2006 IEEE RADAR Conference
April 24-27
Verona, NY - USA

Cancellation of clutter and multipath in passive radar using a sequential approach

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The authors acknowledge the support of Finmeccanica through SESM

PCL Radar

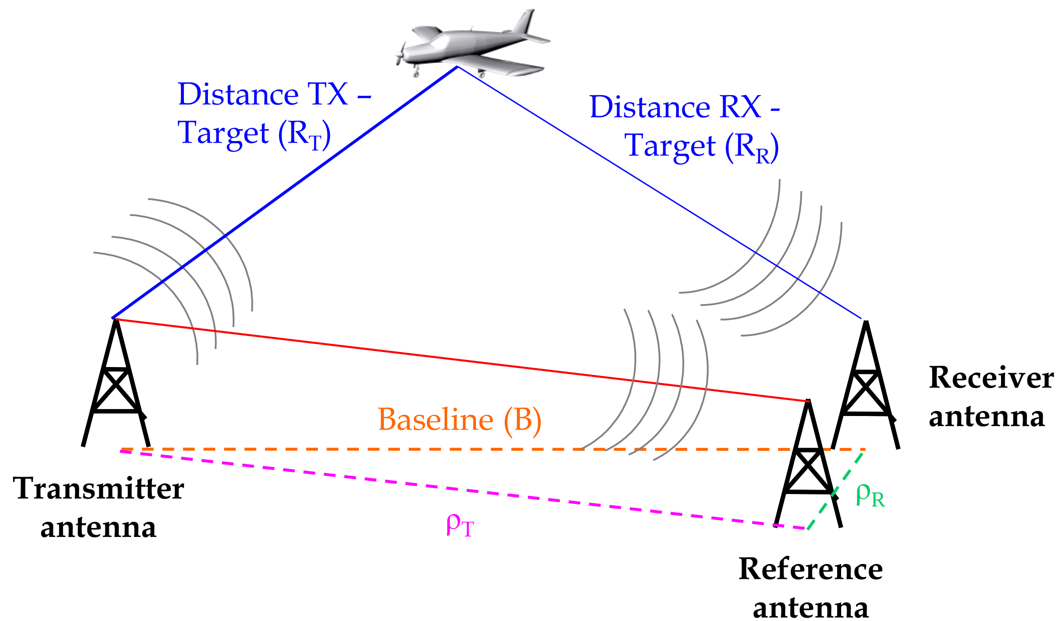
Passive radars exploit existing transmitters as illuminators of opportunity to perform target detection and localization.

<u>Advantages</u>:

-low cost,

-covert operation, low vulnerability,

-reduced impact on the environment



<u>Drawbacks:</u>

The transmitted waveform is not under control of the radar designer:

- continuous wave and low power levels → long integration time
- high sidelobes of the ambiguity function with a time-varying structure

Target echoes can be masked by:

- the fraction of the direct signal received by the side/backlobe of the receiver antenna
- strong clutter/multipath echoes, and
- echoes from other strong targets at short ranges.

Outline

A sequential approach is presented for disturbance cancellation and target detection based on projections of the received signals in a subspace orthogonal to both disturbance and previously detected targets.

- Signal model and reference scenario
- Least square technique for extensive cancellation of clutter and direct signal
- Sequential disturbance cancellation technique
- Ordering strategy and stop criterion
- Extension of the sequential approach to the detection of multiple targets in order to remove the masking effect of the strong targets on the others
- Conclusions

Signal model

- Complex envelope of the signal at the Rx channel:

$$s_R(t) = A_R(t)d(t) + \sum_{m=1}^{M} a_m d(t-\tau_m) e^{j2\pi f_{dm} t} + \sum_{i=1}^{N_c} c_i(t)\, d(t-\tau_{ci}) + n_R(t) \qquad 0 \le t < T_0$$

direct signal: delayed and complex scaled replica of the transmitted signal

m-th **target** contribution: delayed, complex scaled and Doppler shifted replica of the direct signal

i-th **stationary scatterer** contribution:

$$c_i(t) = \sum_{p=-P}^{P} c_{ip} e^{j2\pi p \frac{t}{T}}$$

thermal noise

- Complex envelope of the signal at the Ref channel:

$$s_{ref}(t) = A_{ref}\, d(t) + n_{ref}(t)$$

Hp: 1) targets and clutter echoes are negligible;
2) reference signal free of multipath.

Signal model

- Complex envelope of the signal at the Rx channel:

$$\mathbf{s}_R = \left[s_R(t_0) \quad s_R(t_1) \quad \cdots \quad s_R(t_{N-1}) \right]^T$$

(N×1)

The samples collected at the RX and Ref channels at time instants $t_i = iT$ are arranged in column vectors

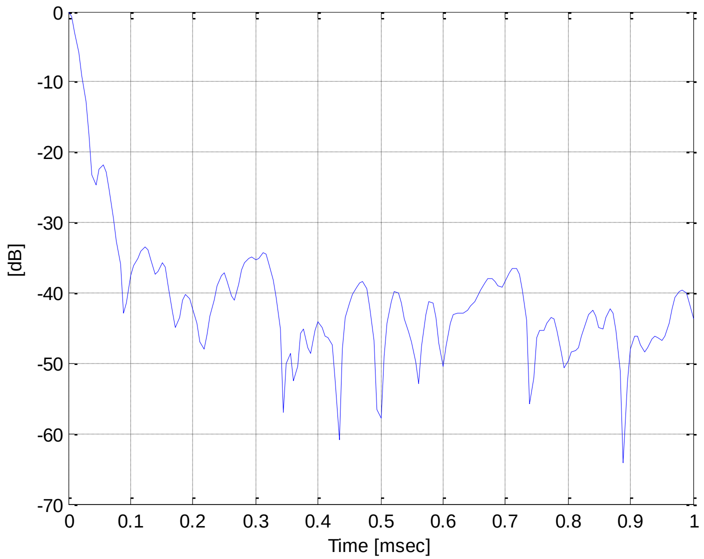
- Complex envelope of the signal at the Ref channel:

$$\mathbf{s}_{ref\_L} = \left[s_{ref}(t_{-R+1}) \quad \cdots \quad s_{ref}(t_0) \quad \cdots \quad s_{ref}(t_{N-1}) \right]^T$$

(N+R-1×1)

Reference scenario

- ✓ FM radio Tx (88÷108 MHz) with BW=150 kHz;
- ✓ N_c=4 clutter spikes with different RCS∈[35÷40]dB, spread on 3 adjacent Doppler bins (*P*=1) and different delays within the first 45 range bins;
- ✓ *M*=3 targets with different RCS.



