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QIntern2025

QWORLD

July - August 2025

INTERNATIONAL YEAR OF Quantum Science and Technology

QUANTUM CIRCUITS FOR OPEN SYSTEM DYNAMICS: SIMULATING THE MASTER EQUATION

PROJECT SUMMARY

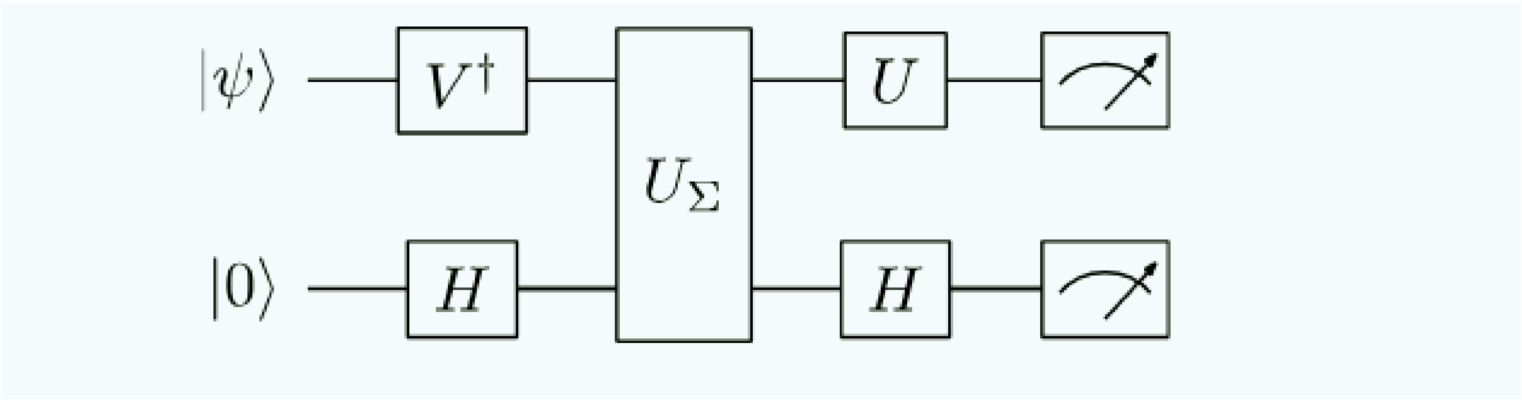
- Simulated open quantum systems (amplitude damping, qubits in bosonic reservoirs, thermal decay) using quantum circuits.
- Decomposed Kraus operators via SVD + unitarization in operator-sum representation for circuit implementation.
- Analyzed expectation values of Pauli operators, scaling of qubits, and general simulation methodologies for OQS.
- Experimented with swapping SVD and Choi decompositions, achieving comparable fidelity in results.
- Explored multi-qubit interaction profiles to study scalability of open quantum dynamics simulations.

AD Channel Damping

Quantum simulation through Kraus operator decomposition for Open Quantum System

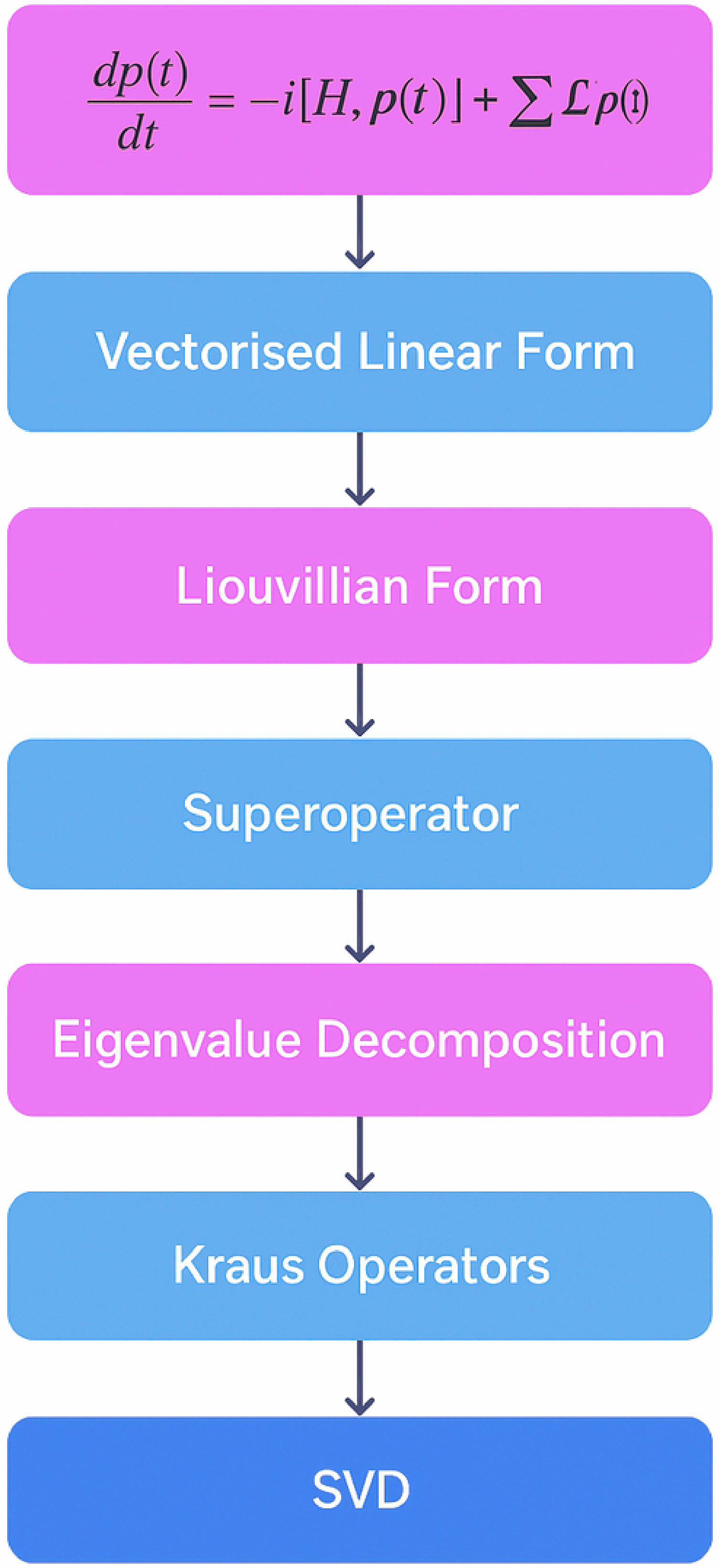
Implementation follows a method for simulating amplitude damping channels using quantum circuits by:

- Decomposing Kraus operators via SVD
- Implementing the decomposition on quantum hardware
- Comparing analytical vs. quantum circuit results



Open system Dynamics $\rho(t) = U(t)\rho(0)U^\dagger(t)$

Referrence Research - Exact Non-Markovian Quantum Dynamics on the NISQ Device Using Kraus Operators



FROM LIOVILLIAN TO THE KRAUS OPERATORS

Vectorised Liovillian Form

$$\mathrm{vec}(\rho(t)) = \mathcal{L}(t)\mathrm{vec}(\rho(0))$$

Choi Matrix Framework

$$C := \sum_{i,j=1}^{N} (E_{i,j} \otimes I)\mathcal{L}(I \otimes E_{i,j})$$

Eigen Value Decomposition

$$C = \mathcal{U}\Sigma\mathcal{U}^{\dagger} = \sum_{k=1}^{N^2} \sigma_k u_k u_k^{\dagger}$$

Vectorised Operator Sum

$$\mathrm{vec}(M_k) = \sqrt{\sigma_k}\, u_k$$

$$\rho_s(t) = \sum M_k(t)\rho_s(0)M_k^{\dagger}(t) := \mathcal{E}_{t,0}(\rho_s(0))$$

$$M_k = U\Sigma V^{\dagger}$$

Generalised Singular Value Decomposition

UNITARIZATION OF THE KRAUS OPERATORS

$$U_{\Sigma} = \begin{pmatrix} \Sigma_+ & 0 \\ 0 & \Sigma_- \end{pmatrix}$$

where

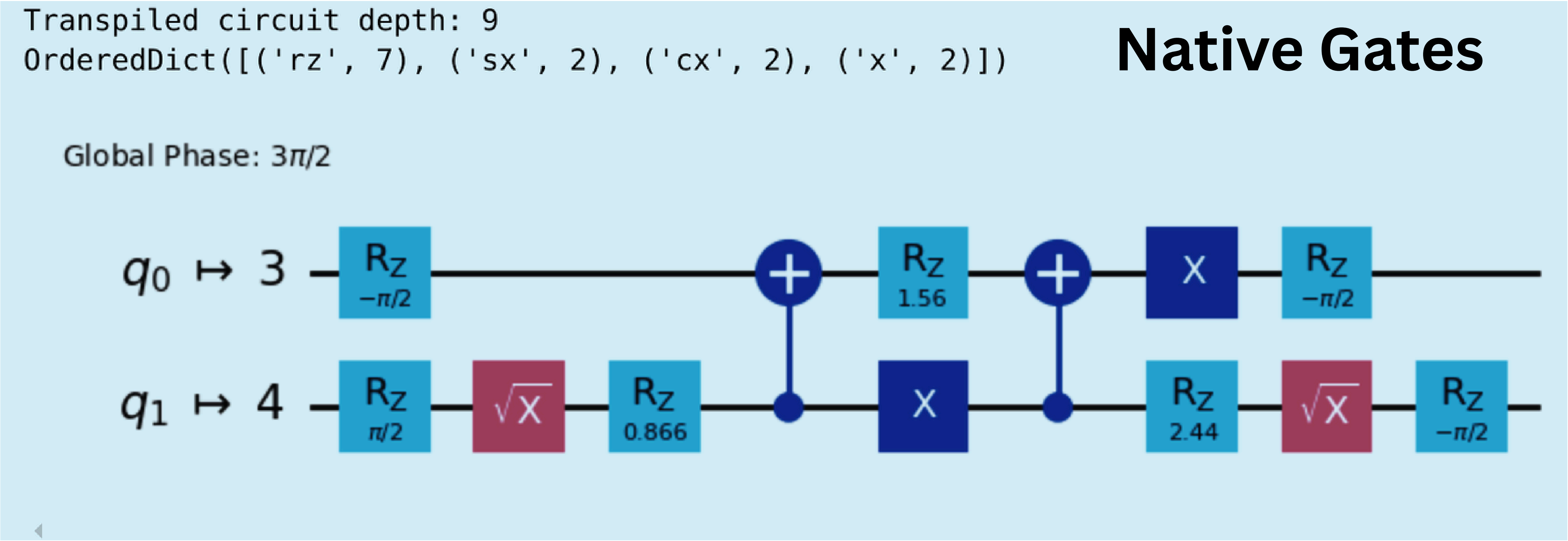
$$\Sigma_{\pm_j} = \sigma_j \pm i\sqrt{1 - \sigma_j^2}$$

