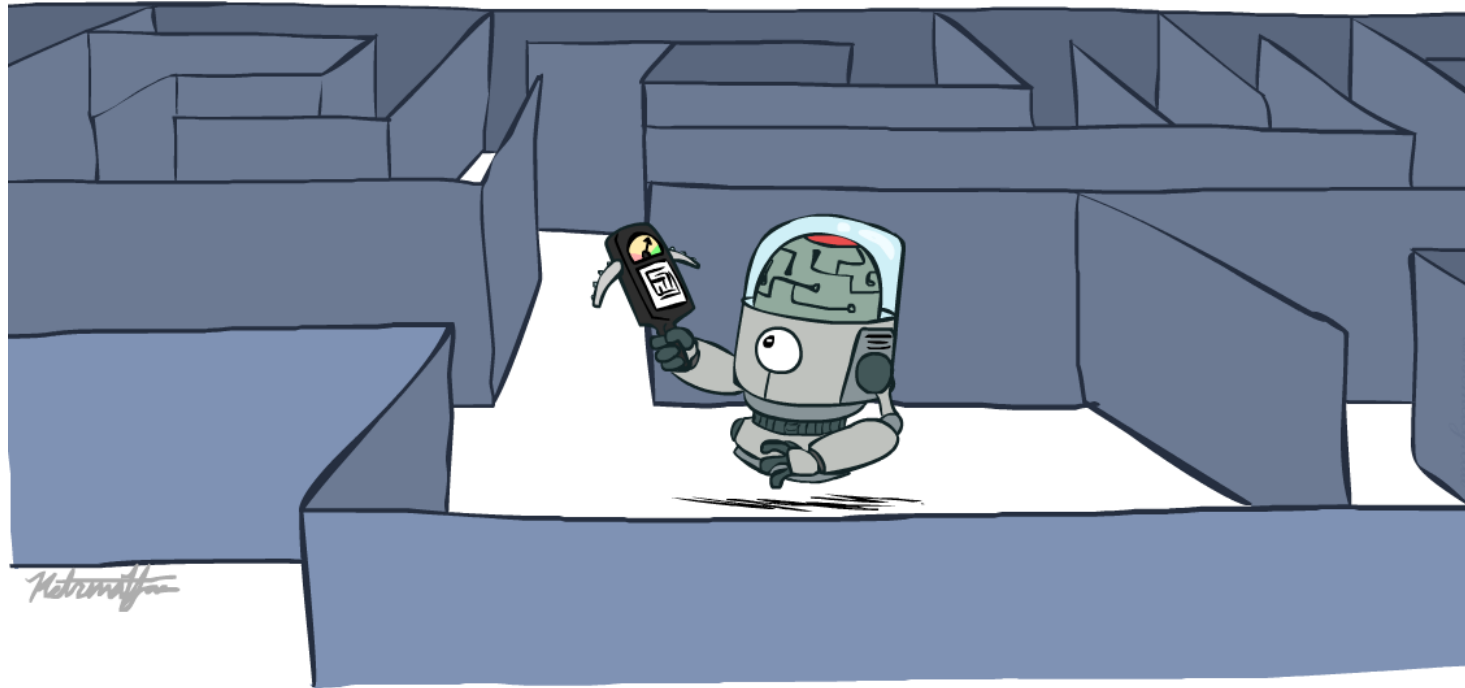# Announcements

- Project 0: Python Tutorial
  - Due Friday before midnight
- Homework 1
  - Due Aug 29th before midnight
  - Covers this lecture (we probably will take two days to cover it).
  - You can start today!
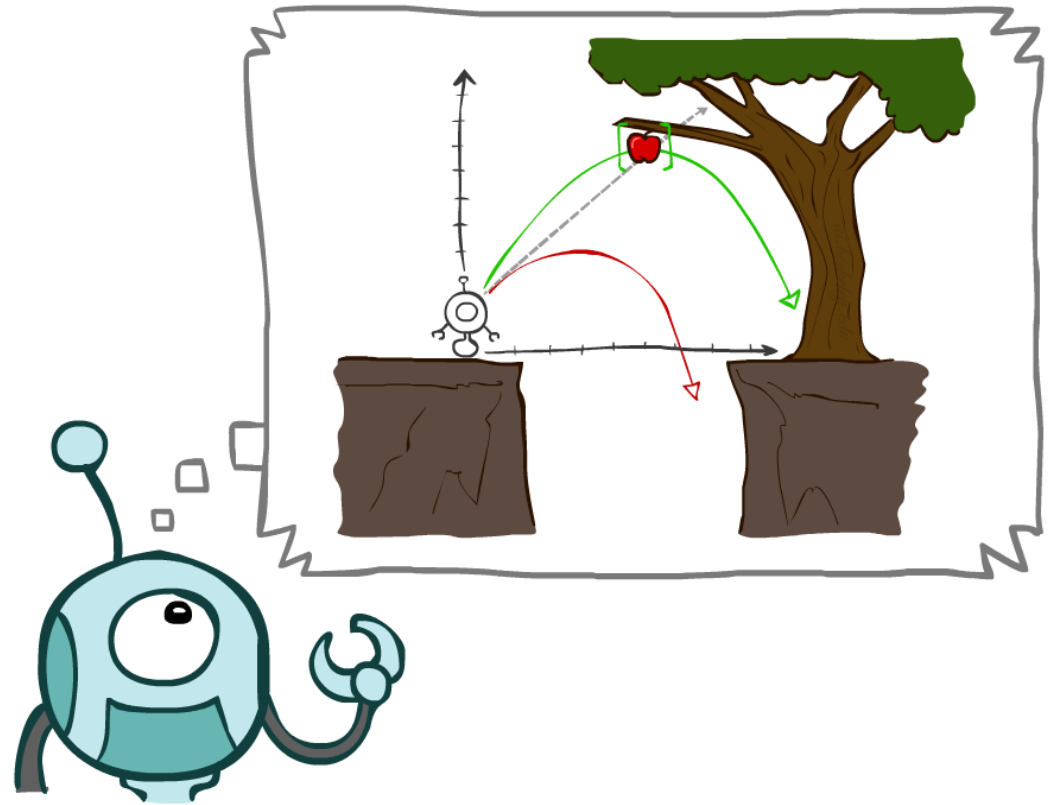 - Look at the practice problems first if you're stuck!

CS 4300/6300: Search

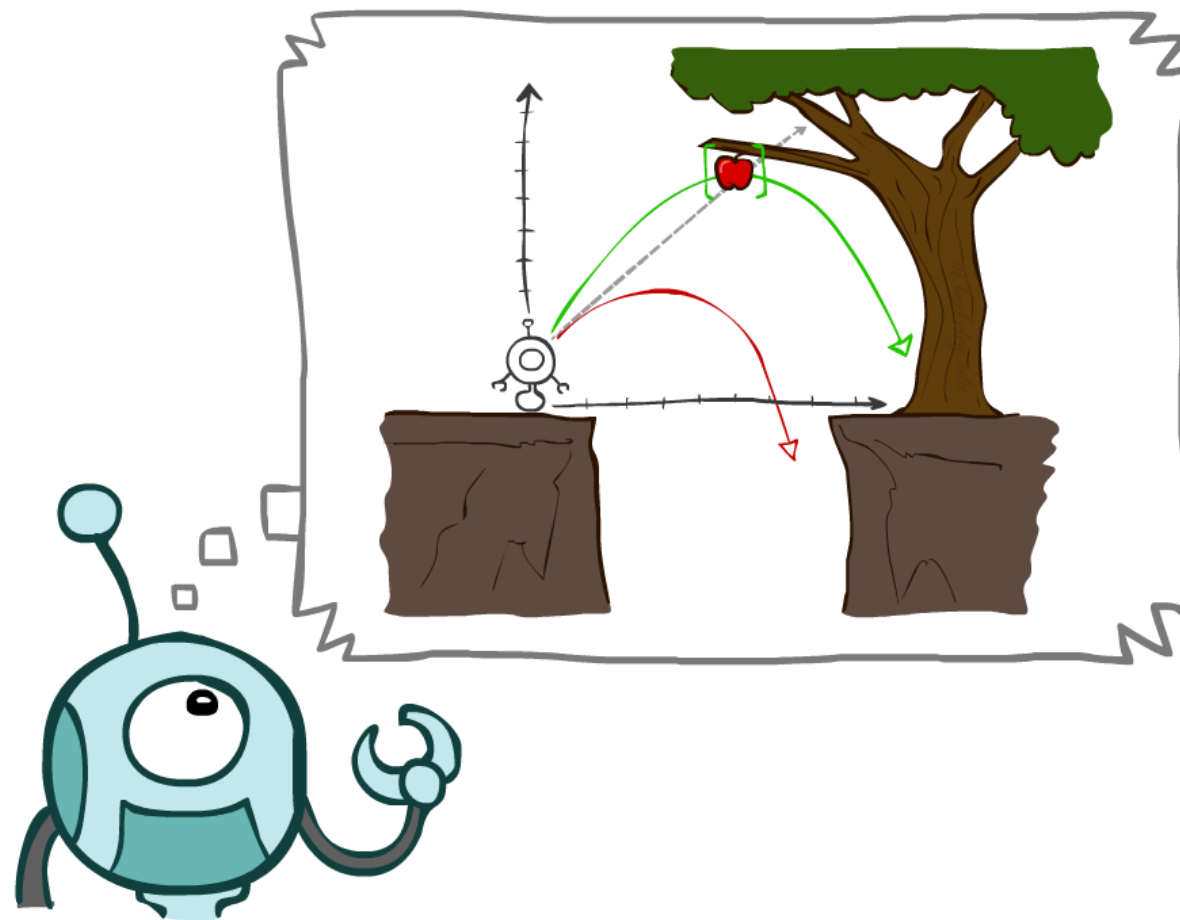
Instructor: Daniel Brown

University of Utah

Today

- Agents that Plan Ahead

- Search Problems

- Uninformed Search Methods

- Informed (heuristic) Search

Agents that Plan

Planning Agents

- **Planning agents:**
  - Ask "what if"
  - Decisions based on (hypothesized) consequences of actions
  - Must have a model of how the world evolves in response to actions
  - Must formulate a goal (test)
  - Consider how the world WOULD BE

- **Optimal Planning**
  - Returns a least cost solution.

- **Complete Planning**
  - If there exists a solution it will find it.

- **Planning vs. replanning**
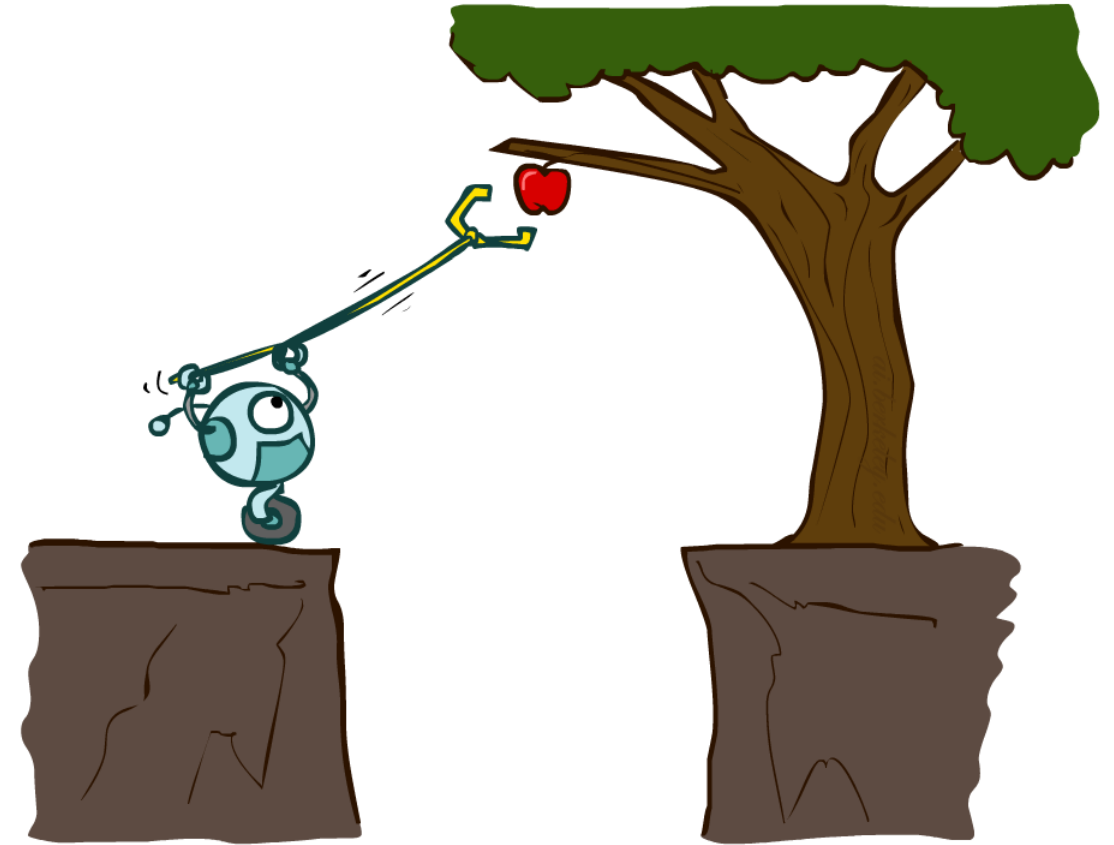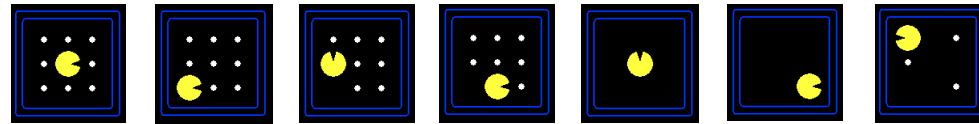
Video of Demo Mastermind
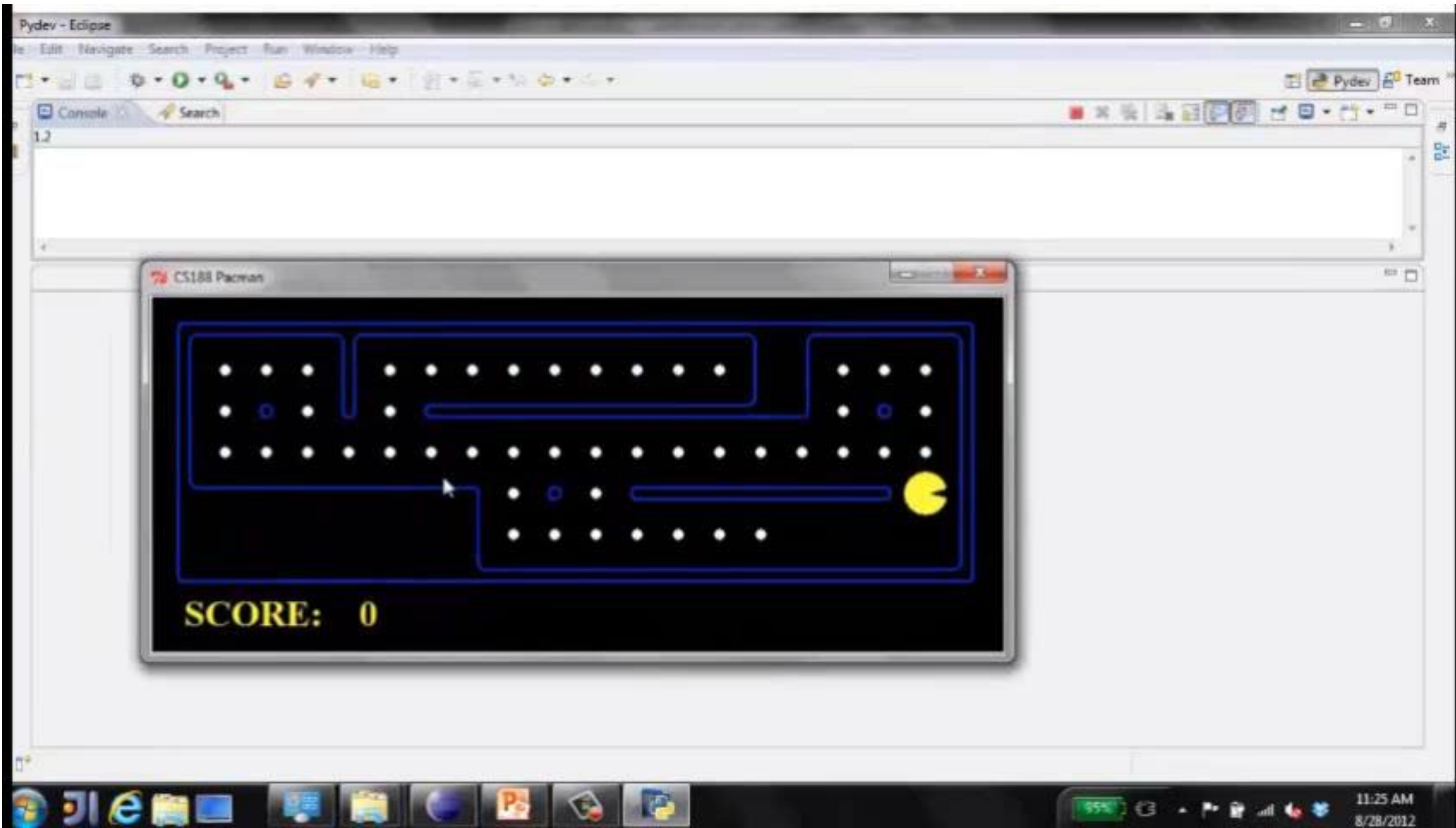
Video of Demo Replanning
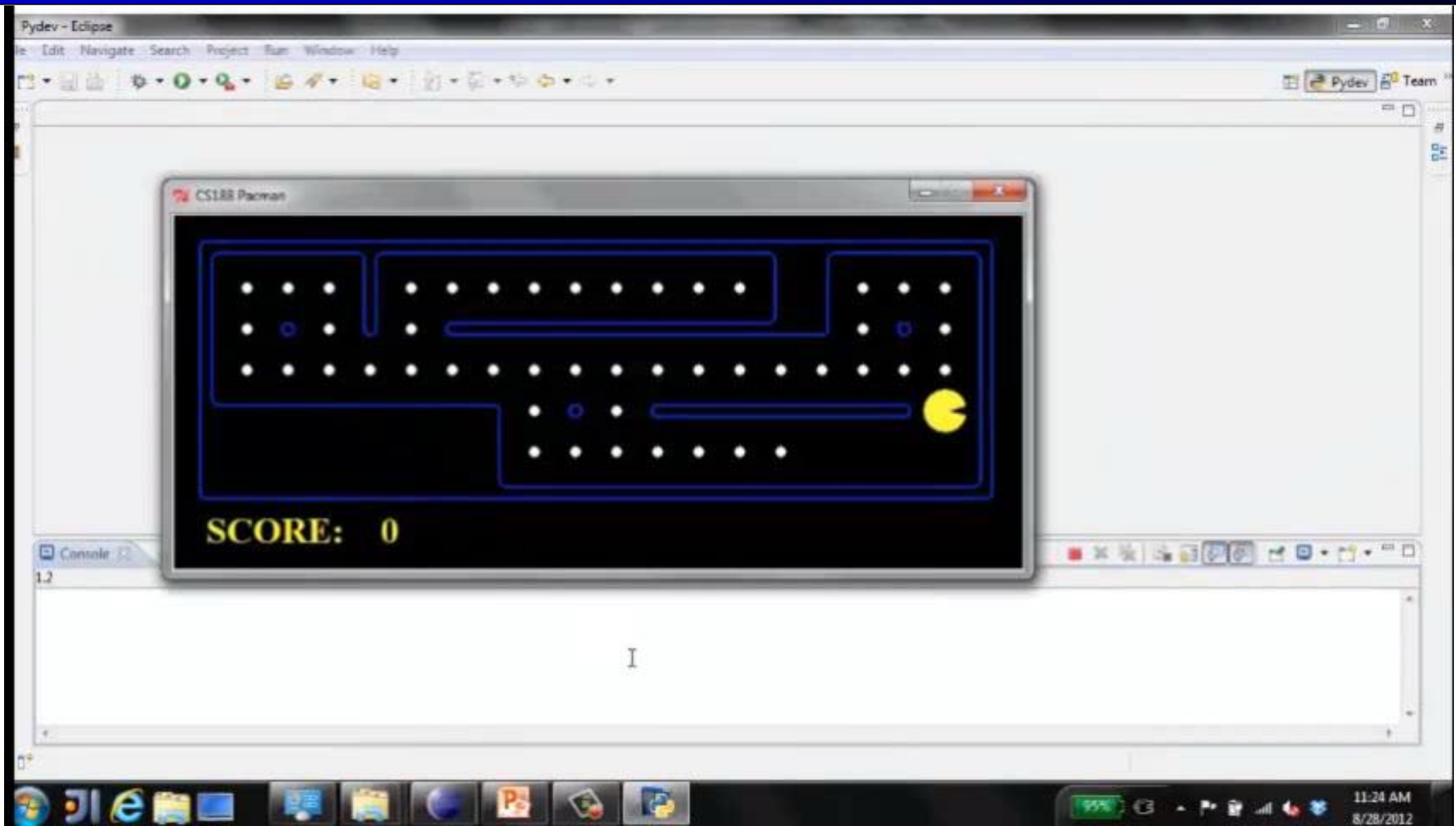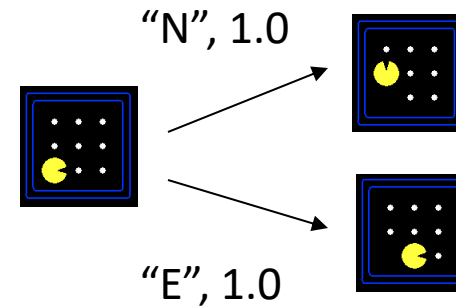
Search Problems

