DSBA Calculus HW3

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1. Find the following limits:

   (a) $\lim_{n\to\infty} \frac{n^2+4n-11}{3n^3-4n^2+5n-2}$

   $$\lim_{n\to\infty} \frac{n^2+4n-11}{3n^3-4n^2+5n-2} = \lim_{n\to\infty} \frac{\frac{1}{n}+\frac{4}{n^2}-\frac{11}{n^3}}{3-\frac{4}{n}+\frac{5}{n^2}-\frac{2}{n^3}} = \left(\frac{0}{c}\right) = 0$$

   (b) $\lim_{n\to\infty} \frac{5n^4-6n+2}{-n^2+n-1}$

   $$\lim_{n\to\infty} \frac{5n^4-6n+2}{-n^2+n-1} = \lim_{n\to\infty} \frac{5-\frac{6}{n^3}+\frac{2}{n^4}}{-\frac{1}{n^2}+\frac{1}{n^3}-\frac{1}{n^4}} = \left(\frac{c}{0}\right) = -\infty$$

   Denominator approaches zero from negative values, because $-\frac{1}{n^2}$ has negative coefficient.

   (c) $\lim_{n\to\infty} \frac{\sqrt{3n^4+2n^3+5}}{n^2+7}$

   $$\lim_{n\to\infty} \frac{\sqrt{3n^4+2n^3+5}}{n^2+7} = \lim_{n\to\infty} \frac{\sqrt{3+\frac{2}{n}+\frac{5}{n^4}}}{1+\frac{7}{n^2}} = \sqrt{3}$$

   (d) $\lim_{n\to\infty} \frac{1-n+2n^2}{2+4+...+2n}$
   By sum of arithmetic progression:

   $$\lim_{n\to\infty} \frac{1-n+2n^2}{2+4+...+2n} = \lim_{n\to\infty} \frac{1-n+2n^2}{\frac{2+2n}{2}n} = \lim_{n\to\infty} \frac{1-n+2n^2}{n+n^2} = \lim_{n\to\infty} \frac{\frac{1}{n^2}-\frac{1}{n}+2}{\frac{1}{n}+1} = 2$$

   (e) $\lim_{n\to\infty} \frac{n^{10}-1}{1+n\cdot 1.1^n}$
   Let's use this fact $\lim_{n\to\infty} \frac{n^k}{a^n} = 0,\ |a| > 1$

   $$\lim_{n\to\infty} \frac{n^{10}-1}{1+n\cdot 1.1^n} = \lim_{n\to\infty} \frac{\frac{n^{10}}{1.1^n}-\frac{1}{1.1^n}}{\frac{1}{1.1^n}+n} = \left(\frac{0}{\infty}\right) = 0$$

(f) $\lim_{n\to\infty}\left(\frac{-5n+4n^2-4}{n-5}-\frac{4n^2-3}{n+4}\right)$

$$\lim_{n\to\infty}\left(\frac{-5n+4n^2-4}{n-5}-\frac{4n^2-3}{n+4}\right)=$$
$$\lim_{n\to\infty}\frac{-5n^2+4n^3-4n-20n+16n^2-16-4n^3+3n+20n^2-15}{(n-5)(n-4)}=$$
$$\lim_{n\to\infty}\frac{31n^2-21n-31}{n^2-9n+20}=\lim_{n\to\infty}\frac{31-\frac{21}{n}-\frac{31}{n^2}}{1-\frac{9}{n}+\frac{20}{n^2}}=31$$

2. Find the following limits:

(a) $\lim_{n\to\infty}\frac{-5n^8+n-6}{\sqrt{6n^{16}+7n-6}+\sqrt{7n^8-3}}$

$$\lim_{n\to\infty}\frac{-5n^8+n-6}{\sqrt{6n^{16}+7n-6}+\sqrt{7n^8-3}}=$$
$$\lim_{n\to\infty}\frac{-5+\frac{1}{n^7}-\frac{6}{n^8}}{\sqrt{6+\frac{7}{n^{15}}-\frac{6}{n^{16}}}+\sqrt{\frac{7}{n^8}-\frac{3}{n^{16}}}}=\frac{-5\sqrt{6}}{6}$$

(b) $\lim_{n\to\infty}\sqrt[n]{\frac{n^2+4^n}{n+5^n}}$

As we already know $\lim_{n\to\infty}\frac{n^k}{a^n}=0,\ |a|>1$

$$\lim_{n\to\infty}\sqrt[n]{\frac{n^2+4^n}{n+5^n}}=\lim_{n\to\infty}\sqrt[n]{\frac{\frac{n^2}{5^n}+(\frac{4}{5})^n}{\frac{n}{5^n}+1}}=\frac{4}{5}$$

3. Find the following limits:

(a) $\lim_{n\to\infty}\frac{2\cdot5^n-5\cdot4^n+3}{4\cdot5^n-3\cdot4^n+2}$
For sure $\lim_{n\to\infty}a^n=0,\ 0<|a|<1$

$$\lim_{n\to\infty}\frac{2\cdot5^n-5\cdot4^n+3}{4\cdot5^n-3\cdot4^n+2}=\lim_{n\to\infty}\frac{2-5\cdot(\frac{4}{5})^n+\frac{3}{5^n}}{4-3\cdot(\frac{4}{5})^n+\frac{2}{5^n}}=\frac{1}{2}$$

(b) $\lim_{n\to\infty}\frac{2\cdot6^{-n}+5\cdot5^{-n}}{4\cdot5^{-n}-3\cdot6^{-n}}$

$$\lim_{n\to\infty}\frac{2\cdot6^{-n}+5\cdot5^{-n}}{4\cdot5^{-n}-3\cdot6^{-n}}=\lim_{n\to\infty}\frac{2\cdot(\frac{6}{5})^{-n}+5}{4-3\cdot(\frac{6}{5})^{-n}}=\lim_{n\to\infty}\frac{2\cdot(\frac{5}{6})^n+5}{4-3\cdot(\frac{5}{6})^n}=\frac{5}{4}$$

4. Find the following limits:

(a) $\lim_{n\to\infty} \sqrt{n}(\sqrt{n+2} - \sqrt{n-1})$

$$\lim_{n\to\infty} \sqrt{n}(\sqrt{n+2}-\sqrt{n-1}) = \lim_{n\to\infty} \sqrt{n}\frac{3}{\sqrt{n+2}+\sqrt{n-1}} = \lim_{n\to\infty} \frac{3}{\sqrt{1+\frac{2}{n}}+\sqrt{1-\frac{1}{n}}} = \frac{3}{2}$$

(b) $\lim_{n\to\infty}(\sqrt{n^2+3n-1} - \sqrt{n^2-n})$

$$\lim_{n\to\infty} (\sqrt{n^2+3n-1} - \sqrt{n^2-n}) = \lim_{n\to\infty} \frac{4n-1}{\sqrt{n^2+3n-1}+\sqrt{n^2-n}} =$$
$$\lim_{n\to\infty} \frac{4-\frac{1}{n}}{\sqrt{1+\frac{3}{n}-\frac{1}{n^2}}+\sqrt{1-\frac{1}{n}}} = 2$$