HW #1 | Daniel Benjamin | 01/20/2021

1 Decimal representation from binary

1. $1111\ 1011_2$ (signed)

$\underline{1}111\ 1011_2$

MSB
↑
negative number

2's complement:

$$\begin{array}{r} 0000\ 0100 \\ +\ \ \ \ \ \ \ \ \ \ \ \ \ 1 \\ \hline 0000\ 0101_2 \end{array}$$

$0000\ 0\underset{2^2}{1}\underset{2^1}{0}\underset{2^0}{1}_2$

weighted sum $= 1 \cdot 2^2 + 1 \cdot 2^0 = 4 + 1 = 5$

Answer is: $\boxed{-5_{10}}$

2. $0110\ 0100_2$ (signed)

$\underset{2^7}{0}\underset{2^6}{1}\underset{2^5}{1}\underset{2^4}{0}\ \underset{2^3}{0}\underset{2^2}{1}\underset{2^1}{0}\underset{2^0}{0}_2 \Rightarrow$ weighted sum $= 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^2$

$= \boxed{100_{10}}$ ⇐ answer

3. $1001\ 1010_2$ (unsigned)

$\underset{2^7}{1}\underset{2^6}{0}\underset{2^5}{0}\underset{2^4}{1}\ \underset{2^3}{1}\underset{2^2}{0}\underset{2^1}{1}\underset{2^0}{0}_2 \Rightarrow 1 \cdot 2^7 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^1 =$

$128 + 16 + 8 + 2 = \boxed{154_{10}}$ ⇐ answer

2. Minimum number of binary bits needed

1. 65437_{10}

Converting from decimal to binary:

2	65437	LSB
2	32718	r. 1
2	16359	r. 0
2	8179	r. 1
2	4089	r. 1
2	2044	r. 1
2	1022	r. 0
2	511	r. 0
2	255	r. 1
2	127	r. 1
2	63	r. 1
2	31	r. 1
	15	r. 1

2	15	
2	7	r. 1
2	3	r. 1
2	1	r. 1
	0	r. 1

↑ MSB

$\underset{15}{1}\underset{14}{1}\underset{13}{1}\underset{12}{1}\ \underset{11}{1}\underset{10}{1}\underset{9}{1}\underset{8}{1}\ \underset{7}{1}\underset{6}{0}\underset{5}{0}\underset{4}{1}\ \underset{3}{1}\underset{2}{1}\underset{1}{0}\underset{0}{1}_2$

Bit #

16 binary-bits are needed to represent decimal number

2.

2. 10361_{10}

Converting to binary:

2	10361	LSB
2	5180	r. 1
2	2590	r. 0
2	1295	r. 0
2	647	r. 1
2	323	r. 1
2	161	r. 1
2	80	r. 1
2	40	r. 0
2	20	r. 0
2	10	r. 0
2	5	r. 0
2	2	r. 1
2	1	r. 0
	0	r. 1 MSB

10100001111001_2

bit #s: 13 12 11 10 9 8 7 6 5 4 3 2 1 0

14 binary bits are needed to represent 10361_{10}

3. -4177_{10}

converting to binary:

2	4177	LSB
2	2088	r. 1
2	1044	r. 0
2	522	r. 0
2	261	r. 0
2	130	r. 1
2	65	r. 0
2	32	r. 1
2	16	r. 0
2	8	r. 0
2	4	r. 0
2	2	r. 0
2	1	r. 0
	0	r. 1 MSB

$= 0001000001010001_2$

= two's complement:

1110111110101110
\+ 1

1110111110101111

$1110\ 1111\ 1010\ 1111_2$

signed number system range:

$$-2^{(n-1)} \le -4177 \le 2^{(n-1)} - 1$$

16 binary bits are need to represent -4177_{10} in two's-complement number signed number system

3. Hexadecimal representation of following binary numbers?

1. $\underset{2^3 2^2 2^1 2^0}{1011}\ \underset{2^3 2^2 2^1 2^0}{1001}\ \underset{2^3 2^2 2^1 2^0}{1001}\ \underset{2^3 2^2 2^1 2^0}{1100}{}_2$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

$11_{10} = 8+2+1 \quad 8+1 = 9_{10} \quad 8+1 = 9_{10} \quad 12_{10} = 8_{10}+4_{10}$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

