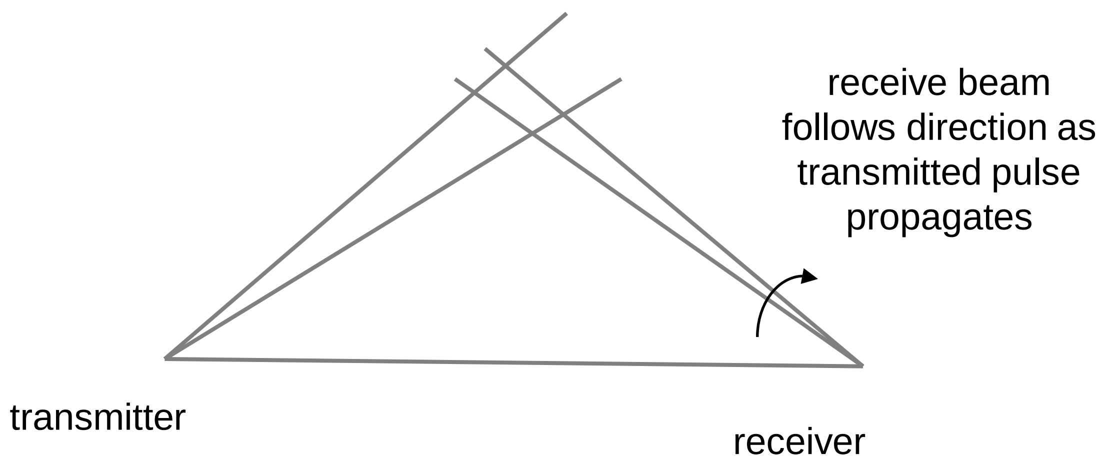
β	δ	f_D	*condition*
0	–	$(2v/\lambda)\cos\delta$	*monostatic*
0	0	$(2v/\lambda)$	*monostatic*
180	–	0	*forward scatter*
–	± 90	0	$v \perp$ *to bisector*
–	$\pm\beta/2$	$(2v/\lambda)\cos^2(\beta/2)$	$v \Rightarrow$ *tx or rx*
–	0,180	$\pm(2v/\lambda)\cos(\beta/2)$	$v \Rightarrow$ *bisector*
–	$90 \pm \beta/2$	$\mp(v/\lambda)\sin\beta$	$v \perp$ *to tx or rx LOS*

Willis, N.J., ‘Bistatic radars and their third resurgence: passive coherent location’, IEEE Radar Conference, Long Beach, USA, April 2002.

