DSBA Calculus HW7

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22th of October, 2020

1. Find following limits:

$$\text{(a)} \lim_{x\to+\infty}\left(\frac{x+1}{x+2}\right)^{1-x} = \lim_{x\to+\infty}\left(1-\frac{1}{x+2}\right)^{(1-x)\cdot\frac{x+2}{x+2}} = \lim_{x\to+\infty} e^{\frac{x-1}{x+2}}$$

As e^x is a continuous function we can consider a limit of an argument:

$$\lim_{x\to+\infty}\frac{x-1}{x+2} = 1 \Rightarrow \lim_{x\to+\infty}\left(\frac{x+1}{x+2}\right)^{1-x} = e^1 = e$$

$$\text{(b)} \lim_{x\to 0}\left(\frac{2-3x}{5-4x}\right)^{\frac{1}{x^2}} = 0$$

$$\left(\frac{2}{5}\right)^{\frac{1}{x^2}} \to 0 \quad x \to 0$$

$$\text{(c)} \lim_{x\to 0}\left(\frac{1+2x}{1-3x}\right)^{\frac{3}{\sin 2x}} = \lim_{x\to 0}\left(1+\frac{5x}{1-3x}\right)^{\frac{3}{\sin 2x}\cdot\frac{5x}{1-3x}\cdot\frac{1-3x}{5x}} = \lim_{x\to 0} e^{\frac{3}{\sin 2x}\frac{5x}{1-3x}}$$

$$\lim_{x\to 0}\frac{3}{\sin 2x}\frac{5x}{1-3x} = \lim_{x\to 0}\frac{15}{2(1-3x)} = \frac{15}{2} \Rightarrow \lim_{x\to 0}\left(\frac{1+2x}{1-3x}\right)^{\frac{3}{\sin 2x}} = e^{\frac{15}{2}}$$

2. Compute if $a = 0, a = 1, a = +\infty$

$$\text{(a)} \lim_{x\to 0}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}} = \frac{1}{2} \quad \text{(By substitution of } x\text{)}$$

$$\text{(b)} \lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}} = \lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1}{1+\sqrt{x}}} = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$$

(c) $\lim\limits_{x\to+\infty}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}} = \lim\limits_{x\to+\infty}\left(1-\frac{1}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$

$\lim\limits_{x\to+\infty} -\frac{1-\sqrt{x}}{(1-x)(2+x)} = \lim\limits_{x\to+\infty} -\frac{1}{(1+\sqrt{x})(2+x)} = 0 \Rightarrow$

$\lim\limits_{x\to+\infty}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}} = e^0 = 1$

3. Find the following limits:

$$\text{(a) } \lim_{x\to0}\frac{\tan 4x}{7^{3x}-1} = \lim_{x\to0}\frac{4x}{3x\ln 7} = \frac{4}{3\ln 7}$$

$$\text{(b) } \lim_{x\to0}\frac{1-\cos 3x}{x\cdot\arctan 8x} = \lim_{x\to0}\frac{9x^2}{2x\cdot\arctan 8x} = \lim_{x\to0}\frac{9x^2}{16x^2} = \frac{9}{16}$$

$$\text{(c) } \lim_{x\to0}\frac{\ln(1-5x)}{\sqrt[3]{7x+8}-2} = \frac{-5x}{2(\sqrt[3]{\frac{7}{8}x+1}-1)} = \frac{-5x}{2\frac{7}{3\cdot 8}x} = -\frac{60}{7}$$

$$\text{(d) } \lim_{x\to0}\frac{\ln^3(1+2x)}{x(e^{x^2}-1)} = \lim_{x\to0}\frac{8x^3}{x(e^{x^2}-1)} = \lim_{x\to0}\frac{8x^2}{x^2} = 8$$

$$\text{(e) } \lim_{x\to0}\frac{\arcsin(\ln(1+x))}{2x+x^2} = \lim_{x\to0}\frac{x}{2x+x^2} = \lim_{x\to0}\frac{1}{2+x} = \frac{1}{2}$$

$$x\to0 \Rightarrow \ln(1+x)\to0 \Rightarrow \arcsin(\ln(1+x))\to0$$

$$\text{(f) } \lim_{x\to0}\frac{8^x-6^x}{x} = \lim_{x\to0}\frac{8^x-1-(6^x-1)}{x} = \lim_{x\to0}\frac{x\ln 8-x\ln 6}{x} = \ln\frac{4}{3}$$