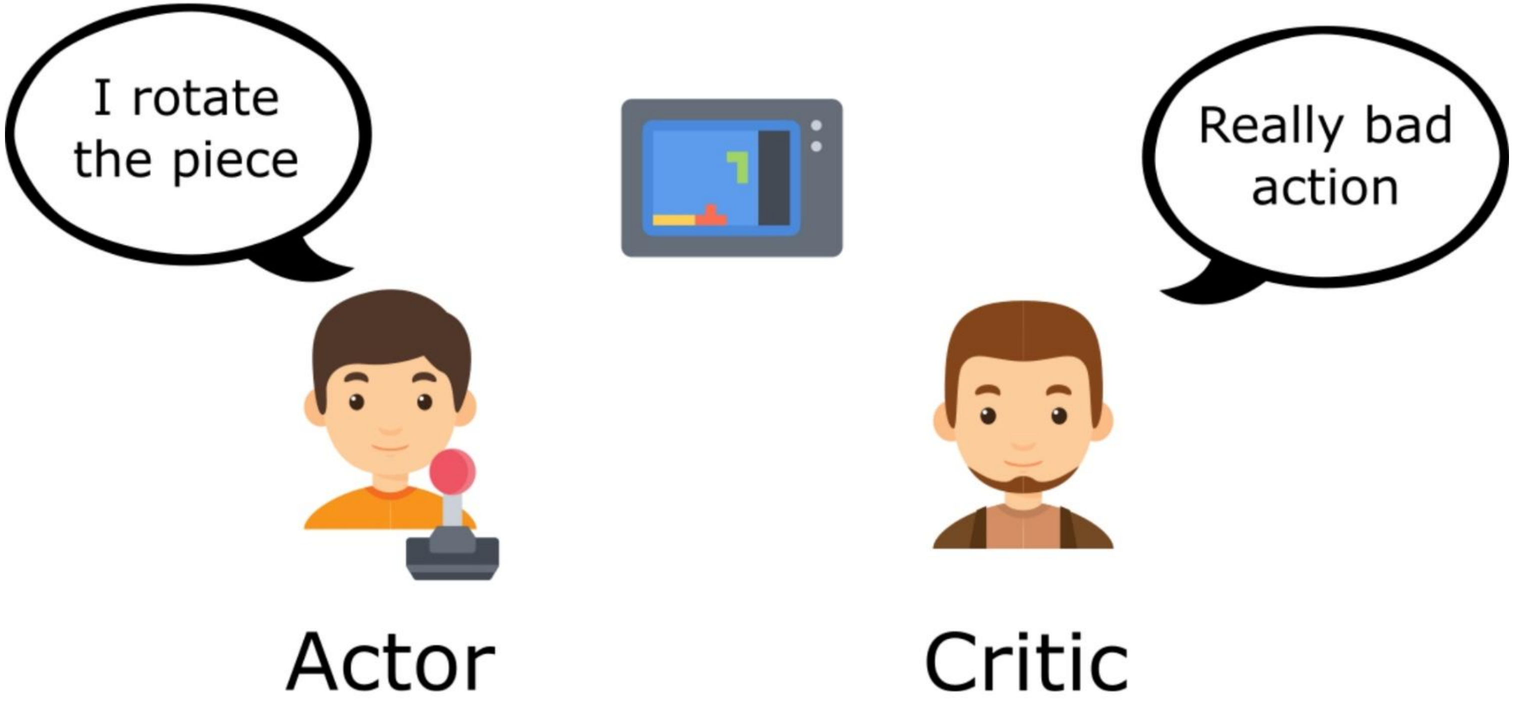
Actor Critic and Proximal Policy Optimization



Instructor: Daniel Brown --- University of Utah

Announcement

- Mid semester feedback. Thank you!
- What y'all like?
  - Exploratory assignments, interesting topics, no exams ☺
  - Experience-based learning
  - Paper reading
- What y'all want to see changed?
  - Zoom options if you are sick.
  - Quizzes: more structure, no paper passing, more frequent, eliminate…
  - End a minute or two early.
  - More reading assignments
  - Record lectures…
  - Less math!
  - More consistency in math.
  - More math!
  - Harder/deeper programming assignments

