Statistical Testing 1 - Classical testing

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Lecture 02.2.1 (v1.1.0)

Signposting

- Last session we covered **regression**.
  - This is something of a pre-requisite for a useful analysis of **testing**.
- We'll cover testing in three sections:
  1. Classical testing (recap)
  2. Resampling methods
  3. Model selection

Intended Learning Outcomes

- ILO2 Be able to **use and apply basic machine learning** tools
- ILO3 Be able to make and report appropriate inferences from the results of applying basic tools to data

Null hypothesis test

- Given some data $\{y\}$:
  - Null Hypothesis **H0**: A statement is true about $\{y\}$.
  - Alternative Hypothesis **H1**: The statement is not true.
- We then compute a **test statistic** $T(\{y\})$ whose distribution is **computable under H0**.
  - By convention, large T is evidence against the null.
- Then compute p-value $p(T \geq T(\{y\}))$, the probability of observing a test statistic at least as large as that observed given H0 is true.
  - Example: H0: $\mathbb{E}(y) = \mu$ with $\mu = 0$. H1: $\mu \neq 0$.
  - This is **not model selection**. We favour H0 and must find evidence against it to accept H1.

Null hypothesis significance testing

- Hypothesis testing is asking: are my data consistent **with this hypothesis** when **using this measure**?
  - If you choose a silly hypothesis, testing will dutifully say “no”
  - If you use a weak measure, testing will dutifully say “yes”
  - Nothing is learned by this!
- The correct use of statistical testing is where:
  1. the **null hypothesis might plausibly be true**, or
  2. it might not be true, but you care how much **power the data has to reject the null**

When to use hypothesis testing

- Some valid use cases include:
  - To **rank hypotheses** by how much evidence there is against them
  - To obtain a **standardised scale** (0-1) for combining evidence
  - When **data are scarce**
- Also when testing plausible nulls, such as:
  - **validating simulations** with a known simulator;
  - **independence** or other non-parametric tests.
  - **broad null hypotheses**, such as testing a range of parameters.

Types of error

- The **p-value** defines *the probability that H0 is true, but is rejected*.
- The **power of the test** is *the probability that H0 is false but is accepted anyway*.
  - Low power situations are to be avoided: see e.g. Andrew Gelman's blog[1].
- Power is a surprisingly important problem because there are many *researcher degrees of freedom*.
  - so if power is low, we tend to find significant results anyway, through the (often unintentional) use of the data to choose the test.

[1] https://andrewgelman.com/2018/02/18/low-power-replication-crisis-learned-since-2004-1984-1964/

Types of error

Error notation

.	H0 true	H0 false
H0 accepted	Correct	Type II error
H0 rejected	Type I error	Correct

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- Under the convention that $H0 = 0 =$ "negative" case and $H1 = 1 =$ "positive case":

Alternative notation

.	H0 holds	H1 holds
H0 accepted	True Negative	False Negative
H0 rejected	False Positive	True Positive

Example: T-test for a difference in mean

- To test if the mean of $\{x\}$ is μ_0, we calculate the **test statistic**:

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}},$$

- where s is the standard deviation and n the sample size. Under H0:

$$t \sim \mathrm{t}(t; \nu = n - 1)$$

- where ν is the degrees of freedom. (See Student's t-distribution).

Example: T-test for a difference in mean

```
## Extract TCP and UDP packet sizes
tcpsize=conndata[conndata[,"proto"]=="tcp","orig_bytes"]
udpsize=conndata[conndata[,"proto"]=="udp","orig_bytes"]
ftpsize=conndata[conndata[,"service"]=="ftp","orig_bytes"]
```

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## Extract TCP and UDP packet sizes
tcpsize=conndata[conndata[,"proto"]=="tcp","orig_bytes"]
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ftpsize=conndata[conndata[,"service"]=="ftp","orig_bytes"]