Reference scenario

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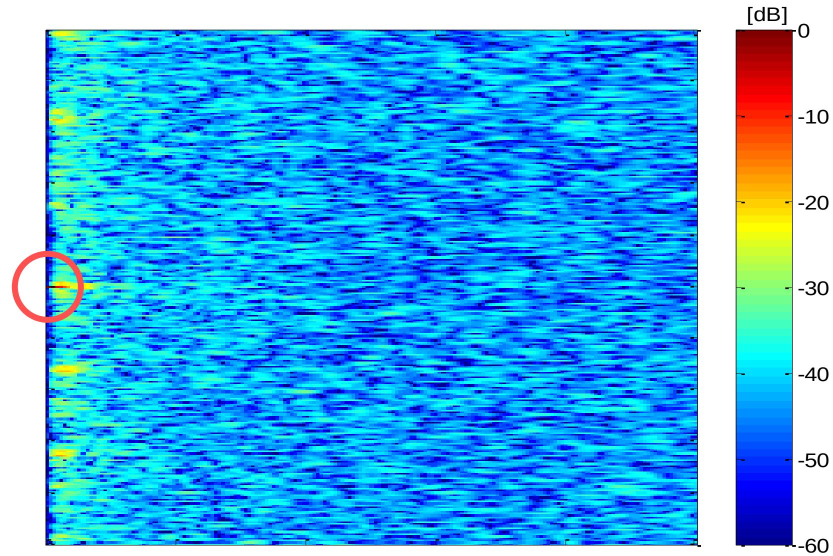
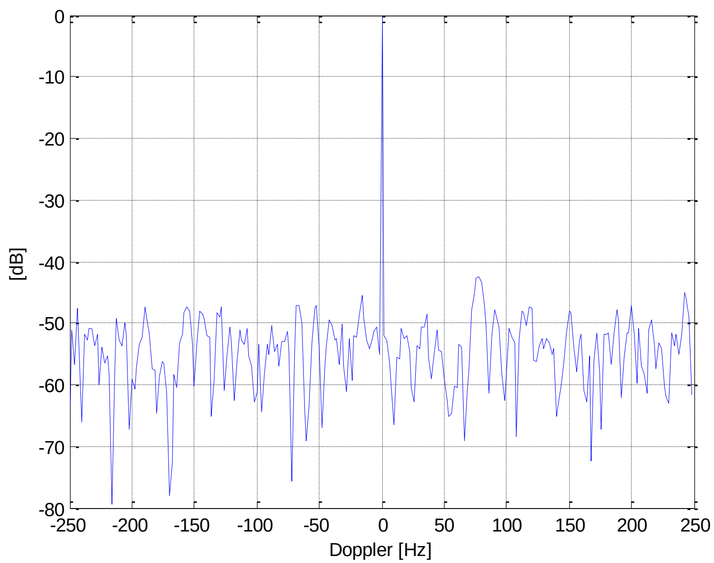
- ✓ M=3 targets with different RCS.

2D matched filter output

$$y_{MF}(\tau, f_d) = \sum_{n=1}^{N} s_R(nT) \cdot s_{ref}^{*}(nT-\tau) e^{-j2\pi f_d nT}$$





A strong peak corresponding to direct signal is present, while both targets and clutter are completely masked by its sidelobes.

Extensive Cancellation Algorithm (ECA)

Hp: the clutter echoes are potentially backscattered from the first K range bins

An effective cancellation filter for the passive radar can be obtained by resorting to the **Least Square (LS) approach**:

minimum residual signal power after cancellation of the disturbance:

$$\boxed{\min_{\boldsymbol{\alpha}} \left\{ \left\| \mathbf{s}_R - \mathbf{X}\boldsymbol{\alpha} \right\|^2 \right\}}$$

where

$$\mathbf{X} = \mathbf{B}\left[\boldsymbol{\Lambda}_{-P}\mathbf{S}_{ref} \quad \cdots \; \boldsymbol{\Lambda}_{-1}\mathbf{S}_{ref} \quad \mathbf{S}_{ref} \quad \boldsymbol{\Lambda}_{1}\mathbf{S}_{ref} \cdots \boldsymbol{\Lambda}_{P}\mathbf{S}_{ref} \right]$$

diagonal matrix that applies the phase shift corresponding to the P-th Doppler bin

incidence matrix that selects only the last N rows of the following matrix

$$\mathbf{S}_{ref} = \left[\mathbf{s}_{ref\_L} \quad \mathbf{D}\mathbf{s}_{ref\_L} \quad \mathbf{D}^2\mathbf{s}_{ref\_L} \quad \cdots \quad \mathbf{D}^{K-1}\mathbf{s}_{ref\_L} \right]$$

Reference signal delayed by K-1 bins

The columns of **X** define a basis for the M-dimensional disturbance subspace (M=(2P+1)K)

$$\boxed{\boldsymbol{\alpha} = \left(\mathbf{X}^H\mathbf{X}\right)^{-1}\mathbf{X}^H\mathbf{s}_R}$$

Extensive Cancellation Algorithm (ECA)

$$\boxed{\min_{\boldsymbol{\alpha}} \left\{ \| \mathbf{s}_R - \mathbf{X}\boldsymbol{\alpha} \|^2 \right\}}$$

$$\mathbf{X} = \mathbf{B}\left[\boldsymbol{\Lambda}_{-P}\mathbf{S}_{ref} \quad \cdots \boldsymbol{\Lambda}_{-1}\mathbf{S}_{ref} \quad \mathbf{S}_{ref} \quad \boldsymbol{\Lambda}_{1}\mathbf{S}_{ref} \cdots \quad \boldsymbol{\Lambda}_{P}\mathbf{S}_{ref} \right]$$

$$\Rightarrow \boxed{\boldsymbol{\alpha} = \left(\mathbf{X}^H \mathbf{X} \right)^{-1} \mathbf{X}^H \mathbf{s}_R}$$