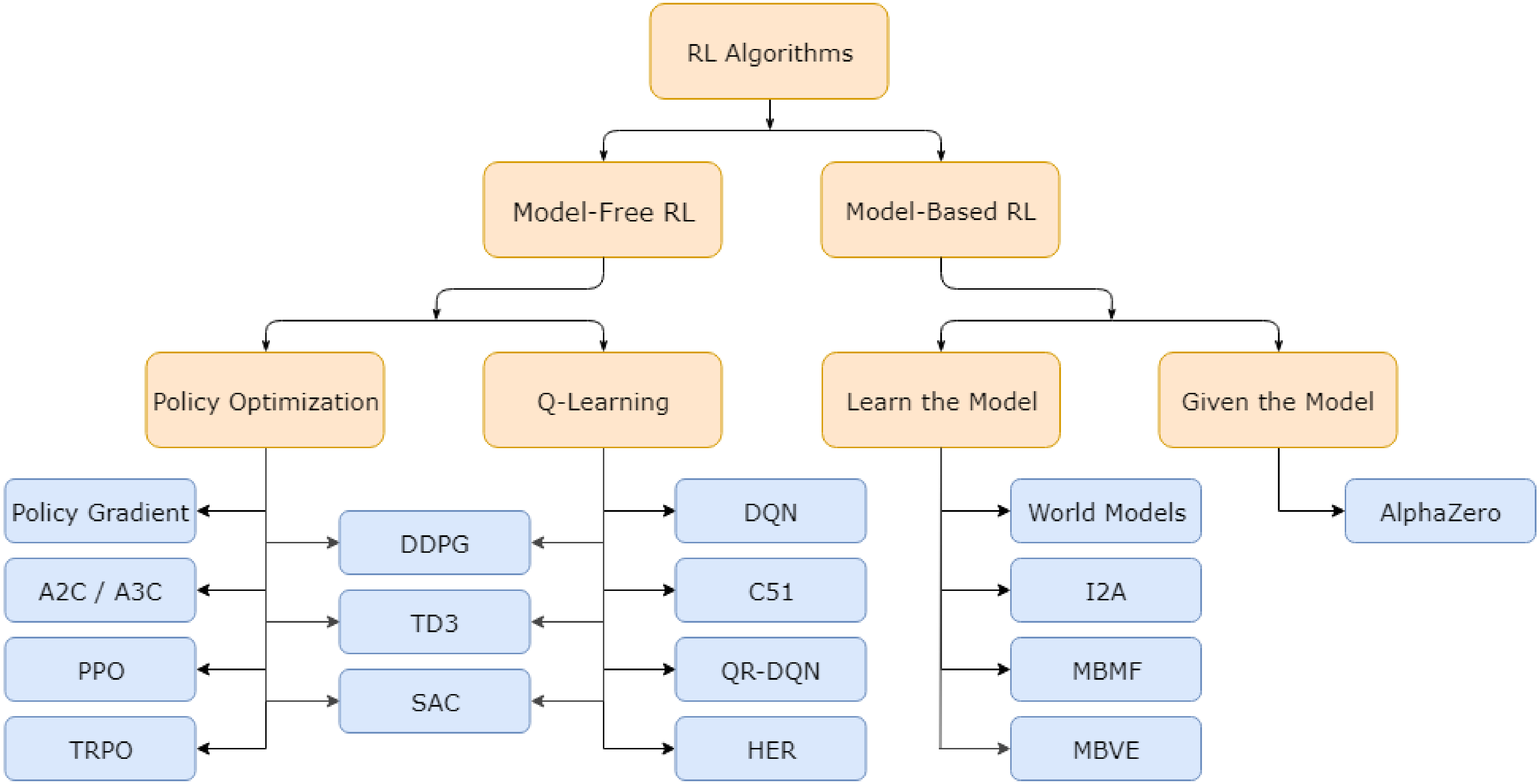
Announcement

- What helps y’all learn?
  - Quizzes, In-class lectures, Recordings, Programming assignments to practice concepts
- What can I do to improve learning?
  - More paper reading assignments, examples of applications.
  - Add subtitles to zoom recordings
  - More interactions and Q&A in lectures
  - Move at a faster pace
  - More structure in homework questions.
  - Harder homework problems
  - Post lecture slides earlier.
  - Discuss pseudo code in lectures
  - More math!
  - Example project ideas.