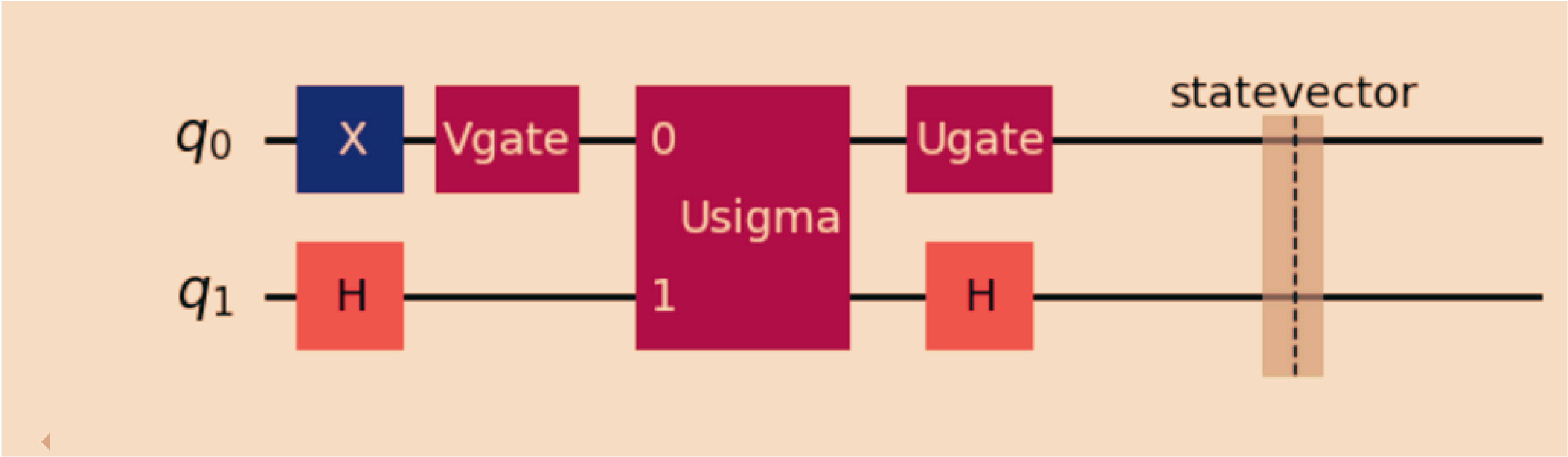
$$\frac{1}{2}\begin{pmatrix} U(\Sigma_+ + \Sigma_-)V^{\dagger}|\psi\rangle \\ U(\Sigma_+ - \Sigma_-)V^{\dagger}|\psi\rangle \end{pmatrix} = \begin{pmatrix} M_k|\psi\rangle \\ |\varphi\rangle \end{pmatrix}$$

ANALYSIS OF QUANTUM CIRCUIT

Circuit Decomposition

Qubits	2	3
Transpiled Circuit Depth	9	76
CNOT Gate	2	33
Rotation Z ‘ rz’	7	49
Sqrt Rotation X ‘sx’	2	26
Pauli X	2	4





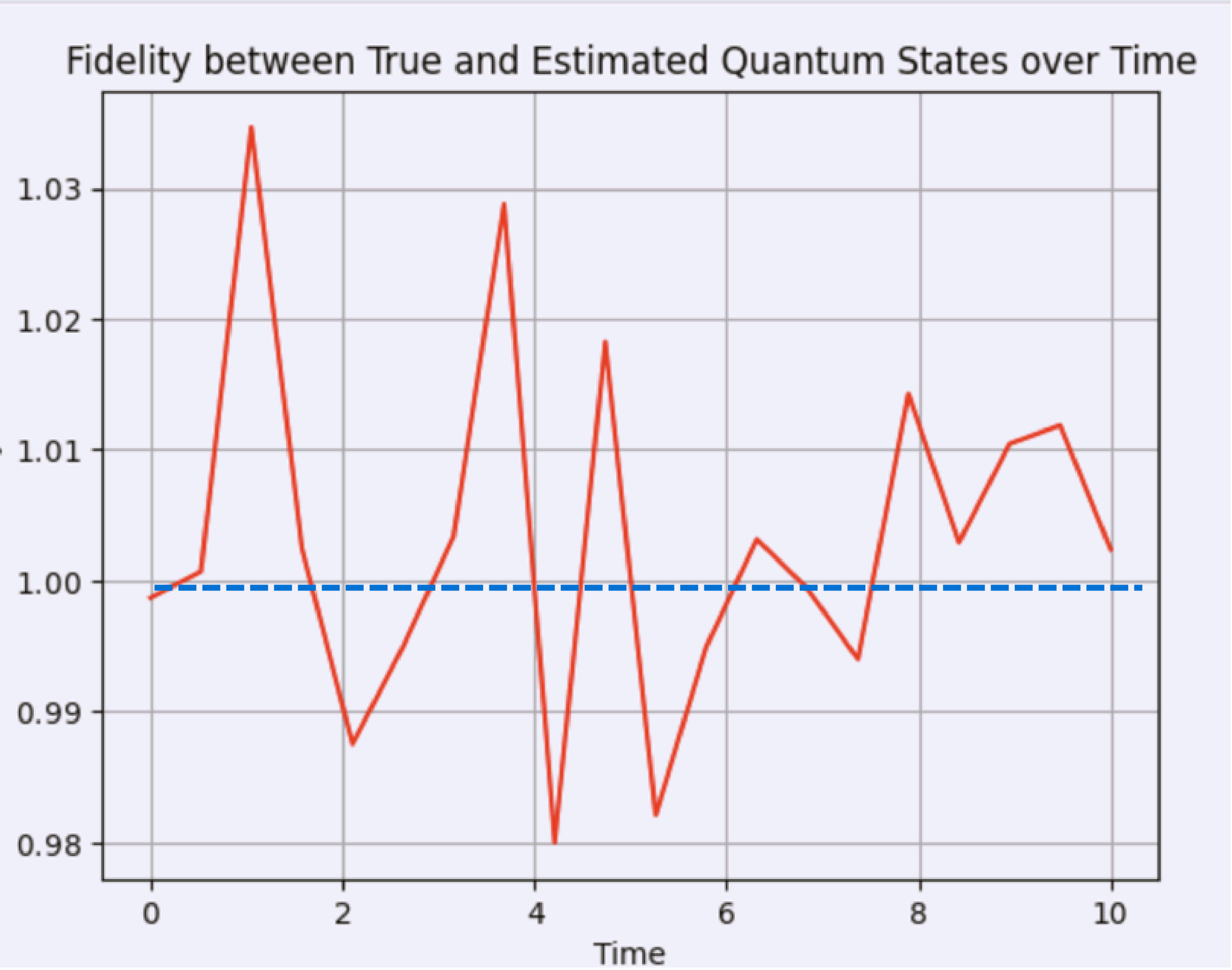
Quantum Circuit Realization:

- 2-qubit implementation with auxiliary qubits
- Controlled unitary operations for U, Σ, V components
- Hadmard gates for creating necessary superposition states

