E9 231: Digital Array Signal Processing

Scribe: Nadir Cazi
Dept. of EE & ECE
Indian Institute of Science
Bangalore 560 012, India
nadir@ee.iisc.ernet.in

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1 Minutes of Last Lecture

- Null Steering
- Asymmetric Beams: DOA estimation
- Minimum Redundancy Array
- Perfect MRA
- Beam pattern design algorithm

2 Today's topics

1. Beamspace Processing
2. Broadband Arrays
3. Planar Arrays

3 Beamspace Processing

In element-space processing, computing W may be difficult due to

- Large arrays(N large)
- Adaptive processing

Consider an N-element Uniform Linear Array and steer beams at a spacing of $\frac{2}{N}$

$$B_m(u) = \frac{1}{N}\frac{\sin\left(\frac{N\pi}{2}\left(u - \frac{2m}{N}\right)\right)}{\sin\left(\frac{\pi}{2}\left(u - \frac{2m}{N}\right)\right)}, \quad -\frac{N-1}{2} \leq m \leq \frac{N-1}{2} \tag{1}$$

$$B_m(u) = \mathbf{W}_m{}^{\mathbf{H}}\mathbf{V}(u) \tag{2}$$

$$\mathbf{W}_m = \frac{1}{N}\mathbf{V}\left(\frac{2m}{N}\right) \tag{3}$$

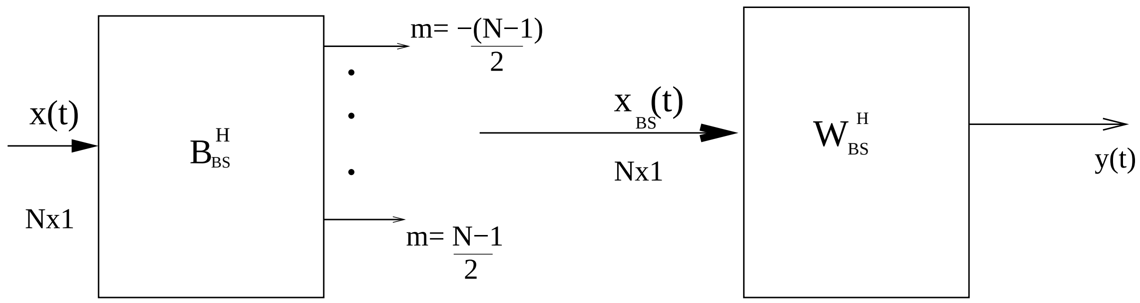


Figure 1: *Butler Beamformer*

$$\mathbf{B}_{BS} = \left[\mathbf{W}_{-\frac{N-1}{2}}, \mathbf{W}_{-\frac{N-1}{2}+1}, ..., \mathbf{W}_{\frac{N-1}{2}}\right]_{N\times N} \tag{4}$$

$$y(t) = \mathbf{W}_{BS}{}^{\mathbf{H}}\mathbf{B}_{BS}{}^{\mathbf{H}}\underline{x}(t) \tag{5}$$

$$y(t) = \mathbf{W}_{ES}{}^{\mathbf{H}}\underline{x}(t) \tag{6}$$

$$\mathbf{W}_{ES} = \mathbf{B}_{BS} * \mathbf{W}_{BS} \tag{7}$$

$\mathbf{W}_{ES}$ is the element-space combining weights

- In a lot of applications, $\mathbf{W}_{BS}$ turns out to be a sparse vector.
- Although $\mathbf{W}_{BS}$ may be sparse, $\mathbf{W}_{ES}$ may not sparse.
- $\mathbf{B}_{BS}{}^{\mathbf{H}} = \left[\mathbf{W}_{-\frac{N-1}{2}}, \mathbf{W}_{-\frac{N-1}{2}+1}, ..., \mathbf{W}_{\frac{N-1}{2}}\right]^{\mathbf{H}}$ = DFT matrix if the array is a standard ULA. It is called the Butler matrix.
- If we want nulls in time, then element-space processing is better as we know the timing information.
- If we need to place nulls in space, beamprocessing is better.

4 Broadband Arrays

Arrays that need to process signals over a broad frequency band.