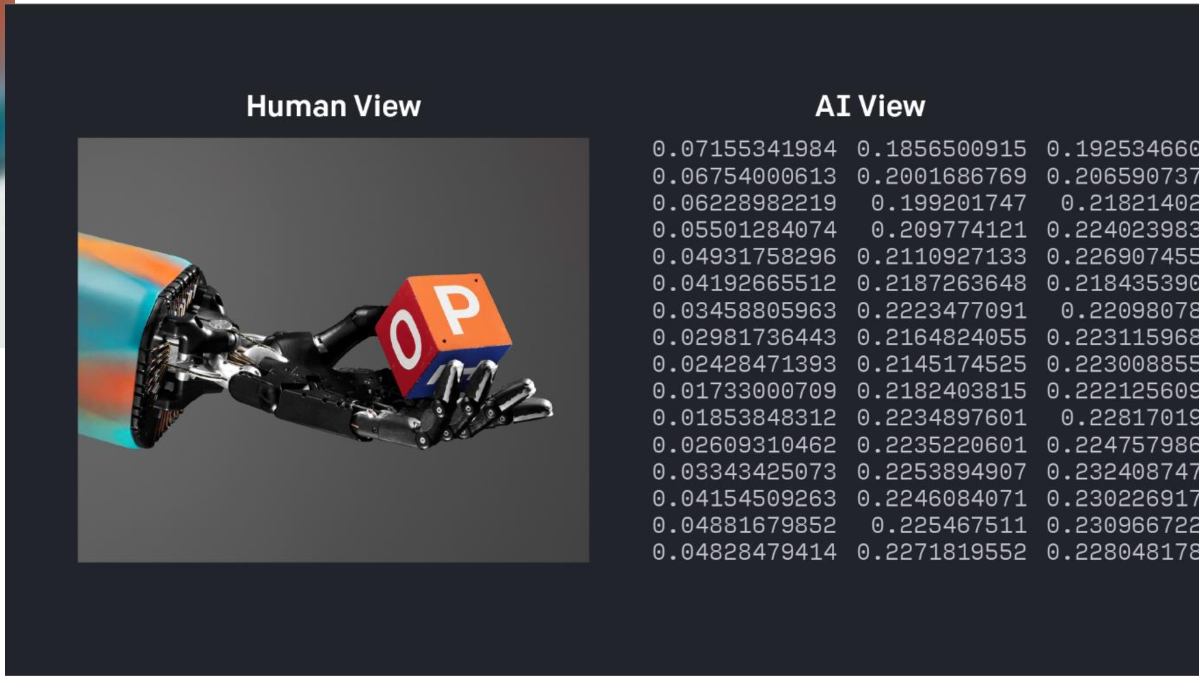
Announcement

- Homework 5

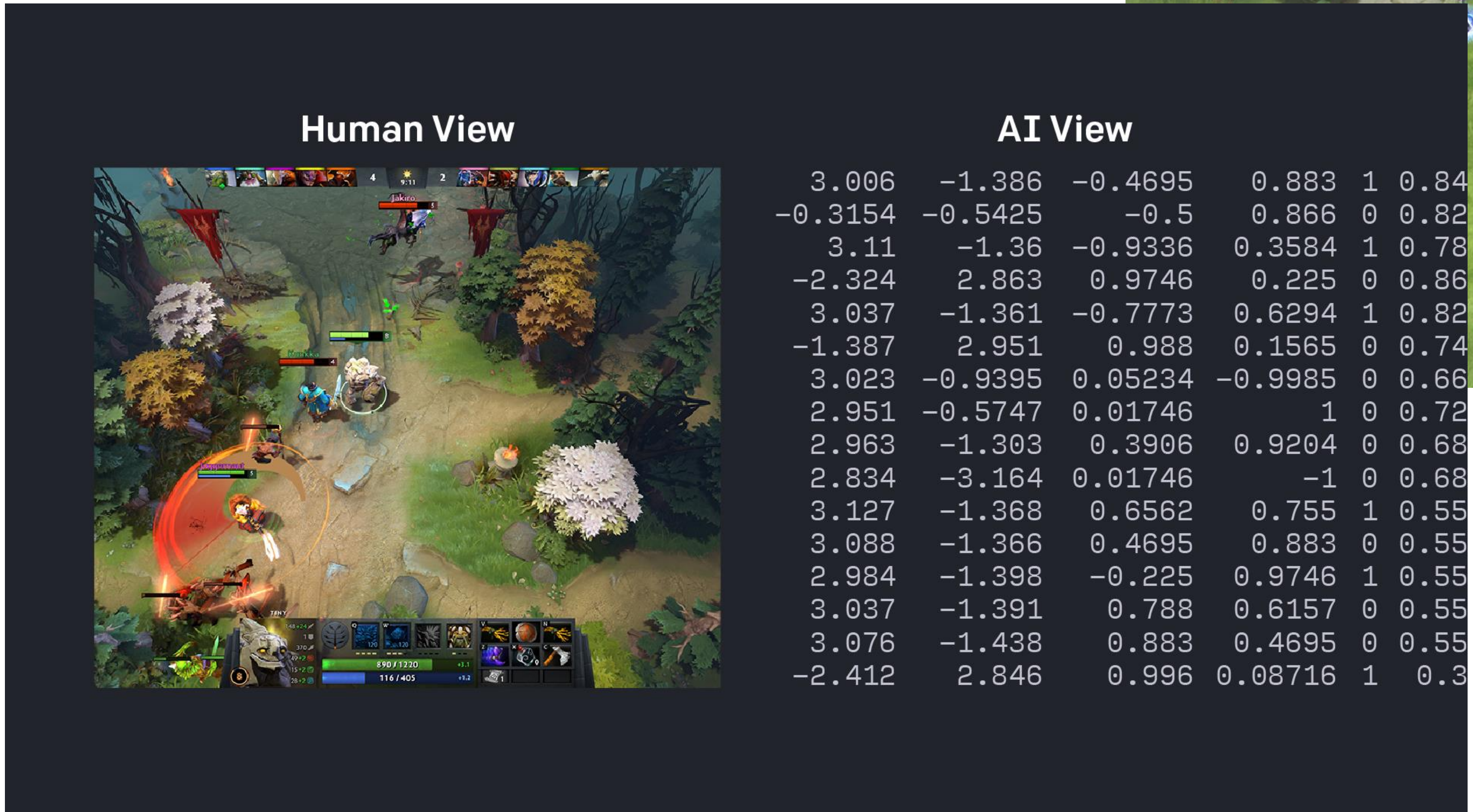
Rough Taxonomy of RL Algorithms



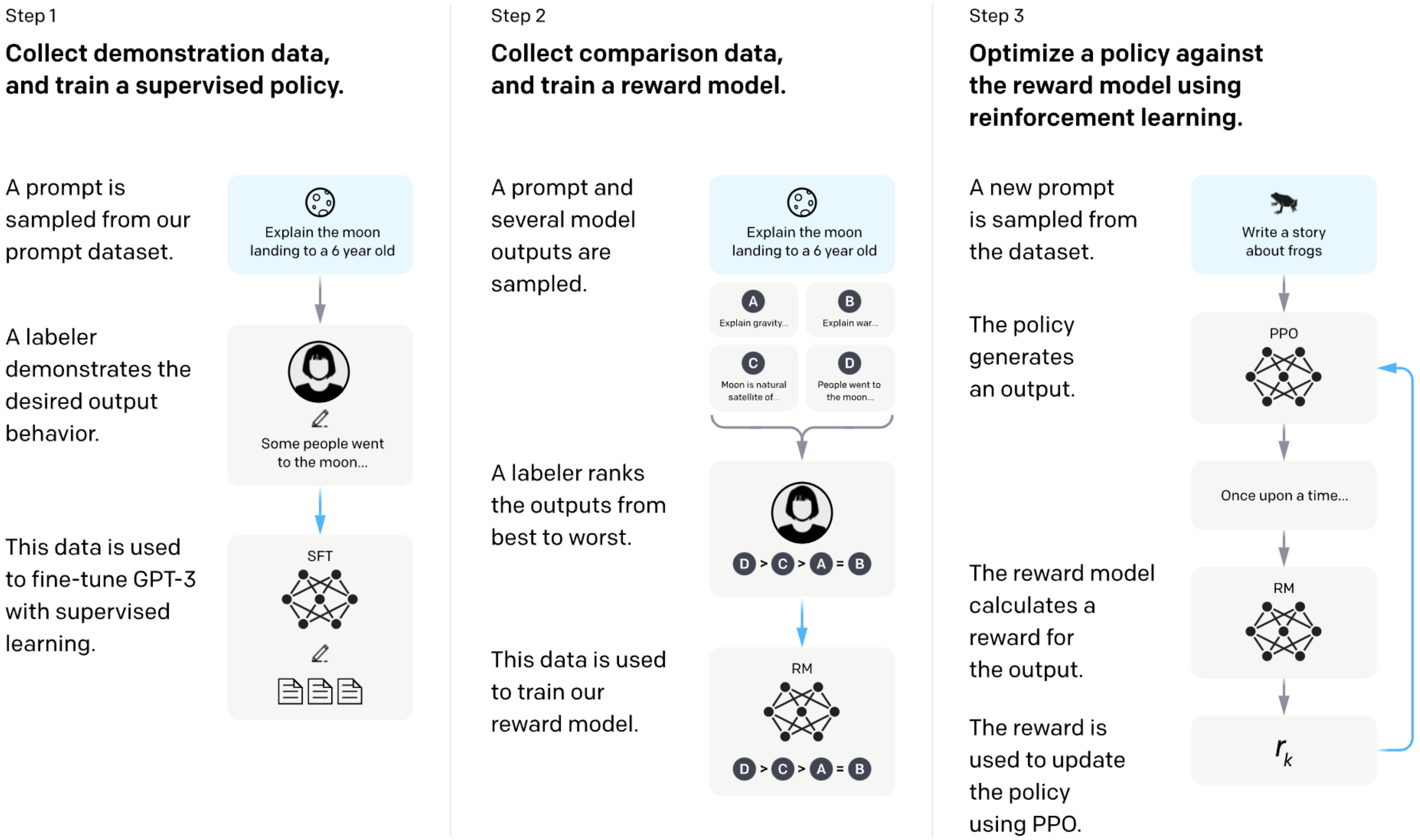
Dexterous Manipulation



OpenAI 5: DOTA 2



RLHF in ChatGPT



What is the goal of RL?

- Find a policy that maximizes expected utility (discounted cumulative rewards)

$$\pi^* = arg\max_{\pi} E_{\pi}\left[\sum_{t=0}^{\infty} \gamma^t R(s, \pi(s), s')\right]$$

The Policy Gradient (REINFORCE)

- We can now perform gradient ascent to improve our policy!

$$\theta_{k+1} \leftarrow \theta_k + \alpha \nabla_\theta J(\pi_\theta)\Big|_{\theta_k}$$

$$\nabla_\theta J(\pi_\theta) = E_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\ R(\tau)\right]$$

Estimate with a sample mean over a set D of policy rollouts given current parameters

$$\approx \frac{1}{|D|} \sum_{\tau \in D} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\ R(\tau)$$

Policy Gradient RL Algorithms

- We can directly update the policy to achieve high reward.
- Pros:
  - Directly optimize what we care about: Utility!
  - Naturally handles continuous action spaces!
  - Can learn specific probabilities for taking actions.
  - Often more stable than value-based methods (e.g. DQN).
- Cons:
  - On-Policy -> Sample-inefficient we need to collect a large set of new trajectories every time the policy parameters change.
  - Q-Learning methods are usually more data efficient since they can reuse data from any policy (Off-Policy) and can update per sample.

Many forms of policy gradients

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathrm{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t|s_t)\Phi_t\right]$$

What we derived: $\Phi_t = R(\tau),$

Follows a similar derivation: $\Phi_t = \sum_{t'=t}^{T} R(s_{t'}, a_{t'}, s_{t'+1}),$

https://medium.com/@thechrisyoon/deriving-policy-gradients-and-implementing-reinforce-f887949bd63

- What is better about the second approach?
  - Focuses on rewards in the future!
  - Less variance -> less noisy gradients.

Many forms of policy gradients

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathrm{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\Phi_t\right]$$

$$\Phi_t = \sum_{t'=t}^{T} R(s_{t'}, a_{t'}, s_{t'+1}),$$

Looks familiar….