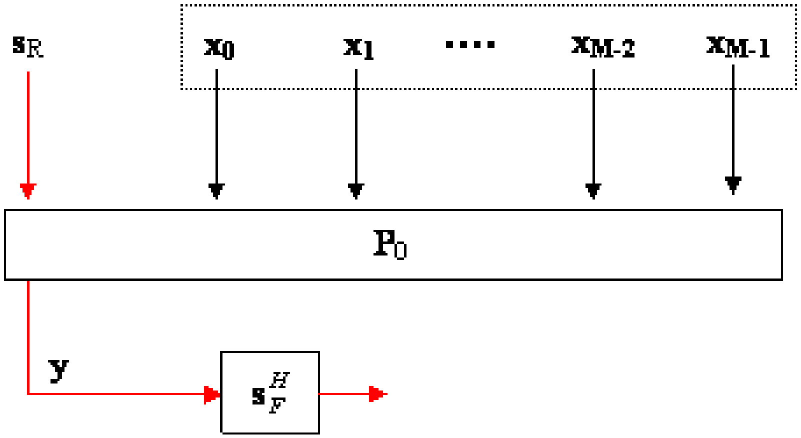
Received signal after cancellation

$$\mathbf{y} = \mathbf{s}_R - \mathbf{X}\boldsymbol{\alpha} = \left[\mathbf{I}_N - \mathbf{X}\left(\mathbf{X}^H \mathbf{X} \right)^{-1} \mathbf{X}^H \right] \mathbf{s}_R = \mathbf{P}_0 \, \mathbf{s}_R$$

projection matrix

Matched filtering

$$y_{MF} = \mathbf{s}_F^H \mathbf{P}_0 \, \mathbf{s}_R$$



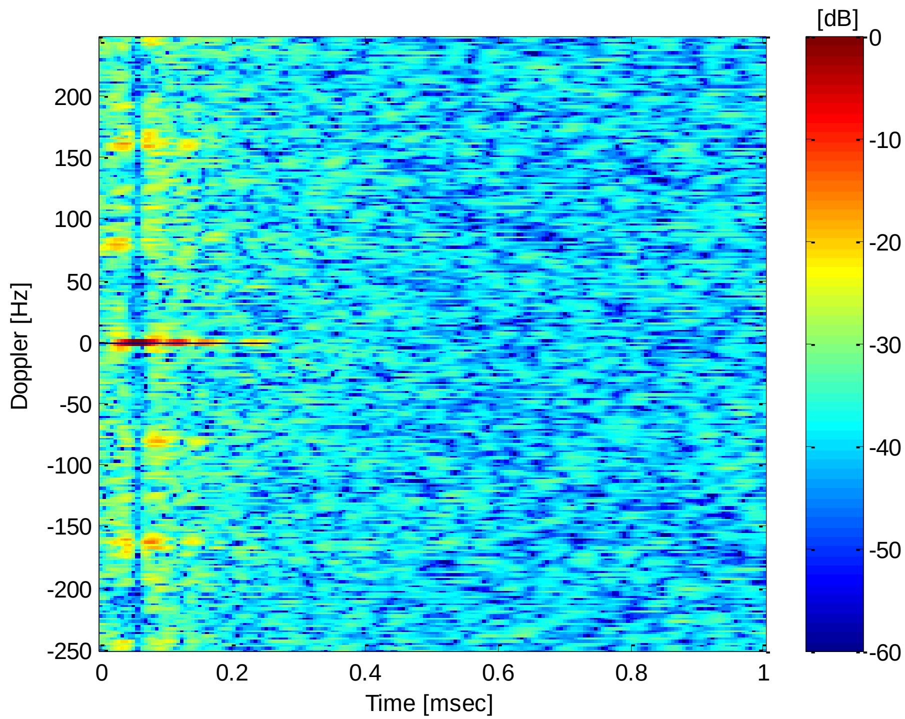
ECA Performance

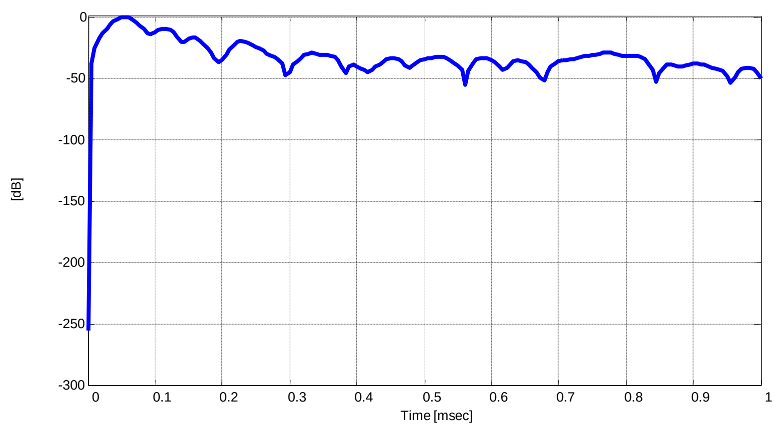
2D matched filter output when only the direct signal is cancelled.

$\mathbf{X}=\mathbf{B}\mathbf{s}_{ref\_L}$ (Nx1)

scalar α

A deep null is present at Doppler zero and delay zero, and the clutter spikes are quite apparent.
Targets are still not visible since they are hidden by the side lobes of the clutter.