Recall:
$Re\{\tilde{f}(t)e^{j2\pi f_c t}\} \underrightarrow{delay} Re\{\tilde{f}(t-\tau)e^{j2\pi f_c(t-\tau)}\} \approx Re\{\tilde{f}(t)e^{-j2\pi f_c \tau}e^{j2\pi f_c t}\}$

To carry this assumption over to wideband signals, split the band into multiple frequencies:
Must ensure

$$f_L < f_1 < .... < f_U \tag{8}$$

$$\frac{d}{\lambda} \leq \frac{1}{2} \quad \forall\lambda \ \text{, for no grating lobe} \tag{9}$$

In particular,

$$\frac{d}{\lambda_{f_U}} \leq \frac{1}{2} \tag{10}$$

Lets look at HPBW

$$HPBW = \alpha \cdot \frac{\lambda}{Nd} \tag{11}$$

$$HPBW = \alpha \cdot \frac{\lambda_{f_L}}{N \cdot \frac{1}{2}\lambda_{f_U}} \tag{12}$$

To satisfy a given HPBW constraint, we need

$$N = \frac{2\alpha}{HPBW} \cdot \frac{\lambda_{f_L}}{\lambda_{f_U}} = \frac{2\alpha}{HPBW} \cdot \frac{f_U}{f_L}, \tag{13}$$

thus N may be very large.

4.1 Potential Solution

Use a nested non-uniform array

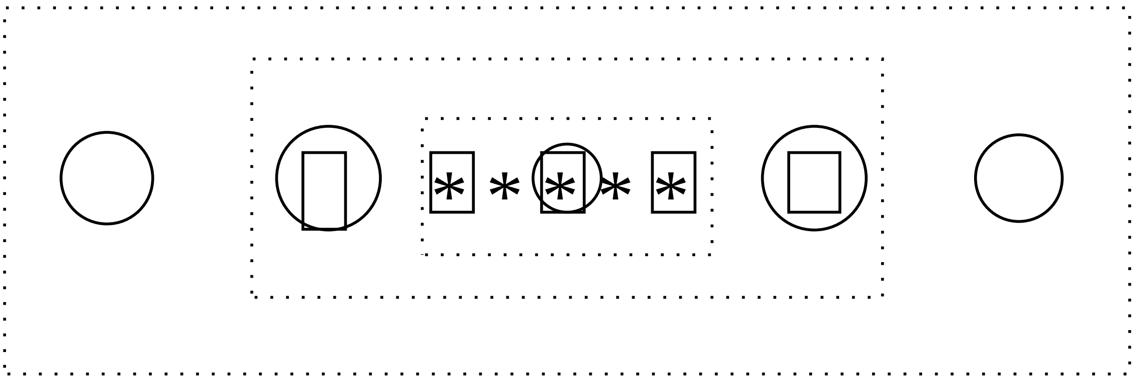


Figure 2: *Nested non-uniform array*

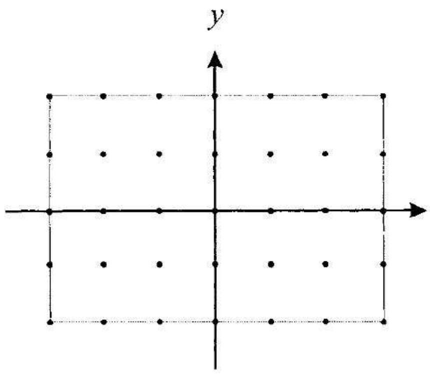
$$\text{Total no. of elements} = N + (\text{no. of levels-1})\left(\frac{N-1}{2}\right) \tag{14}$$

Can process signals in frequency range $\frac{f_U}{8}$ to f_U

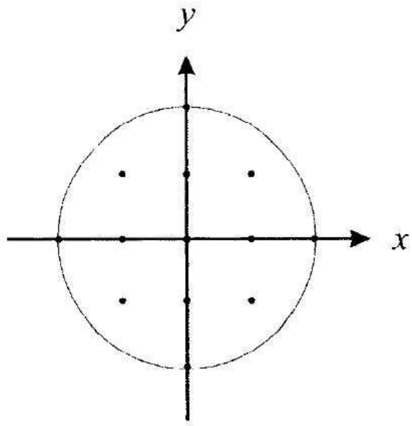
Disadvantage: A limited set of band ratios are achievable.

5 Planar Arrays

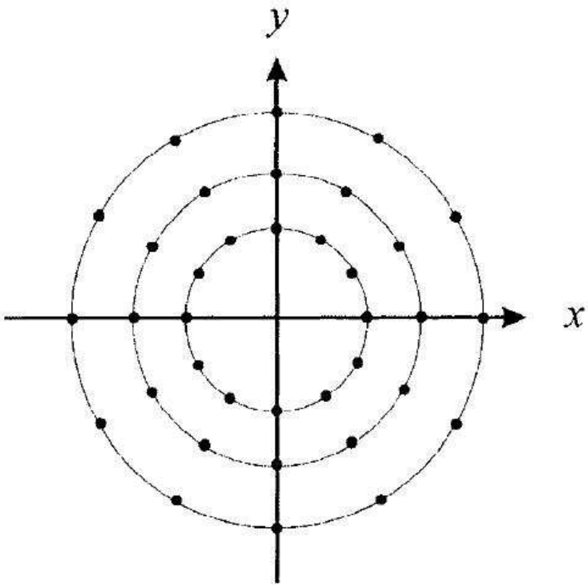
Can distinguish signals in both θ (elevation) and ϕ (azimuth).