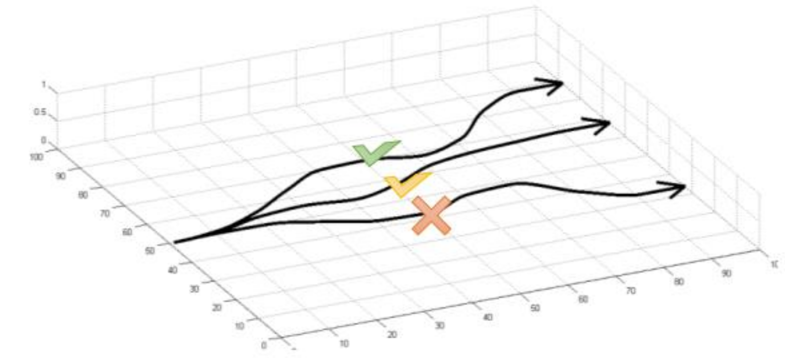
$$\Phi_t = Q^{\pi_\theta}(s_t, a_t)$$

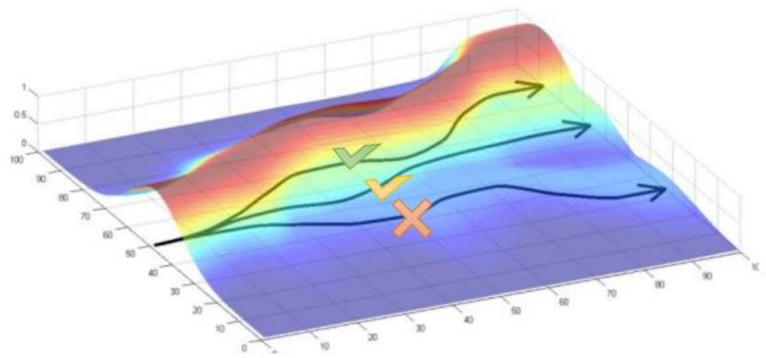
- Now we have an approach that combines a parameterized policy and a parameterized value function!

Baselines

$$\color{green}{\nabla_\theta J(\pi_\theta)} = E_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\ R(\tau)\right]$$

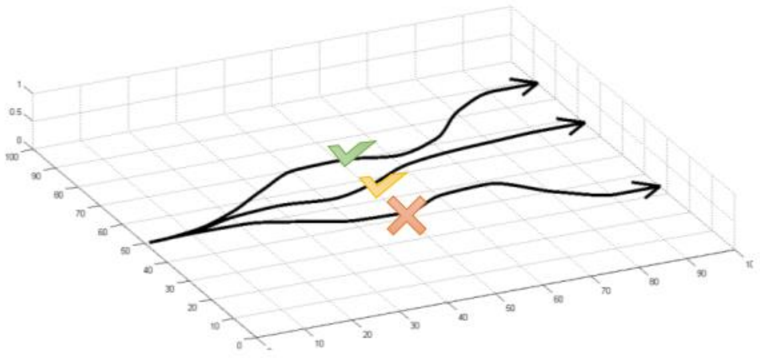
$$\approx \frac{1}{|D|}\sum_{\tau\in D}\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\ R(\tau)$$





Baselines

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\theta \log p_\theta(\tau)[r(\tau) - b]$$

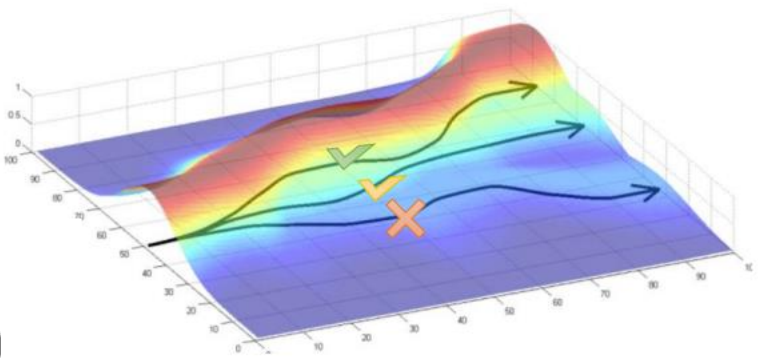


$$b = \frac{1}{N} \sum_{i=1}^{N} r(\tau)$$

But can we do this?

$$E[\nabla_\theta \log p_\theta(\tau) b] = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) b \, d\tau$$

$$= \int \nabla_\theta p_\theta(\tau) b \, d\tau = b \nabla_\theta \int p_\theta(\tau) d\tau = b \nabla_\theta 1 = 0$$



Many forms of policy gradients

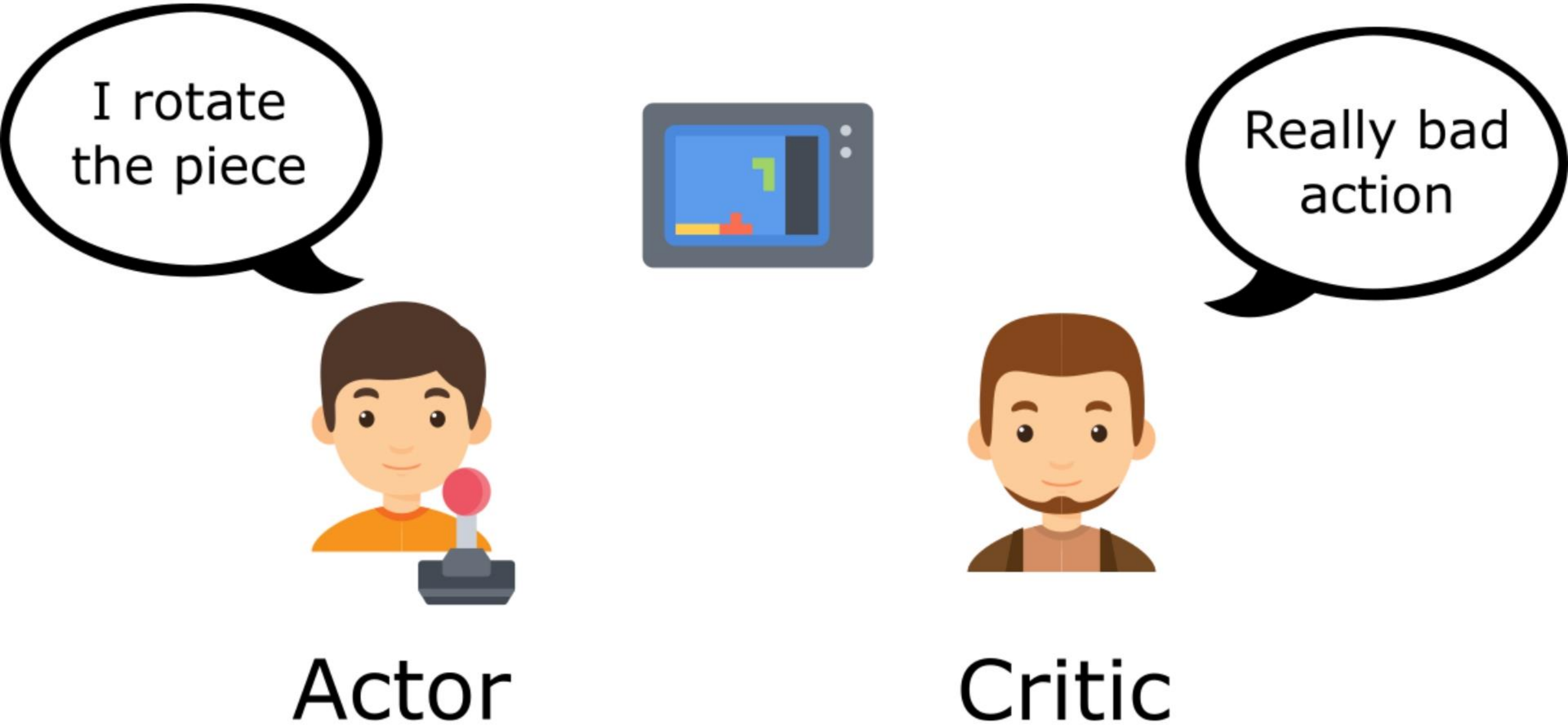
$$\nabla_\theta J(\pi_\theta) = \mathop{\mathrm{E}}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\Phi_t\right]$$

