DSBA Calculus HW6

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1. Give formal definition for the following notions. Construct the negation to each of them:

$$\lim_{x \to a-0} f(x) = L$$
$$\forall \epsilon > 0, \exists \delta > 0 : \forall x \in D(f) \quad a - \delta < x < a \quad |f(x) - L| < \epsilon$$
$$\exists \epsilon > 0 : \forall \delta > 0, \exists x \in D(f) \quad a - \delta < x < a \quad |f(x) - L| \geq \epsilon$$

$$\lim_{x \to -\infty} f(x) = +\infty$$
$$\forall \epsilon > 0, \exists \delta > 0 : \forall x \in D(f) \quad x < -\frac{1}{\delta} \quad f(x) > \frac{1}{\epsilon}$$
$$\exists \epsilon > 0 : \forall \delta > 0, \exists x \in D(f) \quad x < -\frac{1}{\delta} \quad f(x) \leq \frac{1}{\epsilon}$$

2. Find the following one-sided limits:

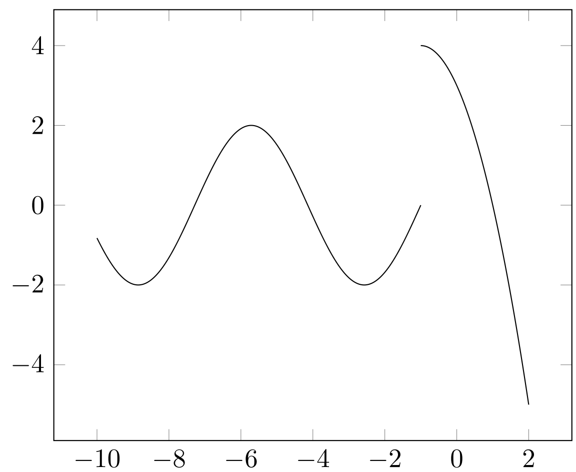
$$\lim_{x \to 7+0} \frac{|x-7|}{x^2 + 5x - 14} = \lim_{x \to 7+0} \frac{x-7}{(x-7)(x+2)} = \lim_{x \to 7+0} \frac{1}{x+2} = \frac{1}{9}$$
$$\lim_{x \to 7-0} \frac{|x-7|}{x^2 + 5x - 14} = \lim_{x \to 7-0} -\frac{x-7}{(x-7)(x+2)} = \lim_{x \to 7-0} -\frac{1}{x+2} = -\frac{1}{9}$$

3. Find the following one-sided limits:

$$\lim_{x \to -1+0} \frac{\sin x + 1}{x + 1}$$
$$\sin x + 1 \geq 0 \quad \forall x \in \mathbb{R}$$
$$x + 1 \geq 0 \quad x \to -1 + 0 \Rightarrow \lim_{x \to -1+0} \frac{\sin x + 1}{x + 1} = +\infty$$
$$\lim_{x \to -1-0} \frac{\sin x + 1}{x + 1}$$
$$x + 1 \leq 0 \quad x \to -1 - 0 \Rightarrow \lim_{x \to -1+0} \frac{\sin x + 1}{x + 1} = -\infty$$

4. Sketch the graph of the piecewise defined function and find limits:

$$f(x) = \begin{cases} 2\sin(x+1), & x \leq -1 \\ 3 - x^2 - 2x, & x > -1 \end{cases}$$