$B_{16} \quad 9_{16} \quad 9_{16} \quad C_{16}$

$\boxed{B99C_{16}}$

2. $\underset{2^3 2^2 2^1 2^0}{1101}\ \underset{2^3 2^2 2^1 2^0}{0110}\ \underset{2^3 2^2 2^1 2^0}{0111}\ \underset{2^3 2^2 2^1 2^0}{0011}{}_2$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

$8+4+1 \quad 4+2 \quad 4+2+1 \quad 2+1 = 3_{10}$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

$13_{10} \quad 6_{10} \quad 7_{10} \quad 3_{10}$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

$D_{16} \quad 6_{16} \quad 7_{16} \quad 3_{16}$

$\boxed{D673_{16}}$

3. $\underset{8421}{0011}\ \underset{8421}{0110}\ \underset{8421}{0001}\ \underset{8421}{1001}{}_2$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

$2+1 \quad 4+2 \quad 1 \quad 8+1$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

$3_{10} \quad 6_{10} \quad 1_{10} \quad 9_{10}$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

$3_{16} \quad 6_{16} \quad 1_{16} \quad 9_{16}$

$\boxed{3619_{16}}$

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4. Decimal value of each hex integer using unsigned notation

1. 4024_{16}

$$\underset{16^3}{\underline{4}}\ \underset{16^2}{\underline{0}}\ \underset{16^1}{\underline{2}}\ \underset{16^0}{\underline{4}} \Rightarrow 4 \cdot 16^3 + 0 \cdot 16^2 + 2 \cdot 16^1 + 4 \cdot 16^0$$
$$= 16384 + 0 + 32 + 4 = \boxed{16420_{10}}$$

2. FEE_{16}

A = 10
B = 11
C = 12
D = 13
E = 14
F = 15

~~151414_{10}~~

$$\underset{16^2}{\underline{F}}\ \underset{16}{\underline{E}}\ \underset{16^0}{\underline{E}} \Rightarrow 15 \cdot 16^2 + 14 \cdot 16^1 + 14 \cdot 16^0$$
$$= 3840 + 224 + 14 = \boxed{4078_{10}}$$

3. $10F3_{16}$

$$\underset{16^3}{\underline{1}}\ \underset{16^2}{\underline{0}}\ \underset{16}{\underline{F}}\ \underset{16^0}{\underline{3}} \Rightarrow 1 \cdot 16^3 + 0 \cdot 16^2 + 15 \cdot 16^1 + 3 \cdot 16^0$$
$$= 4096 + 0 + 240 + 3 = \boxed{4339_{10}}$$

5. What is the 16-bit hexadecimal representation of each decimal integer?

1. -619_{10} (signed number)

$-26B_{16}$

16	619		LSB ↓
16	38	r. 11 ⇒ B	
16	2	r. 6	
	0	r. 2	↑ MSB

Converting $-26B_{16}$ to binary (base-2)

$- \underline{0}\,\underline{0}\,\underline{1}\,\underline{0}\ \ \underline{0}\,\underline{1}\,\underline{1}\,\underline{0}\ \ \underline{1}\,\underline{0}\,\underline{1}\,\underline{1}_2$

$\Rightarrow -0000\ 0010\ 0110\ 1011_2$

Finding two's complement:

```
  1111 1101 1001 0100
+                   1
---------------------
  1111 1101 1001 0101₂
```

Converting back to hexadecimal:

$\underset{2^3 2^2 2^1 2^0}{1111}\ \ \underset{2^3 2^2 2^1 2^0}{1101}\ \ \underset{2^3 2^2 2^1 2^0}{1001}\ \ \underset{2^3 2^2 2^1 2^0}{0101}_2 \Rightarrow \boxed{FD95_{16}}$

15 ⇒ F 13 ⇒ D 9 5

5. 2. -312_{10}

Converting to binary:

2	312	LSB
2	156	r. 0
2	78	r. 0
2	39	r. 0
2	19	r. 1
2	9	r. 1
2	4	r. 1
2	2	r. 0
2	1	r. 0
	0	r. 1
		MSB

$\Rightarrow -0000\ 0001\ 0011\ 1000_2$

Performing 2's complement:

$$\begin{array}{r} 1111\ 1110\ 1100\ 0111 \\ +\qquad\qquad\qquad\qquad 1 \\ \hline 1111\ 1110\ 1100\ 1000_2 \end{array}$$

