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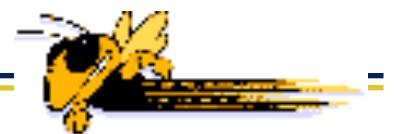
# *Apertures, Part II*

**ECE 6279: Spatial Array Processing  
Fall 2013  
Lecture 5**

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# Where We Are in J&D

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- **Lecture material drawn from:**
  - Sec. 3.3, 3.3.1-3.3.2
- **Might be a good idea to read through Sec. 3.2**
  - Reviews sampling theory

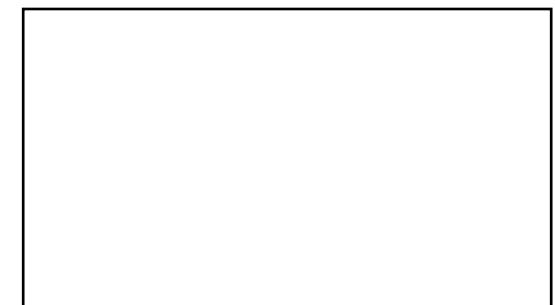
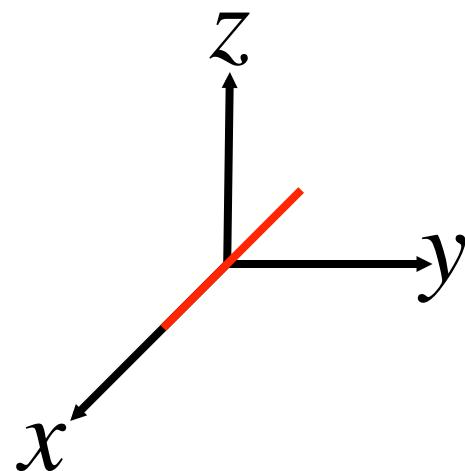


# Last Time: Filled Linear Aperture

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$$b(x) = \begin{cases} 1, & |x| \leq D/2 \\ 0, & \text{otherwise} \end{cases}$$

$$w(\vec{x}) = b(x)\delta(y)\delta(z)$$



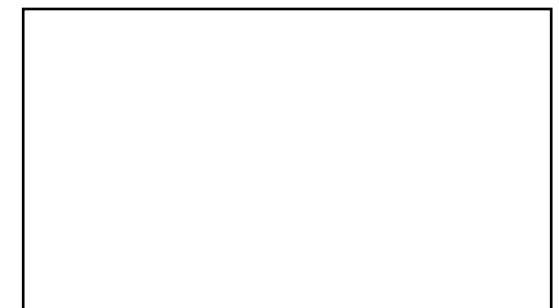
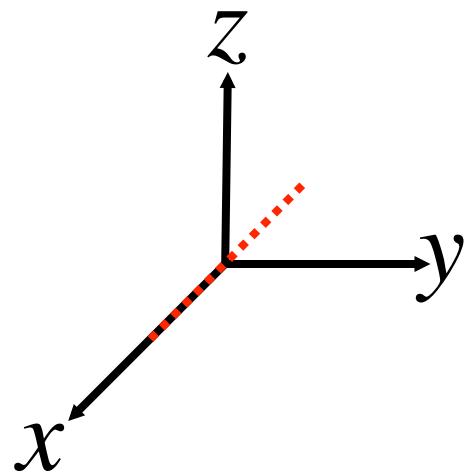
# Regularly Sampled Linear Aperture

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- Odd number of sensors  $M$ , w/spacing  $d$
- Total length  $D=Md$

$$b(x) = \sum_{m=-(M-1)/2}^{(M-1)/2} \delta(x - md)$$

$$w(\vec{x}) = b(x)\delta(y)\delta(z)$$



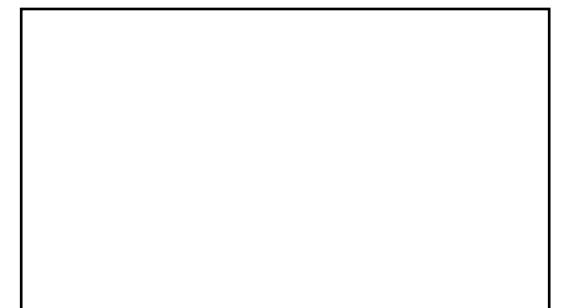
# Aperture Smoothing Function (1)

