E9 231: Digital Array Signal Processing

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Beam Space beamforming

1. If Sx is large then inverse Sx computation is large.
2. Also estimating Sx is large

Another approach to reduce the dimensionality(and complexity) of the processor

- Perform priliminary processing with a set of non adoptive beams

$$B_{bs}^{H} = \begin{bmatrix} b_{bs}^{H}, 1 \\ b_{bs}^{H}, 2 \\ . \\ . \\ b_{bs}^{H}, N_{bs} \end{bmatrix} \tag{1}$$

$$b_{bs}^{H}, m = -\frac{1}{\sqrt{N}}(V_u(u - \frac{(m-N)}{N})2) \tag{2}$$

$$X_{bs} = B_{bs}^{H} * X$$

$$\text{require } B_{bs}^{H} * B_{bs} = 1$$

Beam space array manifold is given as $V_{bs} = B_{bs} * V_{bs}$

$$\text{and } Y = W_{bs}^{H} * X_{bs} = W_{bs} * V_{bs}$$

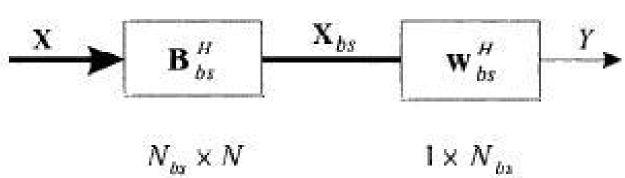

Figure 1: Beam Space Processing

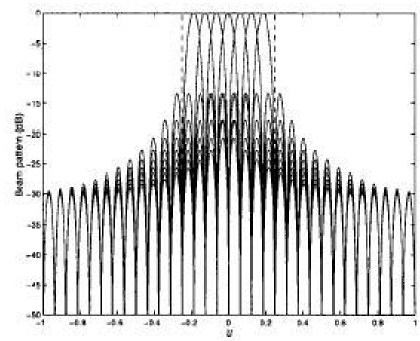

Figure 2: Beam Pattern in Beam Space Processing for 7 beams

Beam pattern is given as $B(\phi) = W_{bs} * V_{bs}(\phi)$

Spatial Spectral Matrix

$$S_{X_b s} = B_{bs}^H S_x B_{bs} \tag{3}$$

for single plane wave desired signal plus interference plus white noise
so

$$S_X = \sigma_s^2 V_s V_s^H + S_c + \sigma_n^2 I \tag{4}$$

$$S_{X_b s} = \sigma_s^2 B_{bs}^H V_s V_s^H B_{bs} + B_{bs}^H S_c B_{bs}^H + \sigma_n^2 I \tag{5}$$

$$S_{X_b s} = \sigma_s^2 V_{bs} V_{bs}^H + S_{cbs} + \sigma_n^2 I \tag{6}$$

Now can use one of the optimum beam former

$$W_{mpdr,bs} = \frac{V_{bs}^H S_{x_b s}^{-1}}{V_{bs}^H S_{x_b s}^{-1} V_{bs}} \tag{7}$$

cost of this beam former is that number of degree of freedom has been reduced so number of interferers that can be null als reduced.
One solution to the reduction in the no. of interferers that can be suppressed to use window but B_{bs} may not orthogonal.
Let us denote non orthogonal B_{bs} as B_{NO}
then

$$S_{X_b s} = \sigma_s^2 V_{bs} V_{bs}^H B_b s + B_{NO}^H S_c B_{NO} + \sigma_n^2 B_{NO}^H B_{NO} \tag{8}$$

so we use noise whitening filter given by equation

$$H_w = (B_{NO}^H B_{NO})^{-\frac{1}{2}} \tag{9}$$

then overall matrix filter would be

$$B_{bs}^H = (B_{NO}^H B_{NO})^{-\frac{1}{2}} B_{NS}^H \tag{10}$$

we use B_{bs} to form the beams.
one can similarly derive Beam space LCMP
few facts regarding beam space beam formation
-computational complexity reduced from

-more robust to array perturbations
-by using low sidelobe beams, we can exchange the sector of interest
-works well if number of interferers less than number of beams
- significant advantage is that estimating $S_{x_b s}$ is easier than estimating S_x