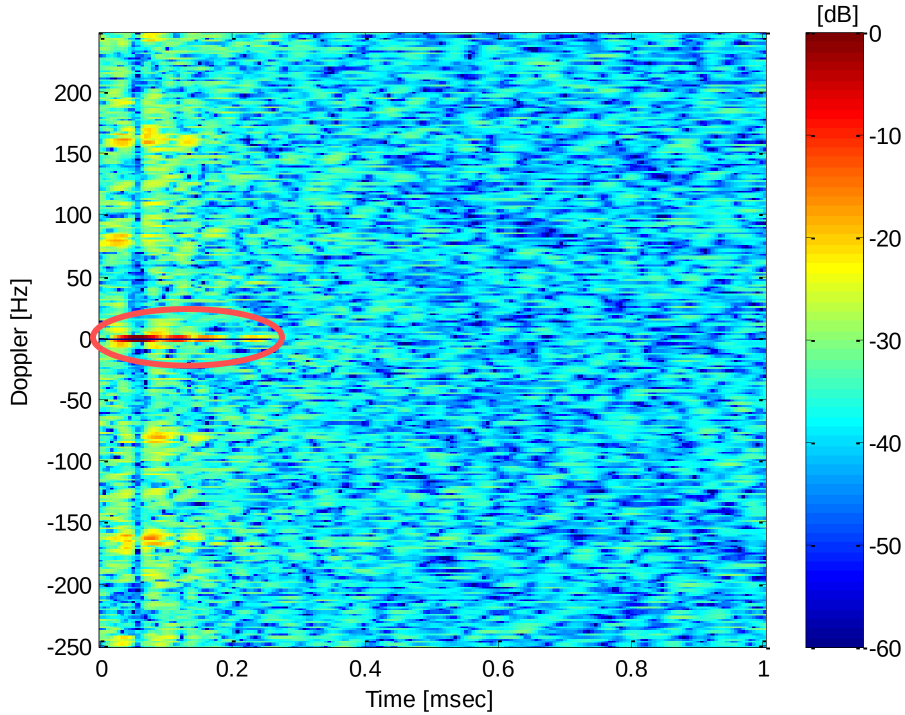
ECA Performance

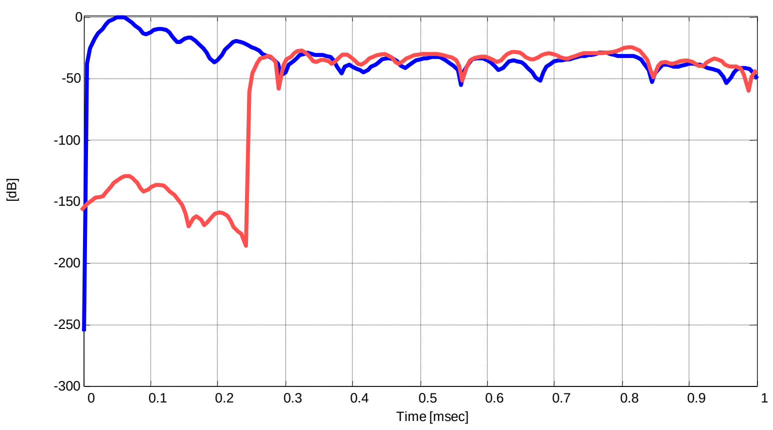
2D matched filter output when both the direct signal and all echoes at zero Doppler from the first *K* range bins (*K*=45 and *P*=0) are cancelled.

$$\mathbf{X}=\mathbf{B}\mathbf{S}_{ref\_L}\ (N\text{x}K)$$

$$\boldsymbol{\alpha}\ (K\text{x}1)$$

A deep null appears at zero Doppler, however clutter echoes are still present at low Doppler frequencies and the target echo cannot be identified.





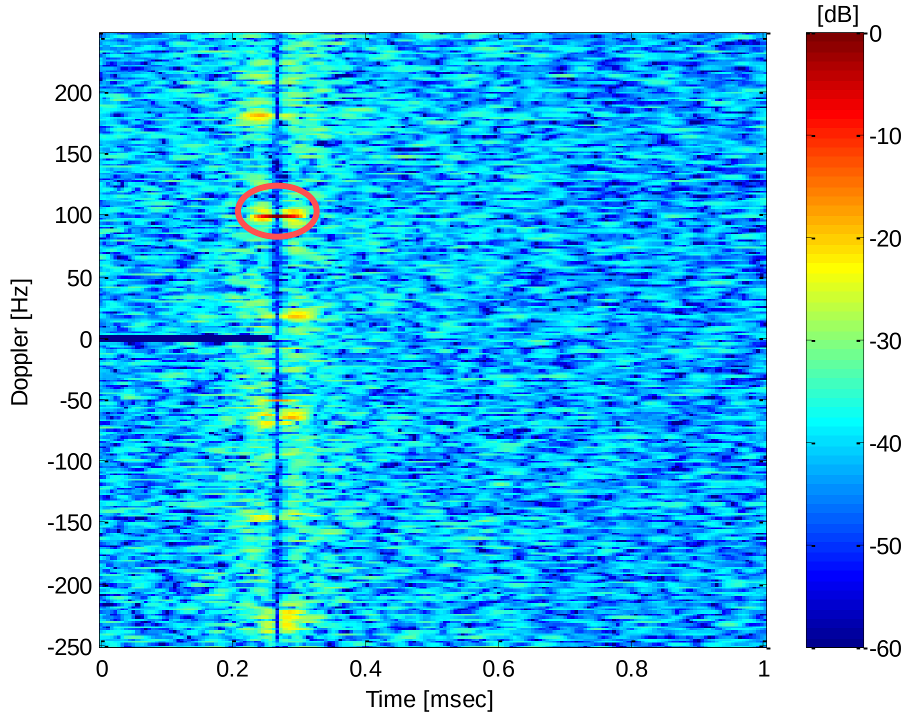
ECA Performance

2D matched filter output when both the direct signal and all echoes at Doppler bins (-1,0,1) from the first K range bins (K=45 and P=1, yielding M=135) are cancelled.

$$\mathbf{X}=\mathbf{B}\mathbf{S}_{ref\_L}\ (N\text{x}M)$$

$$\boldsymbol{\alpha}\ (M\text{x}1)$$

The strongest of the three targets now appears together with all its sidelobe structure, while the other targets cannot be discriminated from the sidelobes of the strong target .



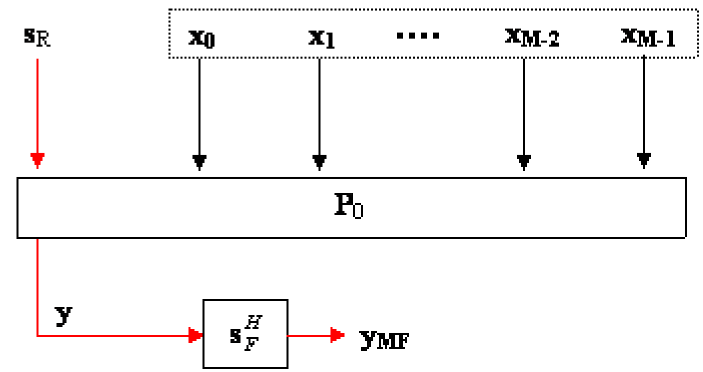
This algorithm assures to minimize the output power, but
-its computational cost is high, since the evaluation of the weight vector $\boldsymbol{\alpha}$ (Mx1) requires the evaluation and the inversion of the matrix $\mathbf{X}^H\mathbf{X}$ with dimensions MxM which corresponds to O[NM^2] complex products;
-the evaluation of vector $\boldsymbol{\alpha}$ requires and the estimation of a number M of coefficients much larger than the actual number of scattered replicas, which implies generally a bad numerical accuracy.