Search Problems

- A search problem consists of:

  - A state space

  - A successor function
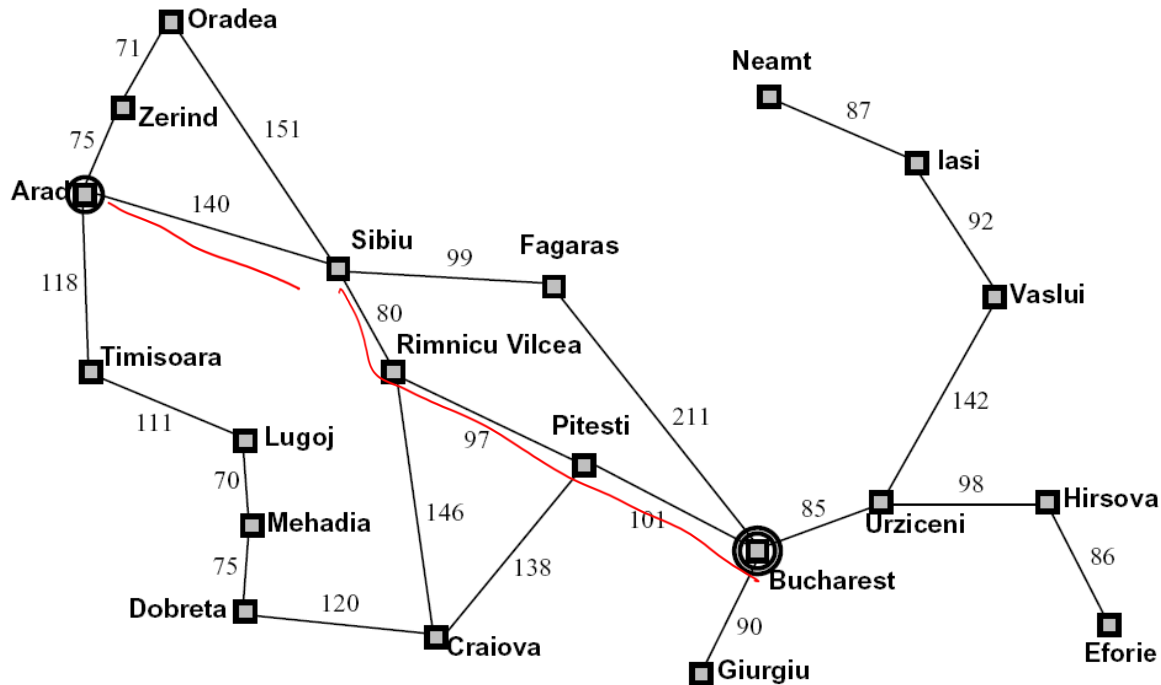   (with actions, costs)

    "N", 1.0

    "E", 1.0

  - A start state and a goal test

- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

Search Problems Are Models
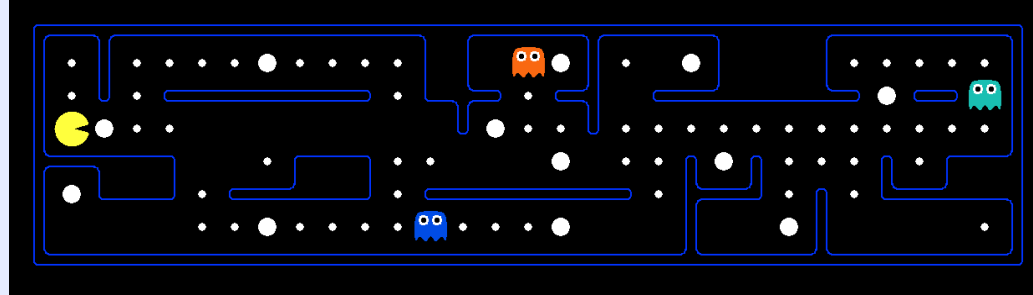
Example: Traveling in Romania

- **State space:**
  - Cities
- **Successor function:**
  - Roads: Go to adjacent city with cost = distance
- **Start state:**
  - Arad
- **Goal test:**
  - Is state == Bucharest?

- **Solution?**

What's in a State Space?

The world state includes every last detail of the environment

A search state keeps only the details needed for planning (abstraction)

- Problem: Pathing (go from location A to B)
  - States: (x,y) location
  - Actions: NSEW
  - Successor: update location only
 - Goal test: is (x,y)=END

- Problem: Eat-All-Dots
  - States: {(x,y), dot booleans}
  - Actions: NSEW
  - Successor: update location and possibly a dot boolean
  - Goal test: dots all false

State Space Sizes?

- **World state:**
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW

- **How many**
  - World states?
    $120 \times (2^{30}) \times (12^2) \times 4$ (~74 trillion)
  - States for pathing?
    120
  - States for eat-all-dots?
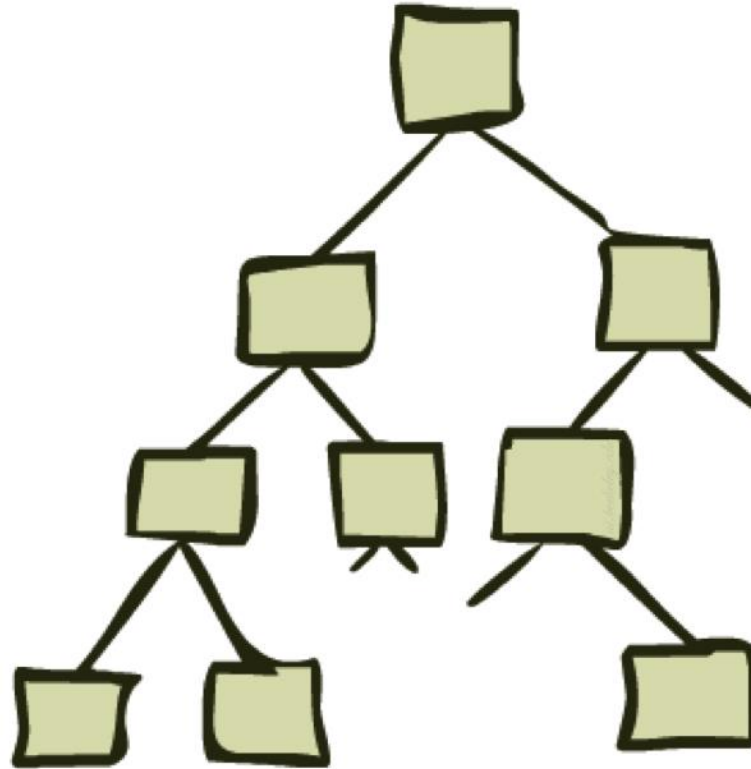 $120 \times (2^{30})$
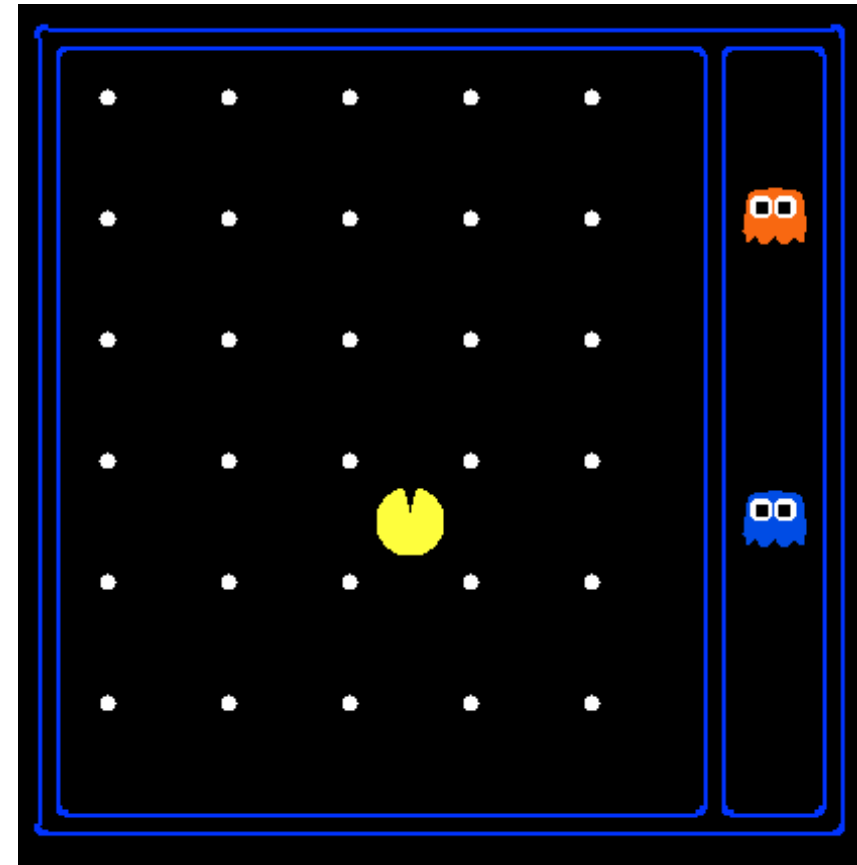
Quiz: Safe Passage
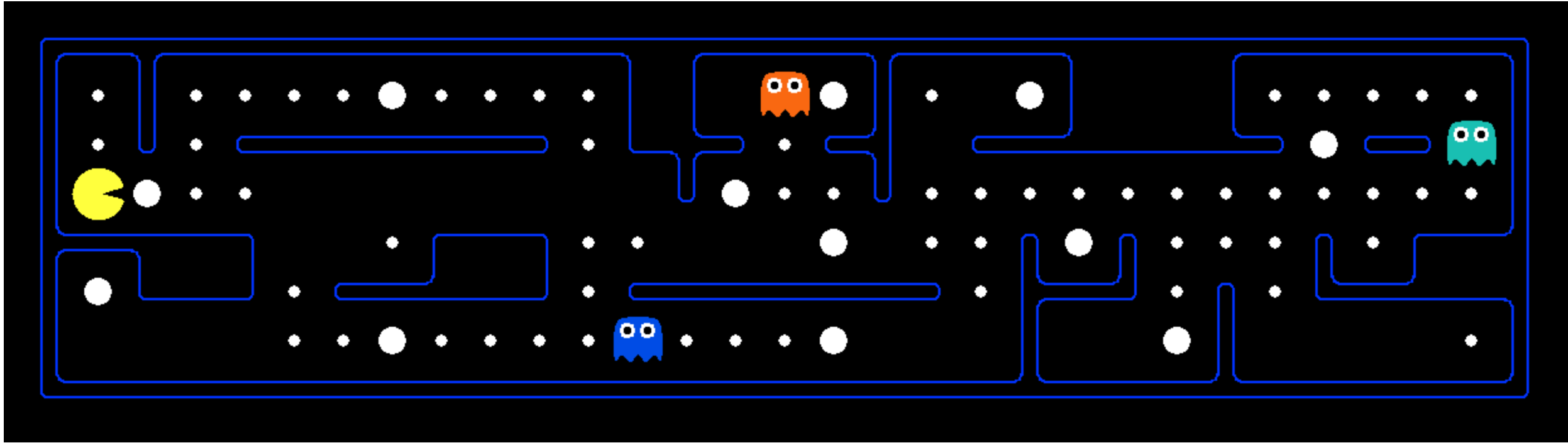
- Problem: eat all dots while keeping the ghosts perma-scared

- What does the state space have to specify?
  - (agent position, dot booleans, power pellet booleans, remaining scared time)

State Space Graphs

- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)

- In a state space graph, each state occurs only once!

- We can rarely build this full graph in memory (it's too big), but it's a useful idea
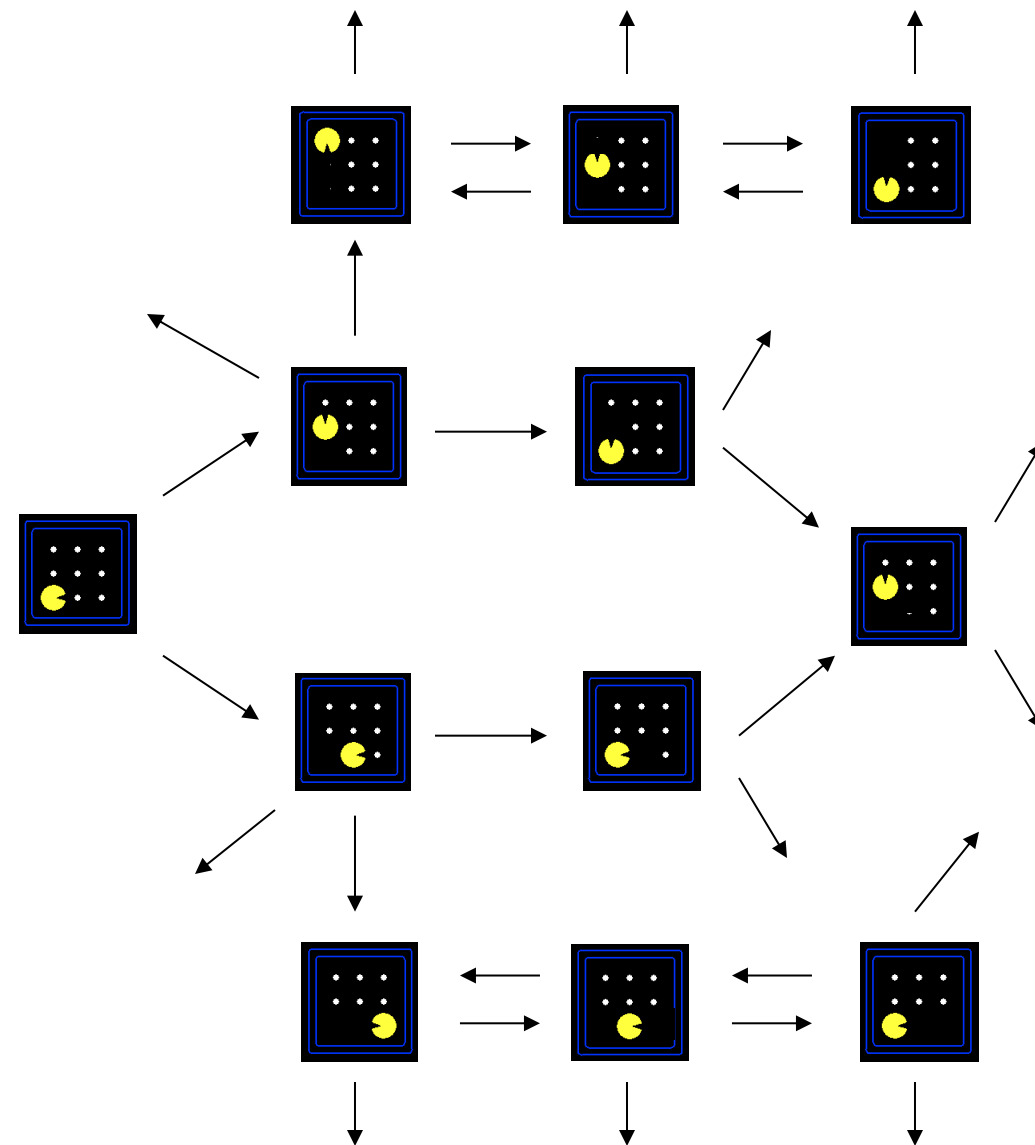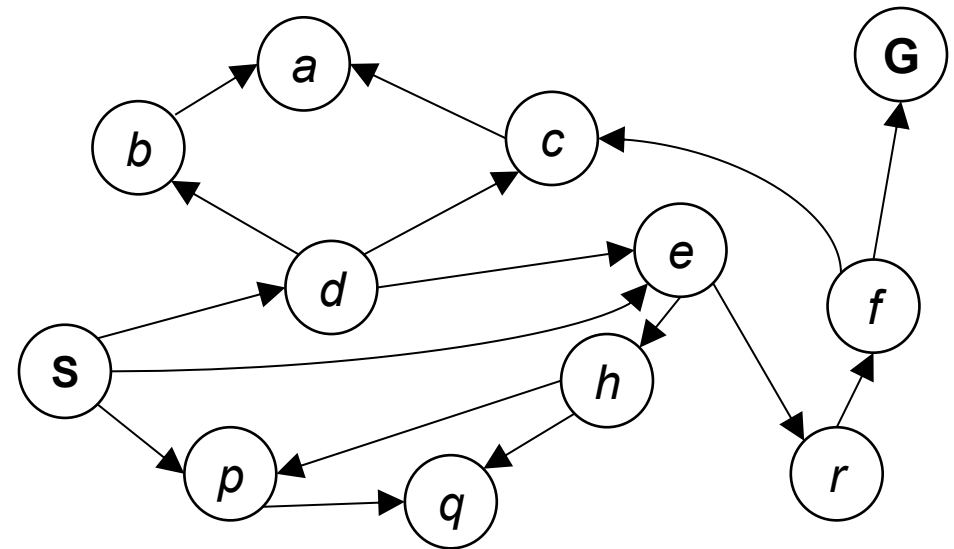
State Space Graphs

- State space graph: A mathematical representation of a search problem
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  - The goal test is a set of goal nodes (maybe only one)

- In a search graph, each state occurs only once!