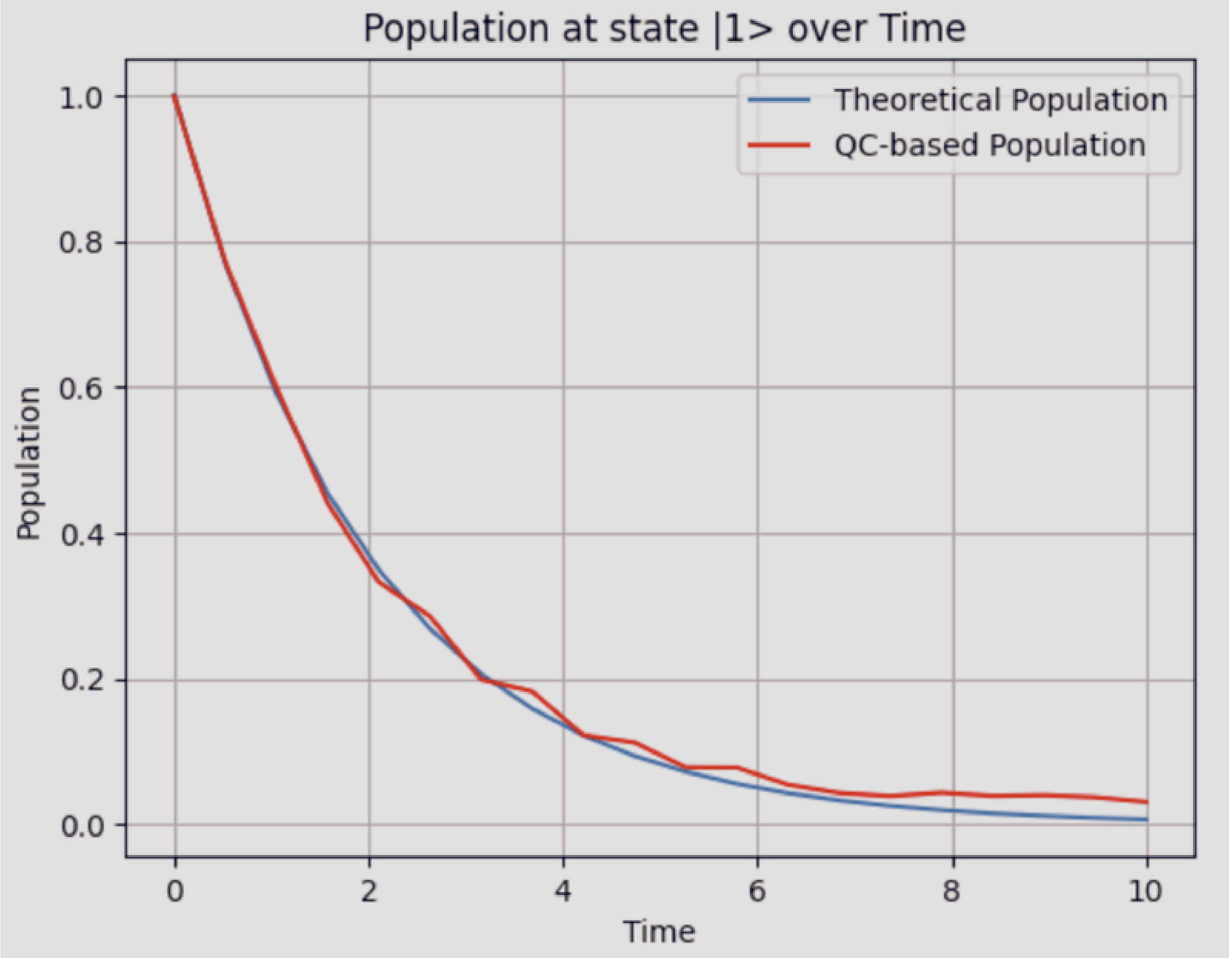
$$M_k = U\Sigma V^\dagger$$

$$U_\Sigma = \begin{pmatrix} \Sigma_+ & 0 \\ 0 & \Sigma_- \end{pmatrix}$$

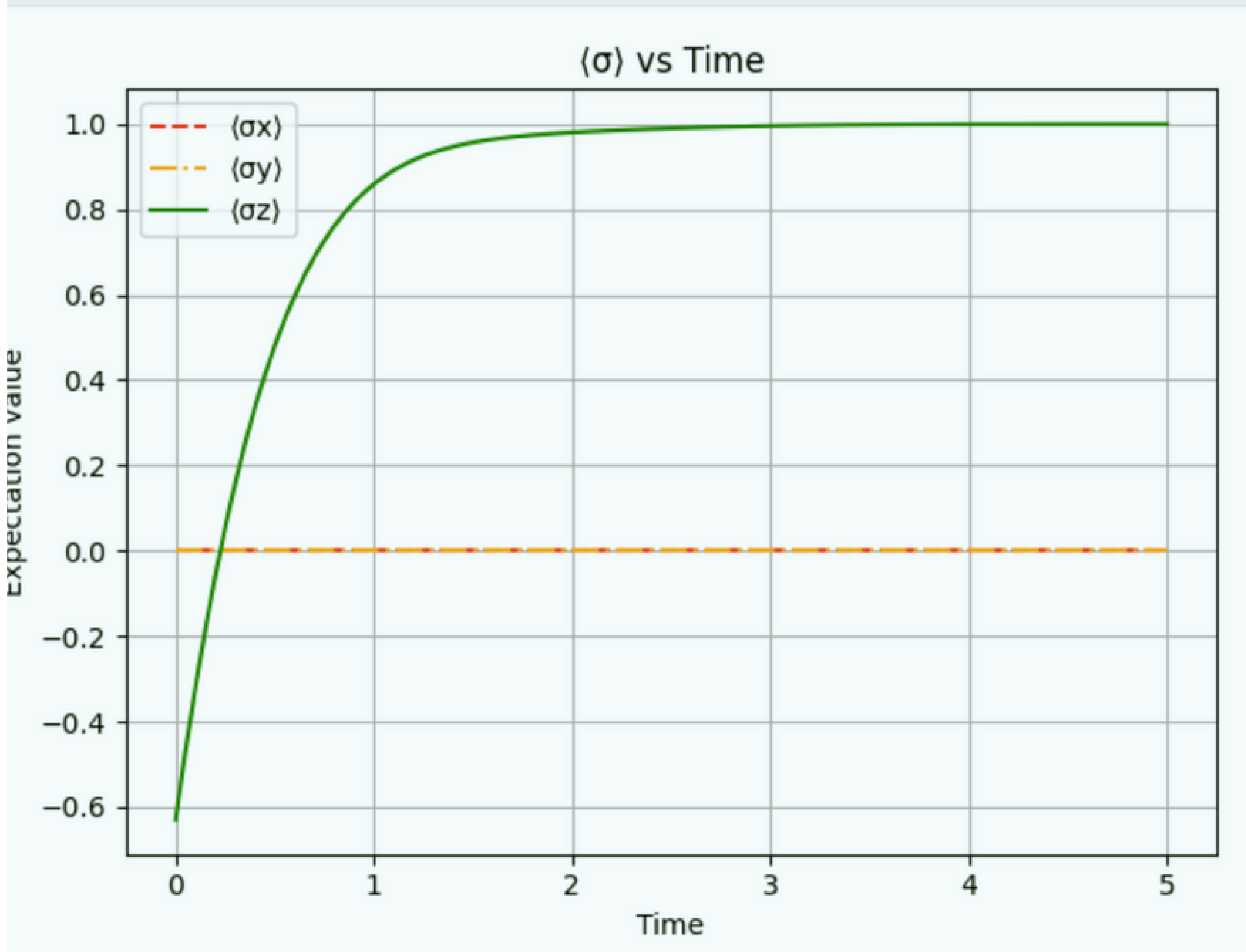
RESULTS FOR PURE AMPLITUDE DAMPING CHANNEL



High Fidelity: The fidelity plots show excellent agreement between analytical and IBM QPU simulated results

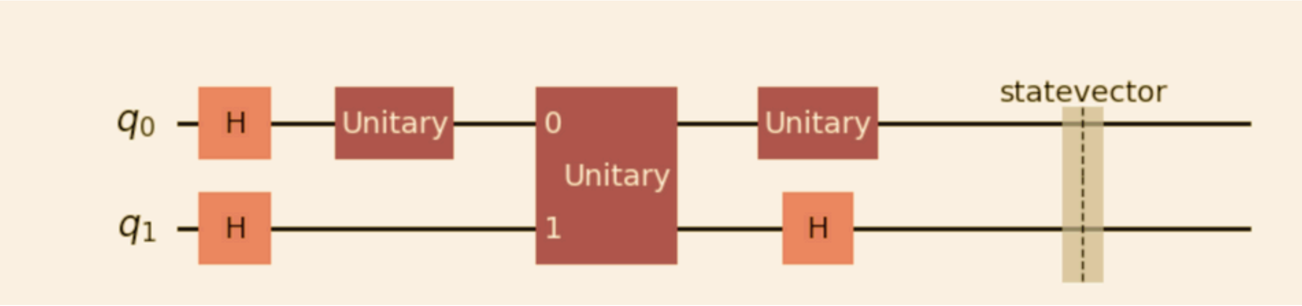


The population decay from |1〉 state follows the expected exponential decay



Expectation values $\langle \sigma_x \rangle$, $\langle \sigma_y \rangle$, $\langle \sigma_u \rangle$ show correct evolution patterns with respect intial state |1〉 and continious decay over time