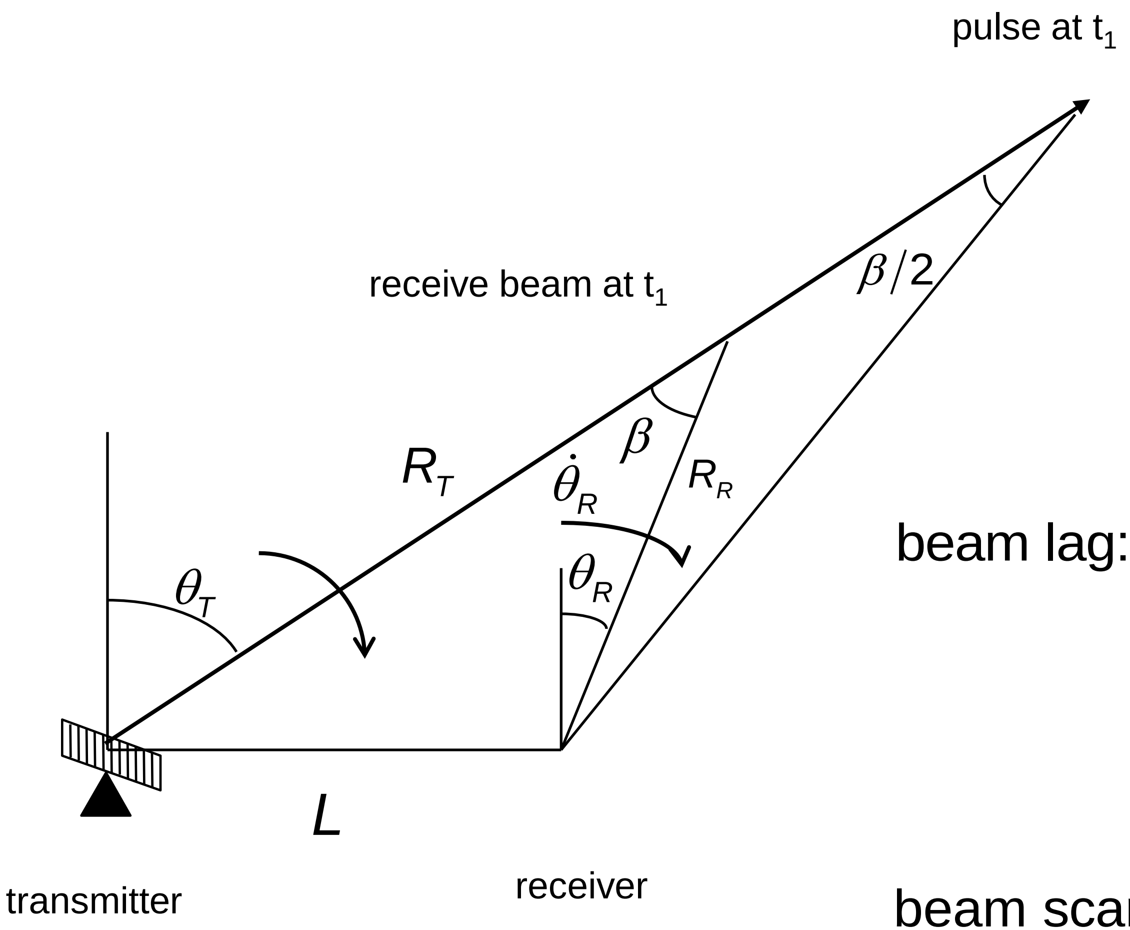
Pulse chasing

If the transmitter scans in azimuth, and if the receiver is to use a directional antenna, then the direction in which the receiver beam must point is a very nonlinear function of time. This means that the receiver must use either a fan of fixed beams, or a scanned beam which, since its scan is nonlinear, will need to be electronically scanned.

The receive beam must point instantaneously in the direction from which an echo will come, which is known as *pulse chasing*.



Pulse chasing



beam lag: $\theta_R = \theta_T - \beta$

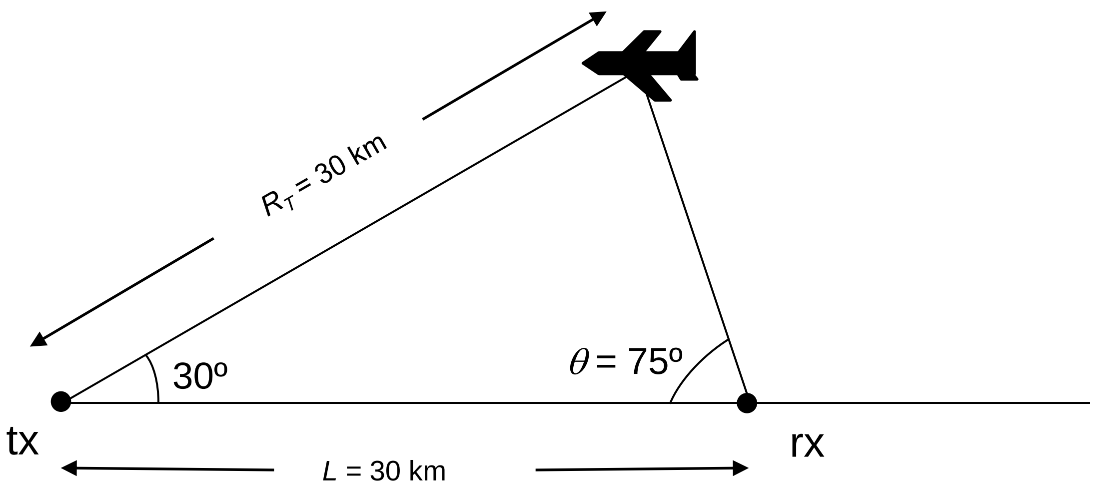
$$= \theta_T - 2\tan^{-1}\left(\frac{L\cos\theta_T}{R_T + R_R - \sin\theta_T}\right)$$

beam scan rate: $\dot{\theta}_R = \dfrac{c\tan(\beta/2)}{R_R}$

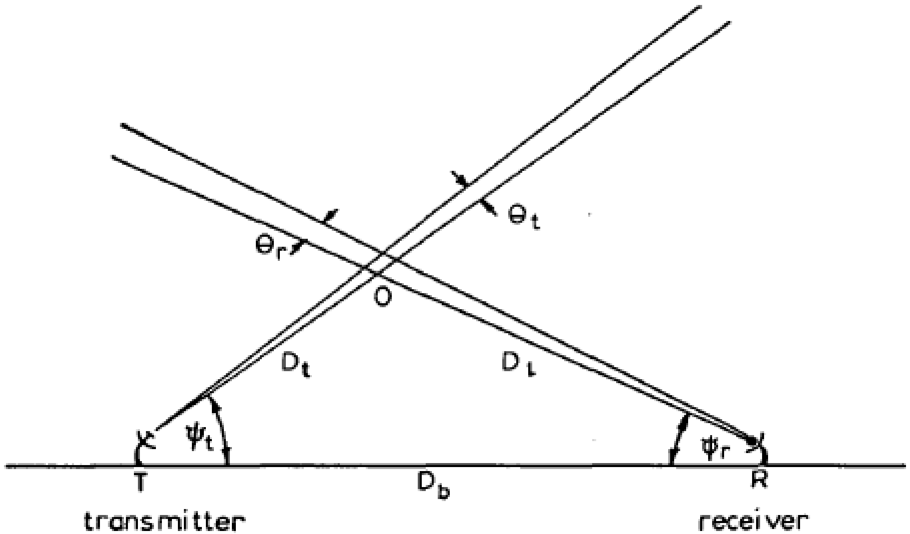
Jackson, M.C., ‘The geometry of bistatic radar systems’; *IEE Proc*., Vol.133, Pt.F, No.7, pp604-612, December 1986.

