KALMAN FILTER IN 2D(Task-5)

The assumptions

There are two significant assumptions when using the Kalman filter:

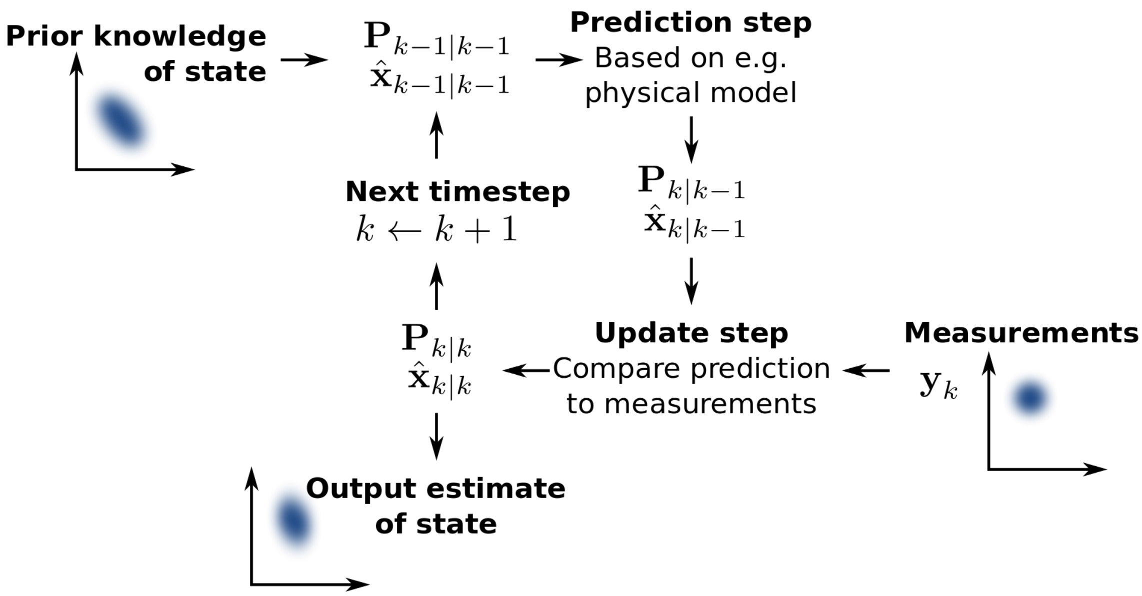
1. The sensor is noisy and its output and noise can be accurately modelled.

   by a **Gaussian probability distribution function (PDF)**.

2. Our initial prior knowledge of the actual state is also a Gaussian PDF.

3. Kinematics in x and y are independent.

Overall Algorithm



Some Terms

State Vector (x): The state of the system at a specific time instance represented as a column vector. It contains the parameters or variables that define the system's current state.

State Transition Model (A):
A matrix that describes how the system's state evolves from one time step to the next. It represents the dynamics of the system and is used to predict the state at the next time step.
In case of 2-D motional without taking into acceleration into account

$$A = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Control Input (u): An optional external control input vector that can be applied to the system to influence its state evolution.

Measurement Vector (z): The measurement obtained from the system at each time step. This measurement is often noisy and imperfect.

In my work this is taken as position vector with dim (2*1).

Measurement Model (H): A matrix that maps the state space to the measurement space. It describes how the state is related to the measurements.

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[[1 0 0 0]
 [0 1 0 0]]
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Measurement Noise (R): The covariance matrix that represents the uncertainty or noise in the measurements.

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[[0.09 0.  ]
 [0.   0.09]]
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Process Noise (Q): The covariance matrix that represents the uncertainty or noise in the state transition process.

$$Q = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\dot{x}} & \sigma_{x\dot{y}} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{y\dot{x}} & \sigma_{y\dot{y}} \\ \sigma_{\dot{x}x} & \sigma_{\dot{x}y} & \sigma_{\dot{x}}^2 & \sigma_{\dot{x}\dot{y}} \\ \sigma_{\dot{y}x} & \sigma_{\dot{y}y} & \sigma_{\dot{y}\dot{x}} & \sigma_{\dot{y}}^2 \end{bmatrix}$$

Kalman Gain (K): A matrix that determines the weight given to the measurement and prediction for updating the state estimate.

$$\boldsymbol{K}_n = \boldsymbol{P}_{n,n-1}\boldsymbol{H}^T\left(\boldsymbol{H}\boldsymbol{P}_{n,n-1}\boldsymbol{H}^T + \boldsymbol{R}_n\right)^{-1}$$

State Prediction (x_hat): The predicted state of the system at the next time step based on the state transition model and, if applicable, the control input.

State Covariance (P): The covariance matrix that represents the uncertainty in the state estimate.

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P = [[Pxx, Pxy, Pvx, Pvy],
[Pxy, Pyy, Pvy, Pvy],
[Pvx, Pvy, Pvvx, Pvvy],
[Pvy, Pvvy, Pvvy, Pvvy]]
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Pxx, Pyy: Variances of the position estimates (x and y).
Pvx, Pvy: Covariance terms between position and velocity estimates.
Pvvx, Pvvy: Variances of the velocity estimates in the x and y directions.

Algorithm:

New state prediction:

$$x_{t+1} = A * x_t$$

$$P = A \cdot P \cdot A' + Q$$

Calculate Kalman gain:

$$\boldsymbol{K}_n = \boldsymbol{P}_{n,n-1}\boldsymbol{H}^T\left(\boldsymbol{H}\boldsymbol{P}_{n,n-1}\boldsymbol{H}^T + \boldsymbol{R}_n\right)^{-1}$$

The Kalman gain represents the weight given to the measurement information and is calculated based on the prediction uncertainty and the measurement uncertainty.

Use measurement data to update state prediction:

$$x = x + (K \cdot y)$$

In the update step of the Kalman filter, we can think of the predicted state estimate as analogous to the prior probability, and the measurement as analogous to the evidence in Bayes' theorem. The Kalman gain acts as the likelihood function that combines the prior probability (predicted state estimate) with the evidence (measurement) to update the state estimate.

Update error parameter

$$P = (I - (K \cdot H)) \cdot P$$

Keep performing the iteration and error will reduce as (I-K.H)<I and So approximation gets better.