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$$\begin{aligned} W(k_x) &= \int_{-\infty}^{\infty} b(x) \exp(+jk_x x) dx \\ &= \int_{-\infty}^{\infty} \sum_{m=-(M-1)/2}^{(M-1)/2} \delta(x - md) \exp(jk_x x) dx \\ &= \sum_{m=-(M-1)/2}^{(M-1)/2} \exp(jk_x md) \times \left[ \exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right) \right] \end{aligned}$$

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$$\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)$$



# Aperture Smoothing Function (2)

$$\sum_{m=-\frac{(M-1)}{2}}^{\frac{(M-1)}{2}} \exp(jk_x m d) \times \left[ \exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right) \right]$$

- Terms from first set:

$$\left\{ a + \frac{1}{2} \right\} \left\{ (a-1) + \frac{1}{2} \right\} \dots \left\{ -(a-1) + \frac{1}{2} \right\} \left\{ -a + \frac{1}{2} \right\}$$

- Subtract terms from second set:

$$\left\{ a - \frac{1}{2} \right\} \left\{ (a-1) - \frac{1}{2} \right\} \dots \left\{ -(a-1) - \frac{1}{2} \right\} \left\{ -a - \frac{1}{2} \right\}$$



# Our Old Friend, the *Sind*

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$$W(k_x) = \frac{\exp\left(\frac{jk_x Md}{2}\right) - \exp\left(-\frac{jk_x Md}{2}\right)}{\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)}$$

$$= \frac{\sin\left(\frac{k_x Md}{2}\right)}{\sin\left(\frac{k_x d}{2}\right)}$$

$$W(0) \equiv M$$



# In Polar Wavenumber Coordinates

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$$\frac{\sin\left(\frac{k_x M d}{2}\right)}{\sin\left(\frac{k_x d}{2}\right)} = \underbrace{\frac{\sin\left(\frac{k_r M d \sin \phi \cos \theta}{2}\right)}{\sin\left(\frac{k_r d \sin \phi \cos \theta}{2}\right)}}_{W(k_r, \phi, \theta)}$$

