Discrete Mathematics HW3

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6. Suppose you are given with the following question when taking a multiple choice test:

   What percentage of answers to this question are correct?

   (a) 50%
   (b) 25%
   (c) 0%
   (d) 50%

   Choosing (a) and (d) is correct because 2 / 4 answers are exactly 50%. Also, choosing (b) will be correct, because 1 / 4 is 25% of answers. All other options are incorrect. Choosing only (a) or (d) cannot be true because it is only 25% of answers. Choosing (c) is self-explanatory, because if we choose something, it cannot be 0%. Other combinations contradict with each other.

7. Prove formally that $init(rev(init([student]))) = [nedut]$.

   By definition of $init(s), \forall s \in S(A)$

$$
\begin{aligned}
init([student]) = s : init([tudent]) = \\
s : (t : init([udent])) = ... = \\
s : (t : (u : (d : (e : (n : init([t])))))) = s : (t : (u : (d : (e : (n : [])))))
\end{aligned}
$$

   By definition of $rev(s), \forall s \in S(A)$

$$
\begin{aligned}
rev(init([student])) = rev(s : (t : (u : (d : (e : (n : [])))))) = \\
app(rev(t : (u : (d : (e : (n : []))))), [s]) = \\
app(app(rev(u : (d : (e : (n : [])))), [t]), [s]) = ... = \\
= app(app(app(app(app(app([], [n]), [e]), [d]), [u]), [t]), [s])
\end{aligned}
$$

Roll it down by definition of $app(s,t), \forall s,t \in S(A)$

$$\begin{aligned} app(app(app(app(app(app([], [n]), [e]), [d]), [u]), [t]), [s]) = \\ app(app(app(app(app([n], [e]), [d]), [u]), [t]), [s]) = \\ app(app(app(app(n : app([], [e]), [d]), [u]), [t]), [s]) = \\ app(app(app(app(n : [e], [d]), [u]), [t]), [s]) = ... = \\ = n : (e : (d : (u : (t : (s : []))))) \end{aligned}$$

Finally, after applying $init$

$$\begin{aligned} init(n : (e : (d : (u : (t : (s : [])))))) = \\ n : init(e : (d : (u : (t : (s : []))))) = ... = \\ n : (e : (d : (u : (t : init([s]))))) = \\ n : (e : (d : (u : (t : [])))) = [nedut] \end{aligned}$$

8. Prove formally that for every $s,t \in S(A)$ the following statements hold: $\forall x \in A$ and $\forall s,t \in S(A)$

   (a) $lh(s) = 0 \iff s = []$
   If $s = []$, then by definition of length function $lh([]) = 0$.
   If $lh(s) = 0$, suppose $s = x : s'$. Then by definition $lh(x : s') = 1 + lh(s') \neq 0$, which contradicts with former condition. Hence $s = []$.

   (b) $lh(app(s,t)) = lh(s) + lh(t)$
   Proof by induction.

   Base $s = []$**:** $lh(app([], t)) \overset{app1}{=} lh(t) \overset{lh1}{=} lh([]) + lh(t) = lh(s) + lh(t)$

   Inductive step: $lh(app(x : s, t)) \overset{app2}{=} lh(x : app(s,t)) \overset{lh2}{=} 1 + lh(app(s,t)) \overset{ind.hyp}{=} 1 + lh(s) + lh(t) \overset{lh2}{=} lh(x : s) + lh(t)$

   (c) $lh(rev(s)) = lh(s)$
   To prove this let's use statement (b). Proof by induction.

   Base $s = []$**:** $lh(rev([])) \overset{rev1}{=} lh([]) \overset{lh1}{=} 0 = lh([]) = lh(s)$

   Inductive step: $lh(rev(x : s)) \overset{rev2}{=} lh(app(rev(s), [x])) \overset{(b)}{=} lh(rev(s)) + lh([x]) \overset{ind.hyp}{=} lh(s) + lh([x]) = lh([x]) + lh(s) \overset{(b)}{=} lh(app([x], s)) \overset{app2}{=} lh(x : app([], s)) \overset{app1}{=} lh(x : s)$

9. Prove that $app(t, s) = app(r, s)$ implies $t = r$ for every $s, t, r \in S(A)$:
   In proof I'm going to use lemmas proved in seminars.

   $app(rev(s), rev(t)) \stackrel{cw1\ 16(a)}{=} rev(app(t, s)) = rev(app(r, s)) \stackrel{cw1\ 16(a)}{=} app(rev(s), rev(r))$
   $app(rev(s), rev(t)) = app(rev(s), rev(r)) \to rev(t) = rev(r) \quad (cw1\ 15)$
   $t \stackrel{cw1\ 16(b)}{=} rev(rev(t)) = rev(rev(r)) \stackrel{cw1\ 16(b)}{=} r$