- We can rarely build this full graph in memory (it's too big), but it's a useful idea

*Tiny state space graph for a tiny search problem*

Search Trees

This is now / start

Possible futures
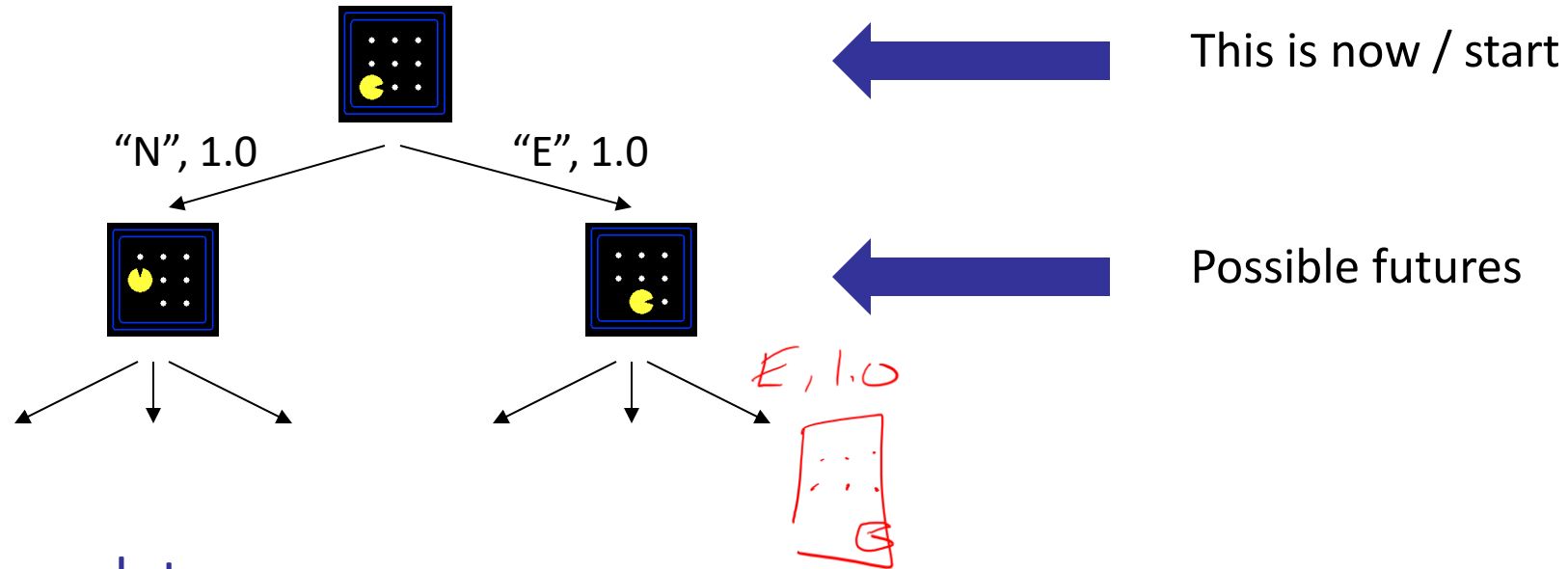
- **A search tree:**
  - A "what if" tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - For most problems, we can never actually build the whole tree
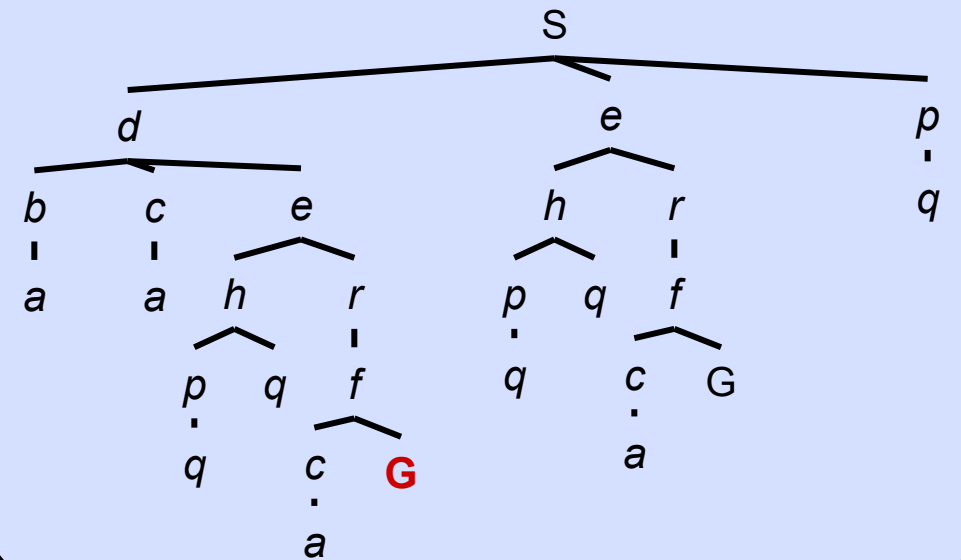
State Space Graphs vs. Search Trees

State Space Graph
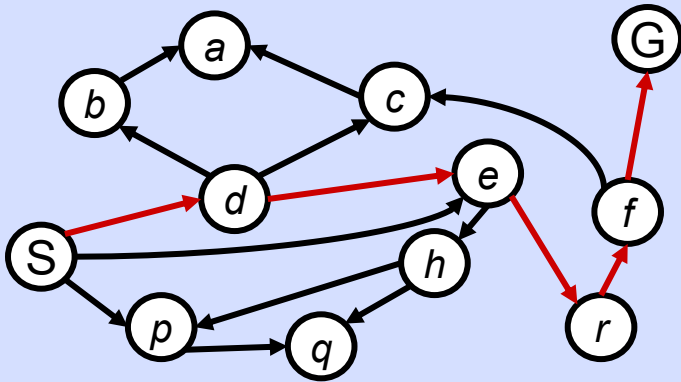
Each NODE in in the search tree is an entire PATH in the state space graph.
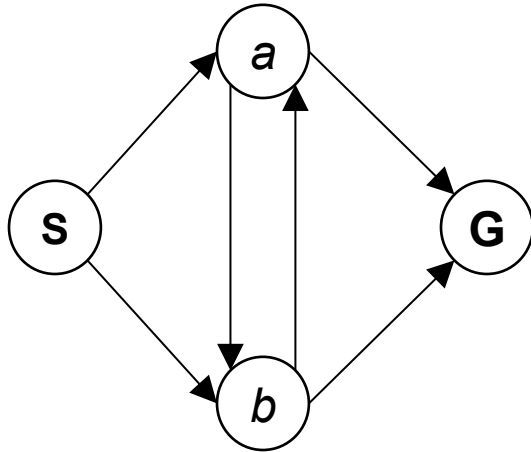
We construct both on demand – and we construct as little as possible.

Search Tree

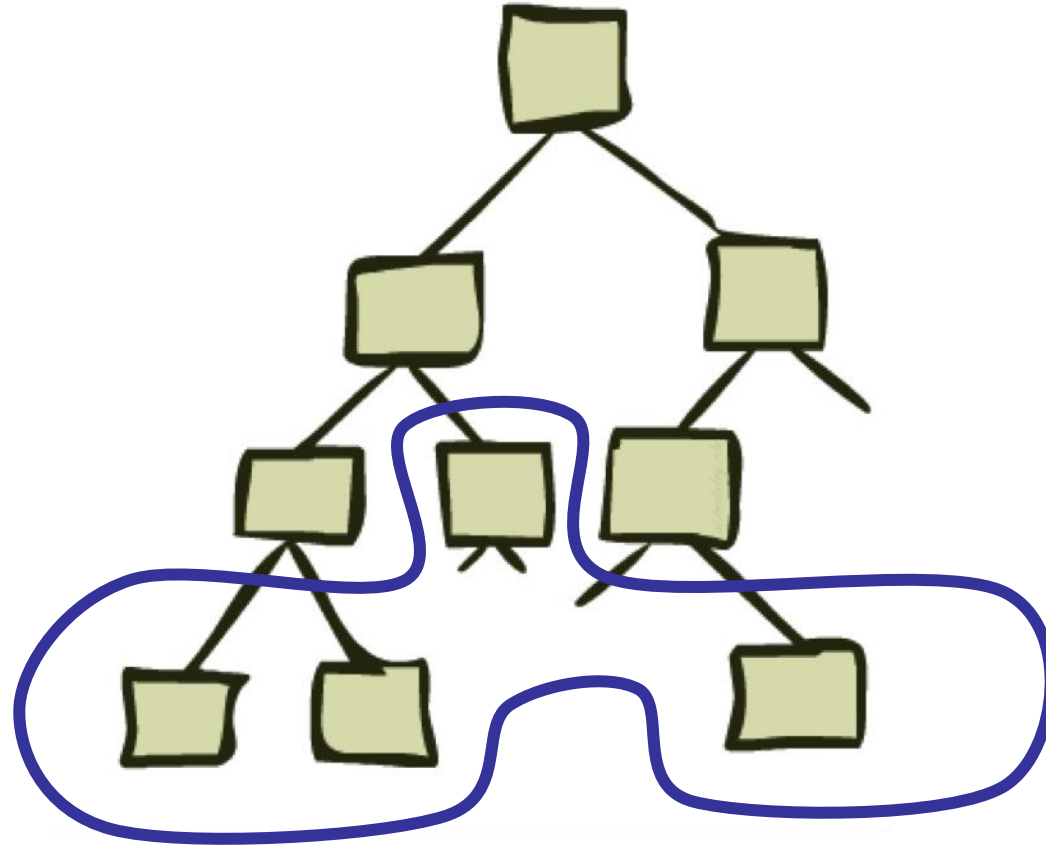# Quiz: State Space Graphs vs. Search Trees

Consider this 4-state graph:
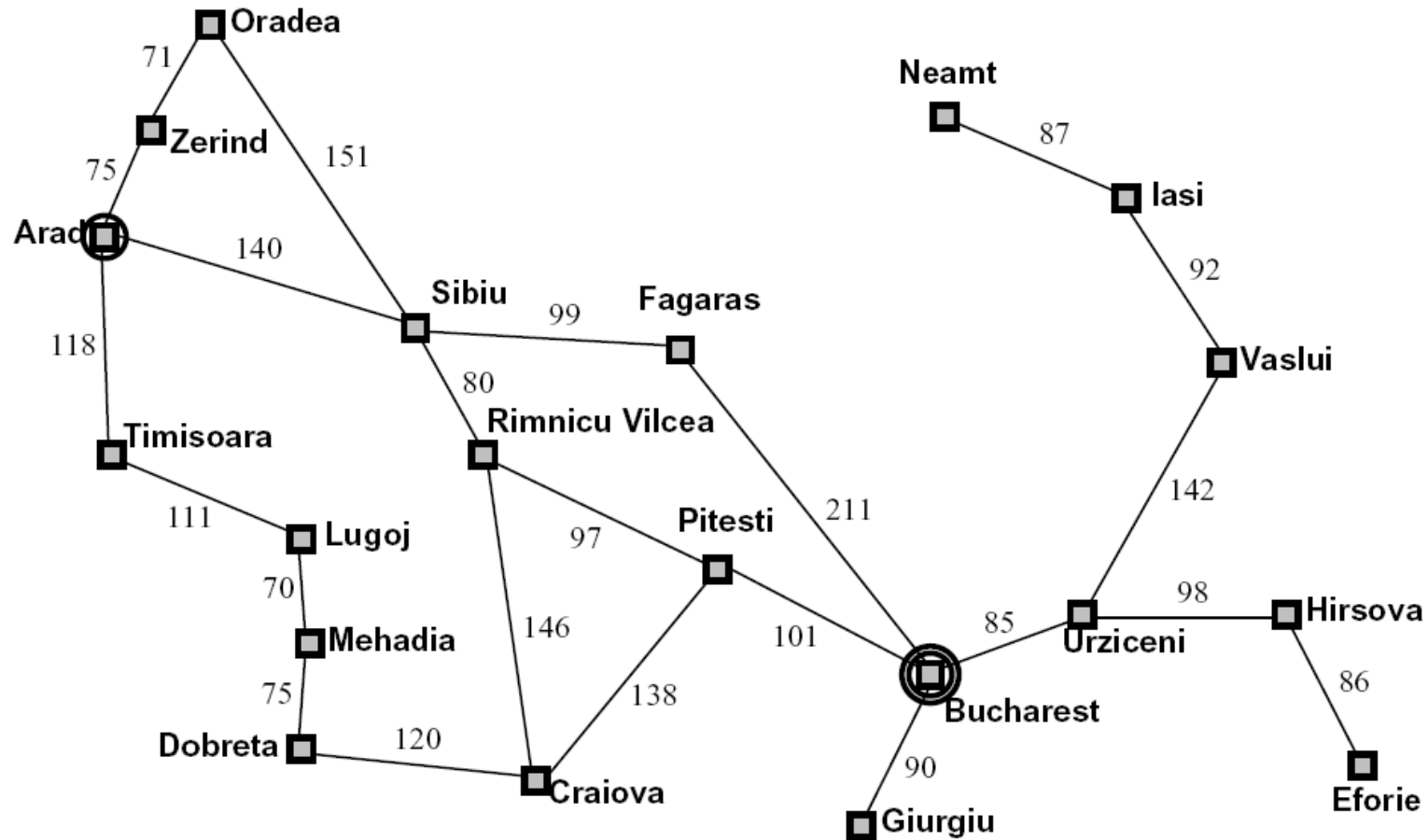
How big is its search tree (from S)?

What does the search tree look like?

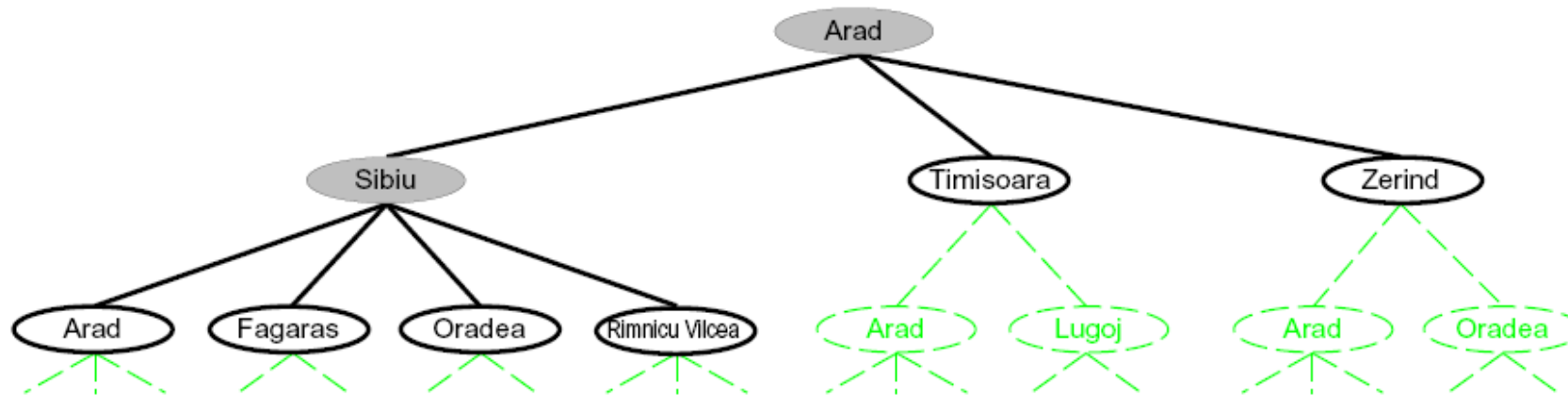Important: Lots of repeated structure in the search tree!

Tree Search

Searching with a Search Tree

- **Search:**
 - Expand out potential plans (tree nodes)
  - Maintain a fringe of partial plans under consideration
  - Try to expand as few tree nodes as possible