$$\Phi_t = R(\tau), \qquad \Phi_t = \sum_{t'=t}^{T} R(s_{t'}, a_{t'}, s_{t'+1}), \qquad \Phi_t = Q^{\pi_\theta}(s_t, a_t)$$

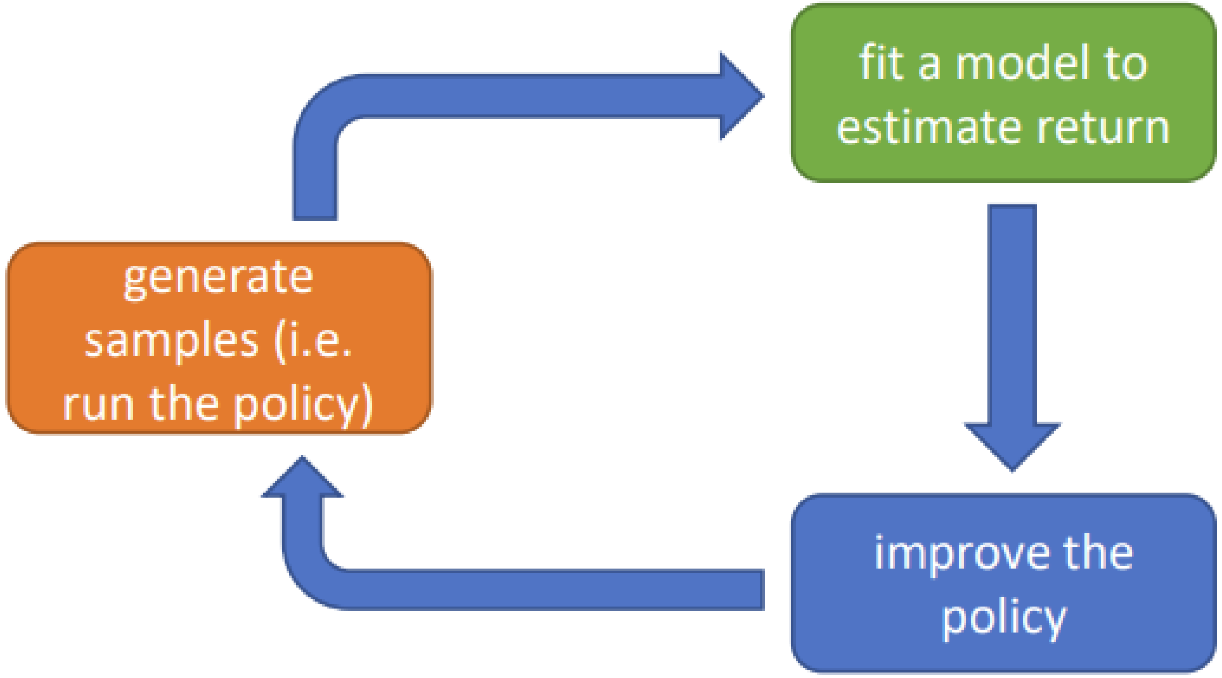
$$\Phi_t = \sum_{t'=t}^{T} R(s_{t'}, a_{t'}, s_{t'+1}) - b(s_t)$$

$$\Phi_t = A^\pi(s_t, a_t) = Q^\pi(s_t, a_t) - V^\pi(s_t)$$

Advantage Function


I rotate the piece
Really bad action
Actor
Critic

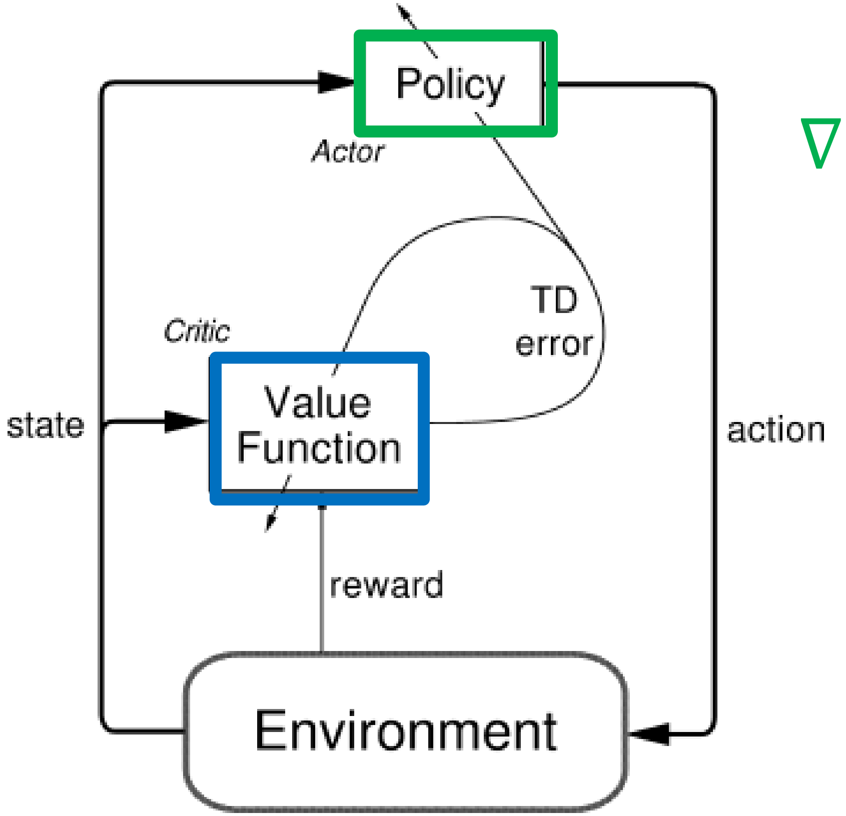
$$\text{fit } \hat{V}_\phi^\pi$$



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

Actor Critic Algorithms

- Combining value learning with direct policy learning
  - One example is policy gradient using the advantage function



$$\nabla_\theta J(\pi_\theta) = E_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\, Q_w^{\pi_\theta}(s_t,a_t)\right]$$