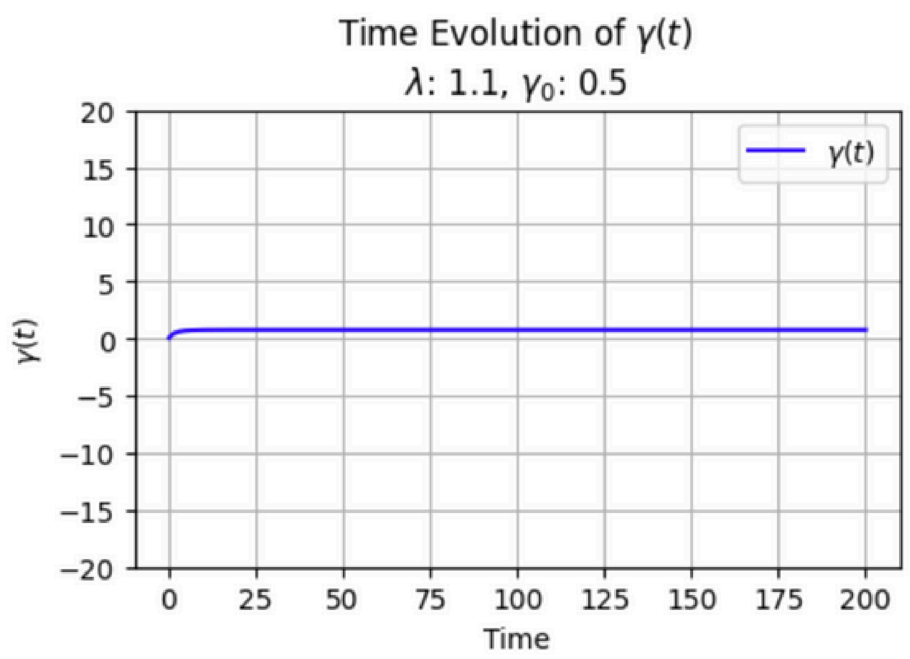
Interaction with Bosonic Reservoir



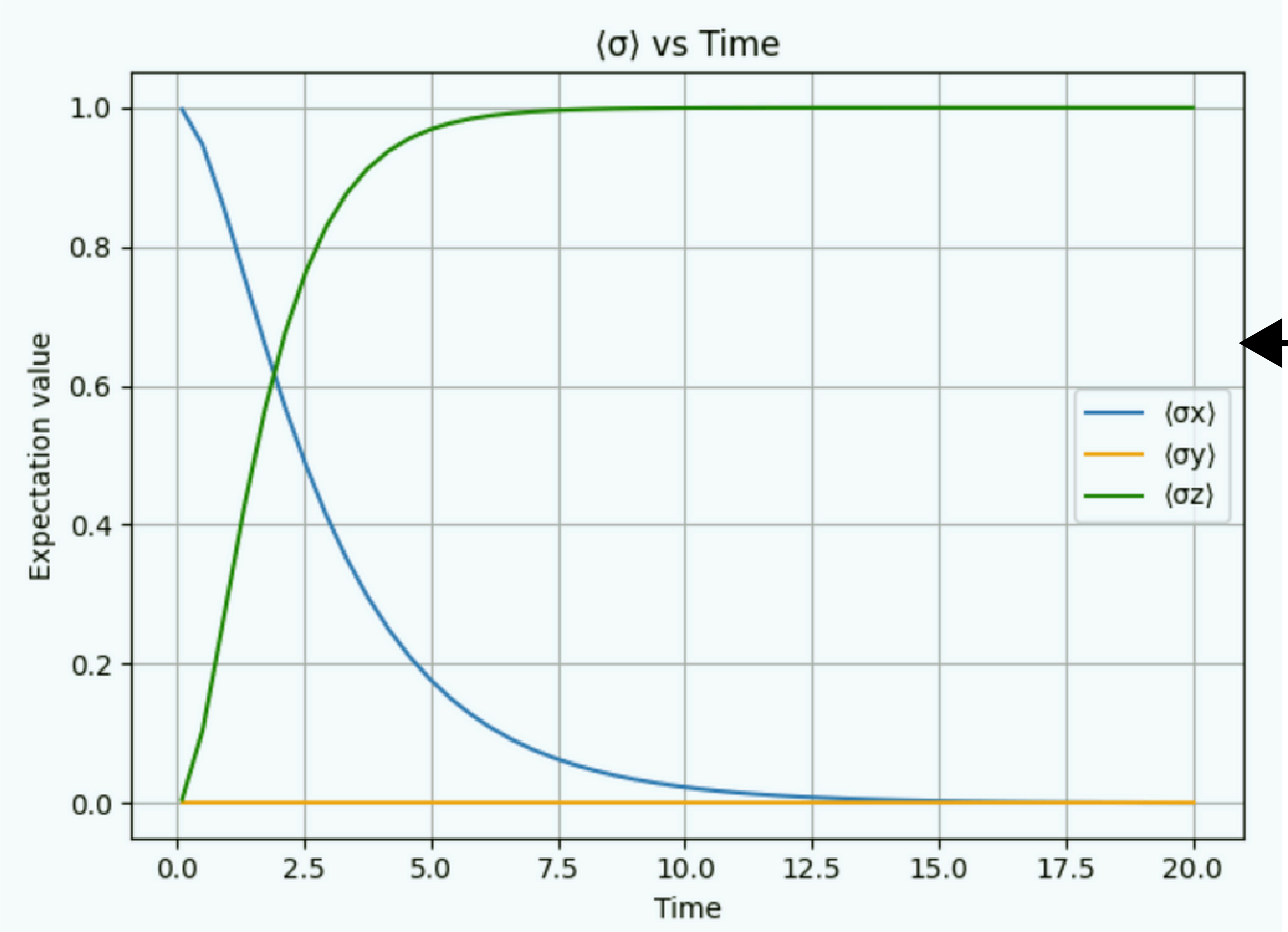
Applied Hadmard Gate to achieve initial state as superposition of both |1⟩ and |0⟩ state

$$\frac{d}{dt}\rho_s(t) = -iS(t)[\sigma^+\sigma^-, \rho_s(t)] + \gamma(t)\left(\sigma^-\rho_s(t)\sigma^+ - \frac{1}{2}\{\sigma^+\sigma^-, \rho_s(t)\}\right)$$

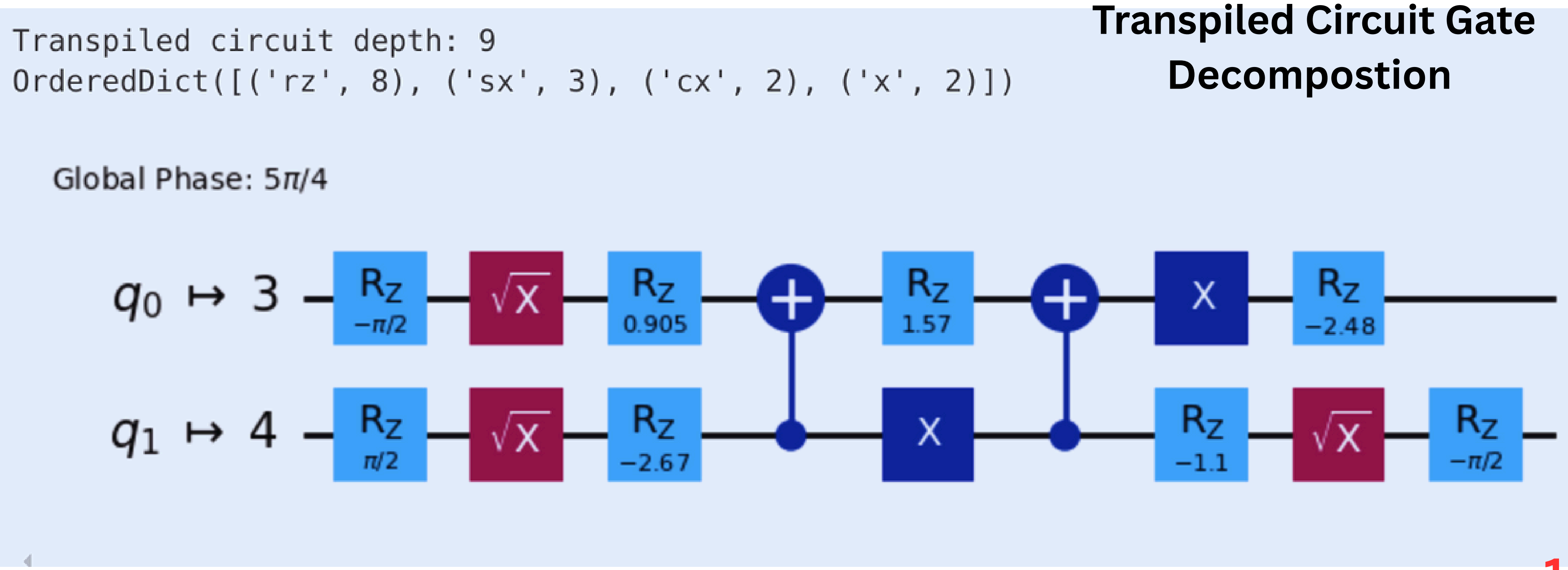
Under Markovian Regime :
λ > 2γ



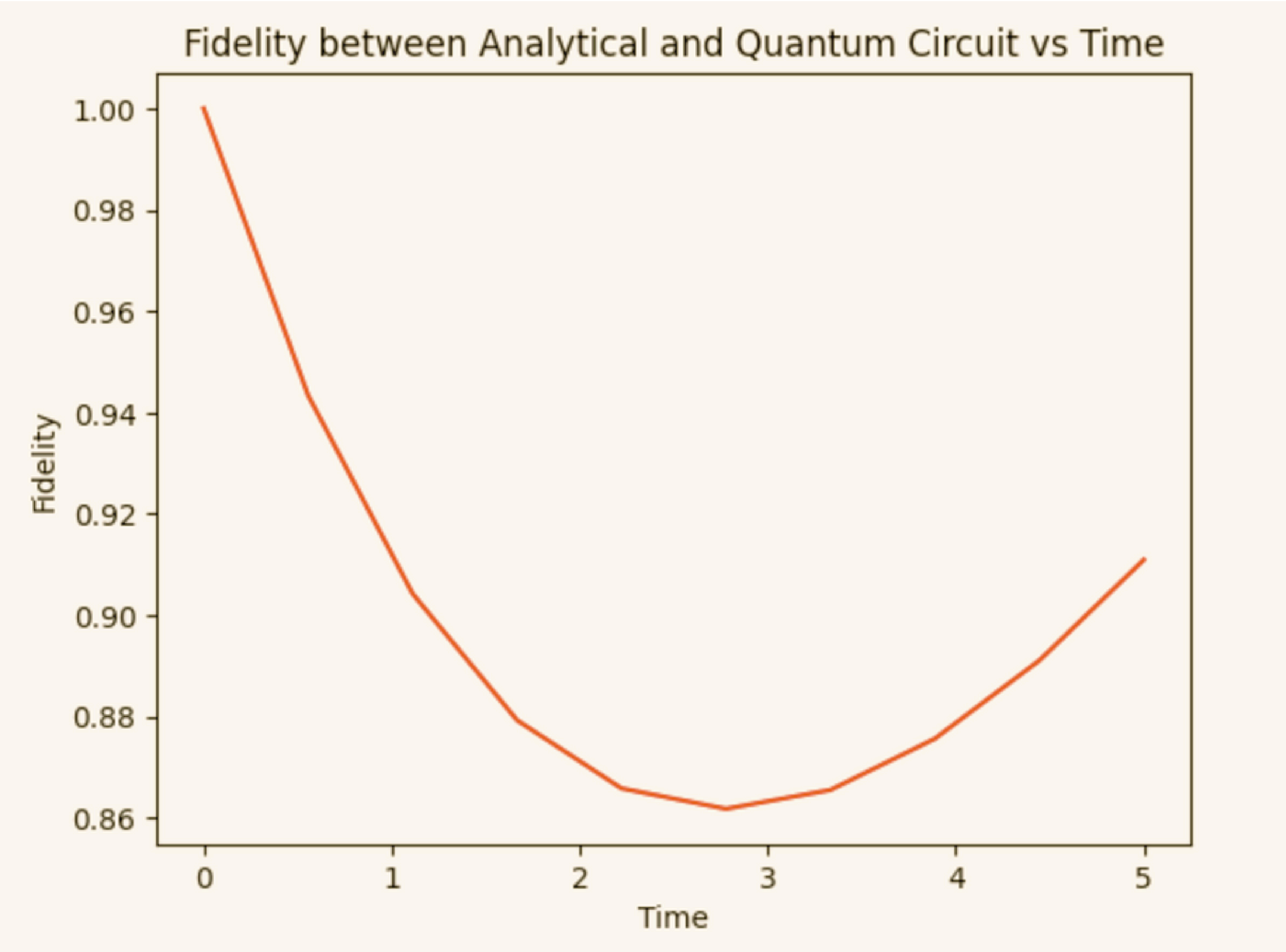
Expectation Values of Pauli Gates over time Changes Cause of system Jump Coefficient Depends on Time