General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy

- Main question: which fringe nodes to explore?
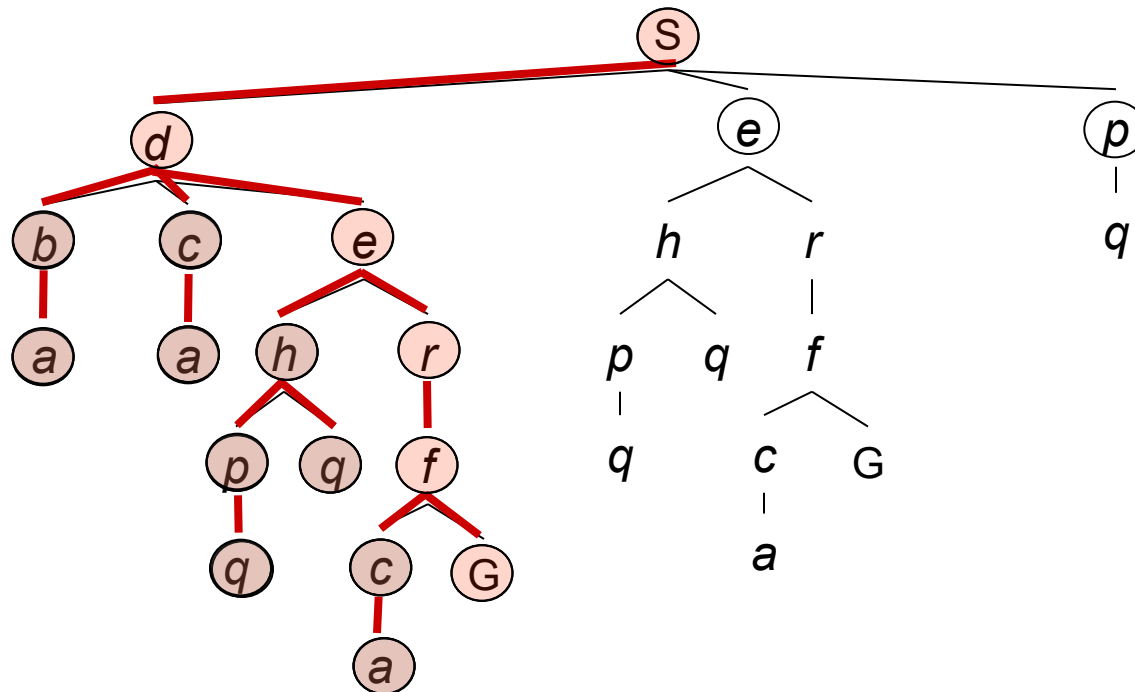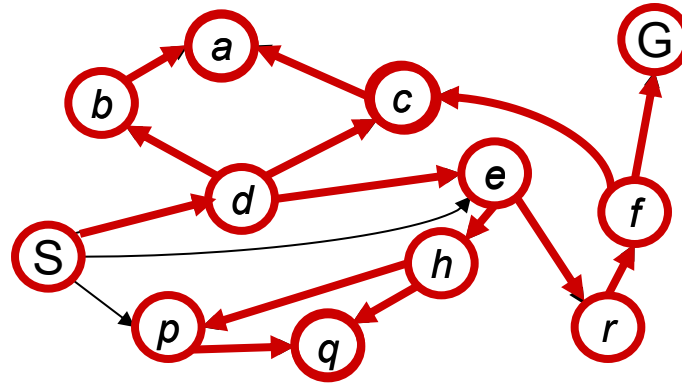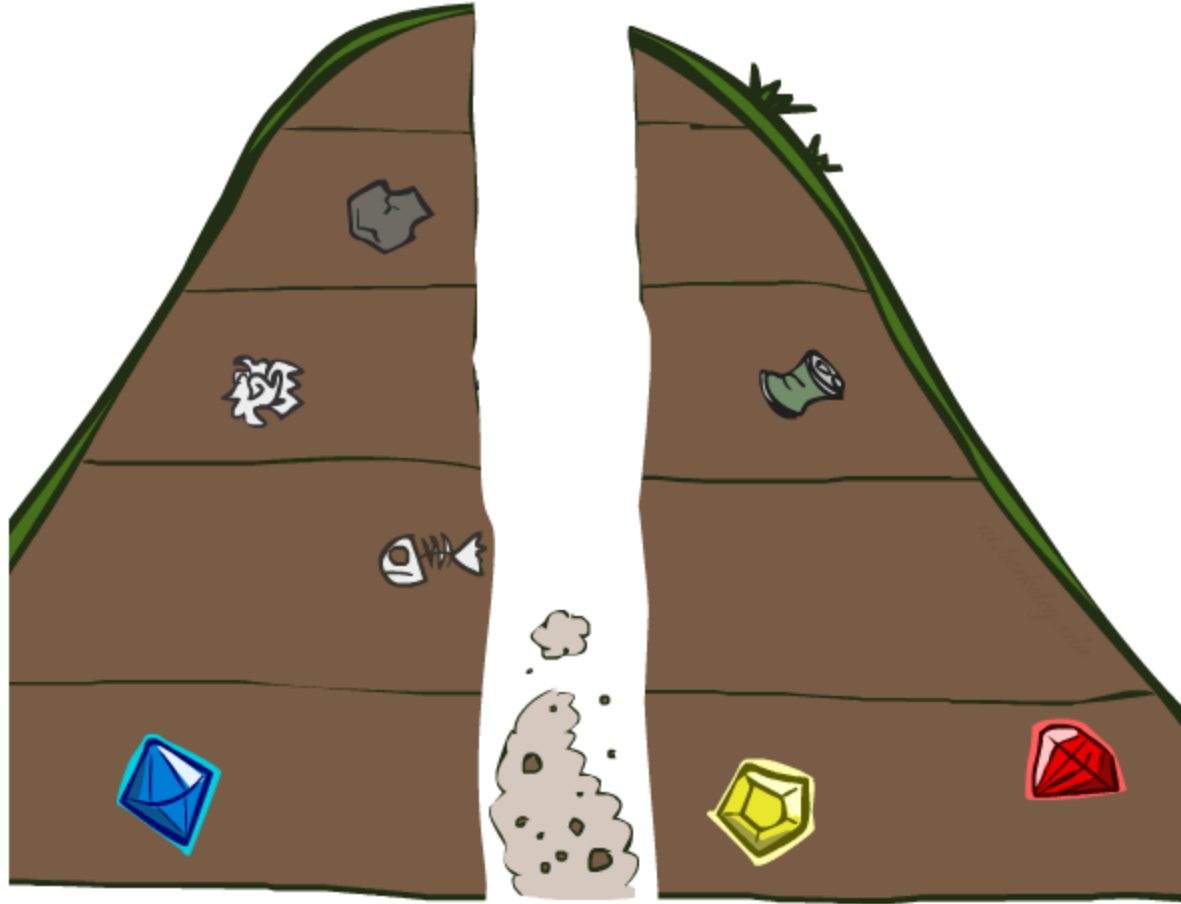
Example: Tree Search
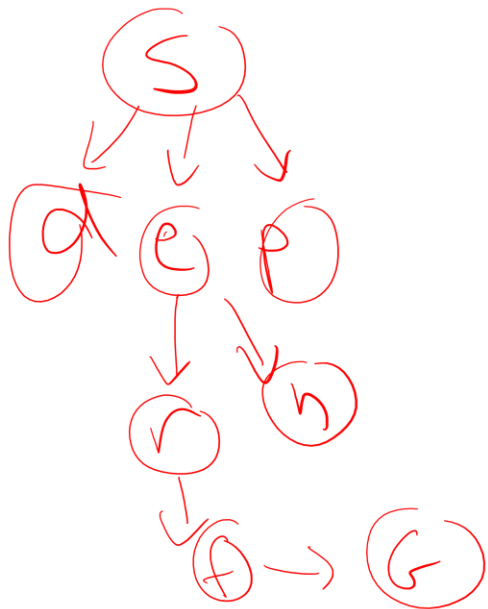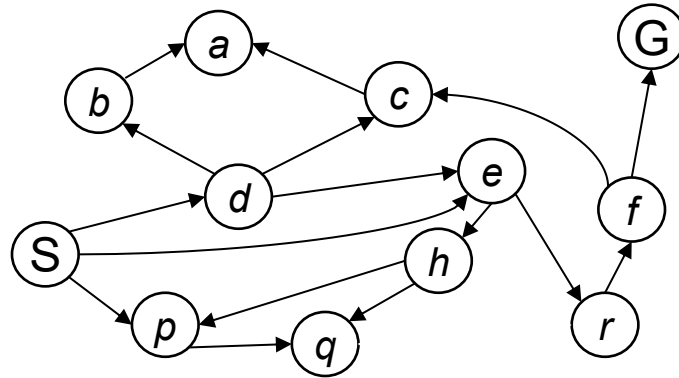
Depth-First Search

Depth-First Search

Strategy: expand a deepest node first

Implementation:
Fringe is a LIFO stack
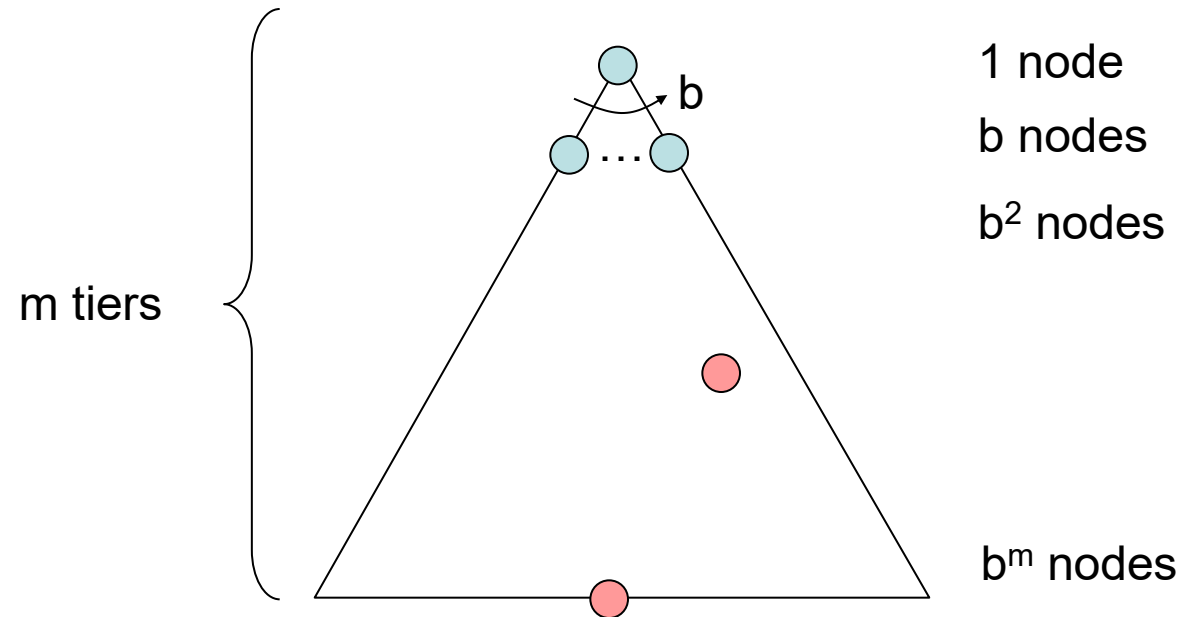
Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?

- Optimal: Guaranteed to find the least cost path?

- Time complexity?

- Space complexity?

- Cartoon of search tree:
  - b is the branching factor
 - m is the maximum depth
  - solutions at various depths

- Number of nodes in entire tree?
  - $1 + b + b^2 + \ldots b^m = O(b^m)$

1 node

b nodes

b^2 nodes

m tiers

b^m nodes

b

Depth-First Search (DFS) Properties

- **What nodes DFS expand?**
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time $O(b^m)$

- **How much space does the fringe take?**
  - Only has siblings on path to root, so $O(bm)$

- **Is it complete?**
  - m could be infinite, so only if we prevent cycles (more later)

- **Is it optimal?**
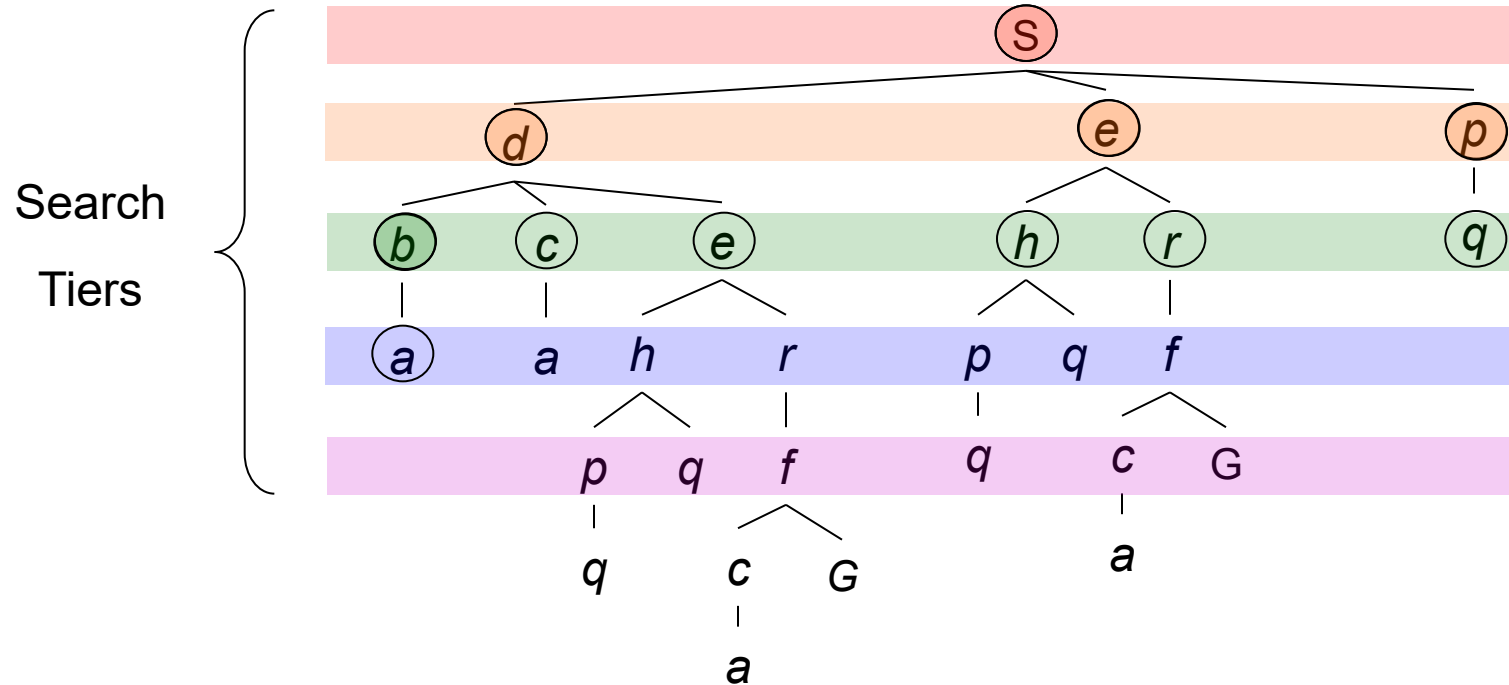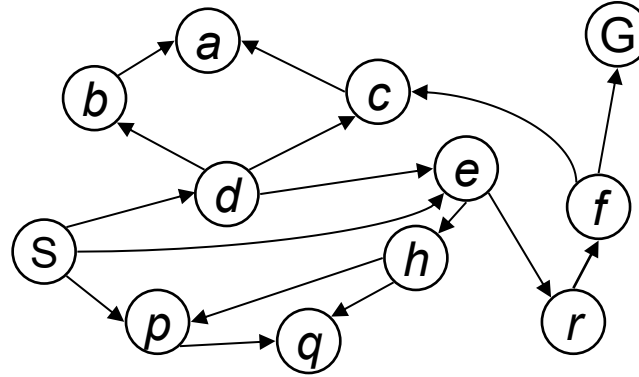 - No, it finds the "leftmost" solution, regardless of depth or cost

1 node

b nodes

b^2 nodes

m tiers

b^m nodes
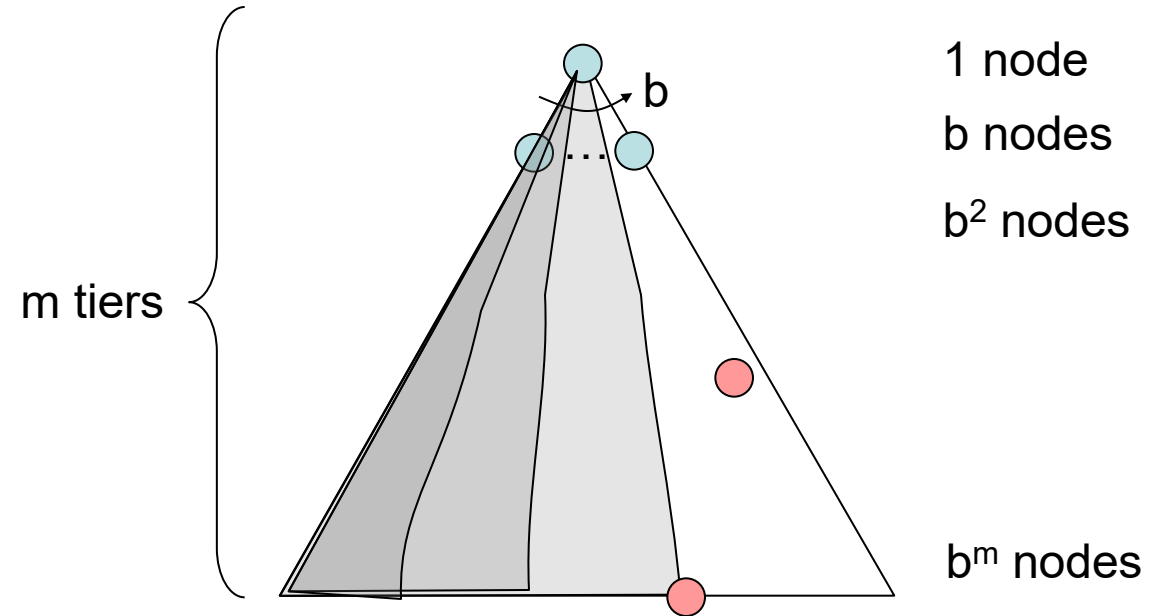
# Breadth-First Search
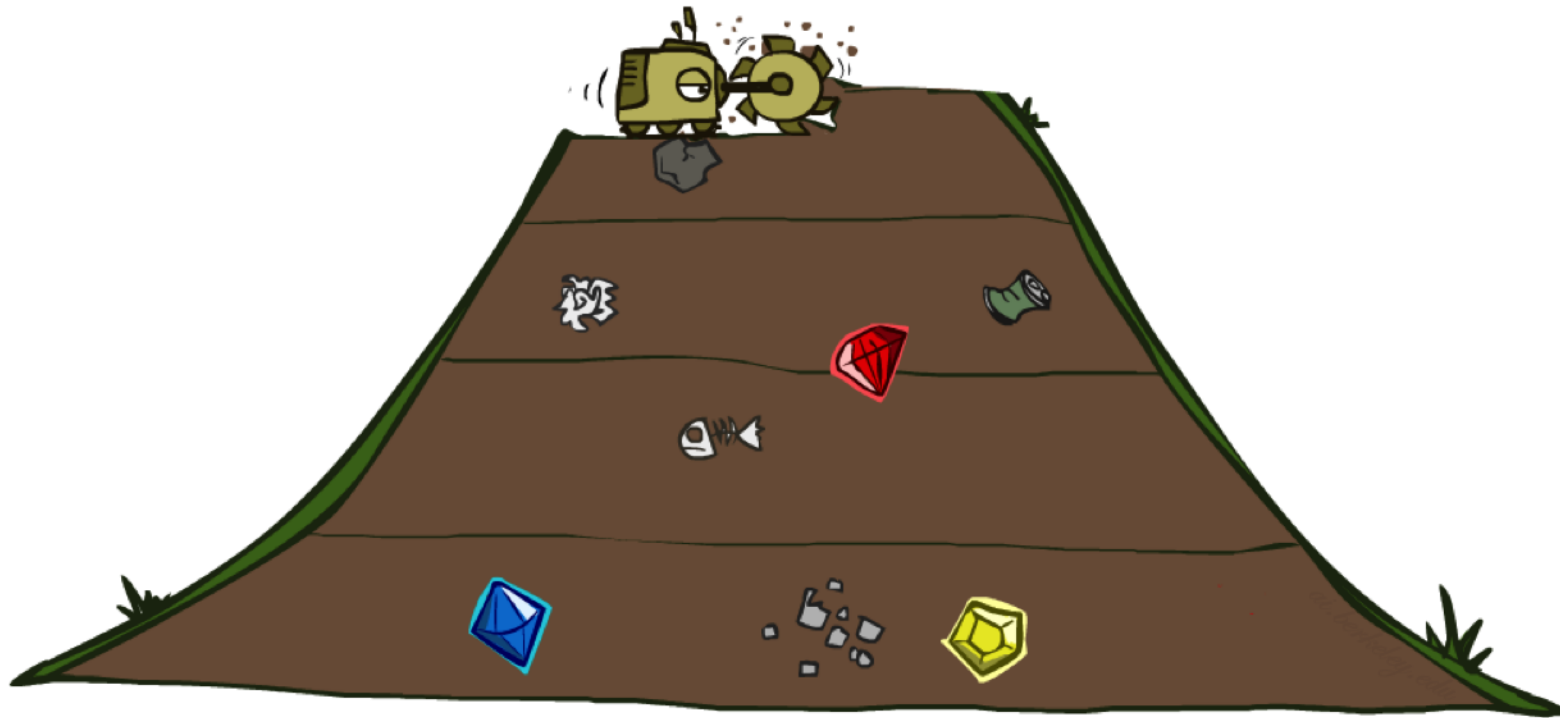
Breadth-First Search

Strategy: expand a shallowest node first

*Implementation: Fringe is a FIFO queue*

Breadth-First Search (BFS) Properties

- **What nodes does BFS expand?**
    - Processes all nodes above shallowest solution
    - Let depth of shallowest solution be s
    - Search takes time $O(b^s)$

- **How much space does the fringe take?**
    - Has roughly the last tier, so $O(b^s)$

- **Is it complete?**
    - s must be finite if a solution exists, so yes!