Sequential Cancellation Algorithm (SCA)

ECA output:

$$\mathbf{y}=\mathbf{s}_R-\mathbf{X}\boldsymbol{\alpha}=\left[\mathbf{I}_N-\mathbf{X}\left(\mathbf{X}^H\mathbf{X}\right)^{-1}\mathbf{X}^H\right]\mathbf{s}_R=\mathbf{P}_0\,\mathbf{s}_R$$

Projection in a subspace orthogonal to the disturbance sub-space



- Matrix decomposition:

$$\mathbf{X}=[\mathbf{x}_0\ \mathbf{X}_1] \Rightarrow \mathbf{X}^H\mathbf{X}=\begin{bmatrix}\mathbf{x}_0^H\mathbf{x}_0 & \mathbf{x}_0^H\mathbf{X}_1\\ \mathbf{X}_1^H\mathbf{x}_0 & \mathbf{X}_1^H\mathbf{X}_1\end{bmatrix}$$

- Projection in a subspace orthogonal to the last columns of **X**:

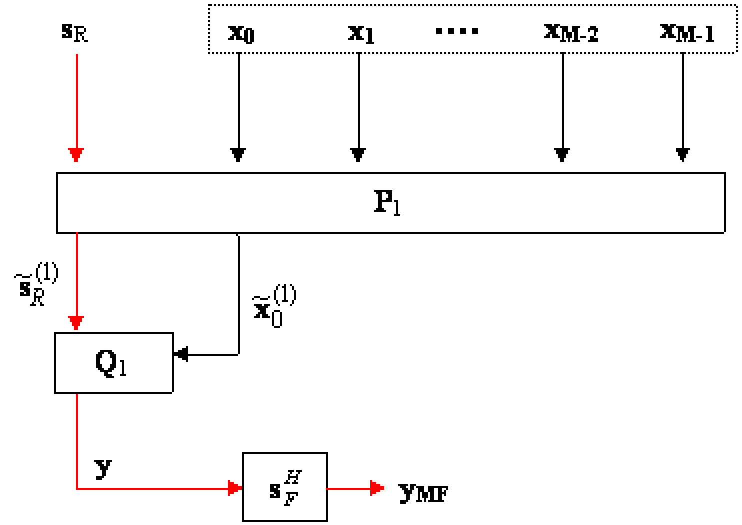
$$\mathbf{P}_1=\mathbf{I}_N-\mathbf{X}_1\left(\mathbf{X}_1^H\mathbf{X}_1\right)^{-1}\mathbf{X}_1^H \Rightarrow \mathbf{P}_0=\mathbf{P}_1-\frac{\mathbf{P}_1\mathbf{x}_0\mathbf{x}_0^H\mathbf{P}_1}{\mathbf{x}_0^H\mathbf{P}_1\mathbf{x}_0}$$

- Definitions:

Projected vectors:
$$\begin{cases}\tilde{\mathbf{s}}_R^{(1)}=\mathbf{P}_1\mathbf{s}_R\\ \tilde{\mathbf{x}}_j^{(1)}=\mathbf{P}_1\mathbf{x}_j \Rightarrow \tilde{\mathbf{X}}^{(1)}=\mathbf{P}_1\mathbf{X}\end{cases}$$

Operators:
$$\mathbf{Q}_1=\left[\mathbf{I}_N-\frac{\tilde{\mathbf{x}}_0^{(1)}\tilde{\mathbf{x}}_0^{(1)H}}{\tilde{\mathbf{x}}_0^{(1)H}\tilde{\mathbf{x}}_0^{(1)}}\right]$$

Sequential Cancellation Algorithm (SCA)



<u>ECA output</u>:

$$\mathbf{y} = \mathbf{s}_R - \mathbf{X}\boldsymbol{\alpha} = \left[\mathbf{I}_N - \mathbf{X}\left(\mathbf{X}^H\mathbf{X}\right)^{-1}\mathbf{X}^H\right]\mathbf{s}_R = \mathbf{P}_0\,\mathbf{s}_R$$

Projection in a subspace orthogon to the disturbance sub-space

- Matrix decomposition:

$$\mathbf{X} = [\mathbf{x}_0\ \mathbf{X}_1] \quad \Longrightarrow \quad \mathbf{X}^H\mathbf{X} = \begin{bmatrix} \mathbf{x}_0^H\mathbf{x}_0 & \mathbf{x}_0^H\mathbf{X}_1 \\ \mathbf{X}_1^H\mathbf{x}_0 & \mathbf{X}_1^H\mathbf{X}_1 \end{bmatrix}$$

- Projection in a subspace orthogonal to the last columns of **X**:

$$\mathbf{P}_1 = \mathbf{I}_N - \mathbf{X}_1\left(\mathbf{X}_1^H\mathbf{X}_1\right)^{-1}\mathbf{X}_1^H \quad \Longrightarrow \quad \mathbf{P}_0 = \mathbf{P}_1 - \frac{\mathbf{P}_1\,\mathbf{x}_0\mathbf{x}_0^H\,\mathbf{P}_1}{\mathbf{x}_0^H\,\mathbf{P}_1\mathbf{x}_0}$$

- Definitions:

Projected vectors:
$$\begin{cases} \tilde{\mathbf{s}}_R^{(1)} = \mathbf{P}_1\mathbf{s}_R \\ \tilde{\mathbf{x}}_j^{(1)} = \mathbf{P}_1\mathbf{x}_j \ \Rightarrow \ \tilde{\mathbf{X}}^{(1)} = \mathbf{P}_1\mathbf{X} \end{cases}$$

Operators:
$$\mathbf{Q}_1 = \left[\mathbf{I}_N - \frac{\tilde{\mathbf{x}}_0^{(1)}\tilde{\mathbf{x}}_0^{(1)H}}{\tilde{\mathbf{x}}_0^{(1)H}\,\tilde{\mathbf{x}}_0^{(1)}}\right]$$