Test your understanding

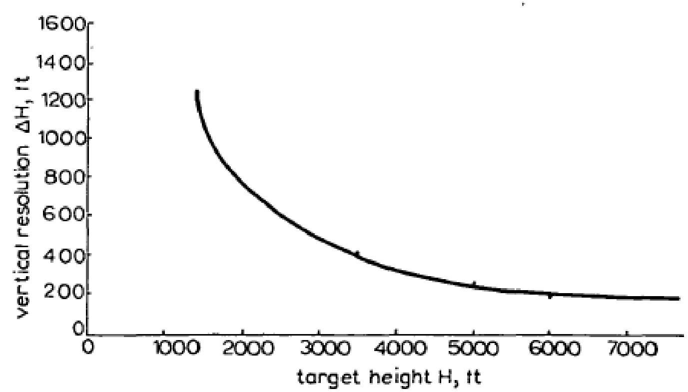
To illustrate this, suppose that a pulse is transmitted at time t = 0. At what value of time t should the receiver beam point in the direction of the target ?



Vertical plane resolution



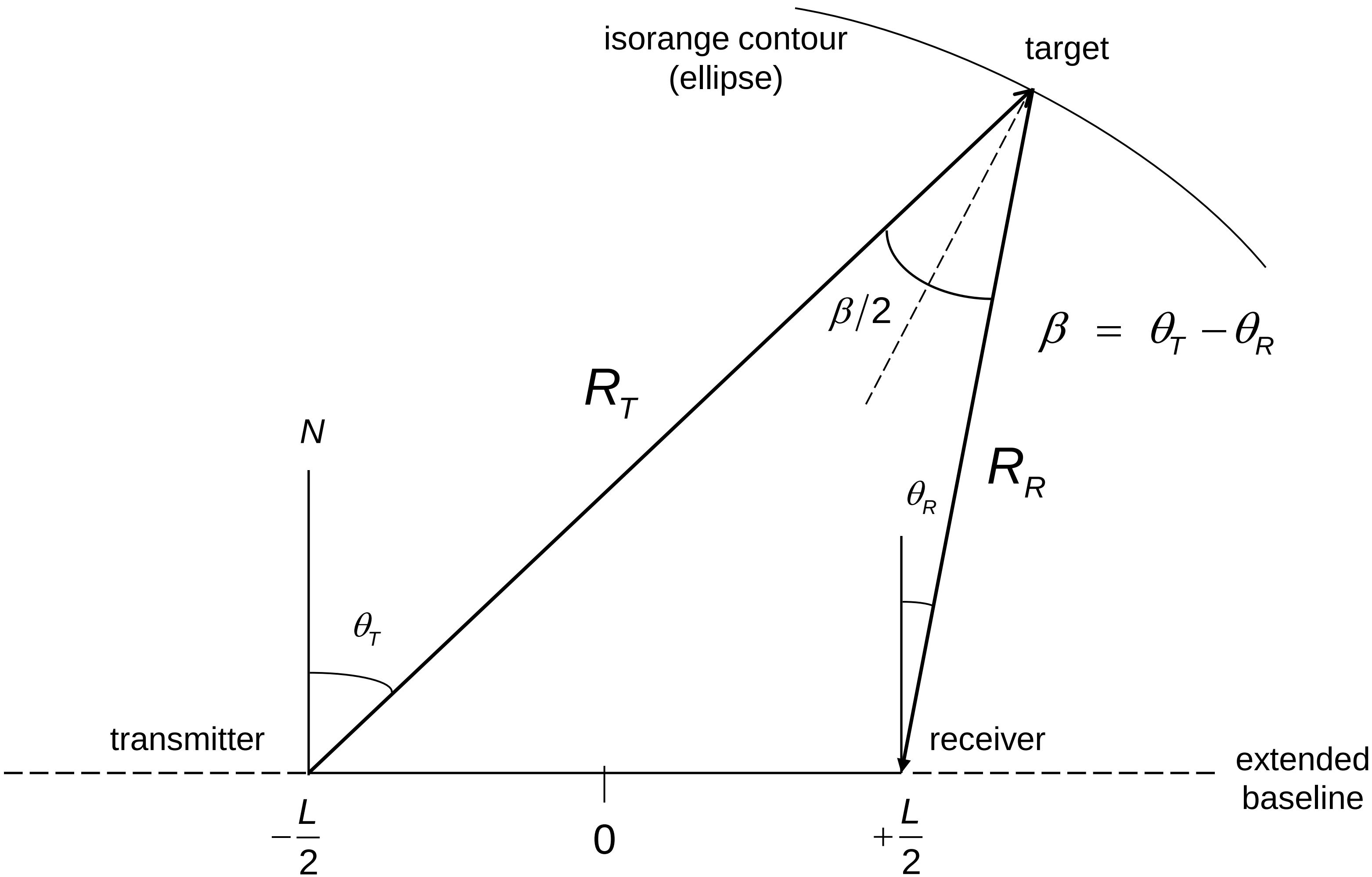
The points T, R and O lie in the vertical plane and the point O is the centre of the beam intersection volume



Variation of vertical resolution cell with elevation
$\theta_r = \theta_t = 3°$ D_b = 20 miles and $\tau = 0.05$ μs.