$$\theta_{k+1} \leftarrow \theta_k + \alpha \nabla_\theta J(\pi_\theta)\Big|_{\theta_k}$$

$$\delta = (r_t + \gamma Q_w^{\pi_\theta}(s_{t+1},a_{t+1}) - Q_w^{\pi_\theta}(s_t,a_t))$$

$$w_{k+1} \leftarrow w_k + \alpha \delta_t \nabla_\theta Q_w^{\pi\_\theta}$$

Q Actor Critic Algorithm Pseudo Code

Algorithm 1 Q Actor Critic

Initialize parameters s, θ, w and learning rates α_θ, α_w; sample $a \sim \pi_\theta(a|s)$.
for $t = 1 \dots T$: **do**
    Sample reward $r_t \sim R(s, a)$ and next state $s' \sim P(s'|s, a)$
    Then sample the next action $a' \sim \pi_\theta(a'|s')$
    Update the policy parameters: $\theta \leftarrow \theta + \alpha_\theta Q_w(s, a) \nabla_\theta \log \pi_\theta(a|s)$; Compute the correction (TD error) for action-value at time t:
$$\delta_t = r_t + \gamma Q_w(s', a') - Q_w(s, a)$$
    and use it to update the parameters of Q function:
$$w \leftarrow w + \alpha_w \delta_t \nabla_w Q_w(s, a)$$
    Move to a $\leftarrow a'$ and s $\leftarrow s'$
end for

Adapted from Lilian Weng's post “Policy Gradient algorithms”

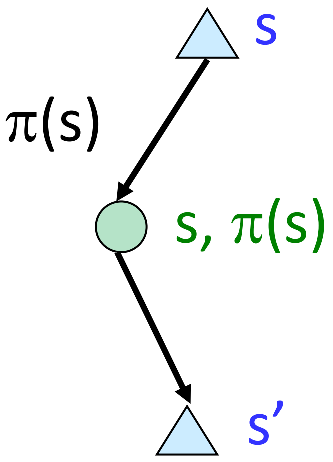
The Advantage Function

$$A(s,a) = \underbrace{Q(s,a)}_{\text{q value for action a in state s}} - \underbrace{V(s)}_{\text{average value of that state}}$$

- Benefits?
- Downsides?

Temporal Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience a transition (s, a, s', r)
  - Likely outcomes s' will contribute updates more often



- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average

Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

The Advantage Function

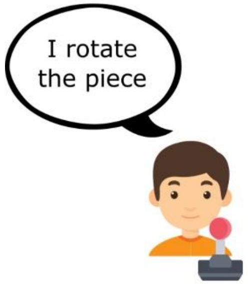
$$A(s,a) = \boxed{Q(s,a)} - V(s)$$

$$r + \gamma V(s')$$

$$A(s,a) = \underbrace{r + \gamma V(s') - V(s)}_{\text{TD Error}}$$

Advantage Actor Critic (A2C)

- Combining value learning with direct policy learning



Actor

Actor

Policy gradient update

$$\nabla_\theta J(\pi_\theta) = \underset{\tau \sim \pi_\theta}{\mathrm{E}}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\Phi_t\right]$$

$$\Phi_t = A^\pi(s_t, a_t) = Q^\pi(s_t, a_t) - V^\pi(s_t)$$

$$\approx r(s_t, a_t) + \gamma V^\pi(s_{t+1}) - V^\pi(s_t)$$

Critic

Critic

TD-Learning update

$$\delta_t = r(s_t, a_t) + \gamma V^\pi(s_{t+1}) - V^\pi(s_t)$$

$$\text{Value} = V^\pi(s_t)$$

$$\text{Target} = r(s_t, a_t) + \gamma V^\pi(s_{t+1})$$

$$w_{k+1} \leftarrow w_k + \alpha MSE(\text{value}, \text{target})$$

https://medium.com/@dixitaniket76/advantage-actor-critic-a2c-algorithm-explained-and-implemented-in-pytorch-dc3354b60b50

Asynchronous Advantage Actor Critic (A3C)

Asynchronous Methods for Deep Reinforcement Learning

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Asynchronous Advantage Actor Critic (A3C)

- Adds a few tricks

    1. Multiple parallel workers to collect rollouts in different copies of the same env and update the global policy and value models asynchronously

    2. n-step returns

    3. Entropy regularization

    4. Share neural network weights for actor and critic

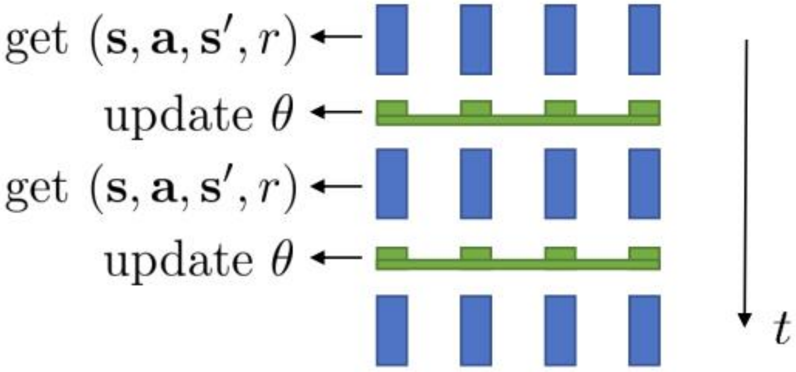
Parallel actors

online actor-critic algorithm:

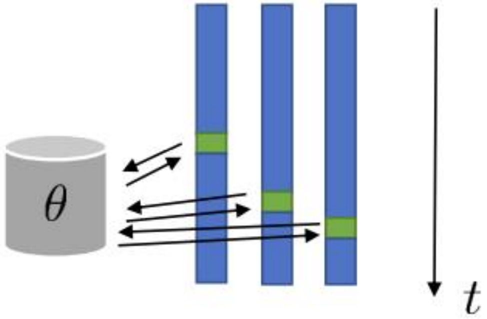
1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}_\phi^\pi$ using target $r + \gamma \hat{V}_\phi^\pi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}') - \hat{V}_\phi^\pi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

works best with a batch (e.g., parallel workers)

synchronized parallel actor-critic



asynchronous parallel actor-critic



N-Step Returns

- At convergence we want $V^\pi(s_t) = E_\pi[r_t + \gamma V^\pi(s_{t+1})]$
- So given experience (s_t, a_t, r_t, s_{t+1}), TD methods push $V^\pi(s_t)$ towards $r_t + \gamma V^\pi(s_{t+1})$
- But why only look one step ahead? [1-step return]
- In practice we have experience that looks like this

$$(s_0, a_0, r_0, s_1, s_2, a_2, r_2, s_3, \dots, s_t, a_t, r_t, s_{t+1}, \dots)$$

What if we pushed $V^\pi(s_t)$ towards $r_t + r_{t+1} + \gamma V^\pi(s_{t+2})$?

Or even pushed $V^\pi(s_t)$ towards $r_t + r_{t+1} + r_{t+2} + \gamma V^\pi(s_{t+3})$?

We can generalize this idea to use n-step returns!