- Matched filter output:

$$y_{MF} = \mathbf{s}_F^H\,\mathbf{P}_0\mathbf{s}_R = \mathbf{s}_F^H\left[\mathbf{P}_1 - \frac{\mathbf{P}_1\,\mathbf{x}_0\mathbf{x}_0^H\,\mathbf{P}_1}{\mathbf{x}_0^H\,\mathbf{P}_1\mathbf{x}_0}\right]\mathbf{s}_R =$$
$$= \mathbf{s}_F^H\,\mathbf{Q}_1\mathbf{P}_1\mathbf{s}_R = \mathbf{s}_F^H\,\mathbf{Q}_1\,\tilde{\mathbf{s}}_R^{(1)}$$

Sequential Cancellation Algorithm (SCA)

By iteratively applying the decomposition of the subspace projection matrix $\mathbf{P}_i$ (i=1,..,M)…

$$\boxed{y_{MF} = \mathbf{s}_F^H \mathbf{Q}_1 \mathbf{Q}_2 \mathbf{Q}_3 \cdots \mathbf{Q}_{M-1} \mathbf{Q}_M \mathbf{s}_R}$$

where
$$\mathbf{Q}_i = \left[\mathbf{I}_N - \frac{\tilde{\mathbf{x}}_{i-1}^{(i)} \tilde{\mathbf{x}}_{i-1}^{(i)H}}{\tilde{\mathbf{x}}_{i-1}^{(i)H} \tilde{\mathbf{x}}_{i-1}^{(i)}} \right]$$

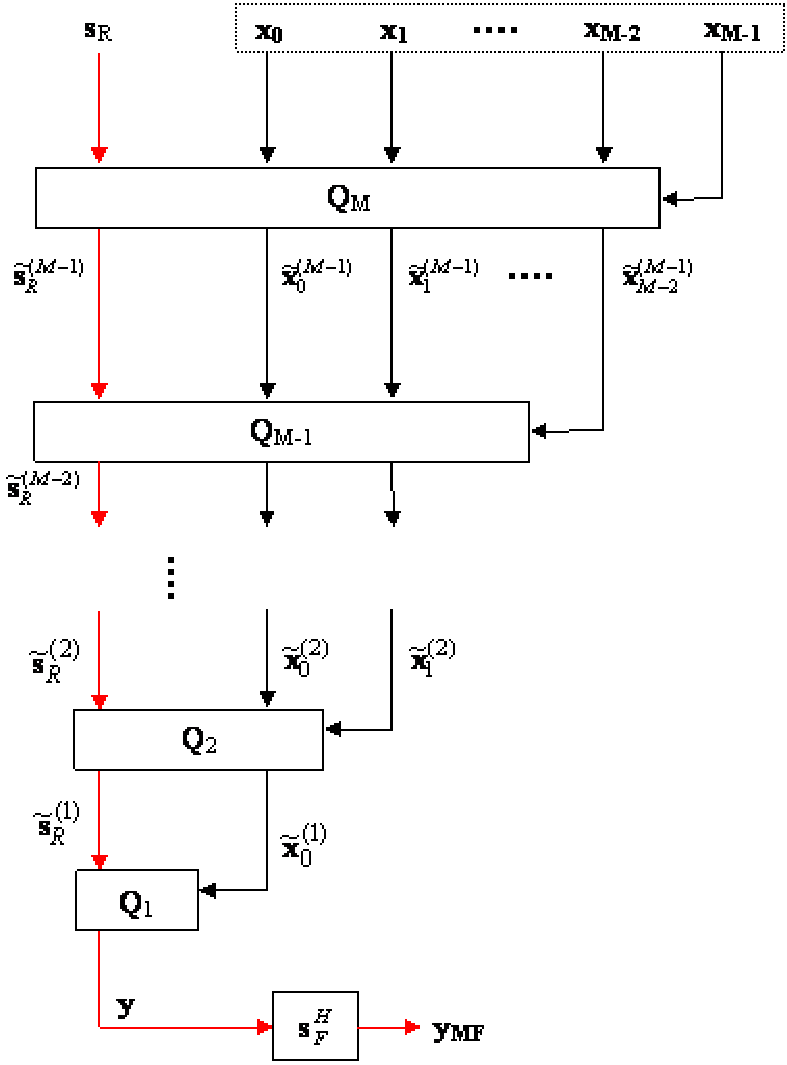
$$\mathbf{P}_i = \mathbf{Q}_{i+1} \mathbf{Q}_{i+2} \cdots \mathbf{Q}_M$$

$$\mathbf{P}_M = \mathbf{I}_N \Rightarrow \mathbf{Q}_M = \left[\mathbf{I}_N - \frac{\tilde{\mathbf{x}}_{M-1}^{(M)} \tilde{\mathbf{x}}_{M-1}^{(M)H}}{\tilde{\mathbf{x}}_{M-1}^{(M)H} \tilde{\mathbf{x}}_{M-1}^{(M)}} \right] =$$

$$= \left[\mathbf{I}_N - \frac{\mathbf{x}_{M-1} \mathbf{x}_{M-1}^H}{\mathbf{x}_{M-1}^H \mathbf{x}_{M-1}} \right]$$