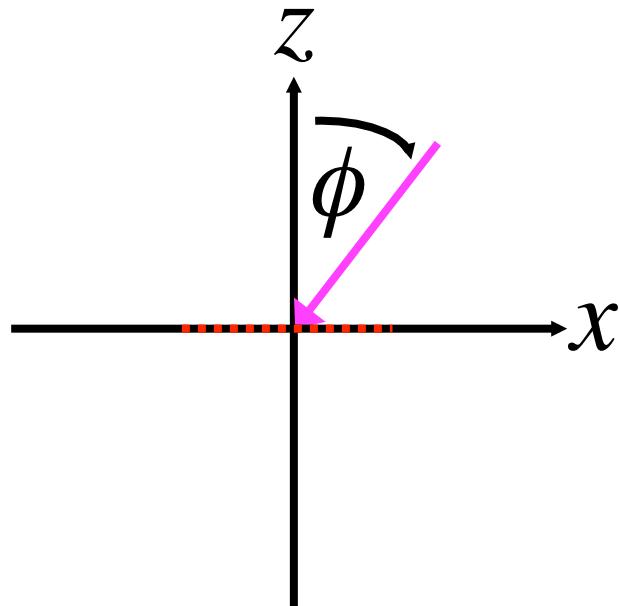
$$k_x = -k_r \sin \phi \cos \theta$$



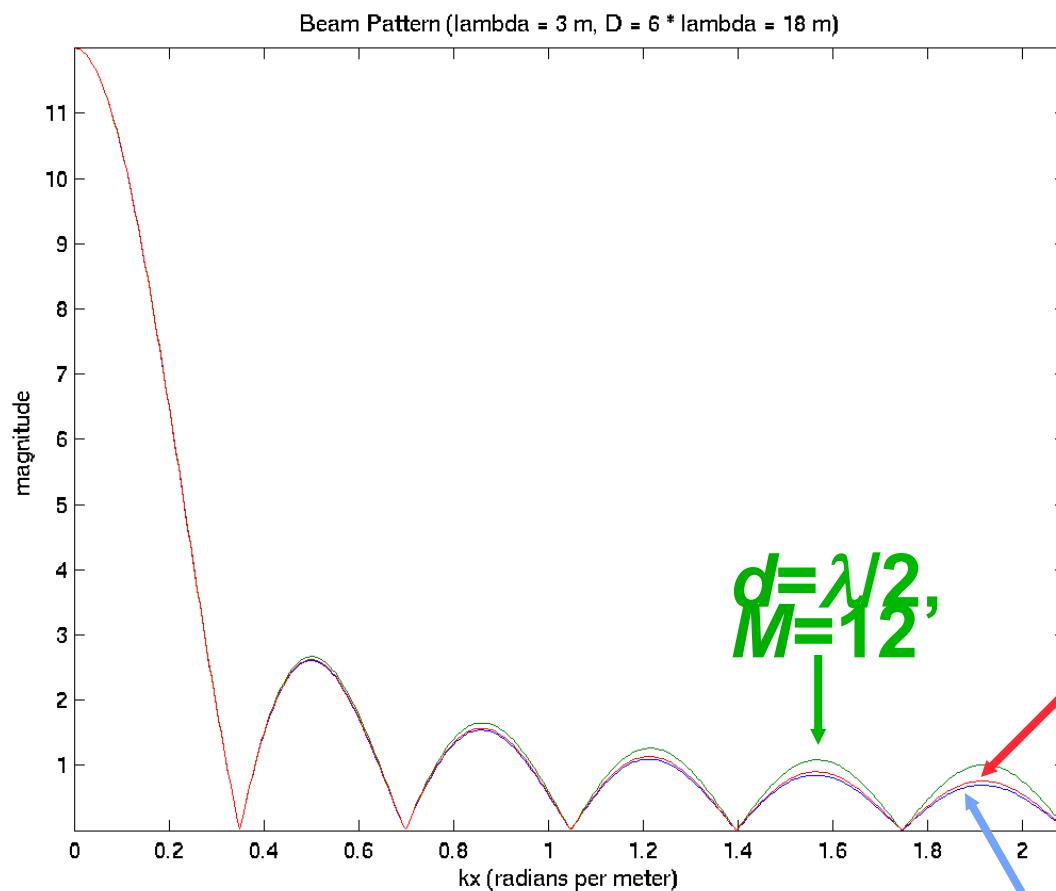
# Limiting to the X-Z Plane

- Suppose  $\theta = 0$

$$W\left(\frac{2\pi}{\lambda}, \phi, 0\right) = \frac{\sin\left(\frac{Md\pi}{\lambda} \sin \phi\right)}{\sin\left(\frac{d\pi}{\lambda} \sin \phi\right)}$$



