Homework 2: Energy conservation with ODE methods

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Introduction

The system described is a simple harmonic oscillator pendulum. Numerical methods for ordinary differential equations are compared in calculating the angle and the energy of the pendulum.

Method

The Euler, Euler-Cromer, and second-order Runge-Kutta (RK2) numerical methods are compared.

Verification of program

The failure of the Euler method to properly represent the pendulum was expected; the method produced a situation in which the energy continuously increased. Meanwhile, the Euler-Cromer method and RK2 produced data that strongly resembled the analytical solution.

Data

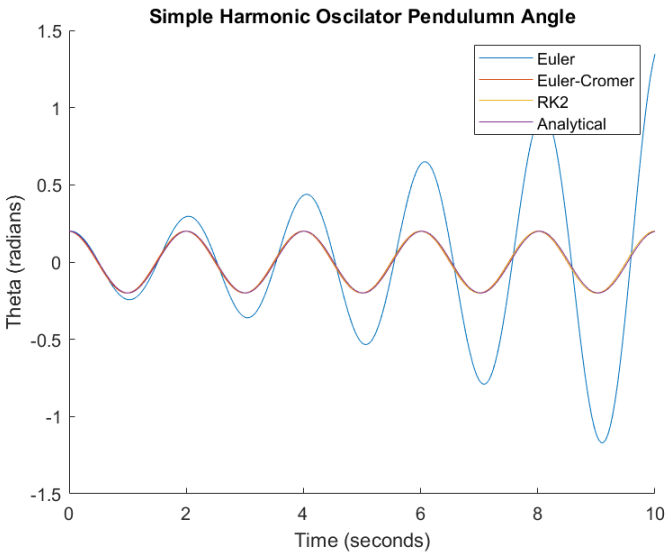

Figure 1: Euler's method fails to resemble the pendulum's motion.

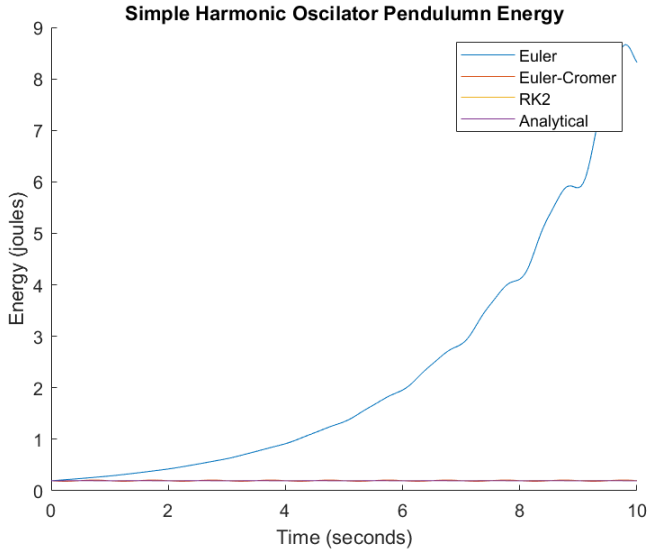

Figure 2: The failure of Euler's method to resemble the pendulum's motion is understood when observing the calculated energy by the method.

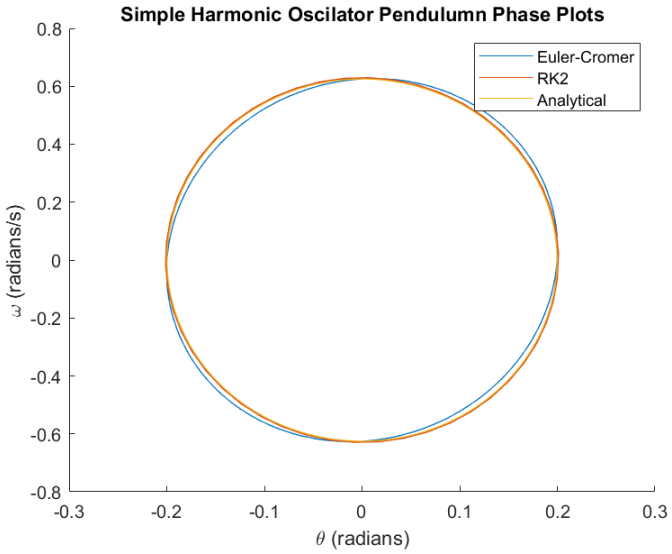

Figure 3: Euler's method is excluded from these phase plots, but it would look like a spiral. Notice the elliptical shape of the Euler-Cromer phase plot in comparison to the circular shape of the RK2 phase plot.

Analysis

Over the course of 10 seconds, the mean differences between the methods and the analytical solution for the angle (in radians) were as follows: 0.2632 for Euler, 0.0093 for Euler-Cromer, 0.0052 for RK2. This implies the failure of the Euler method as well as the higher accuracy of RK2. As seen in **Figure 1** and **Figure 2**, the Euler method fails to properly describe the pendulum, hence the much higher mean difference for the method. Meanwhile, in **Figure 3**, we notice that the RK2 method more closely resembles the circular analytical solution phase plot than the elliptical phase plot of the Euler-Cromer method.

Critique

I have gained a better understanding of the problems that arise when using numerical methods. Not all numerical methods are equal, even if they solve the same types of problems.