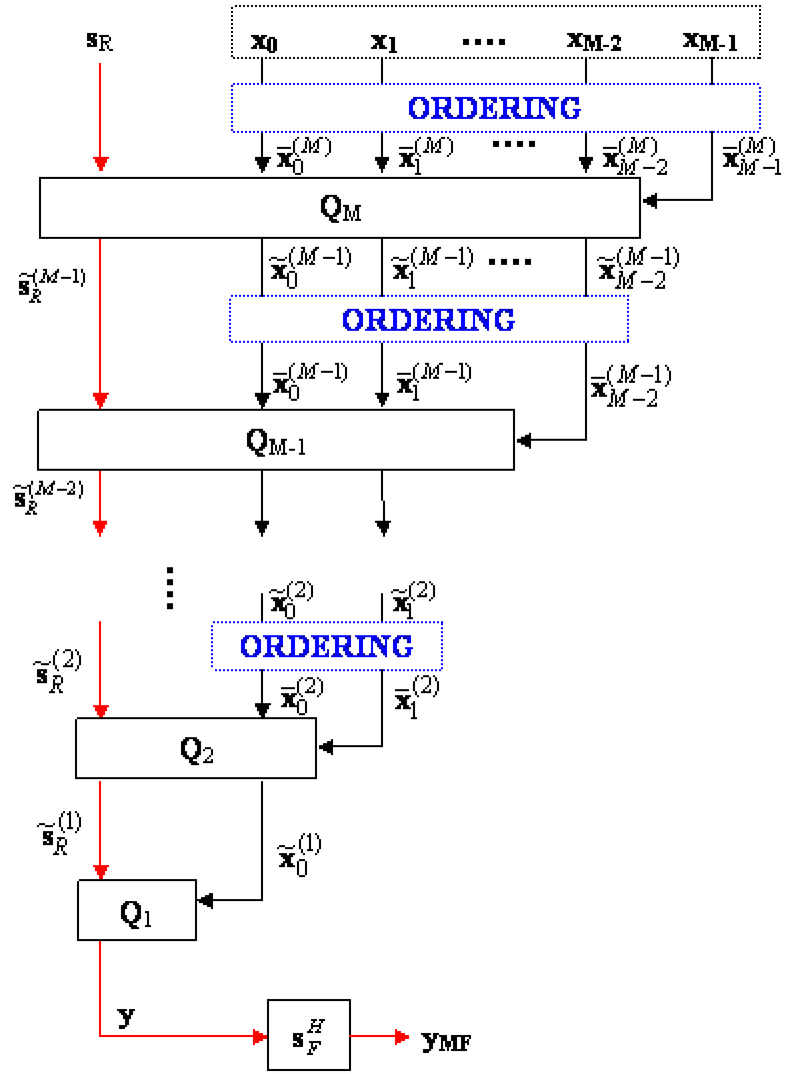
➡ **Sequential version of ECA**:

The column vectors of matrix **X** are **sequentially projected** in a subspace orthogonal to its last columns. At the i-th stage (i=1,..,M), a **1-dimensional cancellation** is performed by projecting the input vector in a subspace orthogonal to the (M-i)-th vector of the base.



Sequential Cancellation Algorithm (SCA)

Notice that:
- the sequential decomposition can be applied to any permutation of the columns of **X**;
- the order is inessential if all the *M* decomposition stages are carried out;
- it is possible to stop the algorithm after a smaller number of stages (partial clutter cancellation) → the vectors can be adequately re-ordered at any stage so that the first projection produces the highest benefit for target detection.

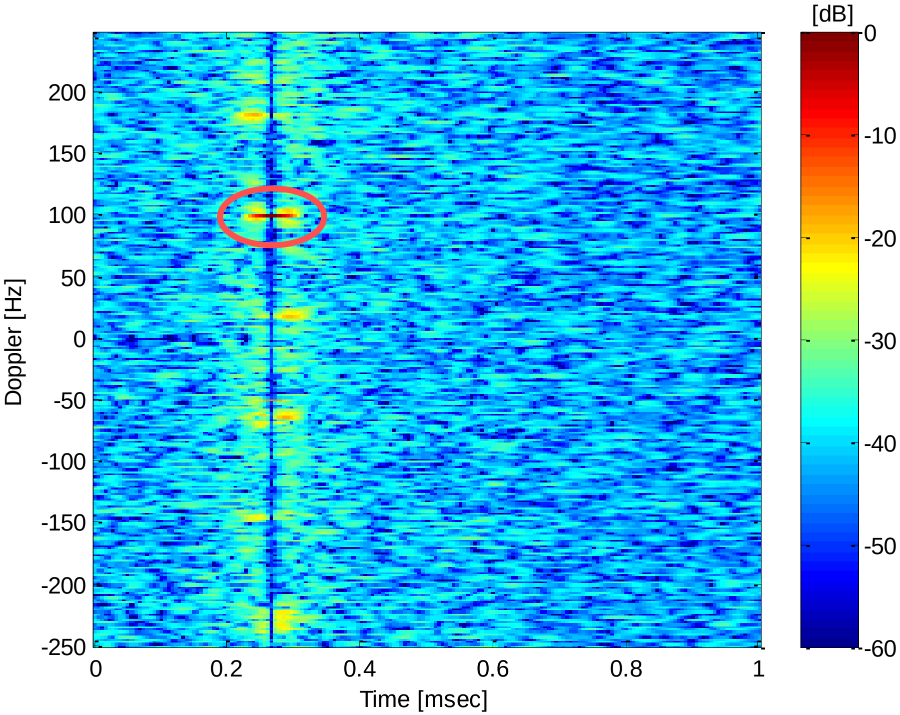


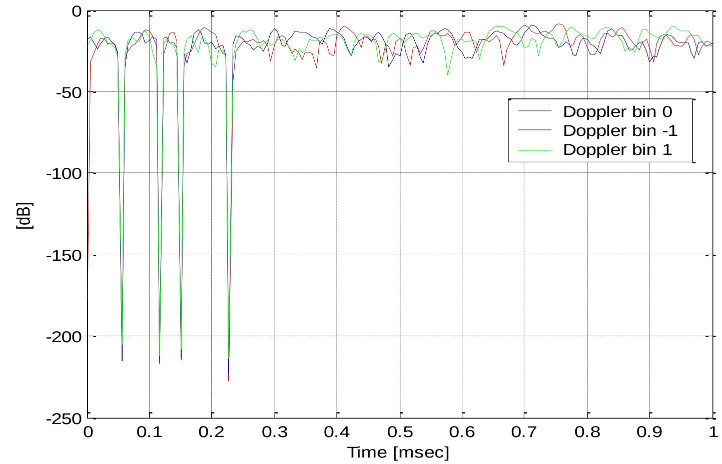
<u>Ordering strategy</u> → largest reduction in the global power (best cancellation performance):

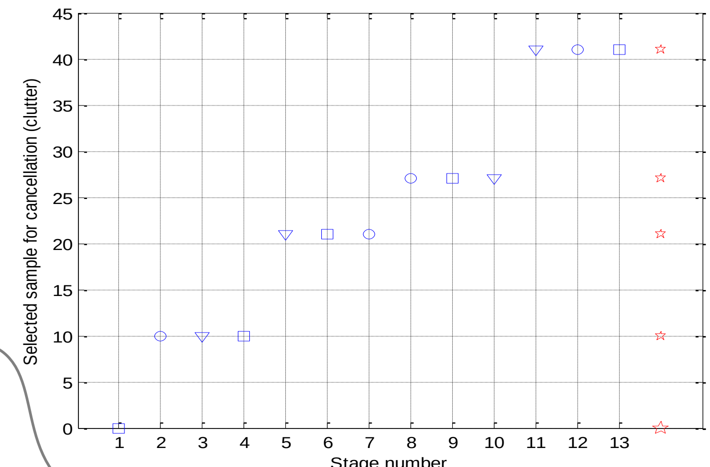
$$\bar{\mathbf{x}}^{(M-q)}_{M-q-1} = \tilde{\mathbf{x}}^{(M-q)}_{\ell} \qquad s.t. \qquad \ell = \underset{\ell=0,\cdots,M-q-1}{\arg\max} \left\{ \left| \tilde{\mathbf{s}}_R^{(M-q)H}\, \tilde{\mathbf{x}}^{(M-q)}_{\ell} \right| \right\}$$

<u>Stop criterion</u> → the algorithm stops when the processed signal power has been reduced to a desired level (when a clutter cancellation of a desired number of dB has been obtained).