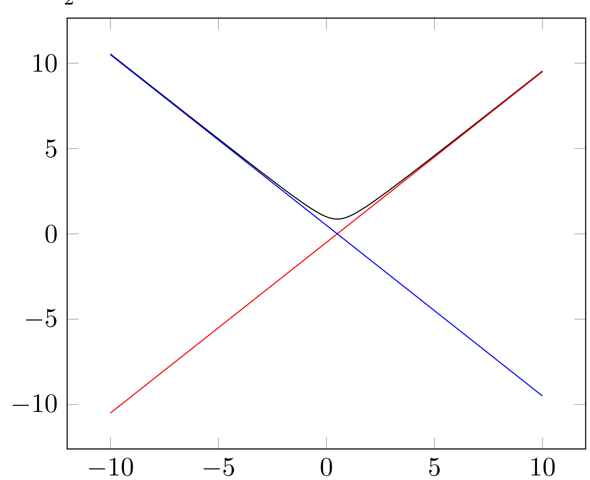
$$\lim_{x \to -1-0} f(x) = \lim_{x \to -1-0} 2\sin(x+1) = 0$$
$$\lim_{x \to -1+0} f(x) = \lim_{x \to -1+0} 3 - x^2 - 2x = 4$$

$\lim_{x \to -1} f(x)$ doesn't exist because left and right one-sided limits are not equal.

5. Find a_i and b_i such that:

$$\lim_{x\to+\infty}(\sqrt{x^2-x+1}-a_1x-b_1)=0$$
$$\lim_{x\to+\infty}(\frac{x^2-x+1-a_1^2x^2-2a_1b_1x-b_1^2}{\sqrt{x^2-x+1}+a_1x+b_1})=0$$
$$\lim_{x\to+\infty}(\frac{(1-a_1^2)x^2-(1+2a_1b_1)x+1-b_1^2}{\sqrt{x^2-x+1}+a_1x+b_1})=0$$
$$\lim_{x\to+\infty}(\frac{(1-a_1^2)x-(1+2a_1b_1)+\frac{1-b_1^2}{x}}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a_1+\frac{b_1}{x}})=0$$
$$1-a_1^2=0\Rightarrow a_1=\pm\sqrt{1}\quad \text{But } \sqrt{x^2-x+1}\to\infty,\quad x\to\infty\Rightarrow$$
$$a_1=1\Rightarrow \lim_{x\to+\infty}(\frac{(1-a_1^2)x-(1+2a_1b_1)+\frac{1-b_1^2}{x}}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a_1+\frac{b_1}{x}})=\lim_{x\to+\infty}\frac{-1-2b_1}{2}=0$$
$$b_1=-\frac{1}{2}$$

For limit when $x\to-\infty$ the same considerations hold with exception that $\sqrt{x^2-x+1}\to-\infty,\quad x\to-\infty\Rightarrow a_1=-1$, therefore $1-2b_1=0\Rightarrow b_1=\frac{1}{2}$



6. Evaluate the following limits:

$$\lim_{x\to 0} \frac{\tan 5x}{\sin 2x} = \lim_{x\to 0} \frac{2x\sin(5x)5x}{2x\cos(5x)\sin(2x)5x} = \lim_{x\to 0} \frac{5x}{2x} = \frac{5}{2}$$

$$\lim_{x\to\frac{\pi}{2}} \cos x \cdot \sin\frac{2}{2x-\pi}$$

$$\cos\frac{\pi}{2} = 0 \quad -1 \le \sin\frac{2}{2x-\pi} \le 1 \Rightarrow \lim_{x\to\frac{\pi}{2}} \cos x \cdot \sin\frac{2}{2x-\pi} = 0$$

$$\lim_{x\to 0} \frac{1-\cos 8x}{5x^2} = \lim_{x\to 0} \frac{2\sin^2 4x}{5x^2} = \lim_{x\to 0} \frac{2\sin^2(4x)16x^2}{5x^2 16x^2} = \frac{32}{5}$$

$$\lim_{x\to 0} \frac{2x-5x^2+x^3}{\sin 3x} = \lim_{x\to 0} \frac{3x(2-5x+x^2)}{3\sin 3x} = \lim_{x\to 0} \frac{2-5x+x^2}{3} = \frac{2}{3}$$

$$\lim_{x\to 0} x\cdot\cot(5x) = \lim_{x\to 0} x\frac{\cos 5x}{\sin 5x} = \lim_{x\to 0} 5x\frac{\cos 5x}{5\sin 5x} = \frac{1}{5}$$