Percolation in a Two-Dimensional Square Lattice

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Introduction

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- For simplicity, we are only simulating the case of the 2D $L \times L$ square lattice.

Method

To simulate this process, cells will be occupied one at a time. The probability p will denote the probability of a given cell of the lattice being occupied. The cells will be assigned one of three numbers.

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- **1:** The cell is occupied but is not a part of the percolating cluster.
- **2:** The cell is occupied and is a part of the percolating cluster.

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- Once the percolating cluster is found, the probability of the cell being filled is recorded at the critical probability p_c.
- If a future cell is found to be neighboring the percolating lattice, BFS will run again, merging the cell and its respective cluster with the percolating cluster.

Analysis

- For an infinitely large 2D lattice (of any type of cell!), we find that $p_c \approx 0.593$. Because we are working with relatively small lattices, we can expect to find on average $p_c \approx 0.6$ with decent room for variation.

Analysis

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- We are also interested in the fraction of the cells contained within the percolating cluster versus the total number of cells in the lattice. Once the percolating cluster spawns, we expect the relationship

$$F = F_0(p - p_c)^{\beta} \tag{1}$$

where for an infinitely large 2D lattice (of any type of cell!), we find $\beta = \frac{5}{36}$. F_0 is a proportionality constant and unknown alongside β.