N-Step Returns for A3C updates

Given $(s_0, a_0, r_0, s_1, s_2, a_2, r_2, s_3, \dots, s_t, a_t, r_t, s_{t+1}, \dots r_{T-1}, s_T)$

Compute advantage for each state. If s_T is a terminal state, then define $V_w^\pi(s_T)$=0

$$A(s_t, a_t) = \sum_{i=0}^{T-t-1} \gamma^i r_{t+i} + \gamma^{T-t} V_w^\pi(s_T) - V_w^\pi(s_t)$$

Accumulate gradients for each state and update policy using policy gradient

$$\nabla_\theta \log \pi_\theta(a_t|s_t)\, A_w(s_t, a_t)$$

Update Value function based on TD-error using MSE loss

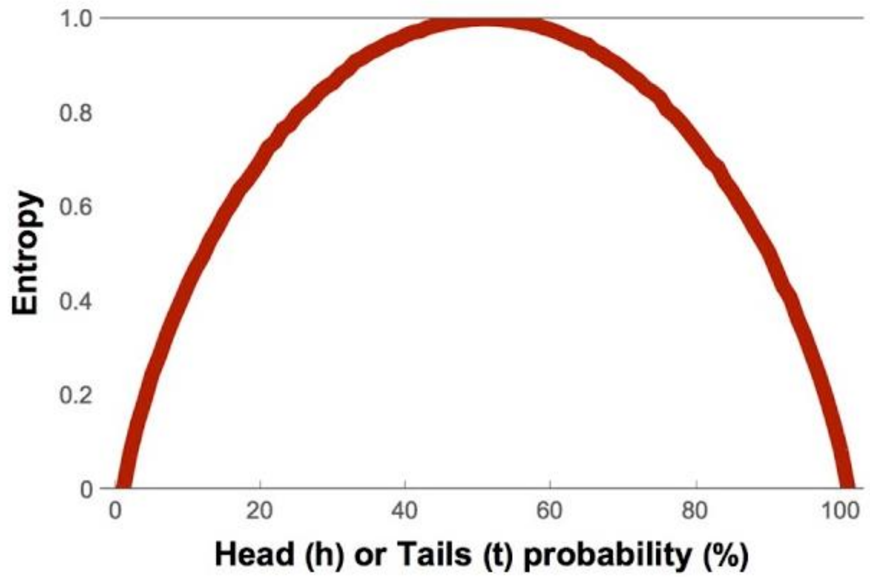
$$\nabla_w \sum_{t=0}^{T-1} \left(\sum_{i=0}^{T-t-1} \gamma^i r_{t+i} + \gamma^{T-t} V_w^\pi(s_T) - V_w^\pi(s_t) \right)^2$$

Shannon Entropy

- Average level of uncertainty associated with a random variable's possible outcomes.

$$P(X = heads) = \frac{1}{2} \qquad P(X = tails) = \frac{1}{2}$$

$$\mathrm{H}(X) := -\sum_{x \in \mathcal{X}} p(x) \log p(x)$$



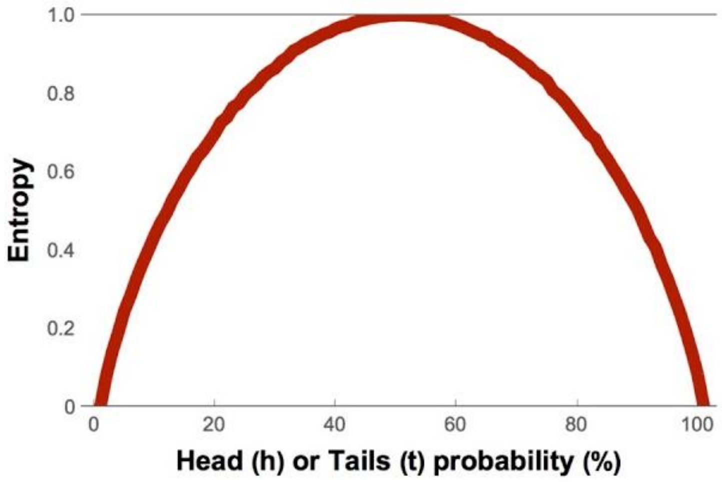
Policy Entropy Bonus

- Improves exploration by discouraging premature convergence to suboptimal deterministic policies.

$$H(\pi) = -\sum_a \pi(a|s) \log \pi(a|s)$$

$$H(\pi) = -\int \pi(a|s) \log \pi(a|s) da$$

$$P(X = heads) = \frac{1}{2} \qquad P(X = tails) = \frac{1}{2}$$

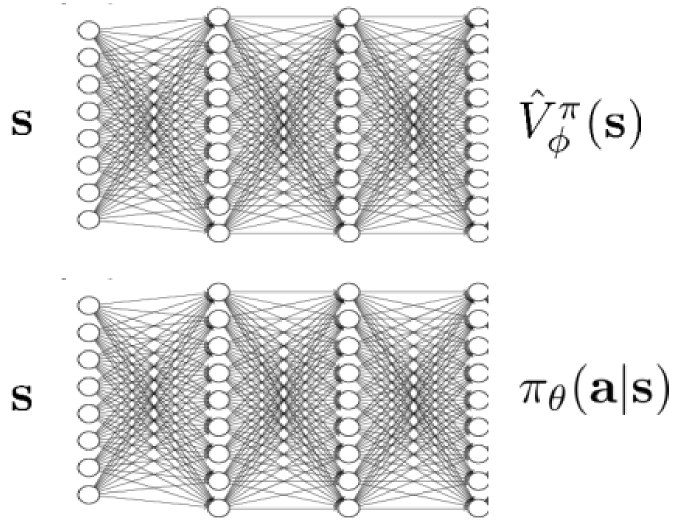


Parameter Sharing

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

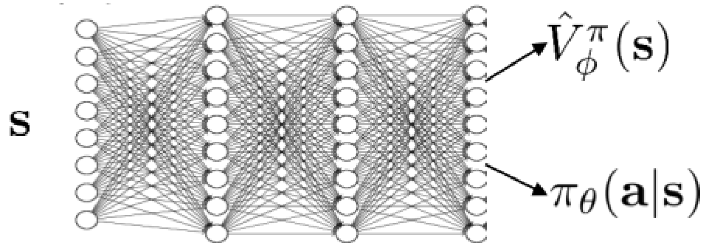
two network design



+ simple & stable

- no shared features between actor & critic

shared network design



Generalized Advantage Estimation (GAE)

Published as a conference paper at ICLR 2016

HIGH-DIMENSIONAL CONTINUOUS CONTROL USING GENERALIZED ADVANTAGE ESTIMATION

John Schulman, Philipp Moritz, Sergey Levine, Michael I. Jordan and Pieter Abbeel
Department of Electrical Engineering and Computer Science
University of California, Berkeley
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- Can we construct all possible n-step returns and average them

$$\hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}_\phi^\pi(\mathbf{s}_t) + \gamma^n \hat{V}_\phi^\pi(\mathbf{s}_{t+n})$$

Smaller n results in lower variance, but higher bias

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{n=1}^{\infty} w_n \hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t)$$

weighted combination of n-step returns

$w_n \propto \lambda^{n-1}$ exponential falloff where $\lambda \in [0,1]$

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} (\gamma\lambda)^{t'-t} \delta_{t'} \qquad \delta_{t'} = r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{t'+1}) - \hat{V}_\phi^\pi(\mathbf{s}_{t'})$$

similar effect as discount!