SCA Performance







As apparent the algorithm stops after stage q=13, when the direct signal and the N_c=4 clutter spikes with Doppler spread of $2P$+1=3 Doppler bins have been cancelled. Also in this case only the strongest target is clearly visible.

STDA performance

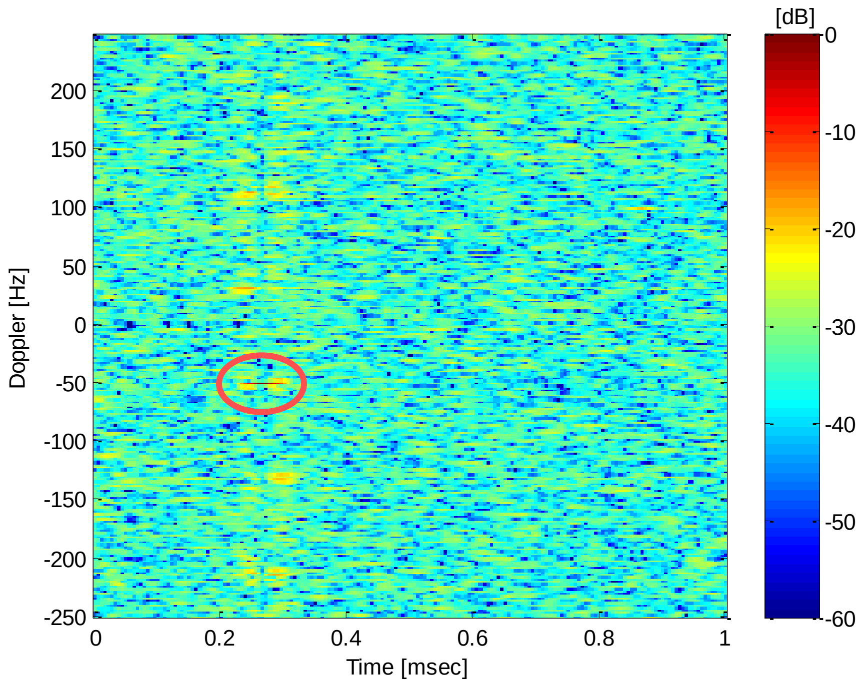
The sequential projection approach can be extended to extract the weak targets present in the scenario.

- A detection is declared at the delay-Doppler location corresponding to the highest peak of the 2D matched filter output;
- The reference signal correspondingly delayed and Doppler shifted is used to create the projector in the orthogonal subspace;
- This is sequentially repeated to detect all targets.

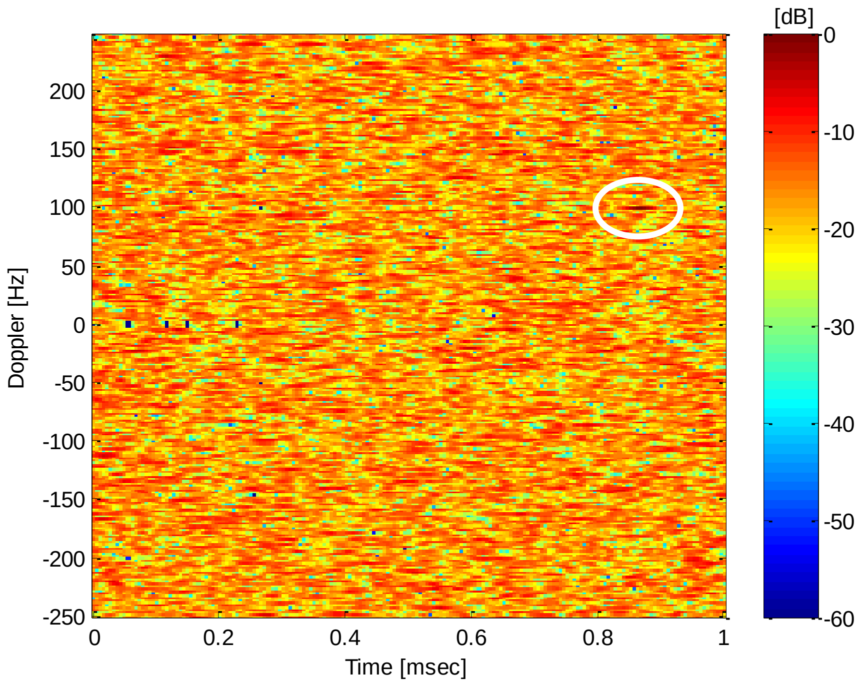
This approach corresponds to an extension of the basis of the clutter space to include the subspace spanned by the strong targets returns.

Sequential Target Detection Algorithm

2D matched filter output



The second strongest target now appears as a strong peak.



The smallest target appears more clearly (dynamic range limited by thermal noise).

When the sequential approach is extended to the cancellation of strong target echoes, the number of stages of the algorithm is increased. However the total computational cost is still smaller than the application of the extensive cancellation algorithm, while it yields better performance.

Sequential Target Detection Algorithm

A sequential approach has been presented for both disturbance cancellation and target detection for PCL radars based on projections of the received signals in a subspace orthogonal to the disturbance and to the previously detected targets.

- The approach has been shown to be effective against typical scenarios with a limited number of iterations.
- The sequential cancellation of the strongest target echoes, allows also the correct detection of weaker target echoes that risk to be masked by the sidelobes of the strong target echoes.
- A preliminary analysis shows that the sequential approach has an average computational load smaller than the cost of the extensive cancellation algorithm**.**

The authors acknowledge the support of Finmeccanica through SESM