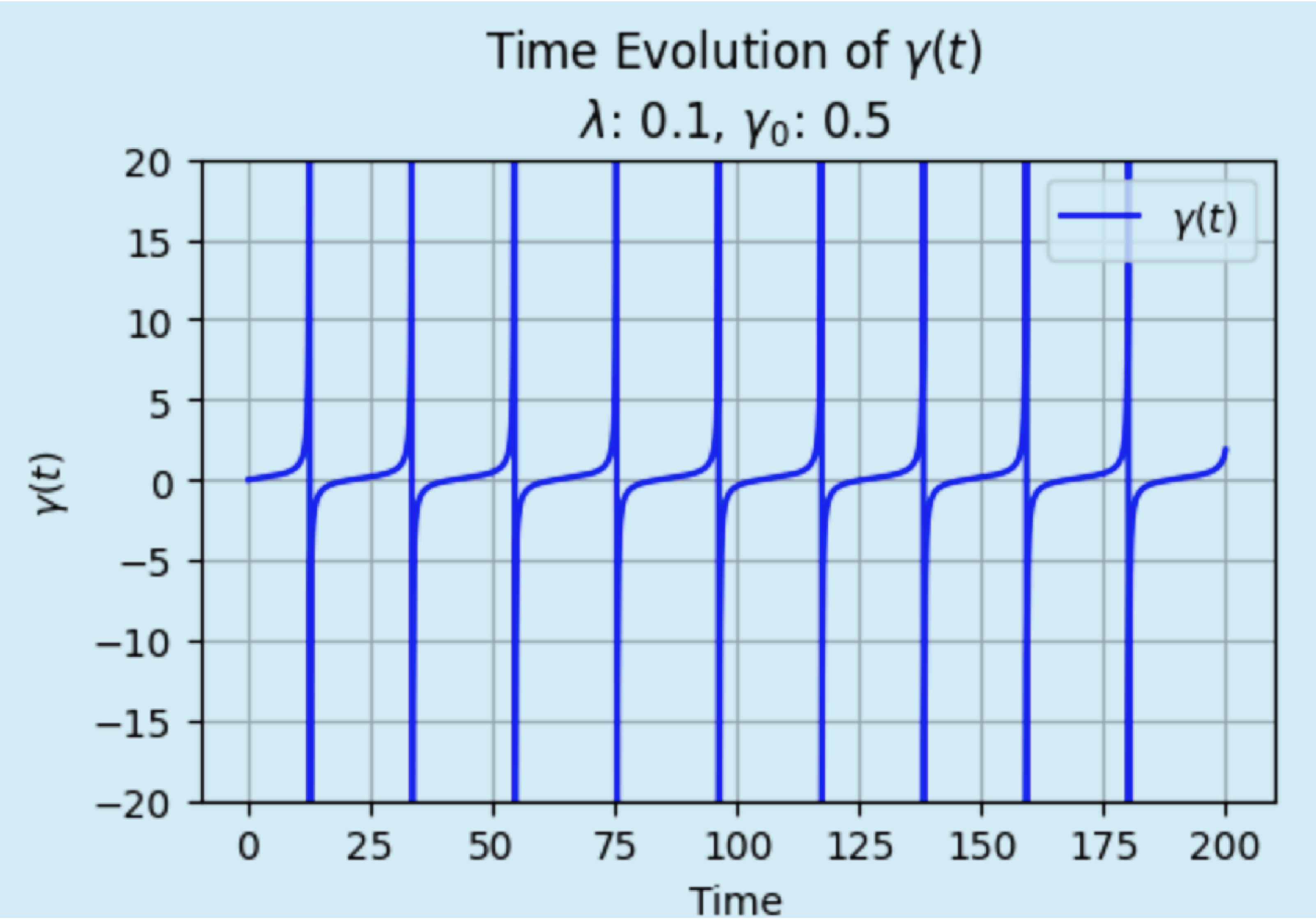


Transpiled Circuit Gate Decompostion



Referrence paper - Impact of non-Markovian evolution on characterizations of quantum thermodynamics



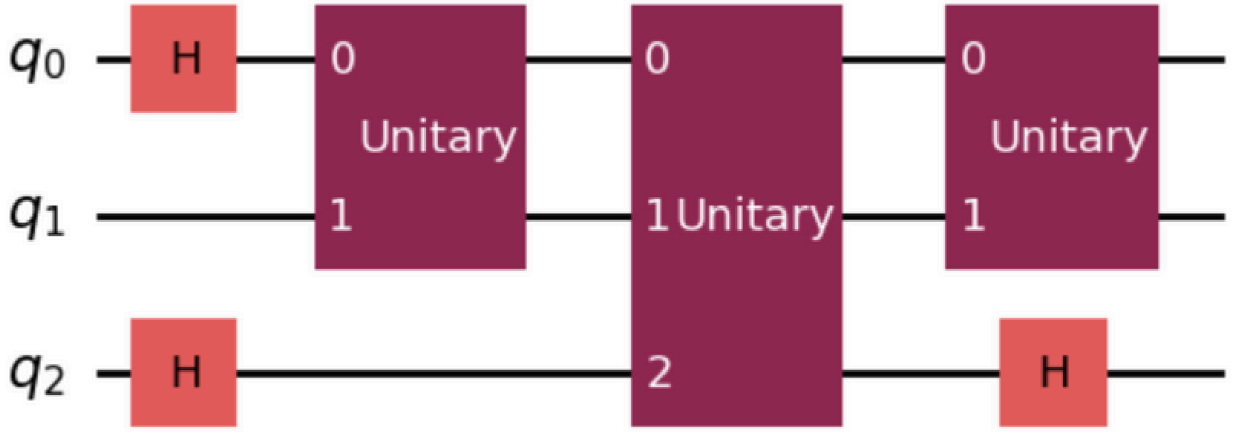


Non-Markovian regime: $\lambda < 2\gamma$

Fidelity for 2 Qubits Decay

Our Future Work Aims to Construct a method for the Non Markovian system, where the Gamma jump coefficient accepts negative values, also the Liouville form of the Master equation Continiously Changes with time

Two Qubits decay with an ancilla qubit to map the Kraus Operator



- Referrence paper - Impact of non-Markovian evolution on characterizations of quantum thermodynamics
- https://github.com/harishaseebat92/Bell-State-Evolution-under-AD-GAD-Channel

For both single-qubit and multiple-qubits scenarios, we are comparing fidelity between QISKIT quantum circuit outputs against 3 different numerical solvers {Mesolve, Strang approximation, and Kraus map}

II. DETAILS ON THE FORMS OF HEAT CURRENTS AND PROOF OF QUANTUM THERMAL VERSION OF KIRCHHOFF'S CURRENT LAW

Consider a general system of n nodes (labeled 1, 2, 3, ..., n), where each node is coupled to all other nodes. Further, each node is weakly coupled to its respective bosonic thermal bath (labeled I, II, III, ..., N). The system's Hamiltonian in this setup is given by

$$H_S = \sum_{k=1}^{n} H_k + \sum_{l,k=1,l<k}^{n} H_{lk}, \qquad (5)$$

where H_k is the Hamiltonian of the node (here, it is $H_k = \frac{\omega_k}{2}\sigma_k^z$) and H_{lk} is the coupling Hamiltonian between the nodes (here, we take $H_{lk} = J_{lk}\left(\sigma_l^x\sigma_k^x + \sigma_l^y\sigma_k^y\right)$ with J_{lk} being the coupling strength). Under the Born-Markov and rotating wave approximations, the dynamics of the system (depicted by ρ) is given by the GKSL master equation