Beam forming in a Correlated environment

$$X = VF + N = V_s F_s + V_I F_I + N \tag{11}$$

$$S_x = V S_f V^H + \sigma_n^2 \tag{12}$$

where

$$S_f = \begin{bmatrix} \sigma_f^2 & \Sigma_{IF} \\ \Sigma_{FI} & S_I \end{bmatrix} \tag{13}$$

causes of correlation are followings
1. Multipath propagation
2. Presence of intelligent jammers

There are two approches to reduce correlation
1. Preprocessing(Spatial smoothning) to reduce correlation between signal and interference followed by MPDR beamforming.
2. Use MMSE beamformer(It is same as MPDR if signal and interference are uncorrelated)
best linear estimate of interference given the signal is given as

$$\hat{F}_I = \Sigma_{IF} \sigma_f^{-2} F_s \tag{14}$$

Let $\tilde{F}_I = F_I - \hat{F}_I$

thus $F_I = \tilde{F}_I + \hat{F}_I = \tilde{F}_I + AF_s$

due to orthogonality principle

$E[\tilde{F}_I F_s^H] = 0$

can now write

$$X = V_s F_s + V_I(\tilde{F}_I + AF_s) + N$$
$$= (V_s + V_I A)F_s + V_I \tilde{F}_I + N$$
$$(V_s + \acute{V}_I) = (V_s + V_I A) \text{ is called virtual direction of arrival}$$

so for MPDR

$$W_{mpdr}^H V_s = 1$$

$$W_{mpdr} = \frac{S_x^{-1} V_s}{V_s^H S_x^{-1} V_s} \tag{15}$$

$$\Sigma_{IF} = 0$$
$$Y_{mpdr} = W_{mpdr}^H X$$
$$Y_{mpdr} = W_{mpdr}^H (V_s + \acute{V}_I) F_s + W_{mpdr}^H V_I \tilde{F}_I + W_{mpdr}^H N$$

$$SNR_o = \frac{E[|W^H (V_s + \acute{V}_I) F_s|^2]}{W^H V_I \Sigma_{\tilde{F}} V_I^H W + \sigma_n^2 W^H W} \tag{16}$$

Single mainlobe Interferers

$$V_I^H V_s = N$$
$$\Sigma_{IF} = \rho * \sigma_s \sigma_I$$

$$SNR_o = \frac{\sigma^2(|1 + \rho * \sigma_I |\sigma_s|^2|)}{\sigma_I^2(1 - |\rho|^2) + \frac{\sigma_n^2}{N}} \tag{17}$$

–could be constructive or destructive.

- if $\rho = 1$ and $\sigma_n^2 = 0, SNR_o = 0$

How good is MPDR beamformer

$$Y_{mpdr} = (1 + W^H \tilde{V}_I)F_s + W^H V_I \tilde{F}_I + W^H N$$
$$E[|Y_{mpdr}|^2] = \sigma_s^2 |1 + W^H \tilde{V}_I|^2 + W^H V_I \Sigma_{\tilde{F}} V_I^H W + \sigma_n^2 W^H W$$

MPDR tries to minimize this such that $W^H V_s = 1$

If signal and interference are uncorrelated, $\acute{V}_I = 0$,
tries to minimze interference plus noise hence it is good.

As correlation increases ,$\Sigma_{\tilde{F}}$ decreases hence $\tilde{F}$ tends to zero.
$\Rightarrow W^H V_I \Sigma_{\tilde{F}} V_I^H W \to 0$

$\Rightarrow E[|Y_{mpdr}|^2]$ dominated by $\sigma_s^2 |1 + W^H \tilde{V}_I|^2$

$\Rightarrow$ will end up choosing W $W^H \tilde{V}_I = -1$ so it is not so good

LMMSE Beam Former

It is a good solution for correlated signal and interference as it exploits all the available(correlation) information.

$$W_{LMMSE} = S_x^{-1} \Sigma_{xf}$$
$$\Sigma_{xf} = E[X F_s^H]$$
$$= [V_s + \tilde{V}_I]\sigma_s^2$$

so

$$W_{LMMSE} = \sigma_s^2 S_x^{-1}[V_s + \tilde{V}_I]$$

first factor of above equation puts nulls in the direction of uncorrelated interferers and second factor is virtual direction

cost of this is estimation of S_x^{-1} and Σ_{xf} [1]

[1] All the figures considered in this lecture notes are taken from *Optimum Array Processing, Part IV of Detection, Estimation And Modulation Theory* by Harry L. Van Trees, 2002. Wiley-InterScience