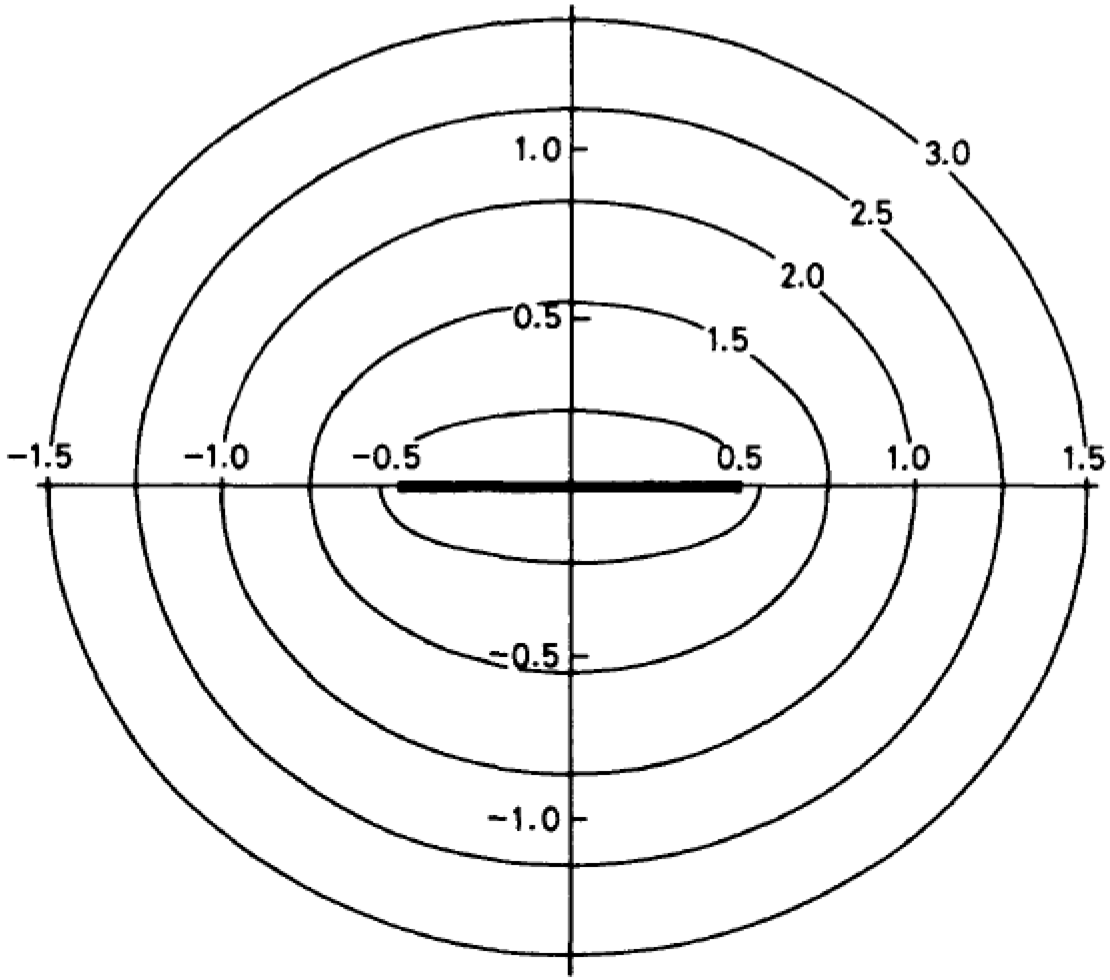
Davies, D.E.N., 'Use of bistatic radar techniques to improve resolution in the vertical plane'; *Electronics Letters*, Vol.4, No.9, pp170-171, 3 May 1968.