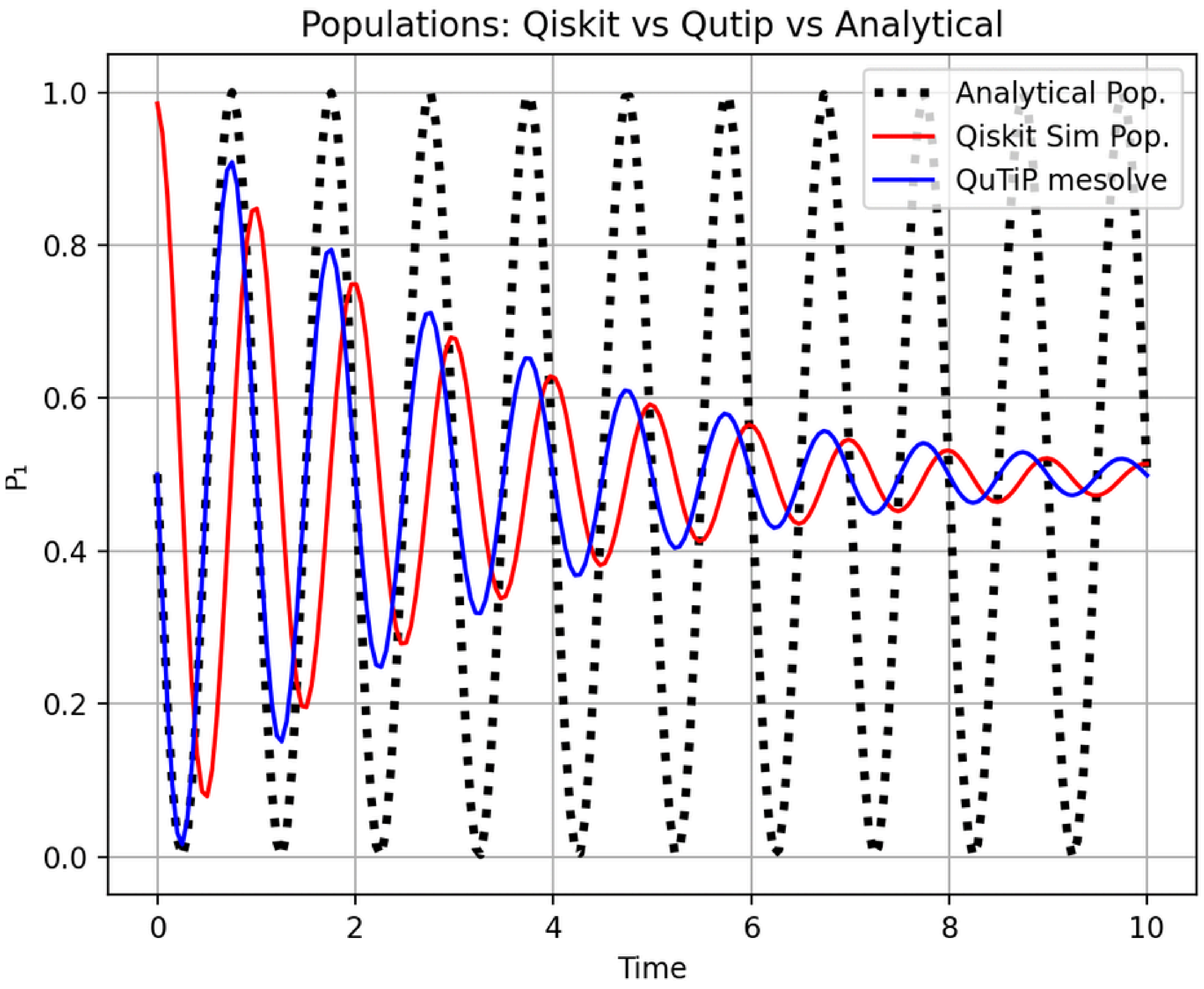
$$\frac{d\rho}{dt} = -i\left[H_S,\rho\right] + \mathcal{D}_{I1}(\rho) + \mathcal{D}_{II2}(\rho) + ... + \mathcal{D}_{Nn}(\rho), \qquad (6)$$

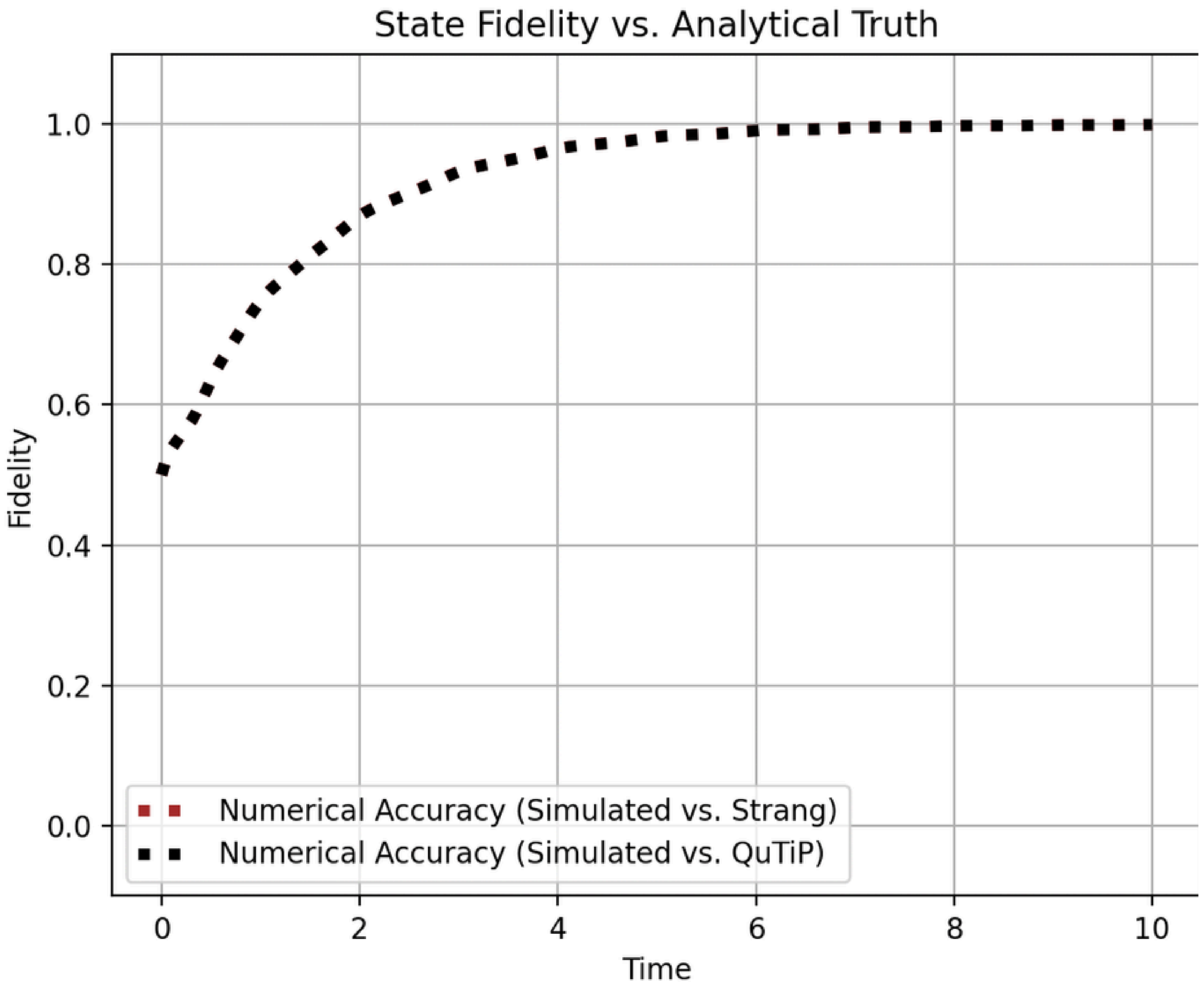
```
# Build Hamiltonian from Eq. (5)
print(f"Building Hamiltonian for {N}-qubit system...")
# Add on-site energy terms: Σ (ω_k/2) σz_k
for k in range(N):
    H += (w[k]/2) * promote_op(sigmaz, k, N)

# Add interaction terms: Σ J_lk (σx_l σx_k + σy_l σy_k)
# which is equivalent to 2 * J_lk * (σ+_l σ-_k + σ-_l σ+_k)
for (l, k), J_val in J.items():
    # Directly build the sx_l @ sx_k and sy_l @ sy_k terms
    sx_l_sx_k = promote_op_pair(sigmax, sigmax, l, k, N)
    sy_l_sy_k = promote_op_pair(sigmay, sigmay, l, k, N)
    H += J_val * (sx_l_sx_k + sy_l_sy_k)
```

For multiple qubits scenario, we need to take more care on quantum circuit gates because the hamiltonian H passed into the circuit is much complicated.

