DSBA Calculus HW8

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1. Find the points of discontinuity of the function and determine their types:

$$f(x) = \frac{x^2+x}{|x|(x-1)}$$

$x = 0$ and $x = 1$ are potential points of discontinuity. Let's check them:

$$\lim_{x\to 0-} \frac{x^2+x}{|x|(x-1)} = \lim_{x\to 0-} \frac{x(x+1)}{-x(x-1)} = \lim_{x\to 0-} -\frac{x+1}{x-1} = 1$$
$$\lim_{x\to 0+} \frac{x^2+x}{|x|(x-1)} = \lim_{x\to 0+} \frac{x(x+1)}{x(x-1)} = \lim_{x\to 0+} \frac{x+1}{x-1} = -1$$

One-sided limits are not equal, therefore $x = 0$ is a jump discontinuity.

$$\lim_{x\to 1-} \frac{x^2+x}{|x|(x-1)} = \lim_{x\to 1-} \frac{x(x+1)}{x(x-1)} = \lim_{x\to 1-} \frac{x+1}{x-1} = -\infty$$
$$\lim_{x\to 1+} \frac{x^2+x}{|x|(x-1)} = \lim_{x\to 1+} \frac{x(x+1)}{x(x-1)} = \lim_{x\to 1+} \frac{x+1}{x-1} = +\infty$$

Limit is equal to infinity, hence it is an essential discontinuity.

$$f(x) = \begin{cases} -\frac{4}{x-5}, & x \le 1 \\ 3x^2+5x-7, & 1 < x \le 9 \\ \frac{9}{x-9}, & x > 9 \end{cases}$$

We don't consider $x = 5$ in $-\frac{4}{x-5}$ because it doesn't lie in its domain.

$$\lim_{x\to 1-} -\frac{4}{x-5} = 1$$
$$\lim_{x\to 1+} 3x^2+5x-7 = 1$$

One-sided limits are equal, therefore $f(x) \in C(1)$.

$$\lim_{x \to 9-} 3x^2 + 5x - 7 = 281$$
$$\lim_{x \to 9+} \frac{9}{x-9} = +\infty$$

When $x \to 9+$, then $f(x) \to +\infty$, hence $x = 9$ is an essential discontinuity.

2. For which value of k will the function $f(x) = \frac{x^2-(k-2)x+8}{x-k}$ have a removable discontinuity at $x = k$?

   Removal discontinuity at $x = x_0$ means that $\exists \lim_{x \to x_0} f(x) \neq f(x_0)$ and the limit is finite.

   Let's divide the numerator by $x - k$ with remainder:

   $$x^2 - (k-2)x + 8 = (x+2)(x-k) + 2k + 8$$

   Then the limit:

   $$\lim_{x \to k} \frac{x^2 - (k-2)x + 8}{x - k} = \lim_{x \to k} \left(x + 2 + \frac{2k+8}{x-k} \right)$$

   If $k = -4$ then the limit is finite and equal to $\lim_{x \to -4}(x + 2 + \frac{0}{x+4}) = -2$ and $f(-4)$ is undefined, therefore $f(x)$ has a removable discontinuity at $x = k$ for $k = -4$.