- **Is it optimal?**
    - Only if costs are all 1 (more on costs later)

s tiers

1 node

b nodes

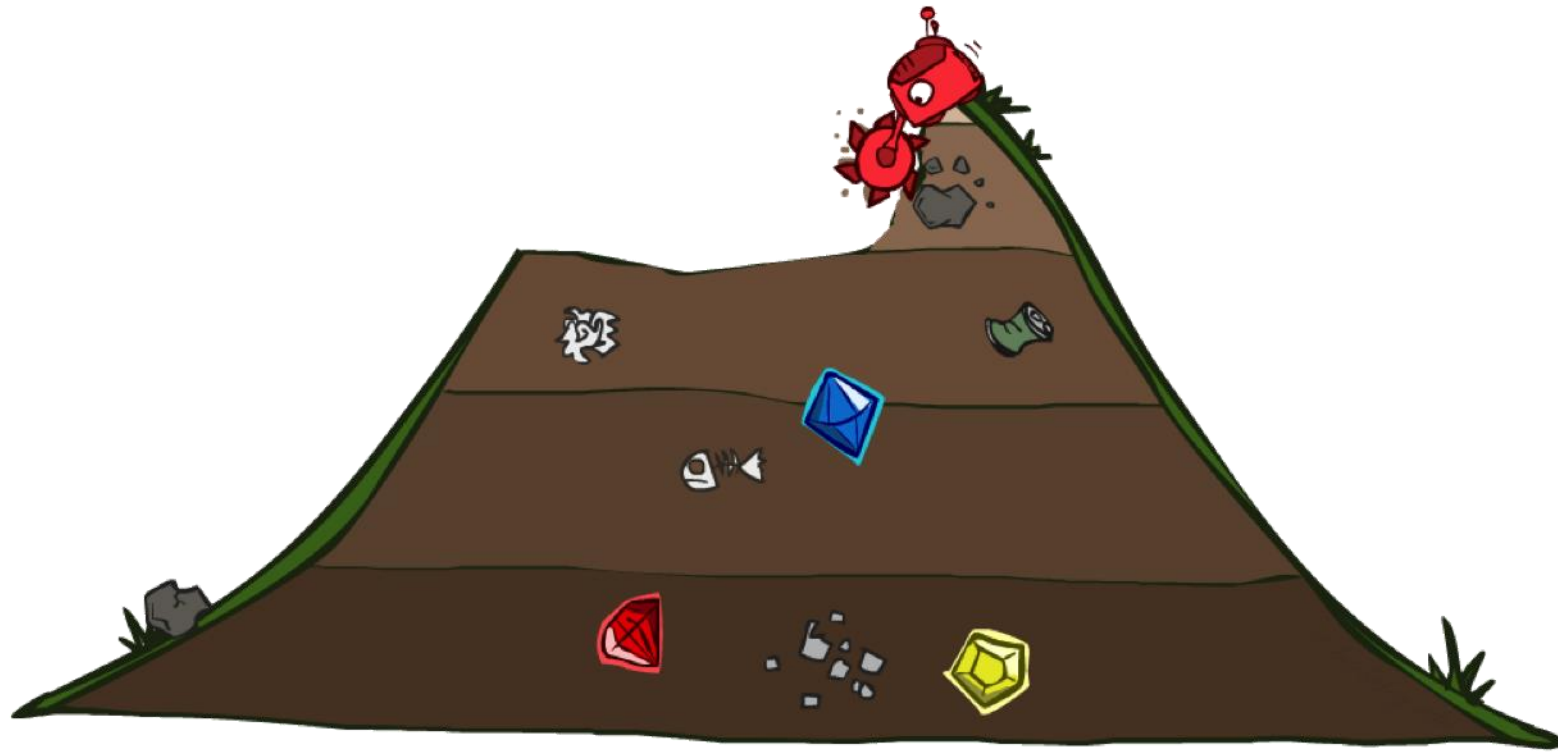b^2 nodes

b^s nodes

b^m nodes
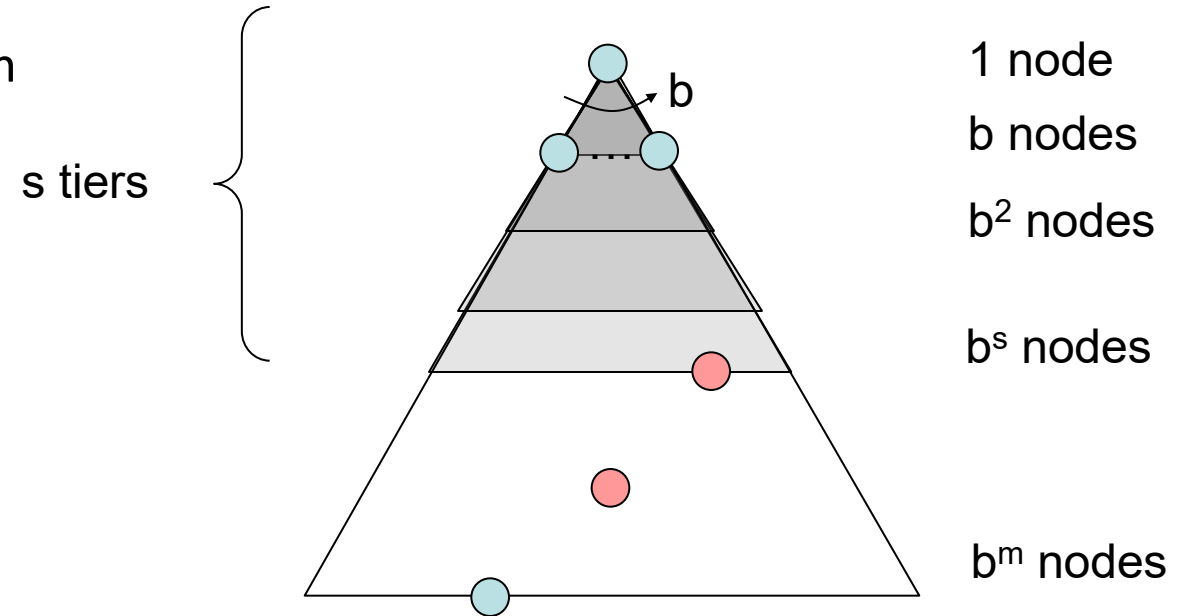
Cost-Sensitive Search
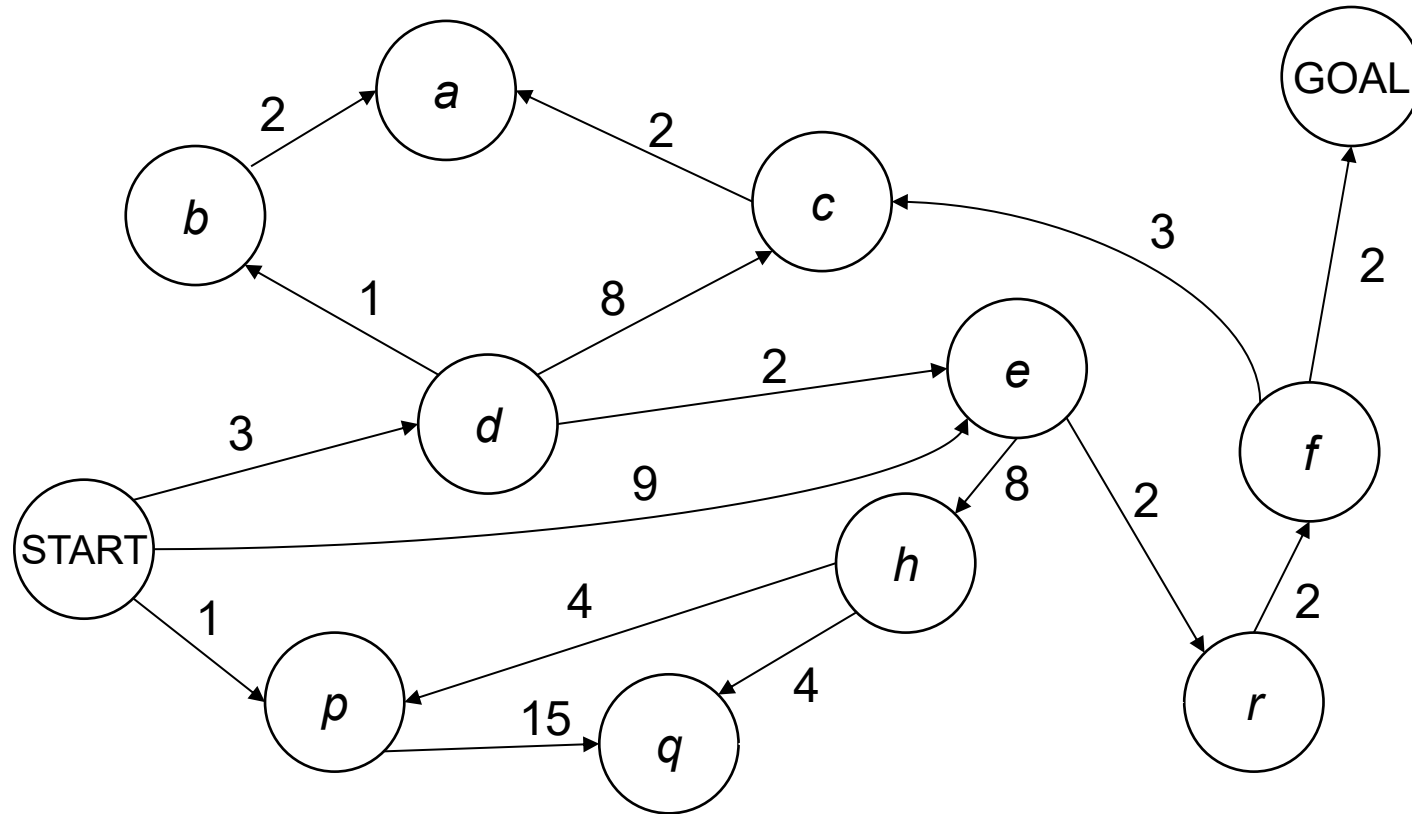
BFS finds the shortest path in terms of number of actions.
It does not find the least-cost path. We will now cover
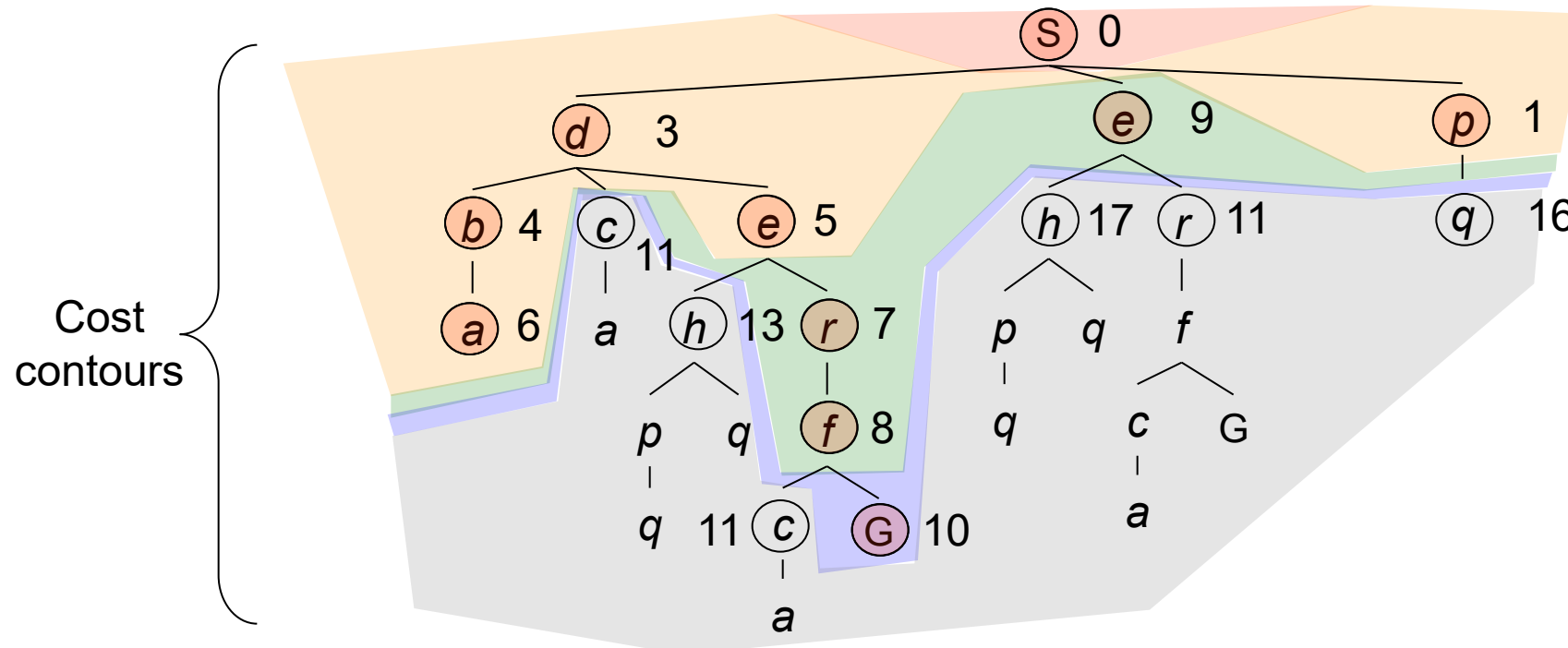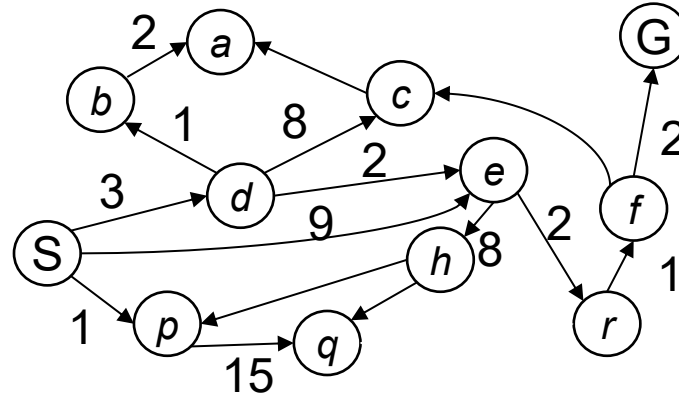a similar algorithm which does find the least-cost path.

Uniform Cost Search

Uniform Cost Search

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

Cost contours

Uniform Cost Search (UCS) Properties

- ## What nodes does UCS expand?

    - Processes all nodes with cost less than cheapest solution!
    - If that solution costs $C*$ and arcs cost at least ε, then the "effective depth" is roughly $C*/\varepsilon$
    - Takes time O(b$^{C*/\varepsilon}$) (exponential in effective depth)

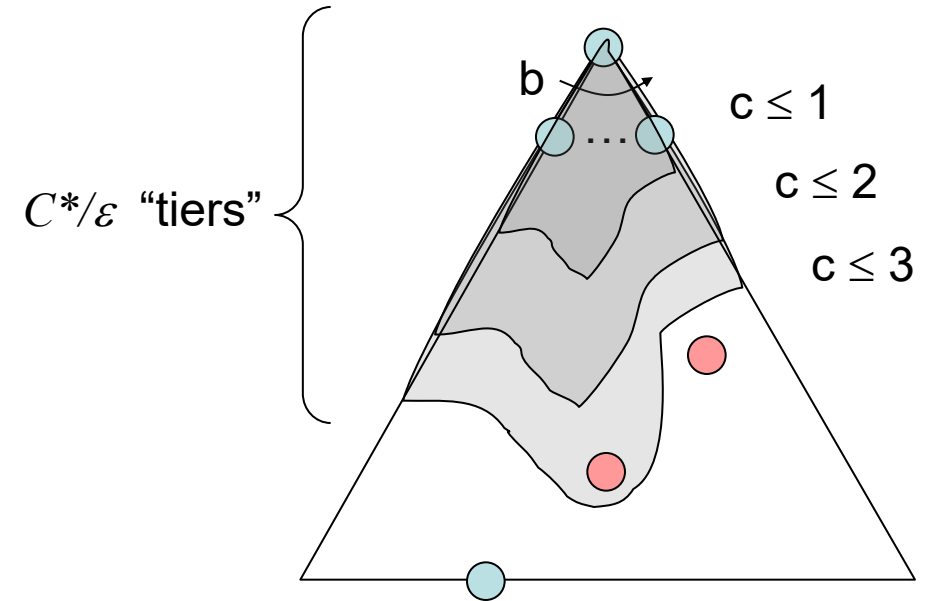- ## How much space does the fringe take?

    - Has roughly the last tier, so O(b$^{C*/\varepsilon}$)

- ## Is it complete?

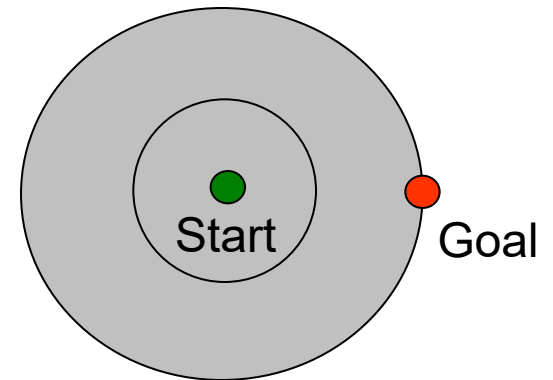   - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
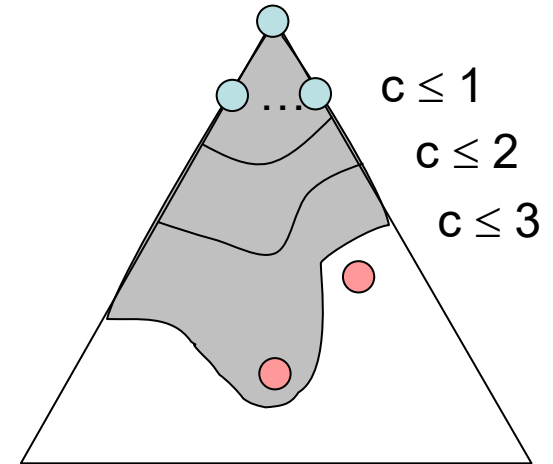
- ## Is it optimal?