SVD is bit expensive ($O(n^3)$) when it comes to scaling to higher number of qubits

Hence we thought about removing SVD + Choi + Kraus operations, and then use trotter / strang approximation, which seems to match/overlap with the output of QUTIP numerical solvers completely (brown versus black curves)

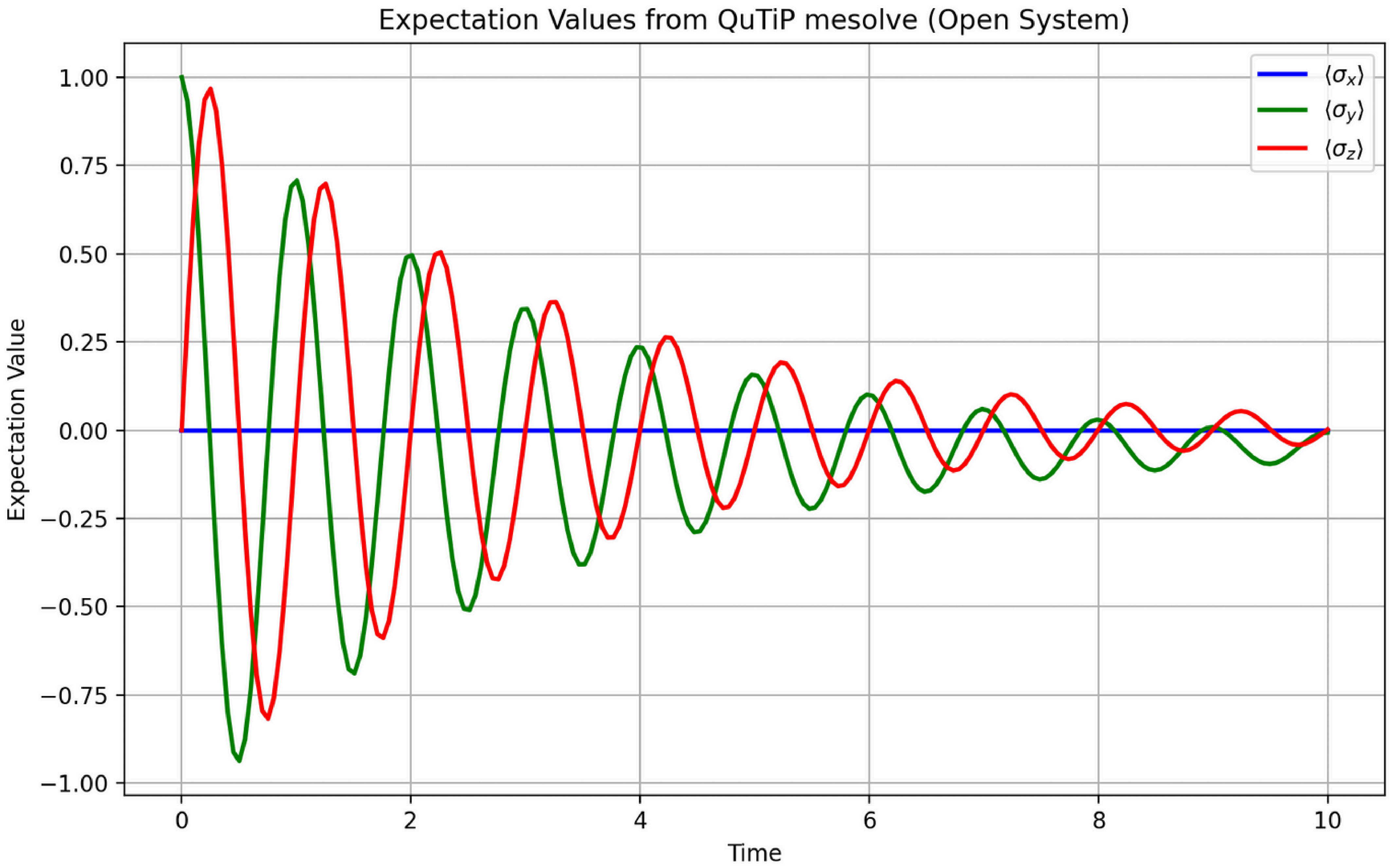




Caveat of doing so:

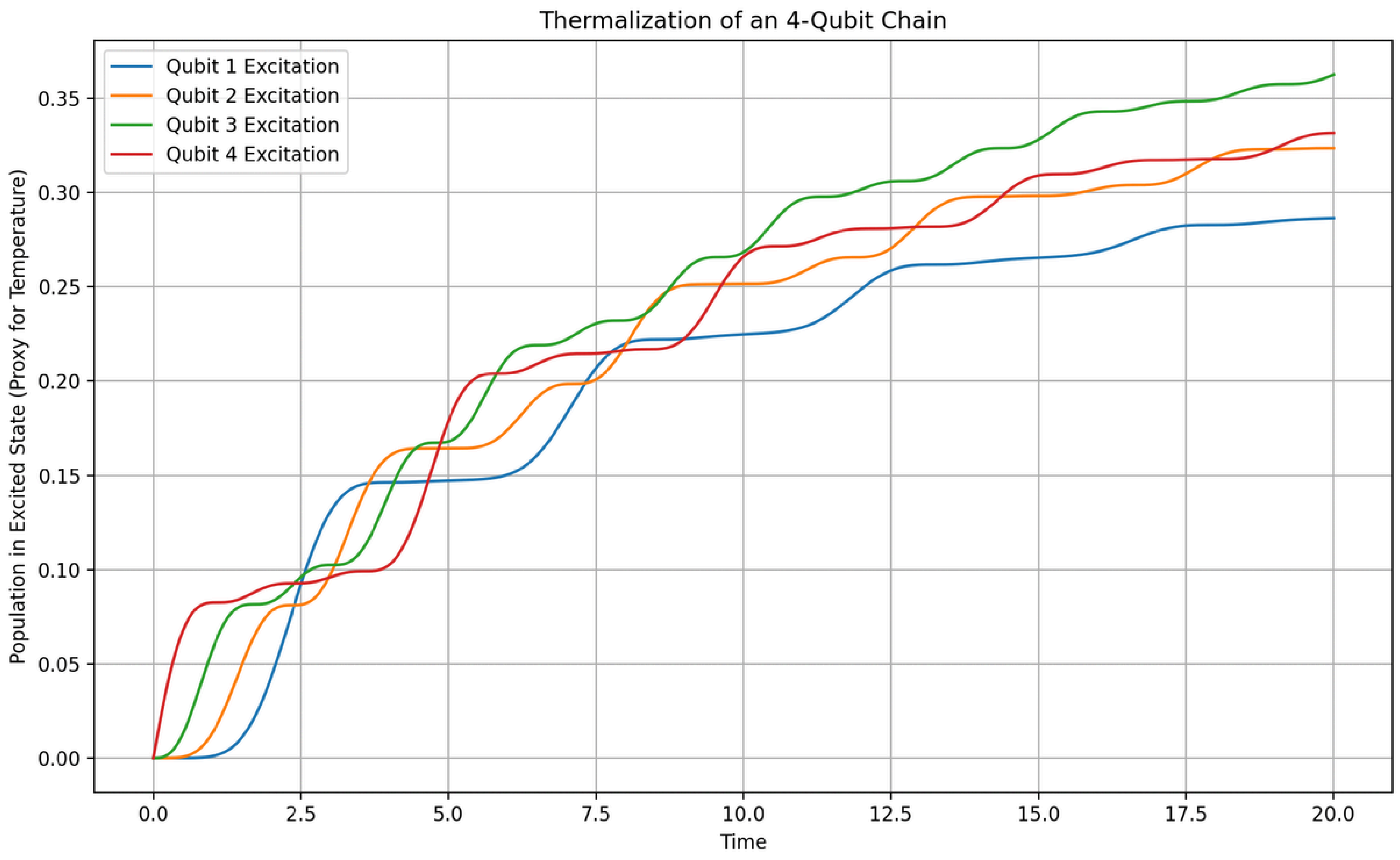
Error approximation : trotter with $O(\Delta t^2)$, Strang with $O(\Delta t^3)$)

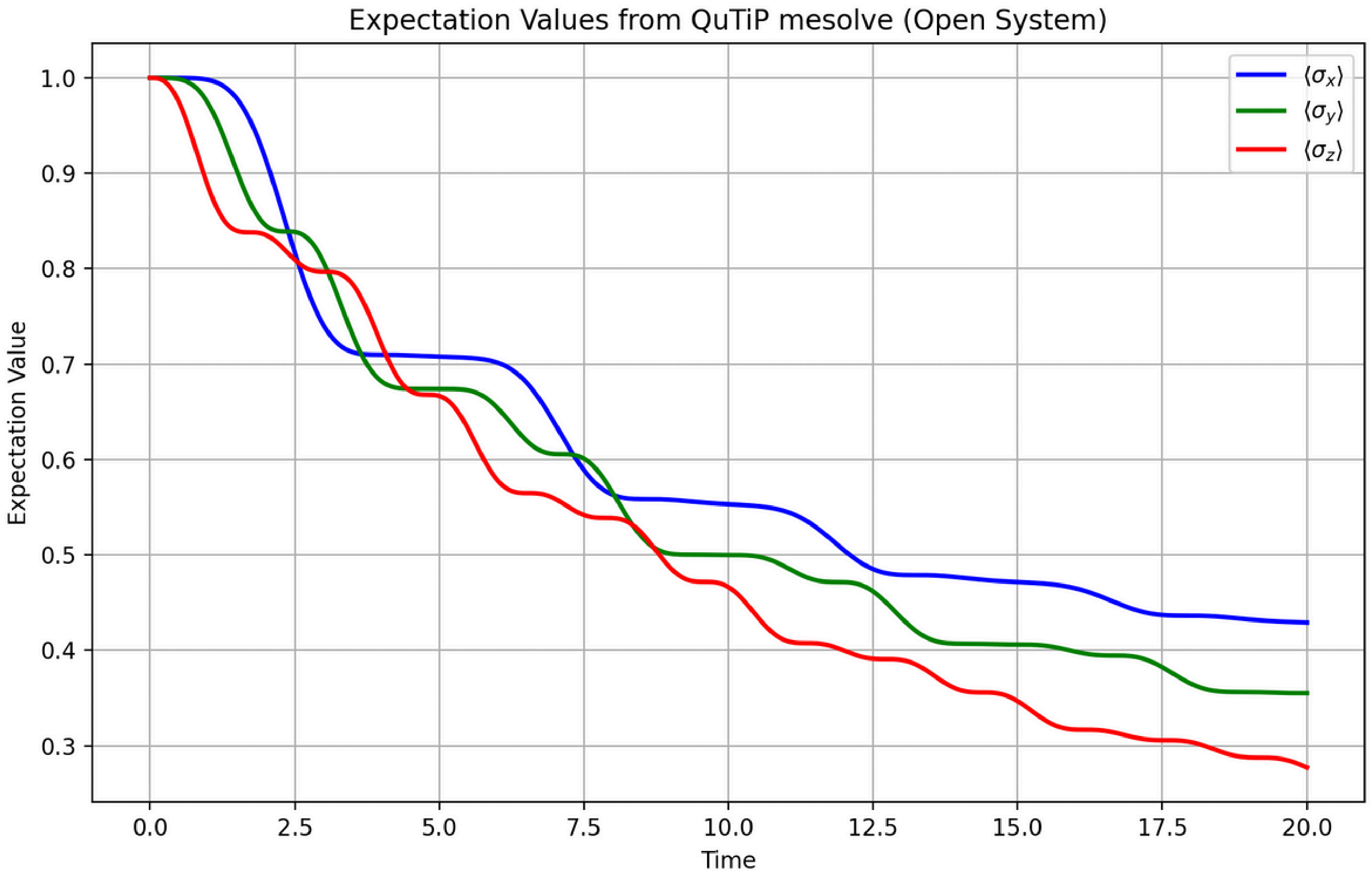
CPTP violation concern since Choi + Kraus guarantee CPTP criterion