# Comparison (Wavenumber)



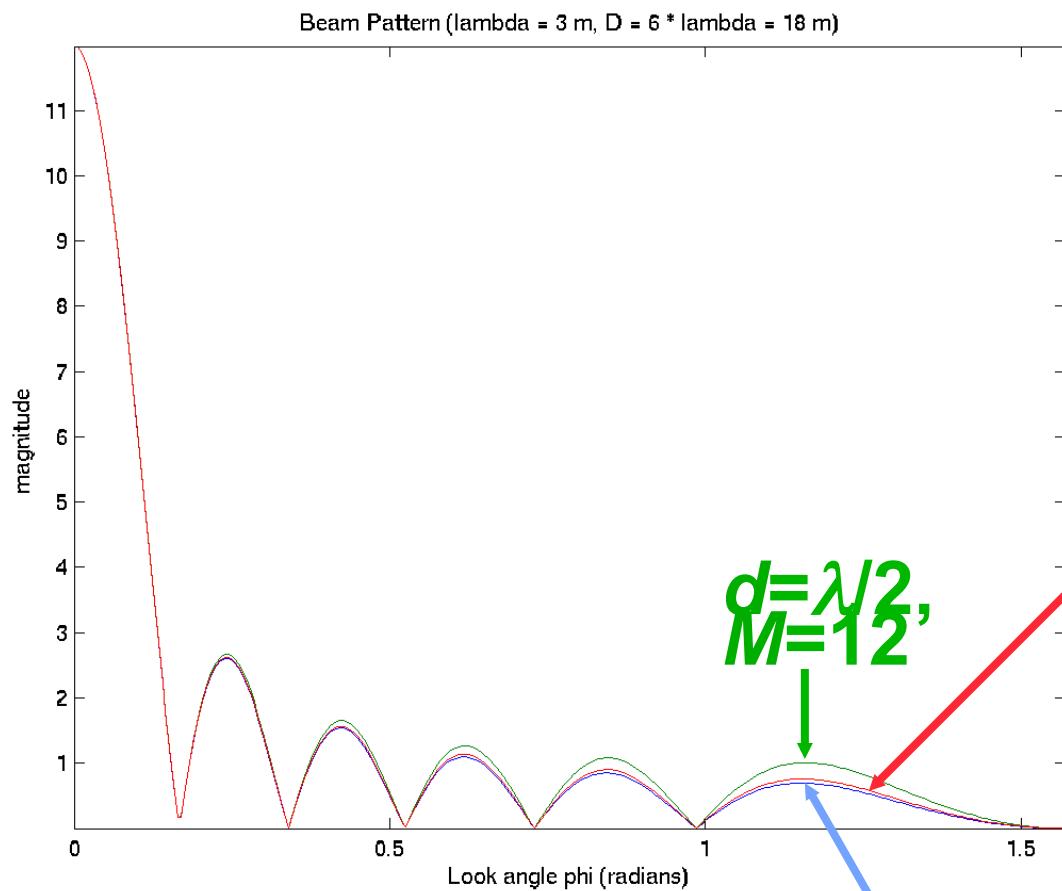
To study changes in overall shape, all plotted curves are artificially normalized to have gain 12 at angle 0

- 100 MHz wave
- 18 meter long aperture

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(sinc)



