DSBA Linear Algebra HW1

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Recall that for $A \in Mat(m, n, \mathbb{R}), B \in Mat(n, p, \mathbb{R})$

$$[A * B](i, j) = \sum_{k=1}^{n} [A](i, k) * [B](k, j)$$
$$\forall i \in [m], \forall j \in [p]$$

1. **Compute the square of a matrix**

   I'll omit calculations if the zero row or column is multiplied.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}^2 =$$

$$\begin{bmatrix} 0 & 0*1+1*0+0*0+0*0 & 0*0+1*2+0*0+0*0 & 0*0+1*0+0*3+0*0 \\ 0 & 0*1+0*0+2*0+0*0 & 0*0+0*2+2*0+0*0 & 0*0+0*0+2*3+0*0 \\ 0 & 0*1+0*0+0*0+3*0 & 0*0+0*2+0*0+3*0 & 0*0+0*0+0*3+3*0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Let A $= \begin{bmatrix} 1 & 5 & 3 \\ 2 & -3 & 1 \end{bmatrix}$ and B $= \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & -2 \\ 3 & -1 & 1 \end{bmatrix}$. Find AB and $AB^2 - 2AB$.

$$AB = \begin{bmatrix} 1*2+5*(-1)+3*3 & 1*(-3)+5*4+3*(-1) & 1*5+5*(-2)+3*1 \\ 2*2+(-3)*(-1)+1*3 & 2*(-3)+(-3)*4+1*(-1) & 2*5+(-3)*(-2)+1*1 \end{bmatrix} =$$

$$\begin{bmatrix} 6 & 14 & -2 \\ 10 & -19 & 17 \end{bmatrix}$$

Notice that $AB^2 - 2AB = (AB)B - 2AB$.

$(AB)B =$

$$\begin{bmatrix} 6*2+14*(-1)+(-2)*3 & 6*(-3)+14*4+(-2)*(-1) & 6*5+14*(-2)+(-2)*1 \\ 10*2+(-19)*(-1)+17*3 & 10*(-3)+(-19)*4+17*(-1) & 10*5+(-19)*(-2)+17*1 \end{bmatrix} =$$

$$\begin{bmatrix} -8 & 40 & 0 \\ 90 & -123 & 105 \end{bmatrix}$$

$$(AB)B - 2AB = \begin{bmatrix} -8-2*6 & 40-2*14 & 0-2(-2) \\ 90-2*10 & -123-2*(-19) & 105-2*17 \end{bmatrix} = \begin{bmatrix} -20 & 12 & 4 \\ 70 & -85 & 71 \end{bmatrix}$$

3. Let A = $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & -1 \end{bmatrix}$.
   Compute ABC and CAB.

$$AB = \begin{bmatrix} 2*3+(-1)*1+1*(-1) \\ 1*3+2*1+1*(-1) \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 4*1 & 4*(-1) \\ 4*1 & 4*(-1) \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 4 & -4 \end{bmatrix}$$

$$C(AB) = \begin{bmatrix} 1*4+(-1)*4 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

4. **Solve the following matrix equation by the brute-force method discussed in seminar**

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \cdot X \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$$

Let X = $\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$

$$\begin{bmatrix} 3x_1+x_3 & 3x_2+x_4 \\ 2x_1+x_3 & 2x_2+x_4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3x_1+x_3+3x_2+x_4 & 9x_1+3x_3+6x_2+2x_4 \\ 2x_1+x_3+2x_2+x_4 & 6x_1+3x_3+4x_2+2x_4 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$$

$$\begin{cases} 3x_1 + 3x_2 + \ \ x_3 + \ \ x_4 = 3 \\ 9x_1 + 6x_2 + 3x_3 + 2x_4 = 3 \\ 2x_1 + 2x_2 + \ \ x_3 + \ \ x_4 = 2 \\ 6x_1 + 4x_2 + 3x_3 + 2x_4 = 2 \end{cases}$$

Subtract third equation from the first: $x_1 + x_2 = 1$. In a third equation we can factor out 2 and plug in the value of $x_1 + x_2$: $2 * 1 + x_3 + x_4 = 2 \Rightarrow x_3 + x_4 = 0$. Then express x_1 from the first equation.

$$\begin{aligned} 3x_1 + 2x_2 &= 1 \\ 3x_1 + 2(1 - x_1) &= 1 \\ \Rightarrow x_1 &= -1, x_2 = 2 \end{aligned}$$

Put these values into fourth equation and replace x_4 with $-x_3$.

$$-6 + 8 + 3x_3 - 2x_3 = 2 \Rightarrow x_3 = 0, x_4 = 0$$

$\mathrm{X} = \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$