Note: We are using both dephasing and decay for the open quantum collapse jump operators

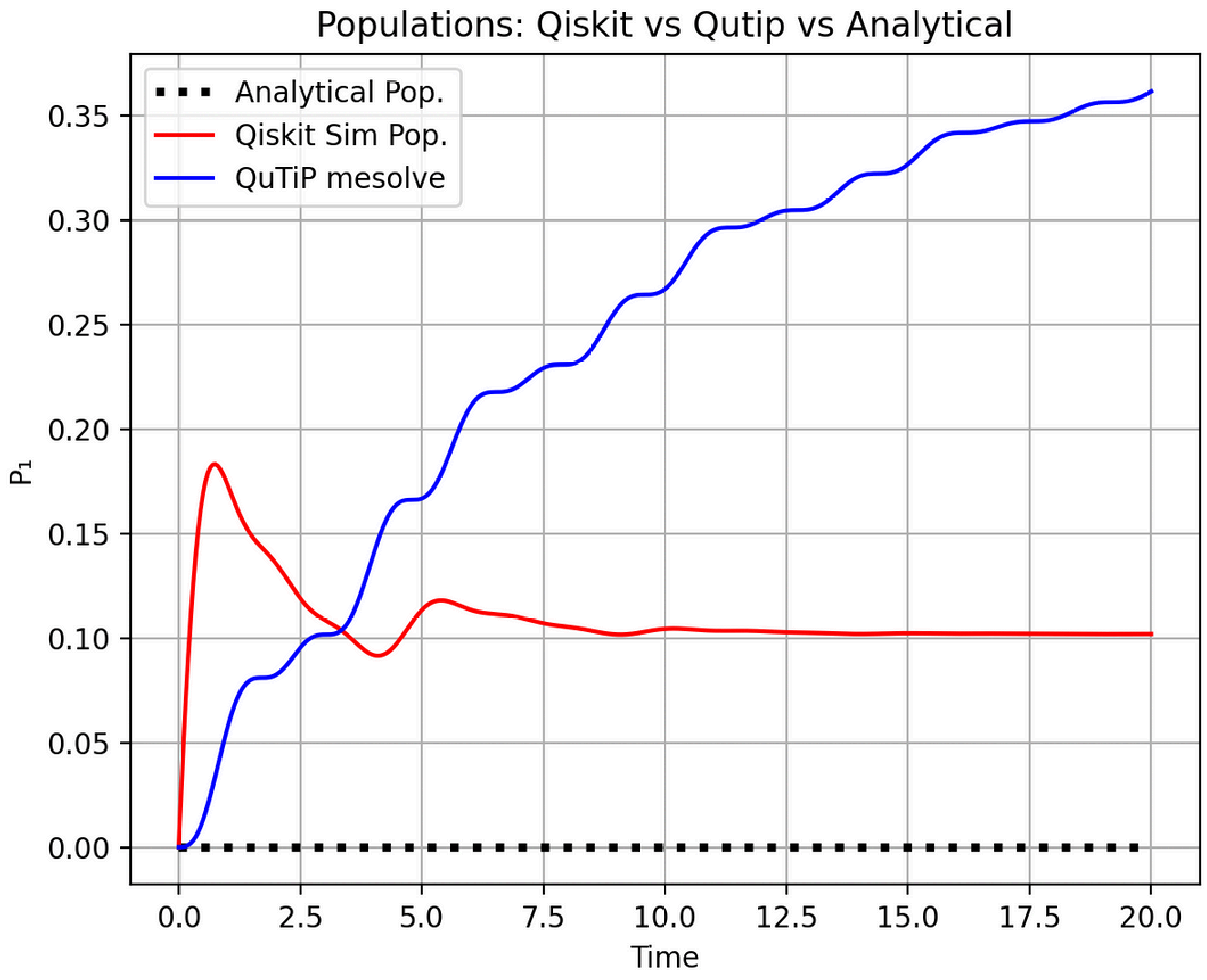
We also tried to experiment with multiple qubits for scaling purposes

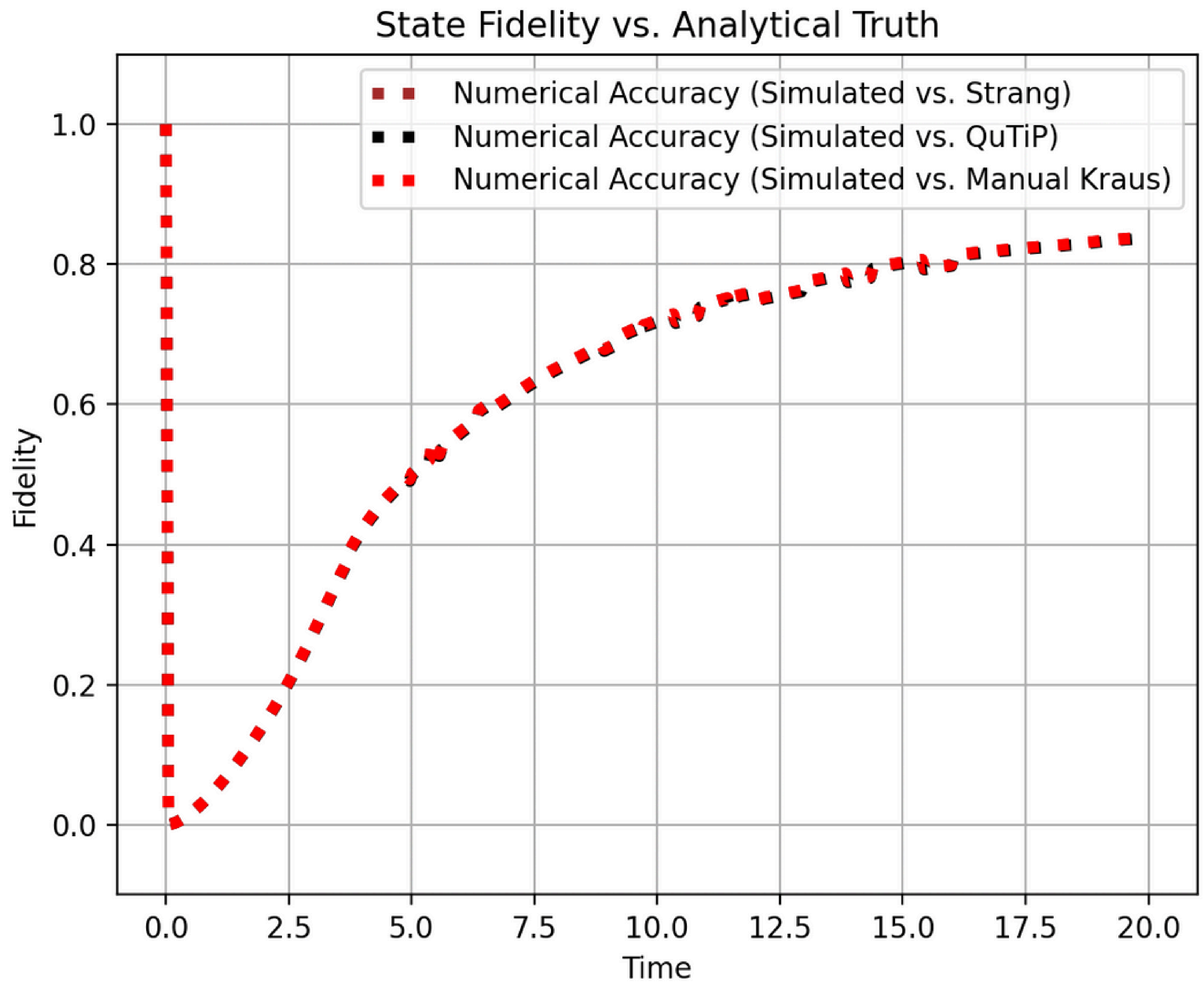




<u>Quantum Thermal Analogs of Electric Circuits: A Universal Approach</u>

For N=4 qubits, all three fidelity plots overlap each other and converge to slightly above 0.8, this means that the physical simulation modeling is correct to some extent.





FUTURE WORK

- We expect to simulate the Open system governed by the Master equation with time dependence.

- Our simulation were confined to Markovian Regime , but Systems we discussed such as bosonic , follows Non Markovian Evolution too , we plan to simulate specifically those part where the jump operator takes negative values too.

- We hope to scale to 100 qubits (currently our result is for max 4 qubits), but that would come with hardware noise nightmare.