The Jackson paper

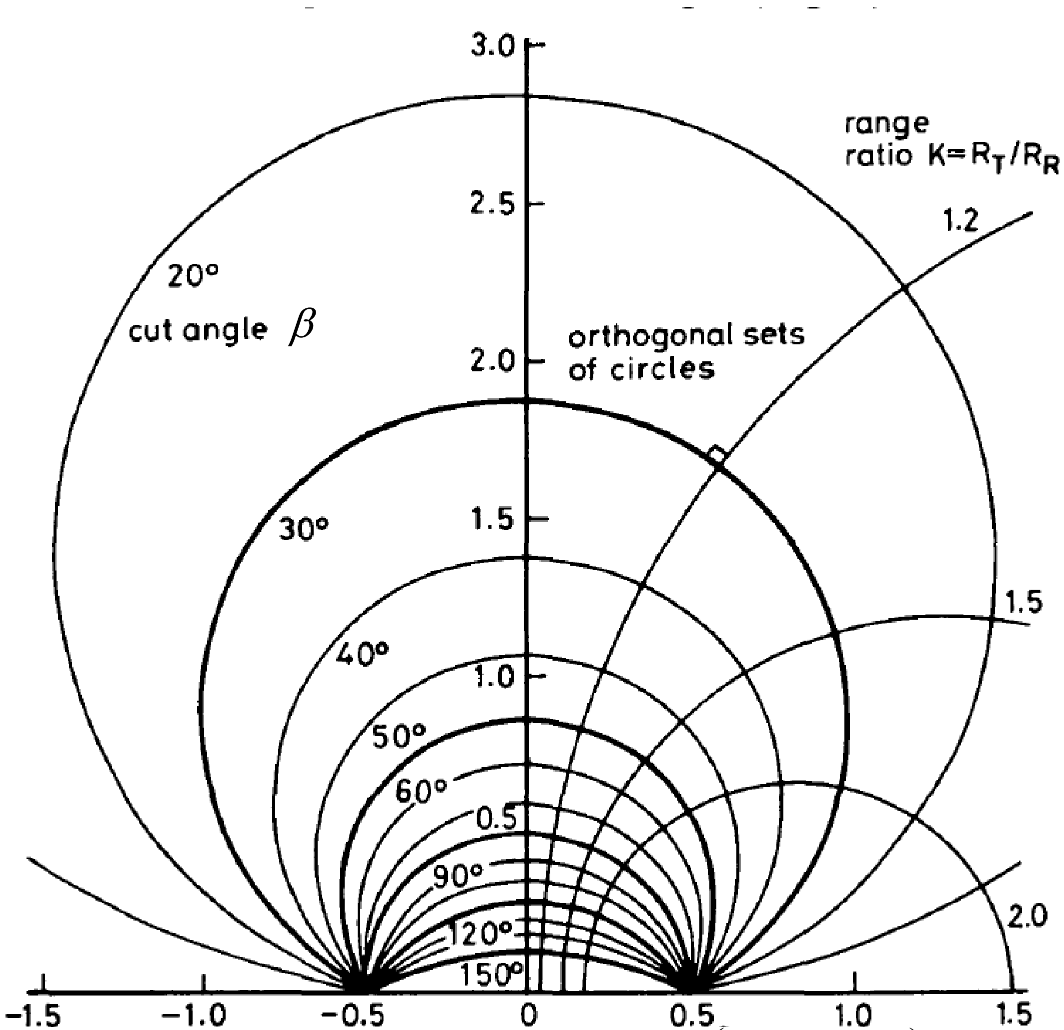


Jackson, M.C., ‘The geometry of bistatic radar systems’; *IEE Proc*., Vol.133, Pt.F, No.7, pp604-612, December 1986.

Contours of constant (R_T + R_R)