Converting binary 2's complement number to 16-bit hexadecimal:

1111 (8 4 2 1)	1110 (8 4 2 1)	1100 (8 4 2 1)	1000_2 (8 4 2 1)
15 ⇒ F	14 ⇒ E	12 ⇒ C	8

$\boxed{FEC8_{16}}$

3. $+1947_{10}$

Converting to binary:

2	1947	LSB
2	973	r. 1
2	486	r. 1
2	243	r. 0
2	121	r. 1
2	60	r. 1
2	30	r. 0
2	15	r. 0
2	7	r. 1
2	3	r. 1
2	1	r. 1
	0	r. 1
		MSB

$0000\ 0111\ 1001\ 1011_2$

Converting to hexadecimal (base-16):

0000 (8 4 2 1)	0111 (8 4 2 1)	1001 (8 4 2 1)	1011_2 (8 4 2 1)
0	7	9	11 ⇒ B

$\boxed{079B_{16}}$

6. 8-bit binary (2's complement) representation of each of the decimal integers?

1. -35_{10}

Converting to binary:

LSB

2 | 35
2 | 17 r. 1
2 | 8 r. 1
2 | 4 r. 0
2 | 2 r. 0
2 | 1 r. 0
0 r. 1

MSB

$-0010\ 0011_2$

two's complement:

$-0010\ 0011_2$

1101 1100

\+ 1

1101 1101 ₂

$\boxed{1101\ 1101_2}$

2. $+103_{10}$

converting to binary: ⇒

LSB

2 | 103
2 | 51 r. 1
2 | 25 r. 1
2 | 12 r. 1
2 | 6 r. 0
2 | 3 r. 0
2 | 1 r. 1
0 r. 1

MSB

⇒ $\boxed{0110\ 0111_2}$

3. -114_{10}

Converting to binary:

LSB

2 | 114
2 | 57 r. 0
2 | 28 r. 1
2 | 14 r. 0
2 | 7 r. 0
2 | 3 r. 1
2 | 1 r. 1
0 r. 1

MSB

$-0111\ 0010_2$

two's complement:

$-0111\ 0010_2$

1000 1101

\+ 1

1000 1110

$\boxed{1000\ 1110_2}$

7. Write ASCII code ~~for~~ the string "COVID" in hexadecimal

C $\Rightarrow \underset{8}{0}\underset{4}{1}\underset{2}{0}\underset{1}{0}\ \underset{8}{0}\underset{4}{0}\underset{2}{1}\underset{1}{1}{}_2 \Rightarrow 43_{16}$

O $\Rightarrow \underset{8}{0}\underset{4}{1}\underset{2}{0}\underset{1}{0}\ \underset{8}{1}\underset{4}{1}\underset{2}{1}\underset{1}{1}{}_2 \Rightarrow 4F_{16}$

V $\Rightarrow \underset{8}{0}\underset{4}{1}\underset{2}{0}\underset{1}{1}\ \underset{8}{0}\underset{4}{1}\underset{2}{1}\underset{1}{0}{}_2 \Rightarrow 56_{16}$

I $\Rightarrow \underset{8}{0}\underset{4}{1}\underset{2}{0}\underset{1}{0}\ \underset{8}{1}\underset{4}{0}\underset{2}{0}\underset{1}{1}{}_2 \Rightarrow 49_{16}$

D $\Rightarrow \underset{8}{0}\underset{4}{1}\underset{2}{0}\underset{1}{0}\ \underset{8}{0}\underset{4}{1}\underset{2}{0}\underset{1}{0}{}_2 \Rightarrow 44_{16}$

$\boxed{434F564944_{16}}$

8. Range ~~of~~ decimal values represented by

1. 7-bit unsigned integer?

Range of 7-bit unsigned integer ~~in binary~~

0_{10} to 127_{10} $\qquad 2^7 - 1 = 128_{10}$ $\qquad \boxed{[0_{10}, 128_{10}]}$

low $\qquad$ high

2. 7-bit signed integer

Range:

$$[-2^{(n-1)}, 2^{(n-1)} - 1]$$

$$[-2^{(7-1)}, 2^{(7-1)} - 1] \Rightarrow [-2^6, 2^6 - 1] \Rightarrow \boxed{[-64_{10}, 63_{10}]}$$