## Convert and omit missing data
tcpsize=as.numeric(tcpsize[tcpsize!="-"])
udpsize=as.numeric(udpsize[udpsize!="-"])
ftpsize=as.numeric(ftpsize[ftpsize!="-"])
tcpsize=tcpsize[tcpsize>0]
udpsize=udpsize[udpsize>0]
ftpsize=ftpsize[ftpsize>0]
```

Example: T-test for a difference in mean

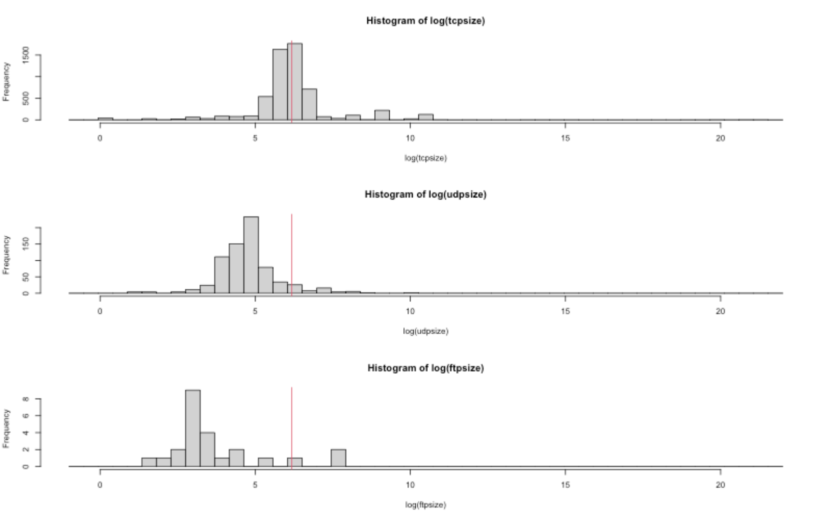
```
mu=mean(log(tcpsize))
t.test(log(udpsize),mu=mu)$p.value
t.test(log(ftpsize),mu=mu)$p.value
```

Example: T-test for a difference in mean

```
mu=mean(log(tcpsize))
t.test(log(udpsize),mu=mu)$p.value
t.test(log(ftpsize),mu=mu)$p.value
```

```
> t.test(log(udpsize),mu=mu)$p.value
[1] 2.733874e-182
> t.test(log(ftpsize),mu=mu)$p.value
[1] 8.334782e-08
```

Example: T-test for a difference in mean



t-tests

- Can be one-tailed (**H0**: $\mu \leq \mu_0$) or two-tailed (**H0**: $\mu = \mu_0$)
- Assumes:
  - independence (note: paired tests are possible) and identically distributed
  - the **data are Normal**
  - the standard deviation is either known (t is then Normal) or estimated from the data (t is then t distributed).
- Used in regression, paired tests, etc.
- *NB Incomplete notes as this is a prerequisite!*

Chi squared test

- The χ^2 test is for categorical data comparing two variables.
- **H0**: No relationship between the variables; **H1** Some relationship between them.
- The **test statistic** for N datapoints from k classes, with x_i observations of type i, with expected value $m_i = Np_i$ where p_i is the expected probabilities, is (under the null):

$$X^2 = \sum_{i=1}^{k} \frac{(x_i - m_i)^2}{m_i} \sim \chi^2(k-1)$$

- This is most often used for **contingency tables** though appears elsewhere.
- See also **Fishers exact test** for small samples.
- *NB Incomplete notes as this is a prerequisite!*

Other important tests

- Nonparametric tests:
  - **Mann-Whitney U or Wilcoxon rank sum** test: are two samples are drawn from the same distribution? by comparing their ranks.
  - **Wilcoxon signed-rank** test - as rank sum test, for paired data.
  - **Kolmogorov-Smirnov** test - are two samples from the same distribution? by comparing the empirical cumulative distribution function.
- There are many online cookbooks which state exactly which circumstances each test should be used in. You should be able to use them.
- *NB Incomplete notes as this is a prerequisite!*

Statistical testing overview

- The tests we have discussed are **classic statistics**, that is, before computers (indeed pre-1950s). The most important types of test for data science are yet to come.

Reflection

- You must be able to:
  - Define and use a null hypothesis significance test,
  - Contrast classical and resampling tests, and judge appropriate uses,
  - Use statistical testing appropriately in projects.
- In Science, why does statistical testing have a bad reputation?
- Does statistical testing have a place in large-scale data science for applied domains?

Signposting

- Further reading:
  - Chapter 4 of Statistical Data Analysis by Glen Cowan
  - Statistical tests, P values, confidence intervals, and power: a guide to misinterpretations by Greenland et al
  - Andrew Gelman's blog has many examples of statistical testing failures in social science and medicine
- Next up: Resampling approaches to testing