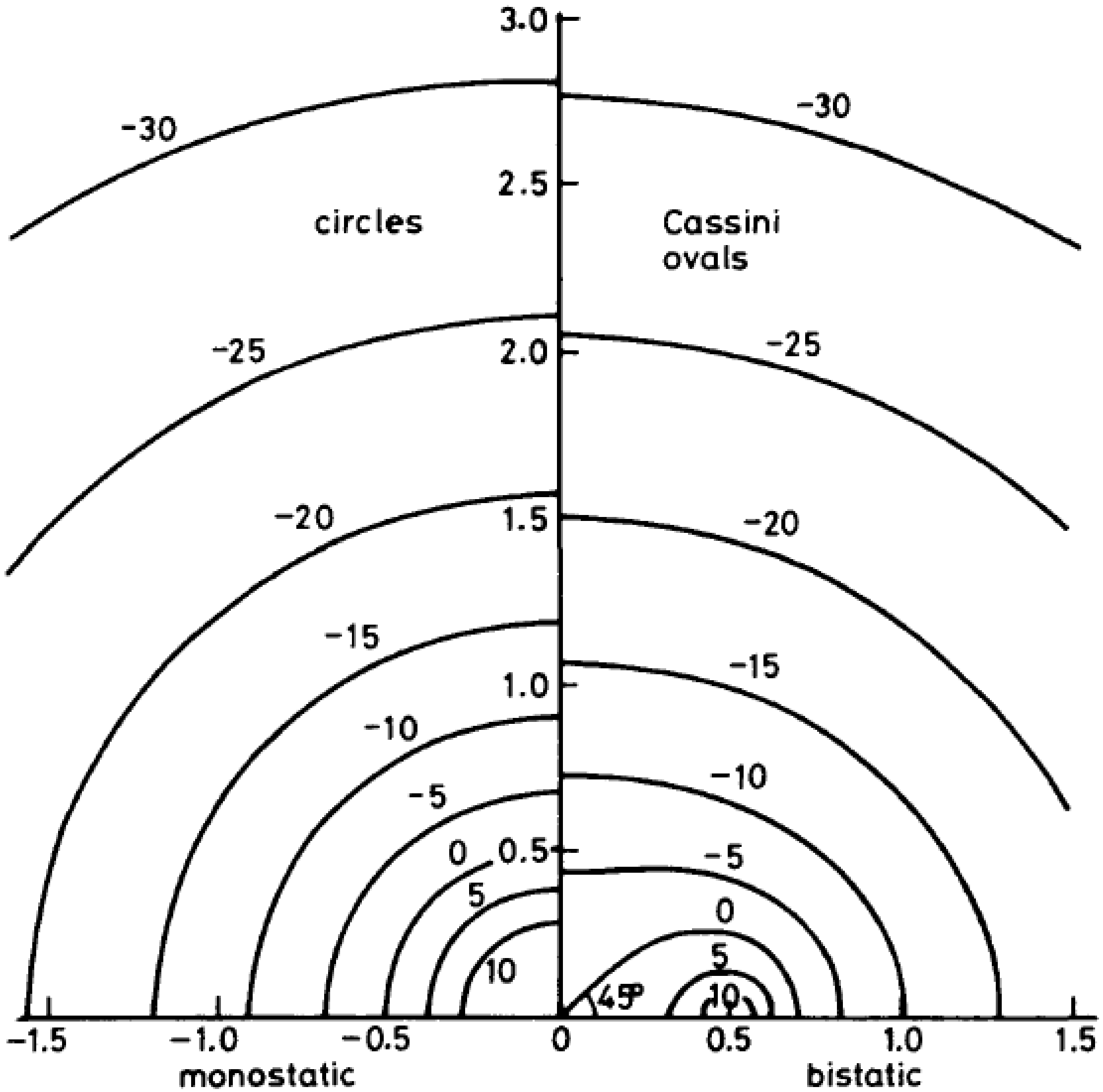


Contours of constant bistatic angle β

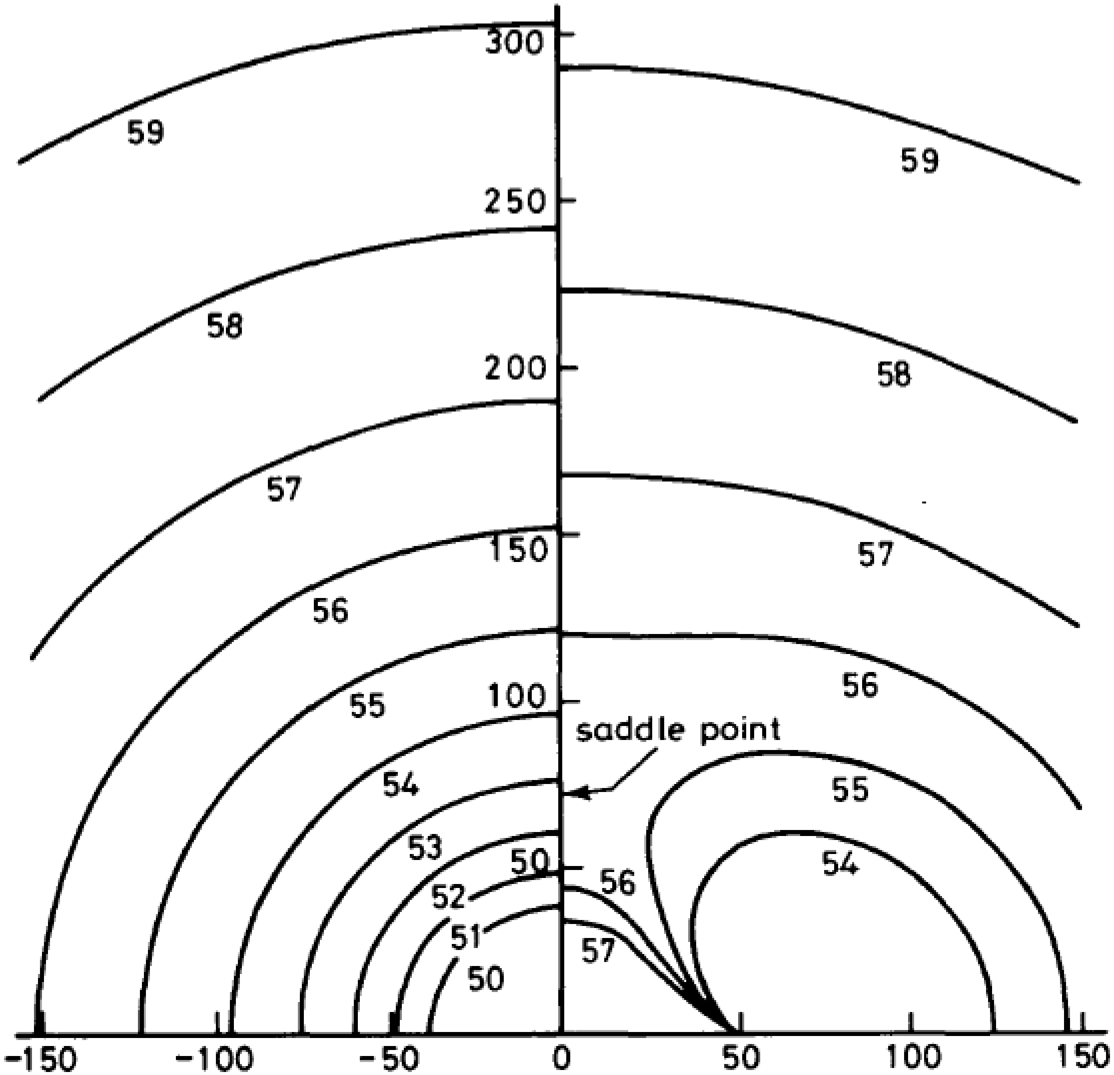


centres $\left(0, \frac{1}{2\tan\beta}\right)$, radii $\frac{1}{2\sin\beta}$ centres $\left(\left(\frac{K^2+1}{K^2-1}\right)/2, 0\right)$, radii $\frac{K}{K^2-1}$

Contours of constant signal level



Resolution cell area



monostatic bistatic

Ambiguity function

- **Expresses the point target response of the waveform as a function of delay τ and Doppler frequency ν (or equivalently, range r and velocity v)**

- **Shows resolution, sidelobe level and structure, and ambiguities, in delay and Doppler.**

$$\left|\chi(\tau,\nu)\right|^2 = \left|\int u(t)u^*(t+\tau)\exp(j2\pi\nu t)\,dt\right|^2$$

Woodward, P.M., *Probability and Information Theory, with Applications to Radar*, Pergamon Press, 1953; reprinted by Artech House, 1980.

Philip Woodward

P.M.Woodward:
Probability and information theory, with applications to radar (1953).

