DSBA Linear Algebra HW2

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24th of September 2020

1. Calculate:

$$\begin{bmatrix} -1 & -2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \cdot \left(tr\left(\begin{bmatrix} 1 & 6 & 2 \\ 2 & 7 & 1 \\ 9 & 8 & -6 \end{bmatrix} \right) \cdot \begin{bmatrix} -3 & 1 & -8 \\ 1 & 3 & 4 \\ 2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -3 \\ 8 & -2 & 3 \\ -9 & 0 & 1 \end{bmatrix} \right)$$

$$tr\left(\begin{bmatrix} 1 & 6 & 2 \\ 2 & 7 & 1 \\ 9 & 8 & -6 \end{bmatrix} \right) = 1 + 7 - 6 = 2$$

$$2 \begin{bmatrix} -3 & 1 & -8 \\ 1 & 3 & 4 \\ 2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -3 \\ 8 & -2 & 3 \\ -9 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 0 & -13 \\ -6 & 8 & 5 \\ 13 & 8 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -7 & 0 & -13 \\ -6 & 8 & 5 \\ 13 & 8 & 1 \end{bmatrix} = \begin{bmatrix} 58 & 8 & 6 \\ 20 & 8 & 14 \end{bmatrix}$$

2. Find two matrices A and B such that

$$C = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & 0 \\ 1 & -2 & 0 \end{bmatrix} = A + B$$

where A is symmetric and B is skew-symmetric.

$$A = A^T \text{ and } B = -B^T \text{ by definition}$$

$$\begin{cases} C = A + B \\ C^T = (A+B)^T = A^T + B^T = A - B \end{cases} \iff \begin{cases} \frac{C+C^T}{2} = A \\ \frac{C-C^T}{2} = B \end{cases}$$

$$A = \begin{bmatrix} 3 & 1.5 & 0 \\ 1.5 & 2 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -0.5 & -1 \\ 0.5 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

3. Find the inverse matrix for $\begin{bmatrix} 2 & 5 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

   By definition the inverse matrix is such matrix X such that $A \cdot X = I_n$

$$\begin{bmatrix} 2 & 5 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} 2x_{11} + 5x_{21} = 1 \\ 2x_{12} + 5x_{22} = 0 \\ 2x_{13} + 5x_{23} = 0 \\ x_{11} + 3x_{21} = 0 \\ x_{12} + 3x_{22} = 1 \\ x_{13} + 3x_{23} = 0 \\ -4x_{31} = 0 \\ -4x_{32} = 0 \\ -4x_{33} = 1 \end{cases} \iff$$

$$\begin{cases} x_{33} = -\frac{1}{4} \\ x_{32} = 0 \\ x_{31} = 0 \\ 2x_{13} + 5x_{23} - 2x_{13} - 6x_{23} = -x_{23} = 0 \\ x_{13} + 0 = 0 \\ 2x_{11} + 5x_{21} - 2x_{11} - 6x_{23} = -x_{21} = 1 \\ x_{11} - 3 = 0 \Rightarrow x_{11} = 3 \\ 2x_{12} + 5x_{22} - 2x_{12} - 6x_{22} = 2 \Rightarrow -x_{22} = -2 \\ x_{12} + 6 = 1 \Rightarrow x_{12} = -5 \end{cases}$$

$$\text{X} = \begin{bmatrix} 3 & -5 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & -\frac{1}{4} \end{bmatrix}$$

4. Is the product of two symmetric matrices necessary symmetric?

   Suppose $A = A^T$ and $B = B^T$.

$$A \cdot B = A^T \cdot B^T = (B \cdot A)^T$$

   Product would be symmetric if $A \cdot B = B \cdot A$, but in general it doesn't work. For example, for matrices:

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ $A \cdot B = \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix}$ and $B \cdot A = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix} \Rightarrow$ $A \cdot B \neq B \cdot A$