Rectangular grid,
rectangular boundary



Rectangular grid,
circular boundary



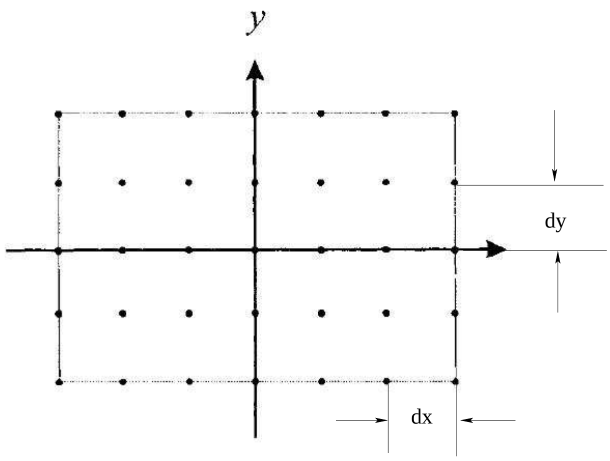
Concentric
Circular array

5.1 Uniform Rectangular Array

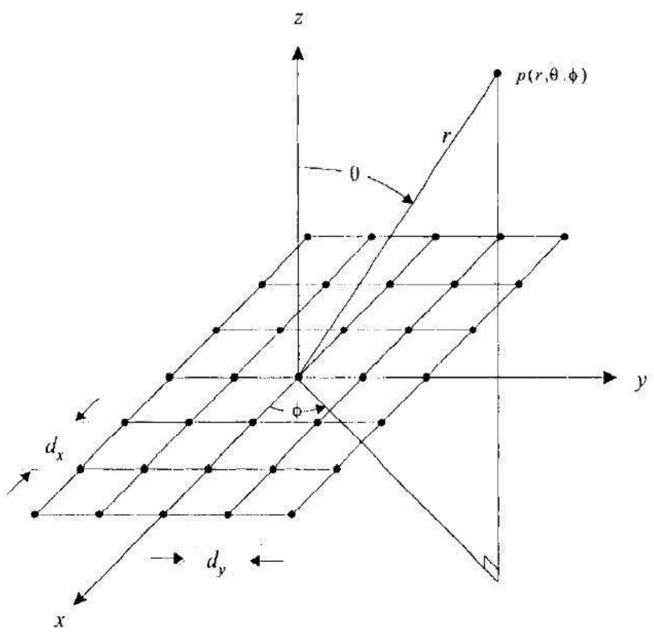
$$
\begin{aligned}
x &= r \sin\theta \cos\phi \\
y &= r \sin\theta \sin\phi \\
z &= r \cos\theta
\end{aligned}
$$

Since the planar array is symmetric with respect to θ about $\theta = 90°$, we consider

$$
\begin{aligned}
0° \leq \theta \leq 90° \\
0° \leq \phi < 360°
\end{aligned}
$$



Rectangular grid,
rectangular boundary



Planar array geometry.

Waves arriving from θ and $180° - \theta$ are indistinguishable.

$$\underline{p}_{nm} = \begin{bmatrix} (n - \frac{N-1}{2})d_x \\ (n - \frac{M-1}{2})d_y \\ 0 \end{bmatrix}, \tag{15}$$

$$\kappa = \frac{-2\pi}{\lambda} \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} \tag{16}$$

$$\omega\tau_{nm} = \kappa^T p_{nm} = \frac{-2\pi}{\lambda}\left(\left(n - \frac{N-1}{2}\right) dx \sin\theta\cos\phi + \left(m - \frac{M-1}{2}\right) dy \sin\theta\sin\phi\right) \tag{17}$$

$$\begin{aligned} u_x &\triangleq \sin\theta\cos\phi \\ u_y &\triangleq \sin\theta\sin\phi \\ \psi_x &\triangleq \frac{2\pi}{\lambda} d_x u_x \\ \psi_y &\triangleq \frac{2\pi}{\lambda} d_y u_y \end{aligned}$$

we can use (θ, ϕ), or (u_x, u_y), or (ψ_x, ψ_y) to describe the array response.

Let ${w_{nm}}^*$ be the weight applied to the $(n, m)^{th}$ sensor. Then

$$B(\psi_x, \psi_y) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} {w_{nm}}^* e^{-j\omega\tau_{nm}} = e^{-j\left(\left(\frac{N-1}{2}\right)\psi_x + \left(\frac{M-1}{2}\right)\psi_y\right)} \left(\underbrace{\sum_{n,m} w_{nm} e^{-j(n\psi_x + m\psi_y)}}\right)^* \tag{18}$$

The underbraced term can be recognized as the 2d - Fourier Transform