PROBABILITY AND INFORMATION THEORY, WITH APPLICATIONS TO RADAR

By

P. M. WOODWARD, B.A.
Principal Scientific Officer, Telecommunications Research Establishment, Ministry of Supply

Philip Woodward
2003

Bistatic ambiguity function

Tsao et al. have looked at the form of the ambiguity function for bistatic radar, and have shown that the bistatic geometry can have a significant effect on the shape of the ambiguity function, since the relationships between ω_D and v, and between τ and *R*, are nonlinear. They propose that the bistatic ambiguity function should be expressed

$$\left|\chi\left(R_{R_H}, R_{R_a}, V_H, V_a, \theta_R, L\right)\right|^2$$

$$= \left| \int_{-\infty}^{\infty} \tilde{f}\left(t-\tau_a\left(R_{R_a},\theta_R,L\right)\right)\tilde{f}^*\left(t-\tau_H\left(R_{R_H},\theta_R,L\right)\right) \exp\left[-j\left(\omega_{D_H}\left(R_{R_H},V_H,\theta_R,L\right)-\omega_{D_a}\left(R_{R_a},V_a,\theta_R,L\right)\right)t\right]dt \right|^2$$

Tsao, T., Slamani, M., Varshney, P., Weiner, D., Schwarzlander, H. and Borek, S. ‘Ambiguity function for bistatic radar’; *IEEE Trans. Aerospace and Electronic Systems*, Vol.AES-33, pp1041-1051,1997.

Bistatic ambiguity function

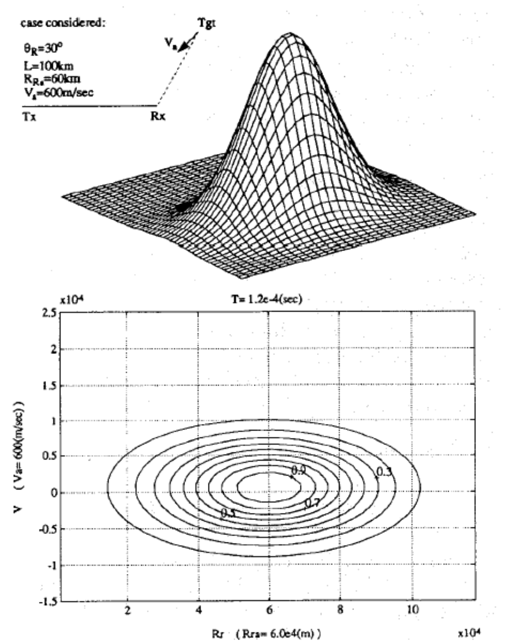


Fig. 4. Bistatic ambiguity function plot of Gaussian pulse for geometry shown.

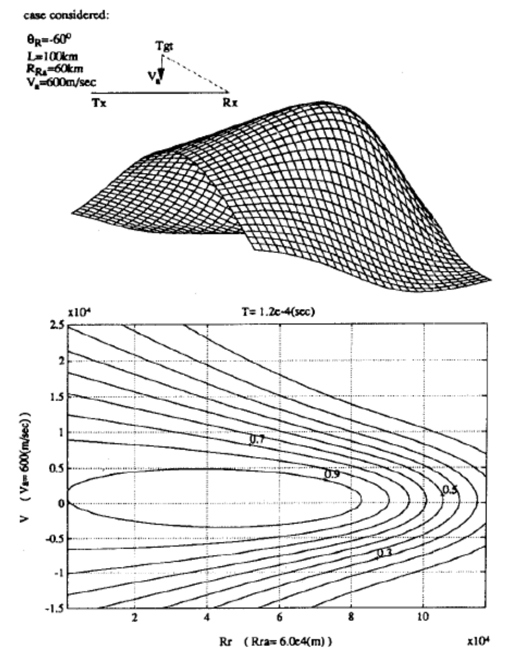


Fig. 5. Bistatic ambiguity function plot of Gaussian pulse for geometry shown.

