E9 231: Digital Array Signal Processing

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DESIGN OF THE WEIGHT VECTOR

The weight vectors are choosen to match the desired beam pattern $B(\psi)$, which is given by the following equation.

$$B(\psi) = w^H[V_\psi(\psi)] \tag{1}$$

where,

$$V_\psi(\psi)= \begin{bmatrix} e^{j(\frac{N-1}{2})\psi} \\ e^{j(\frac{N-1}{2}-1)\psi} \\ . \\ . \\ e^{-j(\frac{N-1}{2})\psi} \end{bmatrix} \tag{2}$$

Consider N spatial points with wavenumbers $\psi_1, ...\psi_N$.Let, $B = [B(\psi_1), ..., B(\psi_N)]$ and $V = [V_\psi(\psi_1), ..., V_\psi(\psi_N)]$.From equation (1), its clear that,

$$w^H V = B \Rightarrow B^H = V^H w \tag{3}$$

If V is invertible, then the weights can be obtained by the following design equation.

$$w = (V^H)^{-1} B^H \tag{4}$$

If ψ_i's are choosen badly, its difficult to invert V. Often one want nulls at $\psi_2, ..., \psi_N$ and 1 at ψ_1, i.,e $B(\psi_i) = 0, i = 2, ..., N$ and $B(\psi_1) = 1$ which can be achieved by the following equation,

$$W = (V_H)^{-1} e_1 \tag{5}$$

where, $e_1 = (10...0)^T$

Consider Uniform linear array with the beam pattern,

$$B(\theta) = \frac{1}{N} \frac{sin(\frac{2\pi d}{\lambda} cos\theta \frac{N}{2})}{sin(\frac{2\pi d}{\lambda} \frac{cos\theta}{2})}, \quad -\pi \leq \theta \leq \pi. \tag{6}$$

Example 1 : Let $\frac{d}{\lambda} = \frac{1}{2}$ and $N = 10$ which implies that the antennas array separation is small.

The resulting beam pattern, $B(\theta) = \frac{1}{N} \frac{sin(5\pi cos\theta)}{sin(\frac{\pi cos\theta}{2})}$

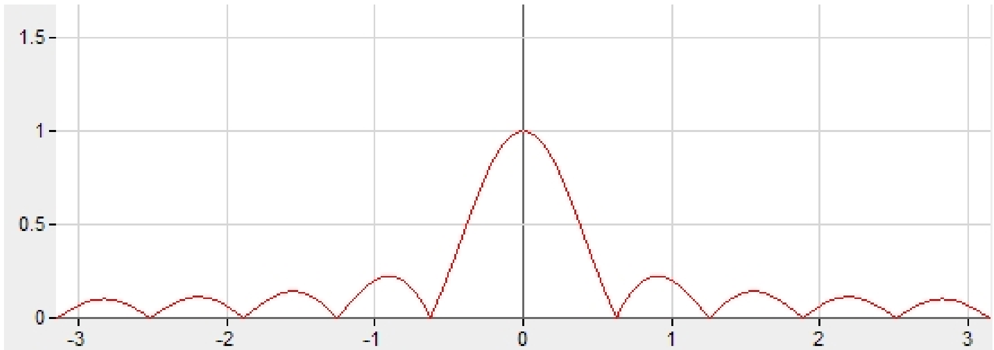

Figure 1: plot of $|B(\psi)|$ vs ψ in radians

Figure 2: plot of $|B(\theta)|$ vs θ in radians

Let, $\Psi = \pi cos\theta$.Then,the beam pattern in ψ space is given by,

$$B(\psi) = \frac{1}{10}\frac{sin(5\psi)}{sin\frac{\psi}{2}}, \qquad -\frac{2\pi d}{\lambda} \leq \psi \leq \frac{2\pi d}{\lambda} \Rightarrow -\pi \leq \psi \leq \pi. \tag{7}$$

The beam pattern is only defined over the range $-\pi \leq \psi \leq \pi$, the visible region.The plot of $|B(\psi)|$ vs ψ and $|B(\theta)|$ vs θ is shown in the figure(1) and (2) respectively.

Example 2: $\frac{d}{\lambda} = 2$(sensors are farther apart) and $N = 10$.
The beam pattern in ψ space is given by,

$$B(\psi) = \frac{sin(5\psi)}{10sin(\frac{\psi}{2})}, \qquad -4\pi \leq \psi \leq \ 4\pi. \tag{8}$$

The plot of $B(\psi)$ vs ψ is shown in Figure(3).Here the grating lobes are mathematically identical to aliasing(due to under-sampling).The polar plot of $B(\theta)$ vs θ is shown in figure(4)(refer class notes).