    - Yes! (Proof via A*)

b

$c \leq 1$

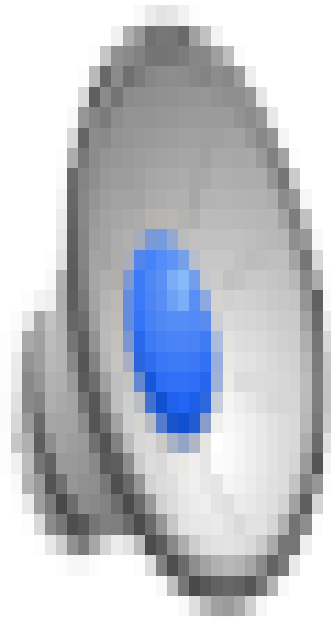$c \leq 2$

$c \leq 3$

$C*/\varepsilon$ "tiers"

Uniform Cost Issues

- **The bad:**
  - Explores options in every "direction"
  - No information about goal location

$c \leq 1$

$c \leq 2$

$c \leq 3$

...

Start

Goal

Video of Demo Empty UCS
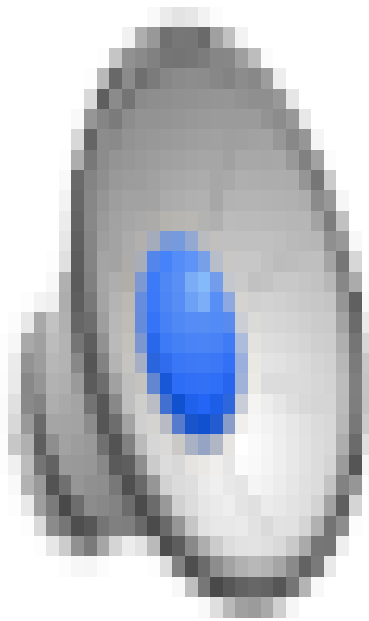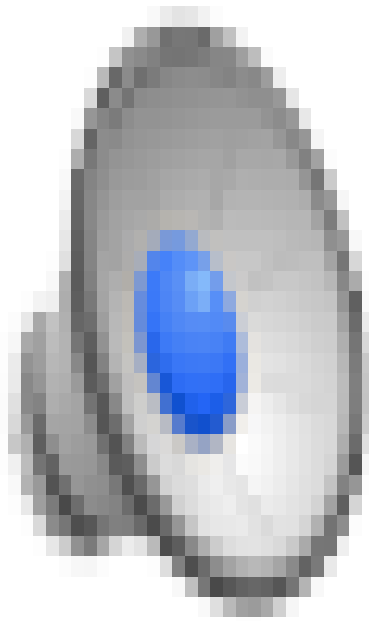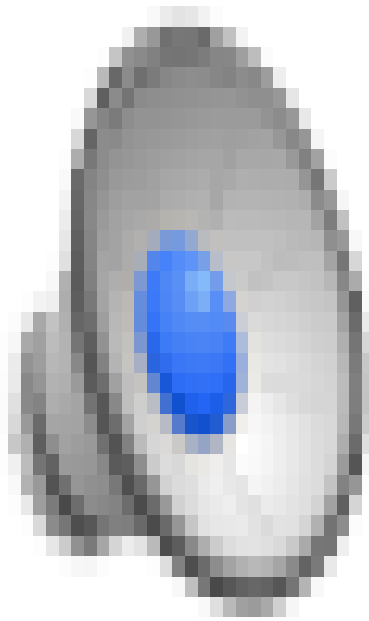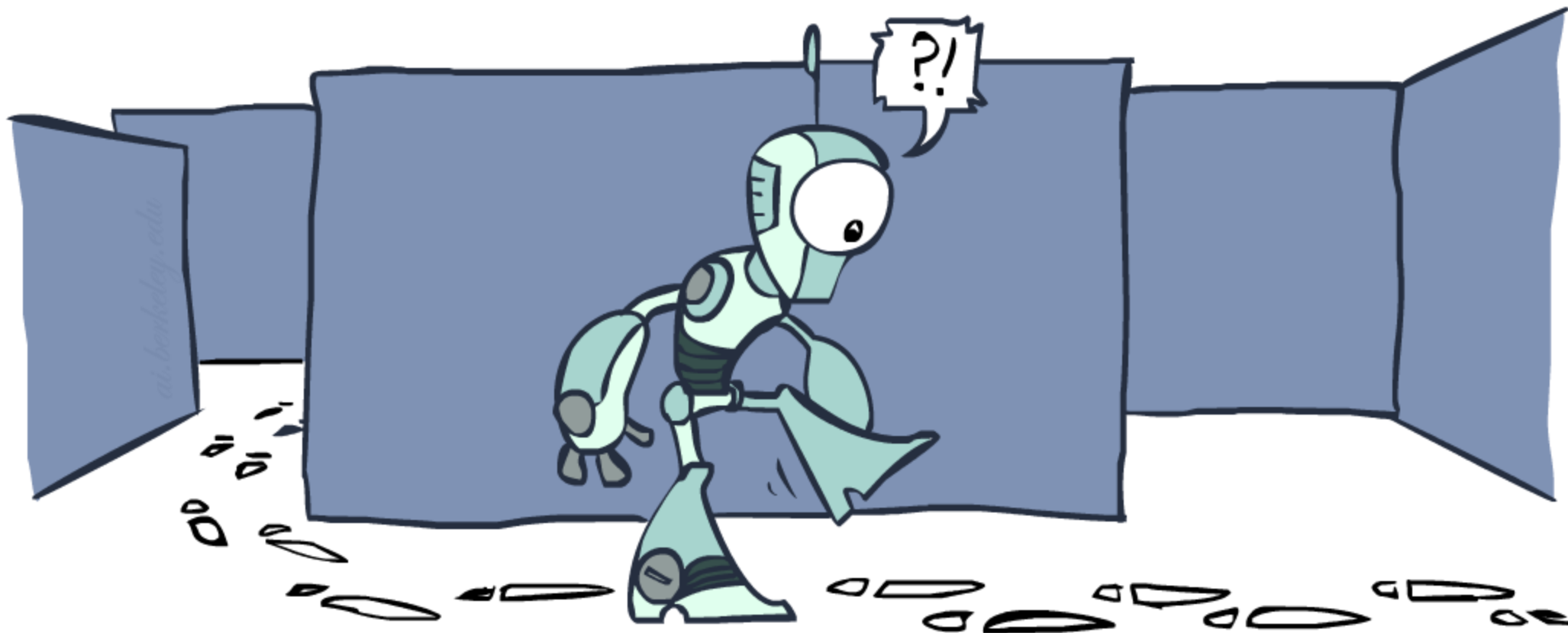
Graph Search

Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.
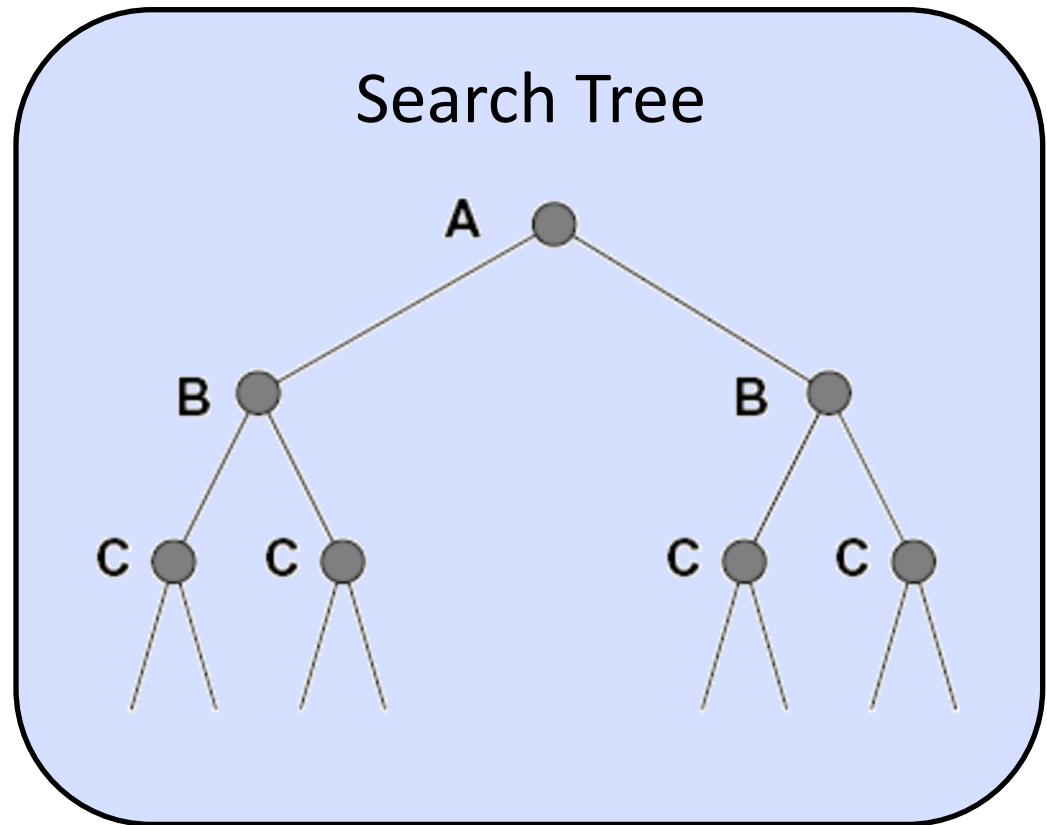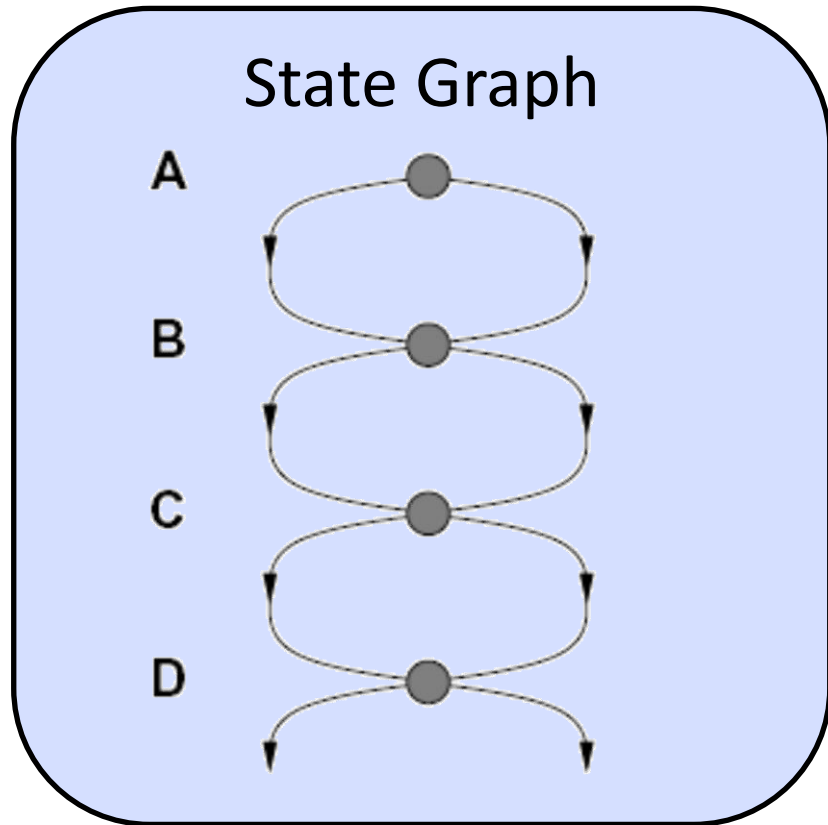
Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)
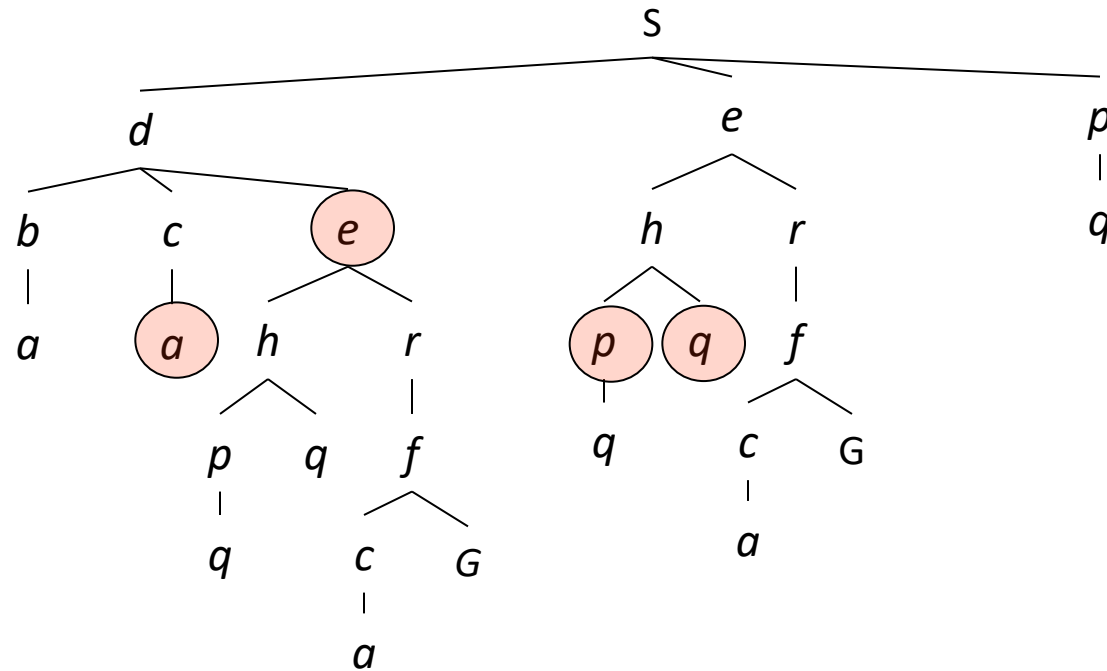
Graph Search

- Idea: never expand a state twice

- How to implement:

  - Tree search + set of expanded states ("closed set")

  - Expand the search tree node-by-node, but…

  - Before expanding a node, check to make sure its state has never been expanded before

  - If not new, skip it, if new add to closed set

- Important: store the closed set as a set, not a list

- Can graph search wreck completeness? Why/why not?

- How about optimality?

Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                if STATE[child-node] is not in closed then   fringe ← INSERT(child-node, fringe)