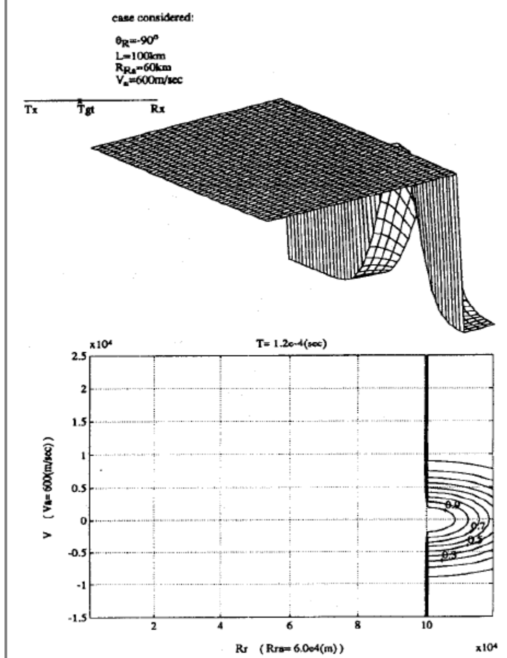
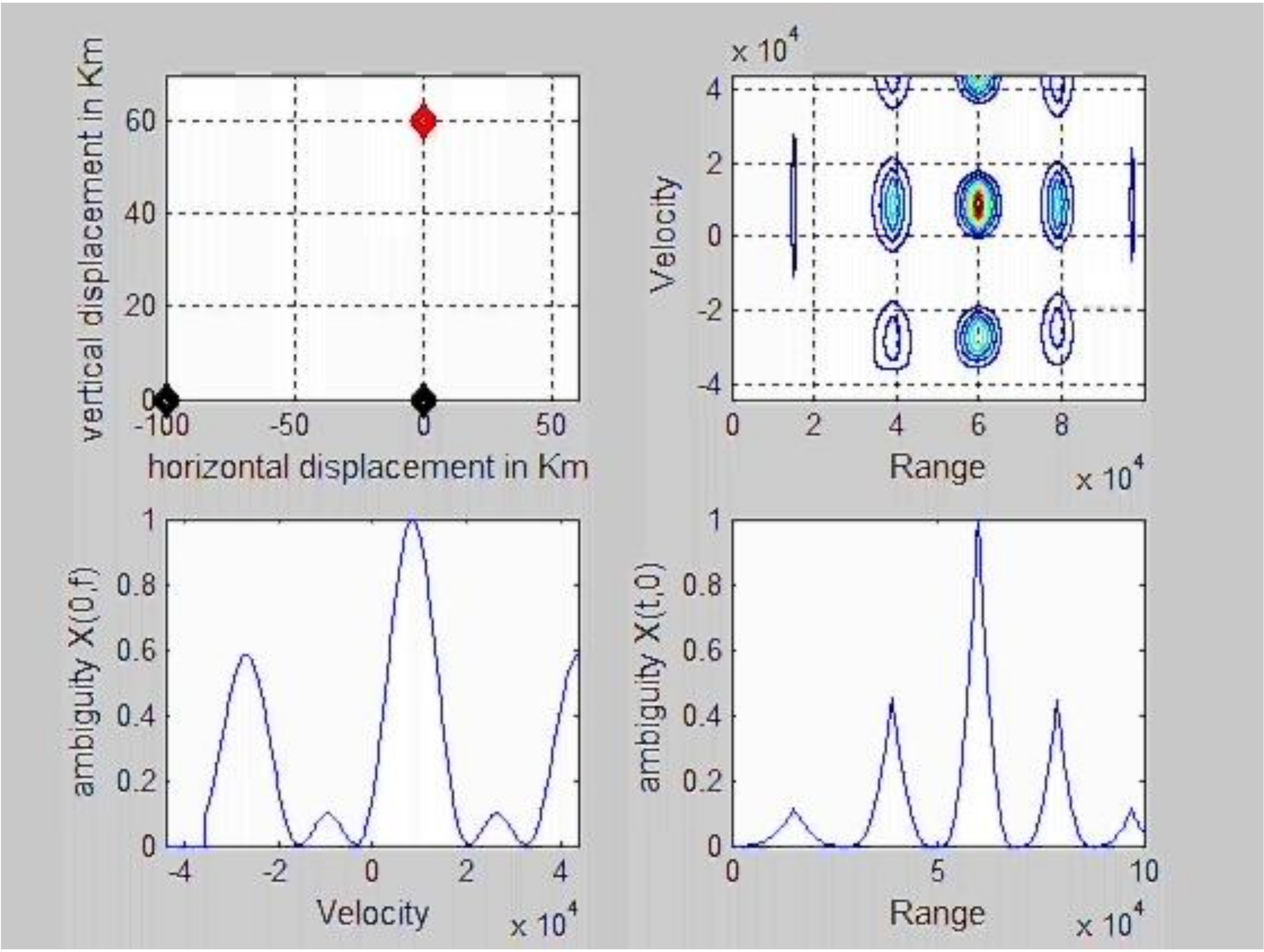


Fig. 6. Bistatic ambiguity function plot of Gaussian pulse for geometry shown.

Tsao, T., Slamani, M., Varshney, P., Weiner, D., Schwarzlander, H. and Borek, S. 'Ambiguity function for bistatic radar'; *IEEE Trans. Aerospace and Electronic Systems*, Vol.AES-33, pp1041-1051,1997.

Bistatic ambiguity function



Test your understanding

A bistatic radar system at a frequency of 10 GHz is configured with the receiver situated 100 km due East of the transmitter. The bistatic range (transmitter-to-target range + target-to-receiver range) of a particular target is measured to be 140 km and the direction of arrival of the echo is 36.9° measured anticlockwise with respect to North.

(i) Draw a diagram showing the geometry of the transmitter, receiver and target, and calculate the range of the target from the receiver.

(ii) The target is moving at a uniform velocity of 212 ms^{-1} towards the baseline along the bisector of the bistatic angle. Calculate the Doppler shift of the echo.

(iii) Explain how the range resolution and Doppler resolution of the bistatic radar vary as the target approaches the baseline, and hence how the ambiguity function of a waveform depends on the bistatic geometry.

(part of UCL MSc exam question, 2012)

Summary

- Many of the properties of bistatic radar are a function of the bistatic geometry.
- Pulse chasing involves a very rapid scan rate, nonlinear with time, so electronic scanning (or rapid switching between a set of fixed beams) is necessary
- In a bistatic radar, the ambiguity function depends not only on the waveform but also on the bistatic geometry. For a target on the baseline the range and Doppler resolution are both infinite, no matter what the waveform is.
- In any calculations, it is wise to check the limiting case $L \rightarrow 0$ or $\beta \rightarrow 0$ to ensure that the results reduce to those for monostatic radar. If not, you've probably made a mistake.