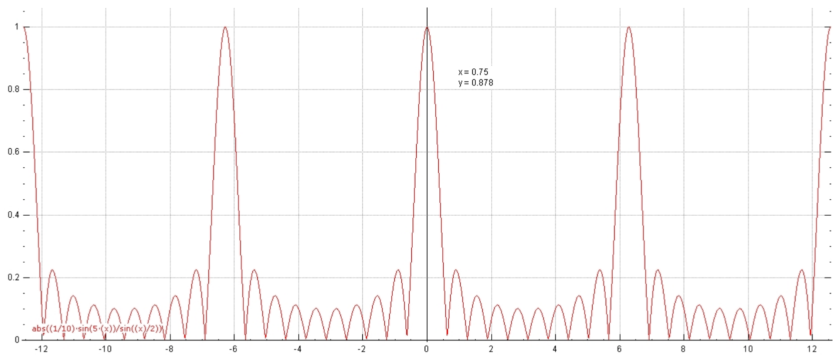

Figure 3: plot of $|B(\psi)|$ vs ψ in radians

Beam Steering:

Its clear from the previous discussion that the beam pattern response is at its best when $\theta = \frac{\pi}{2}$.However, in most of the practical situation θ is no longer eqaul to $\frac{\pi}{2}$.This demands the following two techniques,
1)Mechanical steering(inaccurate!)
2)Electronic steering.

Electronic steering:

The idea is to use a weighting vector at the output of the antenna array to compensate for the phase of arrival.Let w denote the weigting vector.Let $w = V_\psi(\psi_T)$.Then,

$$B_w(\psi) = w^H V = [e^{-j\frac{N-1}{2}\psi_T}...e^{j\frac{N-1}{2}\psi_T}] \begin{bmatrix} e^{j\frac{N-1}{2}\psi_T} \\ . \\ . \\ . \\ e^{-j\frac{N-1}{2}\psi_T} \end{bmatrix} = [e^{-j\frac{N-1}{2}(\psi-\psi_T)}...e^{j\frac{N-1}{2}(\psi-\psi_T)}] \tag{9}$$

Equation(9) is the beam pattern evaluated at $\psi - \psi_T$.For a uniform linear array,

$$B_\psi(\psi, \psi_T) = \frac{1}{N}\frac{sin((\psi-\psi_T)\frac{N}{2})}{sin(\frac{\psi-\psi_T}{2})} \tag{10}$$

In θ-space,

$$B_\psi(\theta, \theta_T) = \frac{1}{N}\frac{sin((cos\theta - cos\theta_T)\frac{N}{2})}{sin(\frac{cos\theta - cos\theta_T}{2})} \tag{11}$$

For no grating lobes, the above equation becomes,

$$B_\psi(\psi) = \frac{1}{N}\frac{sin((\psi)\frac{N}{2})}{sin(\frac{\psi}{2})} \tag{12}$$

$\psi = 2\pi$ is another main lobe which is outside the visible region :i.e., $\frac{d}{\lambda} \leq 1$
No grating lobe if,

$$-2\pi + |\psi_T| \leq -\frac{2\pi d}{\lambda} \tag{13}$$

$$\psi = \frac{2\pi d}{\lambda}u_T \Rightarrow \frac{2\pi d}{\lambda}(1+|\ u_T|) \leq 2\pi \Rightarrow \frac{d}{\lambda} \leq \frac{1}{1+|u_T|}, \quad -1 \leq\ u_T \leq 1 \tag{14}$$

For $\frac{d}{\lambda} \leq \frac{1}{2}$, there will be no grating lobes regardless of u_T.

Half power bandwidth

Let ψ_L and ψ_R be those values which satisfies the following equation:

$$|B(\psi)|^2 = 1/2 \tag{15}$$

where,

$$B(\psi) = \frac{1}{N} \frac{sin\frac{\psi N}{2}}{sin(\frac{\psi}{2})} \tag{16}$$

Then, HPBW $\Delta u_{HPBW} := \psi_R - \psi_L$.
If N is of the order of 10, $\Delta u_{HPBW} \approx \frac{0.891}{Nd}\lambda$ (Home work!).
Then,

$$u_L \approx u_T - \Delta u_{HPBW}/2 \tag{17}$$

$$u_R \approx u_T + \Delta u_{HPBW}/2 \tag{18}$$

Thus, the half power bandwidth is,

$$\theta_H := \theta_R - \theta_L = cos^{-1}[cos\theta_T - \frac{0.450\lambda}{Nd}] + cos^{-1}[cos\theta_T + \frac{0.450\lambda}{Nd}] \tag{19}$$

$$\approx 0.89\frac{\lambda}{Nd}sec(\frac{\pi}{2} - \theta_T) \tag{20}$$

$$= 0.89\frac{\lambda}{Nd\cos\overline{\theta_T}} \quad ;\overline{\theta_T} := \pi/2 - \theta_T \tag{21}$$

The following observations can be made:
1. θ_H increases as you steer the antenna away from $\theta = \pi/2$
2. θ_H varies as $1/N$ for small angles around $\pi/2$.
3. θ_H varies as $1/\sqrt{N}$ for angles close to 0 or π.