# Comparison (Phi)



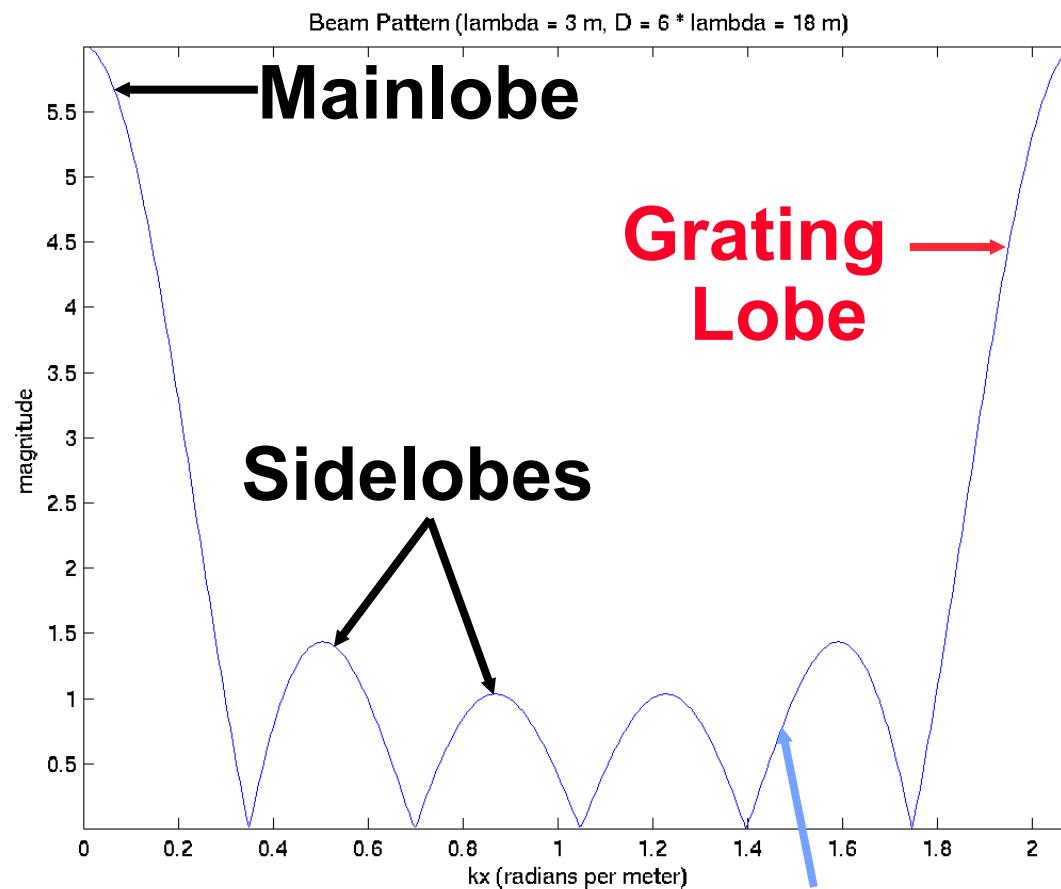
To study changes in overall shape, all plotted curves are artificially normalized to have gain 12 at angle 0