Proximal Policy Optimization (PPO)

Proximal Policy Optimization Algorithms

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, Oleg Klimov
OpenAI
{joschu, filip, prafulla, alec, oleg}@openai.com

Why does the policy gradient work?

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\hat{A}^\pi_{i,t}$$

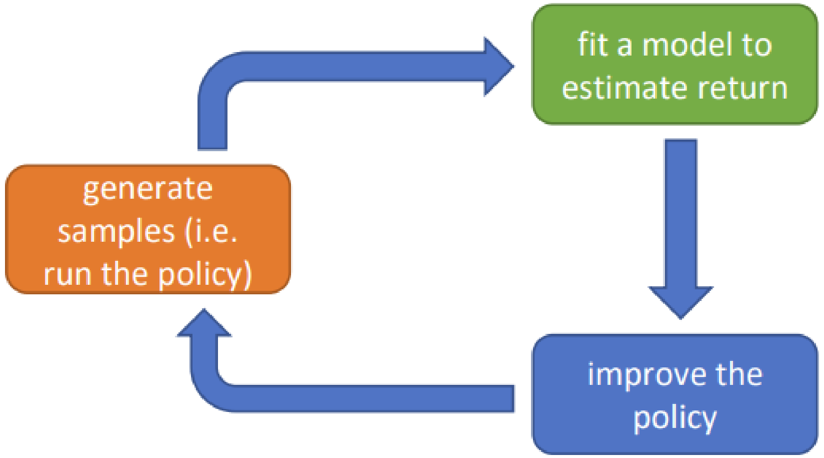
1. Estimate $\hat{A}^\pi(\mathbf{s}_t, \mathbf{a}_t)$ for current policy π
2. Use $\hat{A}^\pi(\mathbf{s}_t, \mathbf{a}_t)$ to get *improved* policy π'

look familiar?

policy iteration algorithm:

1. evaluate $A^\pi(\mathbf{s}, \mathbf{a})$
2. set $\pi \leftarrow \pi'$

$\hat{A}^\pi(\mathbf{x}_t, \mathbf{u}_t)$



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Proximal Policy Optimization (PPO)

- One of the most popular deep RL algorithms
- Used to train ChatGPT and other LLMs

Motivation:

- Many Policy Gradient algorithms have stability problems.
- This can be avoided if we avoid making too big of a policy update.

https://huggingface.co/blog/deep-rl-ppo

https://huggingface.co/blog/deep-rl-ppo

Proximal Policy Iteration (PPO)

- Measure how much we are changing policy compared with previous policy using a ratio:

$$ratio_t(\theta) = \frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_k}(a_t|s_t)}$$

- Clip policy gradient update based on this ratio:

$$\theta_{k+1} = \arg\max_{\theta} \underset{s,a\sim\pi_{\theta_k}}{\mathrm{E}} \left[L(s,a,\theta_k,\theta)\right]$$

$$L(s,a,\theta_k,\theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)} A^{\pi_{\theta_k}}(s,a),\ \ \mathrm{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)}, 1-\epsilon, 1+\epsilon\right) A^{\pi_{\theta_k}}(s,a)\right)$$

Proximal Policy Iteration (PPO)

- Simpler way to write clip objective:

$$L(s, a, \theta_k, \theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)} A^{\pi_{\theta_k}}(s, a), \quad g(\epsilon, A^{\pi_{\theta_k}}(s, a))\right)$$

where

$$g(\epsilon, A) = \begin{cases} (1+\epsilon)A & A \geq 0 \\ (1-\epsilon)A & A < 0 \end{cases}$$

https://spinningup.openai.com/en/latest/algorithms/ppo.html

Proximal Policy Iteration (PPO)

- Simpler way to write clip objective:

$$L(s,a,\theta_k,\theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)} A^{\pi_{\theta_k}}(s,a),\ \ g(\epsilon, A^{\pi_{\theta_k}}(s,a))\right)$$

where

$$g(\epsilon, A) = \begin{cases} \boxed{(1+\epsilon)A \quad A \geq 0} \\ (1-\epsilon)A \quad A < 0 \end{cases}$$

What if the advantage is positive?

$$L(s,a,\theta_k,\theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)}, (1+\epsilon)\right) A^{\pi_{\theta_k}}(s,a)$$

We want to increase $\pi_\theta(a|s)$, but not too much!

Once $\pi_\theta(a|s) > (1+\epsilon)\pi_{\theta_k}(a|s)$ the min kicks in and limits our policy update.

https://spinningup.openai.com/en/latest/algorithms/ppo.html

Proximal Policy Iteration (PPO)

- Simpler way to write clip objective:

$$L(s, a, \theta_k, \theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)} A^{\pi_{\theta_k}}(s,a), \;\; g(\epsilon, A^{\pi_{\theta_k}}(s,a))\right)$$

where

$$g(\epsilon, A) = \begin{cases} (1+\epsilon)A & A \geq 0 \\ \boxed{(1-\epsilon)A \quad A < 0} \end{cases}$$

What if the advantage is negative?

$$L(s, a, \theta_k, \theta) = \max\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)}, (1-\epsilon)\right) A^{\pi_{\theta_k}}(s,a)$$

We want to decrease $\pi_\theta(a|s)$, but not too much!

Once $\pi_\theta(a|s) < (1+\epsilon)\pi_{\theta_k}(a|s)$ the max kicks in and limits our policy update.

https://spinningup.openai.com/en/latest/algorithms/ppo.html

Algorithm 1 PPO-Clip

Input: initial policy parameters θ_0, initial value function parameters ϕ_0

for $k = 0, 1, 2, ...$ **do**

Collect set of trajectories $\mathcal{D}_k = \{\tau_i\}$ by running policy $\pi_k = \pi(\theta_k)$ in the environment.

Compute rewards-to-go $\hat{R}_t$.

Compute advantage estimates, $\hat{A}_t$ (using any method of advantage estimation) based on the current value function V_{ϕ_k}.

Update the policy by maximizing the PPO-Clip objective:

$$\theta_{k+1} = \arg\max_{\theta} \frac{1}{|\mathcal{D}_k|T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \min\left(\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_k}(a_t|s_t)} A^{\pi_{\theta_k}}(s_t, a_t), \quad g(\epsilon, A^{\pi_{\theta_k}}(s_t, a_t)) \right),$$

typically via stochastic gradient ascent with Adam.

Fit value function by regression on mean-squared error:

$$\phi_{k+1} = \arg\min_{\phi} \frac{1}{|\mathcal{D}_k|T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \left(V_\phi(s_t) - \hat{R}_t \right)^2,$$

typically via some gradient descent algorithm.

end for