        end
    end
```

**Use this version for the homeworks, projects, and exams!**

Some Hints for P1

- Implement your closed list (explored set) as a set!

- Nodes are conceptually paths, but better to represent with a state, cost, last action, and reference to the parent node.

- Pseudo code from Russell and Norvig book. Good example of how a child node is created from a parent node.

function CHILD-NODE(*problem*, *parent*, *action*) **returns** a node
  return a node with
    STATE = *problem*.RESULT(*parent*.STATE, *action*),
    PARENT = *parent*, ACTION = *action*,
    PATH-COST = *parent*.PATH-COST + *problem*.STEP-COST(*parent*.STATE, *action*)

The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
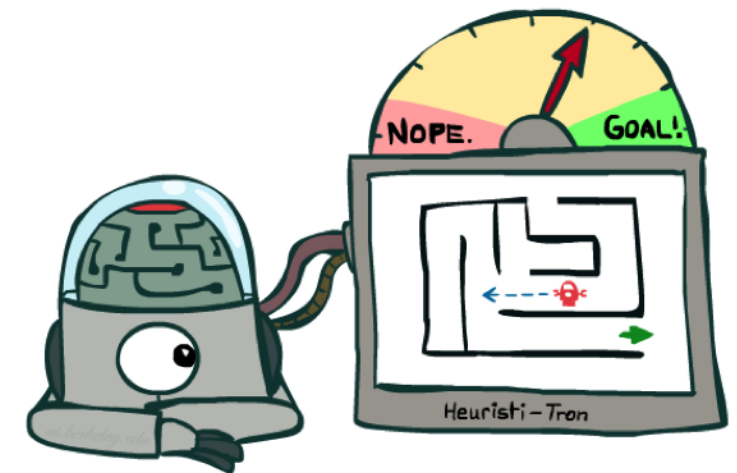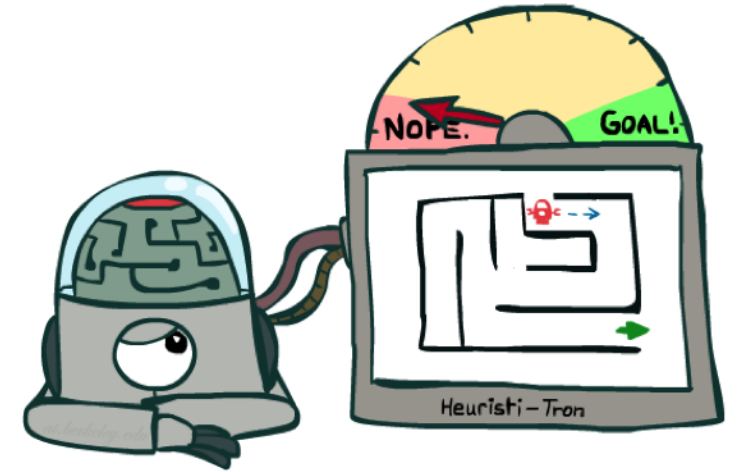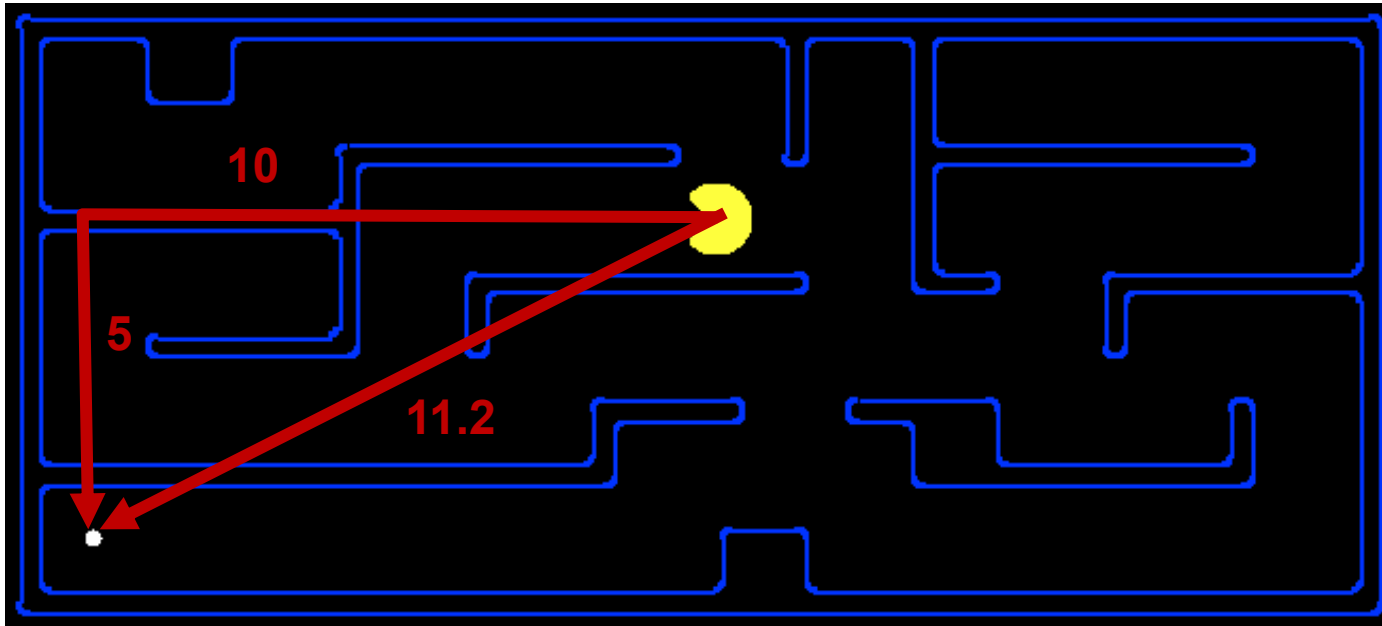 - Can even code one implementation that takes a variable queuing object
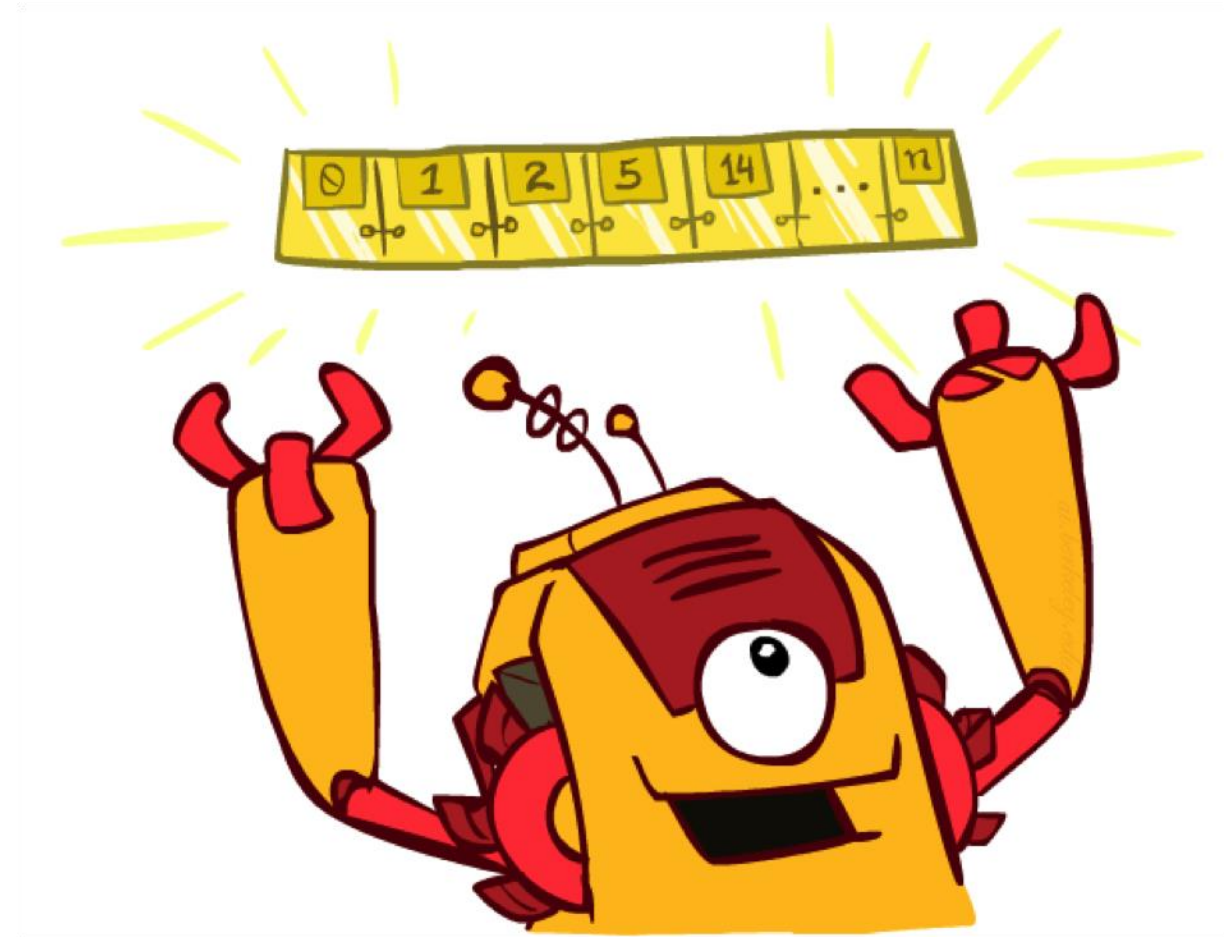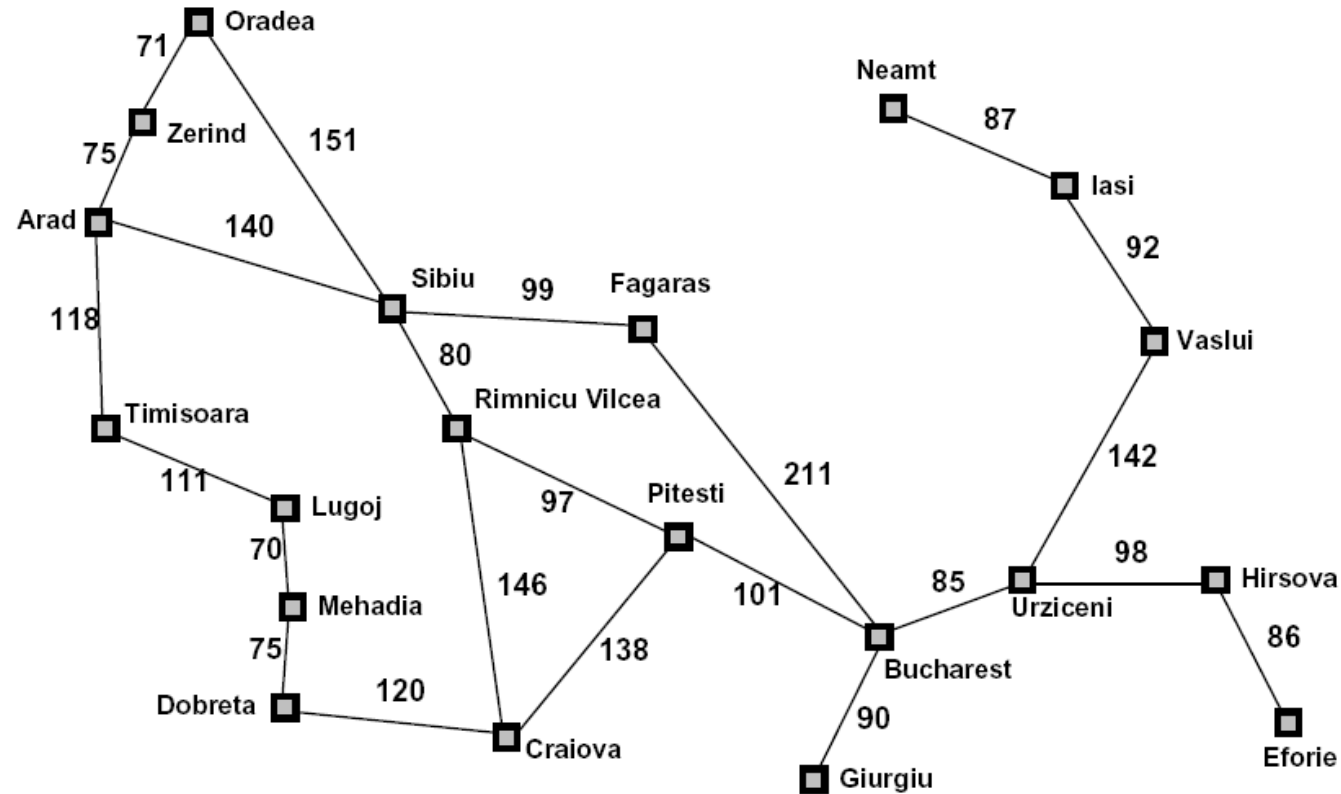
Informed Search
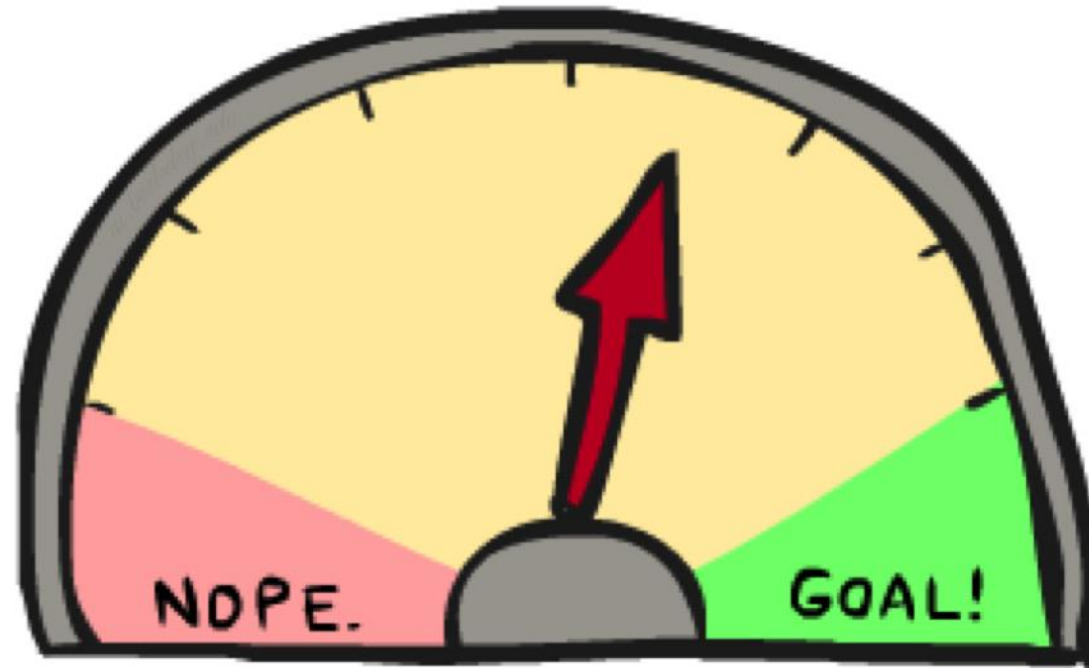
Search Heuristics

- ## A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing

Example: Heuristic Function
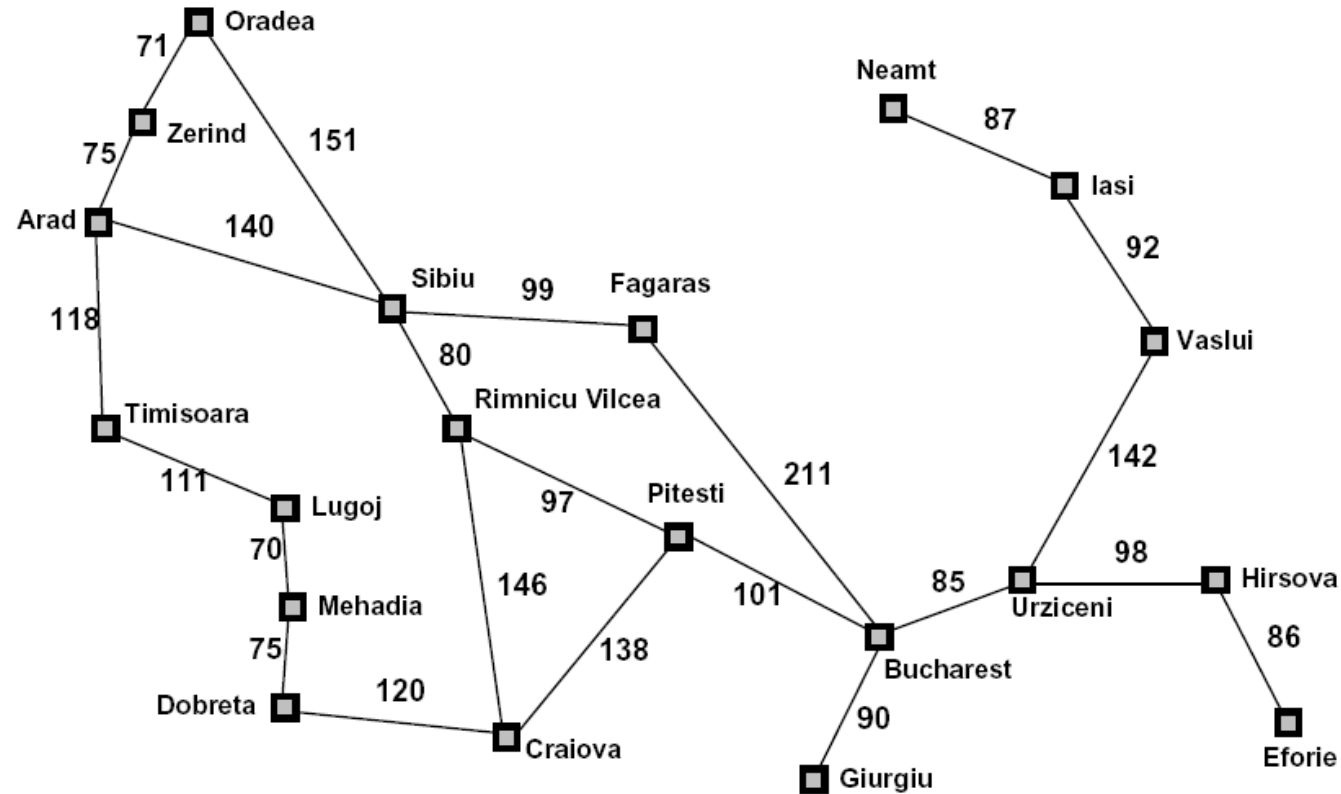

Straight-line distance to Bucharest

City	Distance
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

Greedy Search

Example: Heuristic Function

Straight−line distance to Bucharest