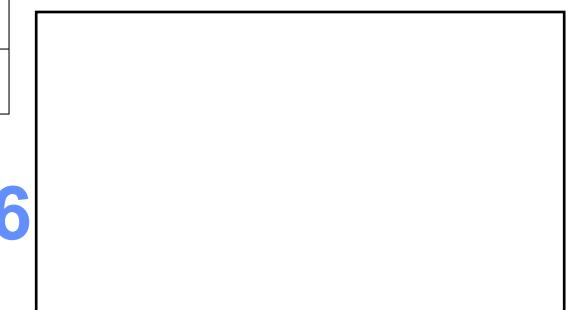
- 100 MHz wave
- 18 meter long aperture



# Grating Lobes



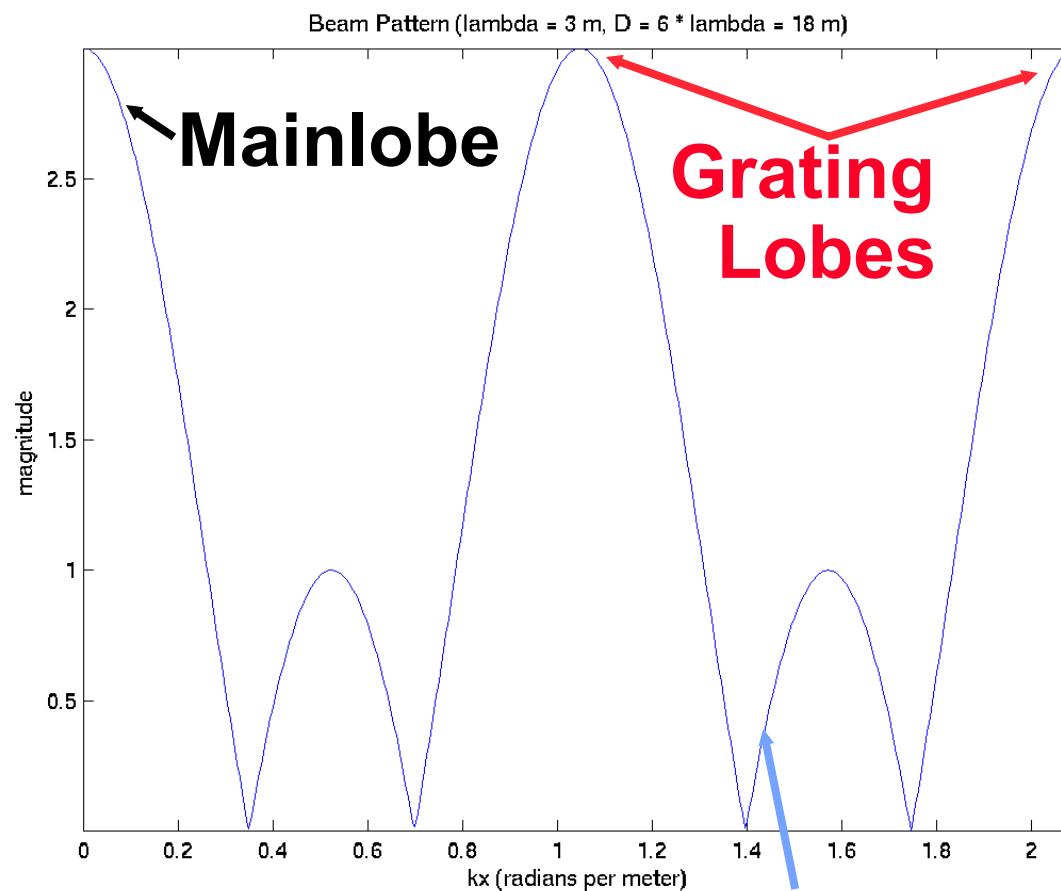
$$d = \lambda, M = 6$$



- 100 MHz wave
- 18 meter long aperture



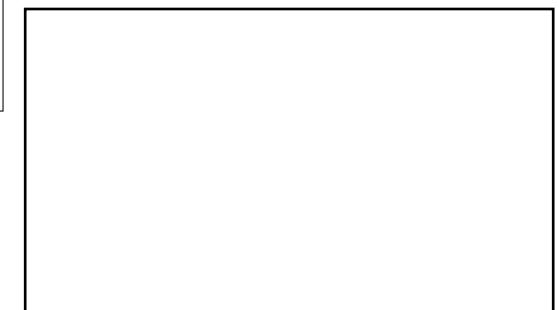
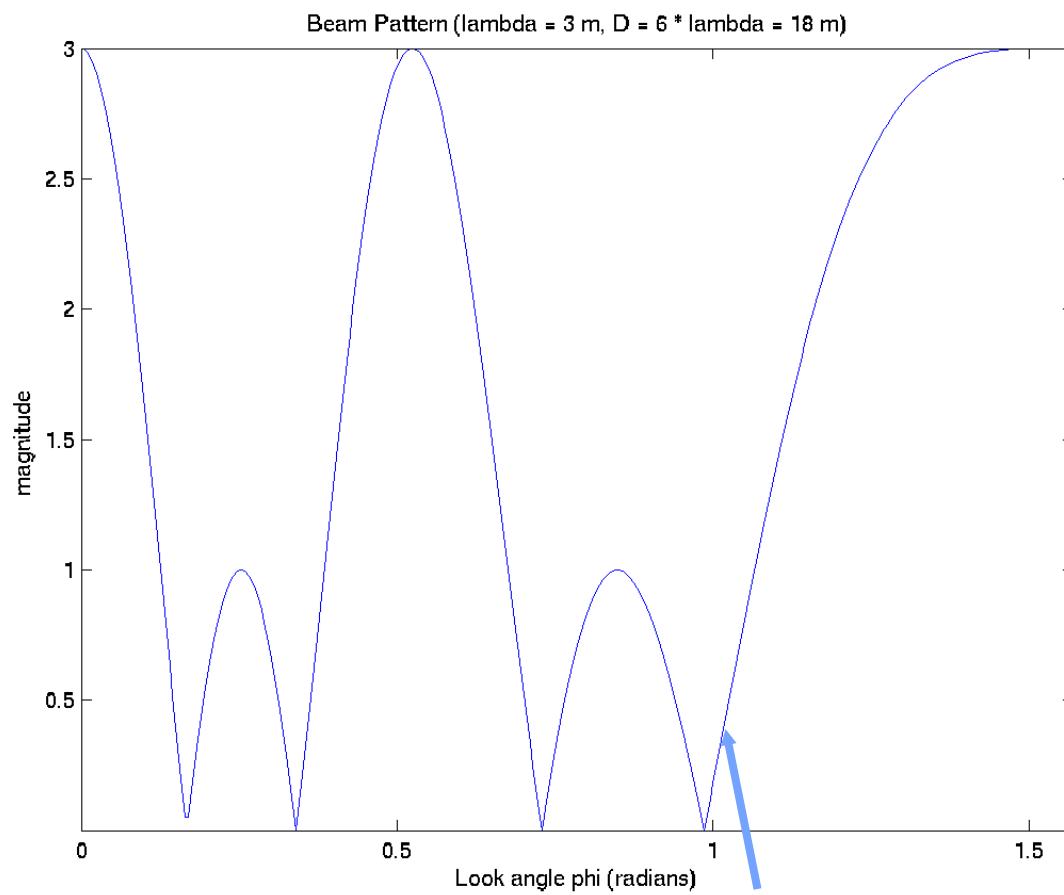
# More Grating Lobes



