

# Discrete Mathematics HW3

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30th of September 2020

6. Suppose you are given with the following question when taking a multiple choice test:

What percentage of answers to this question are correct?

- (a) 50%
- (b) 25%
- (c) 0%
- (d) 50%

Choosing (a) and (d) is correct because 2 / 4 answers are exactly 50%. Also, choosing (b) will be correct, because 1 / 4 is 25% of answers. All other options are incorrect. Choosing only (a) or (d) cannot be true because it is only 25% of answers. Choosing (c) is self-explanatory, because if we choose something, it cannot be 0%. Other combinations contradict with each other.

7. Prove formally that  $init(rev(init([student]))) = [nedut]$ .

**By definition of  $init(s)$ ,  $\forall s \in S(A)$**

$$\begin{aligned} init([student]) &= s : init([tudent]) = \\ &= s : (t : init([udent])) = \dots = \\ &= s : (t : (u : (d : (e : (n : init([t])))))) = s : (t : (u : (d : (e : (n : []))))) \end{aligned}$$

**By definition of  $rev(s)$ ,  $\forall s \in S(A)$**

$$\begin{aligned} rev(init(student)) &= rev(s : (t : (u : (d : (e : (n : []))))) = \\ &= app(rev(t : (u : (d : (e : (n : []))))) , [s]) = \\ &= app(app(rev(u : (d : (e : (n : []))))) , [t]) , [s] = \dots = \\ &= app(app(app(app(app(app([], [n]), [e]), [d]), [u]), [t]), [s]) \end{aligned}$$

**Roll it down by definition of  $app(s, t), \forall s, t \in S(A)$**

$$\begin{aligned}
app(app(app(app(app([], [n]), [e]), [d]), [u]), [t]), [s]) &= \\
app(app(app(app(app([n], [e]), [d]), [u]), [t]), [s]) &= \\
app(app(app(app(n : app([], [e]), [d]), [u]), [t]), [s]) &= \\
app(app(app(app(n : [e], [d]), [u]), [t]), [s]) &= \dots = \\
&= n : (e : (d : (u : (t : (s : []))))))
\end{aligned}$$

**Finally, after applying  $init$**

$$\begin{aligned}
init(n : (e : (d : (u : (t : (s : [])))))) &= \\
n : init(e : (d : (u : (t : (s : [])))))) &= \dots = \\
n : (e : (d : (u : (t : init([s]))))) &= \\
n : (e : (d : (u : (t : [])))) &= [nedut]
\end{aligned}$$

8. Prove formally that for every  $s, t \in S(A)$  the following statements hold:

$\forall x \in A$  and  $\forall s, t \in S(A)$

(a)  $lh(s) = 0 \iff s = []$

If  $s = []$ , then by definition of length function  $lh([]) = 0$ .

If  $lh(s) = 0$ , suppose  $s = x : s'$ . Then by definition  $lh(x : s') = 1 + lh(s') \neq 0$ , which contradicts with former condition. Hence  $s = []$ .

(b)  $lh(app(s, t)) = lh(s) + lh(t)$

Proof by induction.

$$\text{Base } s = []: \quad lh(app([], t)) \stackrel{app1}{=} lh(t) \stackrel{lh1}{=} lh([]) + lh(t) = lh(s) + lh(t)$$

$$\begin{aligned}
\text{Inductive step: } lh(app(x : s, t)) &\stackrel{app2}{=} lh(x : app(s, t)) \stackrel{lh2}{=} 1 + \\
lh(app(s, t)) &\stackrel{ind.hyp}{=} 1 + lh(s) + lh(t) \stackrel{lh2}{=} lh(x : s) + lh(t)
\end{aligned}$$

(c)  $lh(rev(s)) = lh(s)$

To prove this let's use statement (b). Proof by induction.

$$\text{Base } s = []: \quad lh(rev([])) \stackrel{rev1}{=} lh([]) \stackrel{lh1}{=} 0 = lh([]) = lh(s)$$

$$\begin{aligned}
\text{Inductive step: } lh(rev(x : s)) &\stackrel{rev2}{=} lh(app(rev(s), [x])) \stackrel{(b)}{=} lh(rev(s)) + \\
lh([x]) &\stackrel{ind.hyp}{=} lh(s) + lh([x]) = lh([x]) + lh(s) \stackrel{(b)}{=} lh(app([x], s)) \stackrel{app2}{=} \\
lh(x : app([], s)) &\stackrel{app1}{=} lh(x : s)
\end{aligned}$$

9. Prove that  $app(t, s) = app(r, s)$  implies  $t = r$  for every  $s, t, r \in S(A)$ :

In proof I'm going to use lemmas proved in seminars.

$$\begin{aligned}
 app(rev(s), rev(t)) &\stackrel{cw1\ 16(a)}{=} rev(app(t, s)) = rev(app(r, s)) \stackrel{cw1\ 16(a)}{=} app(rev(s), rev(r)) \\
 app(rev(s), rev(t)) &= app(rev(s), rev(r)) \rightarrow rev(t) = rev(r) \quad (cw1\ 15) \\
 t &\stackrel{cw1\ 16(b)}{=} rev(rev(t)) = rev(rev(r)) \stackrel{cw1\ 16(b)}{=} r
 \end{aligned}$$