- 100 MHz wave
- 18 meter long aperture



# Stretching of Grating Lobes



- 100 MHz wave
- 18 meter long aperture



# Sampling Issues

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- When designing a uniform linear array, generally try to pick  $d \leq \lambda / 2$
- Can sometimes get away  $d > \lambda / 2$  with if you have side information
  - Detect a target with a low frequency
    - Long wavelength
  - Then track it with a high frequency
    - Short wavelength
    - Use knowledge from detector to disambiguate aliases
- Most papers will assume  $d \leq \lambda / 2$ 
  - Usually assume  $d = \lambda / 2$



# 2-D & 3-D Apertures

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- **Covered in detail by VT-IV, Chapter 4**
- **Can solve “cone of ambiguity” problem**
  - Careful: some 2-D apertures have their own kinds of cones of ambiguity
    - i.e. circular disk
- **Sampling issues get trickier to analyze**
  - Theory from ECE6258: Digital Image Processing applies here
    - **Hexagonal sampling, etc.**
  - Discussed in Sec. 3.2 of J&D



# Windowing

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- **By weighting an aperture by a window, we can**
  - Reduce sidelobe levels...
  - ...at the expense of a wider mainlobe
- **All your favorites from ECE4270 apply**
  - Bartlett, Dolph-Chevshev, Hamming, Hann, Kaiser, etc.
  - Just use them in space (ECE6279) instead of time (ECE4270)



# Integrating Across Apertures

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- Here's one way aperture smoothing functions show up
- Typically integrate across the aperture

$$z(t) = \int_{-\infty}^{\infty} w(\vec{x}) f(\vec{x}, t) d\vec{x}$$

- “Input” a monochromatic plane wave to the “system”

$$f(x, t) = \exp\{j(\omega_0 t - \vec{k}^0 \cdot \vec{x})\}$$

$$z(t) = \exp(j\omega_0 t) \underbrace{\int_{-\infty}^{\infty} w(\vec{x}) \exp(-jk^0 \cdot \vec{x}) d\vec{x}}_{W(-\vec{k}^0)}$$