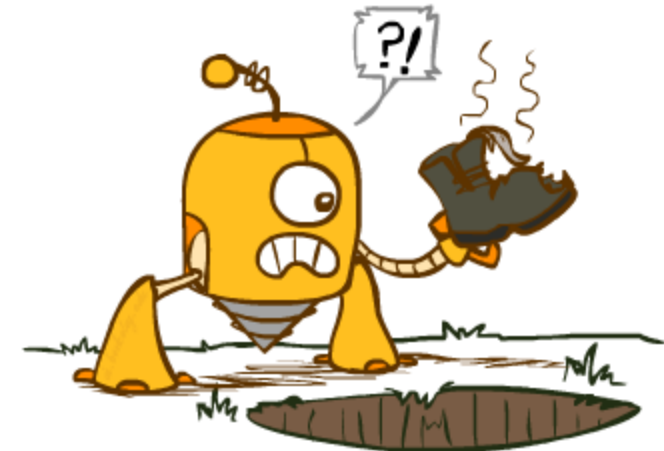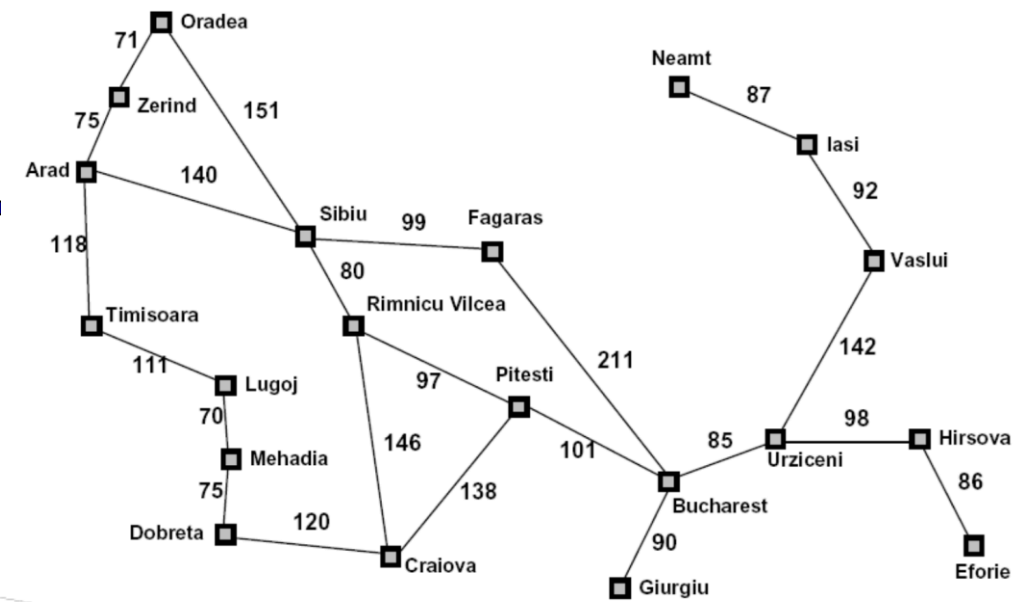|	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

Greedy Search
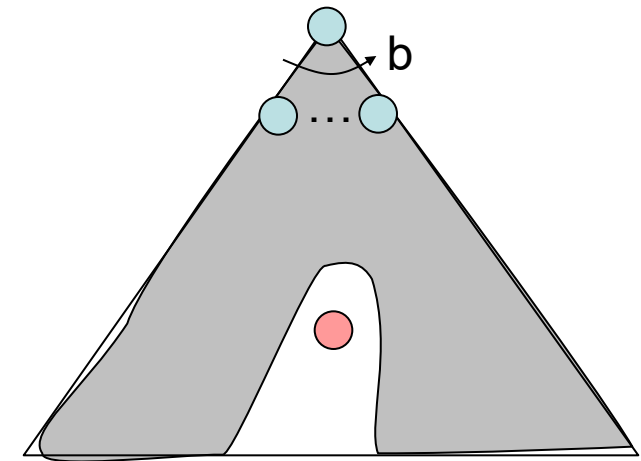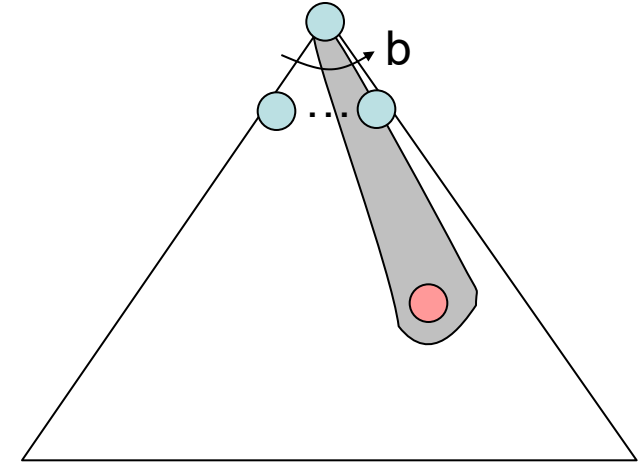

- Expand the node that seems closest…
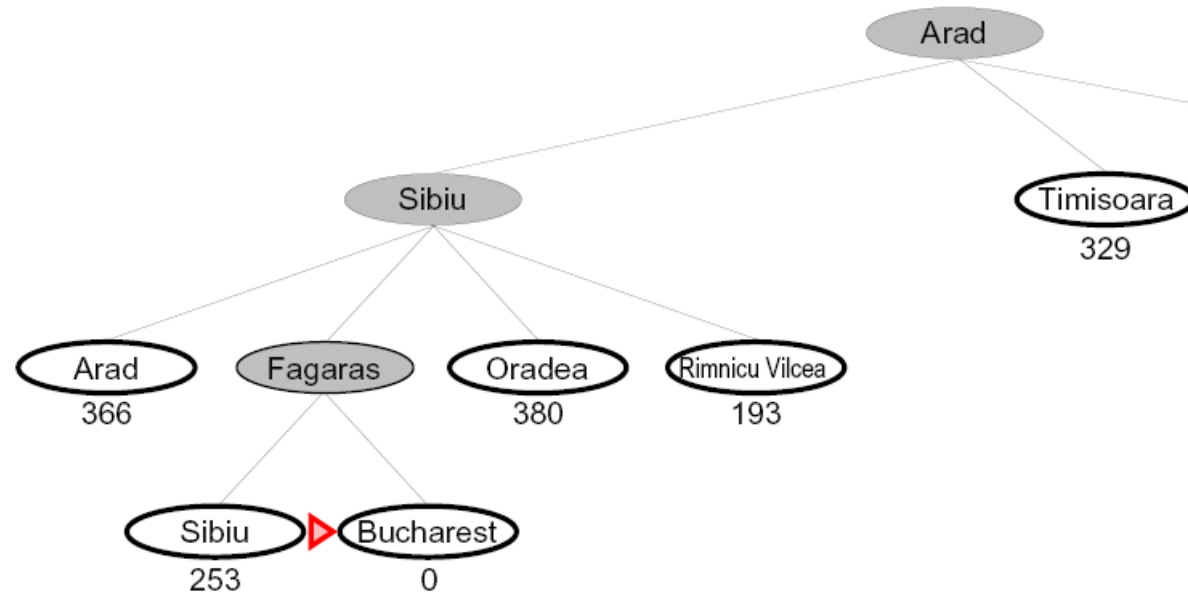
- What can go wrong?
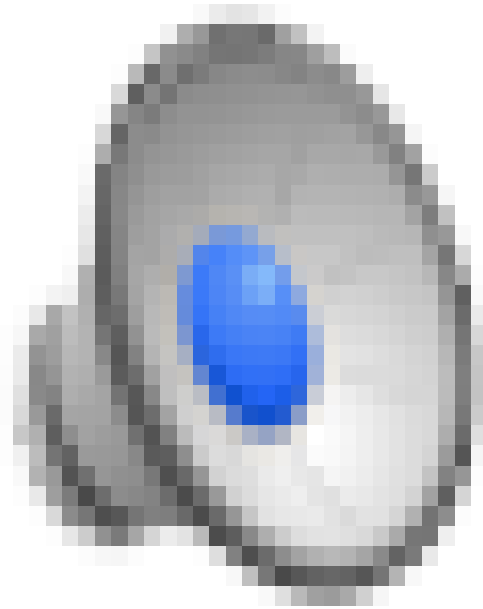
Greedy Search

- **Strategy: expand a node that you think is closest to a goal state**
  - Heuristic: estimate of distance to nearest goal for each state

- **A common case:**
  - Best-first takes you straight to the (wrong) goal

- **Worst-case: like a badly-guided DFS**
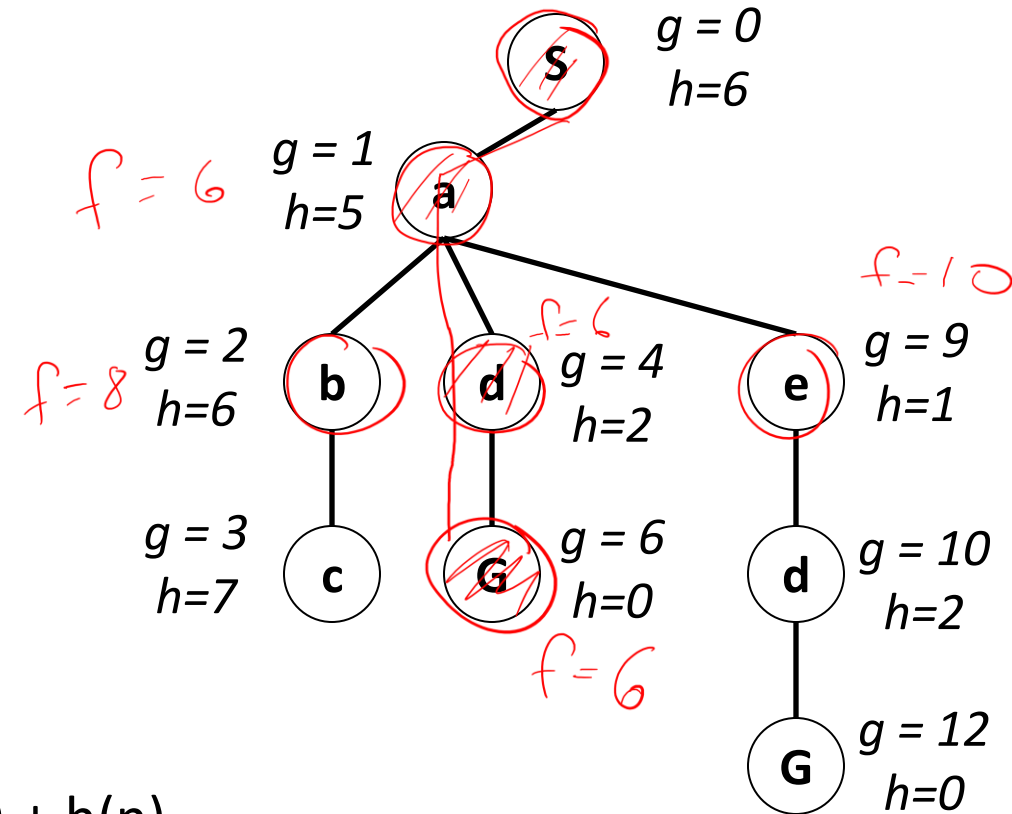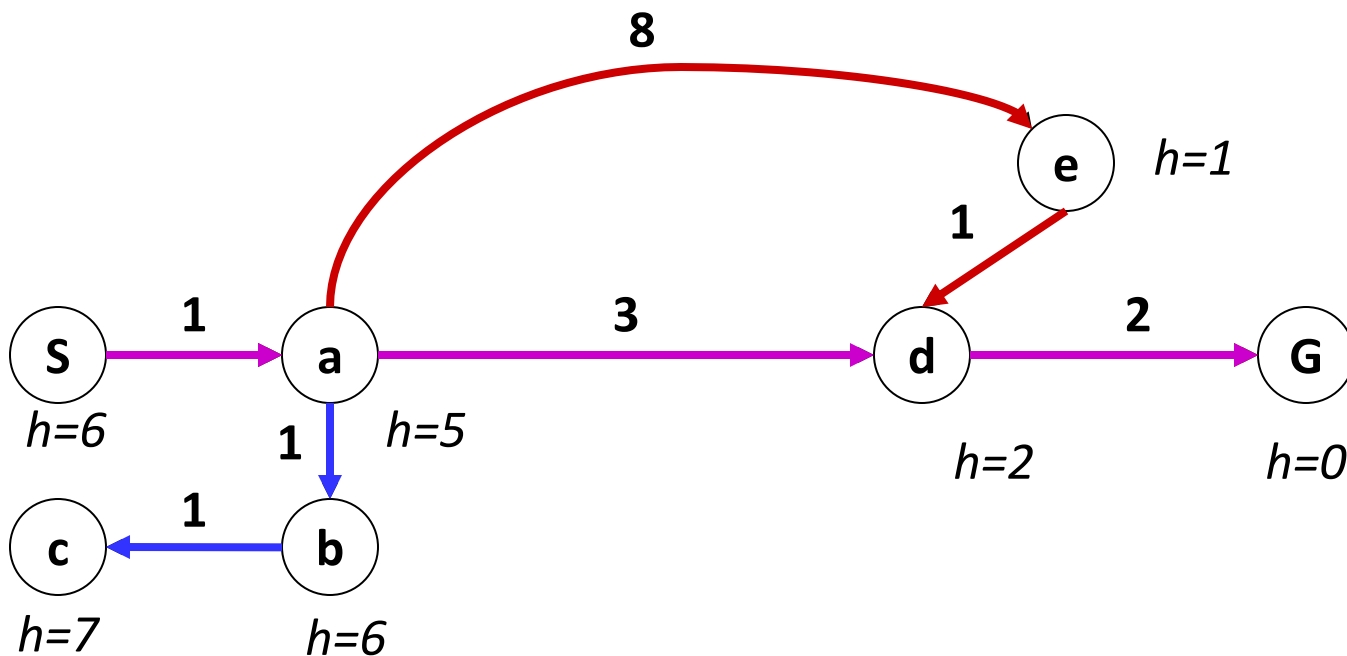
Video of Demo Contours Greedy (Pacman Small Maze)

A* Search

A* Search

Combining UCS and Greedy
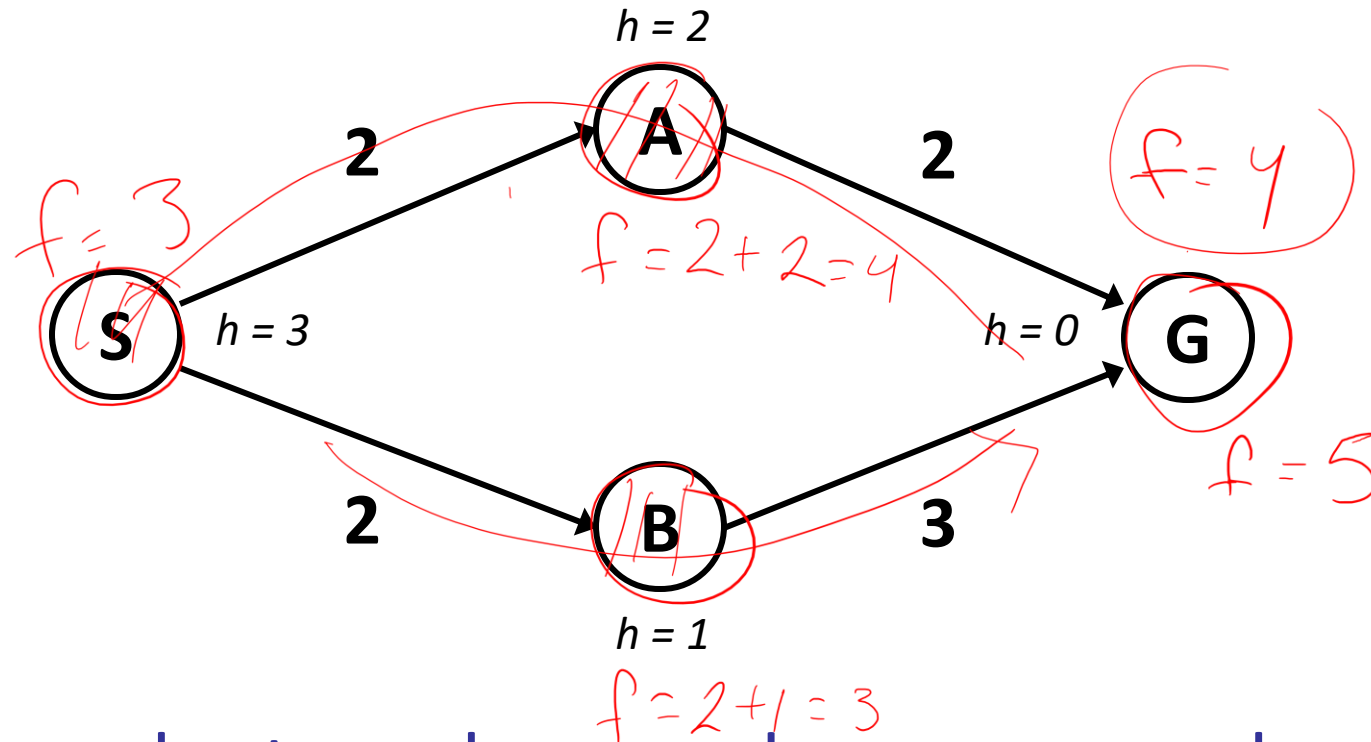
- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)

- A* Search orders by the sum: f(n) = g(n) + h(n)

Example: Teg Grenager

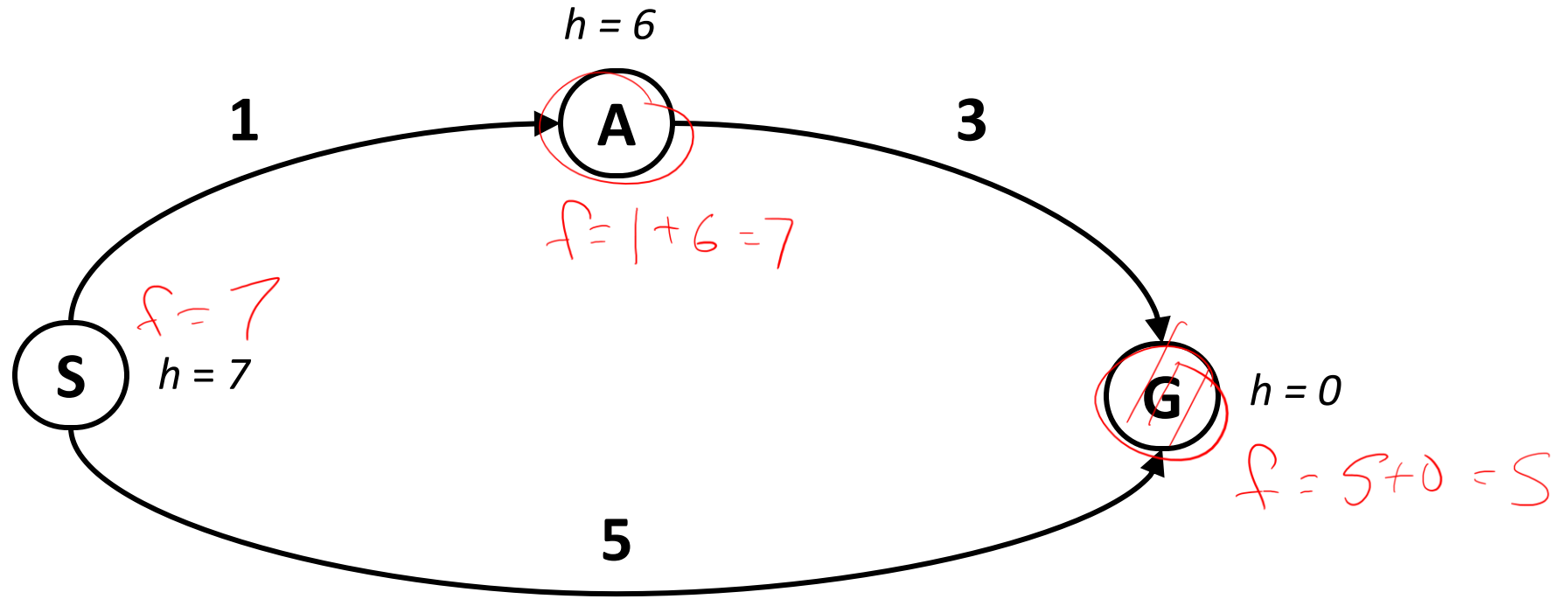# When should A* terminate?

- Should we stop when we enqueue a goal?

- No: only stop when we dequeue a goal

Is A* Optimal?

h = 6

S   h = 7   1   A   3   G   h = 0

f = 7

f = 1 + 6 = 7

f = 5 + 0 = 5

5

- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!
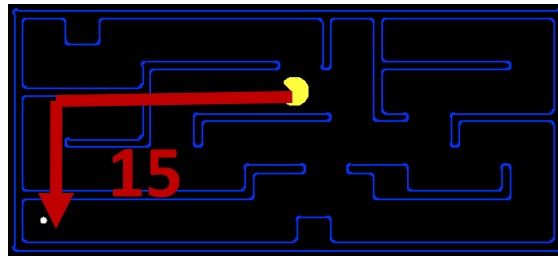
Admissible Heuristics
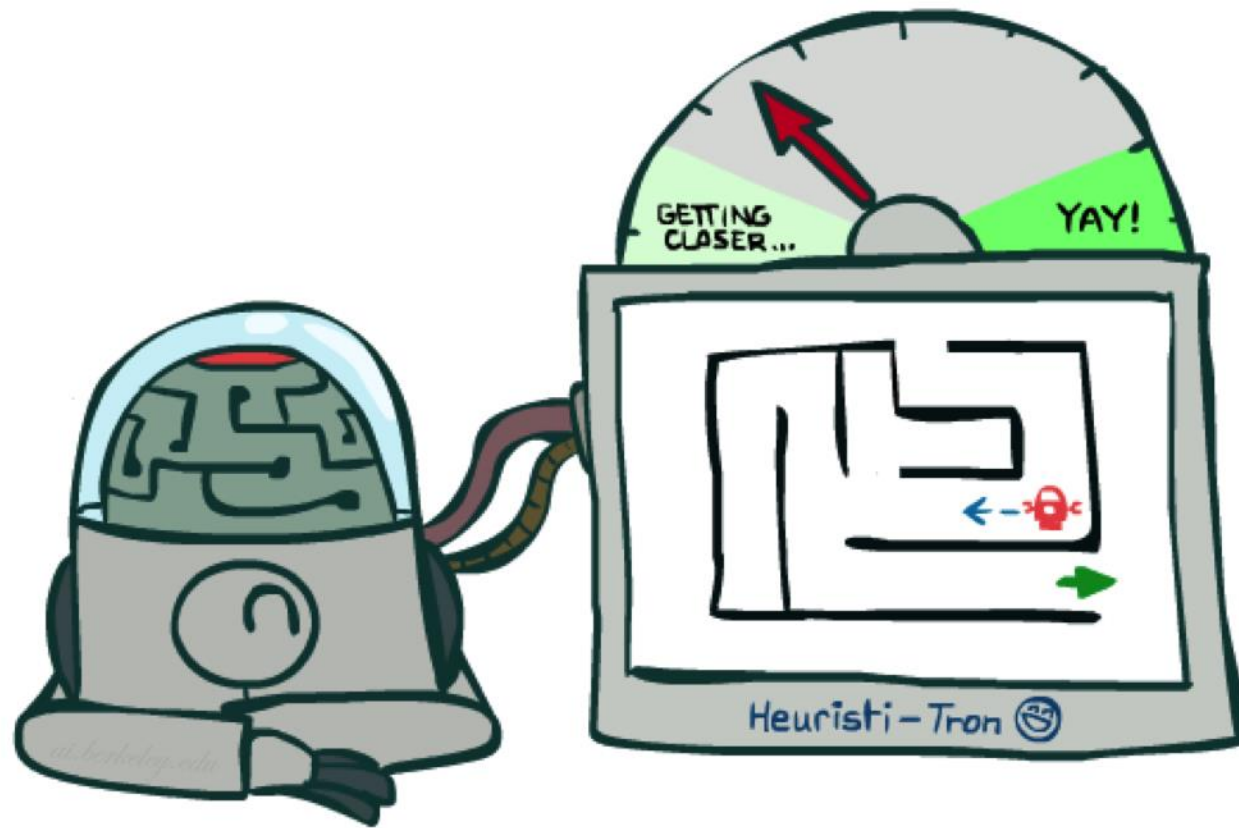
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:
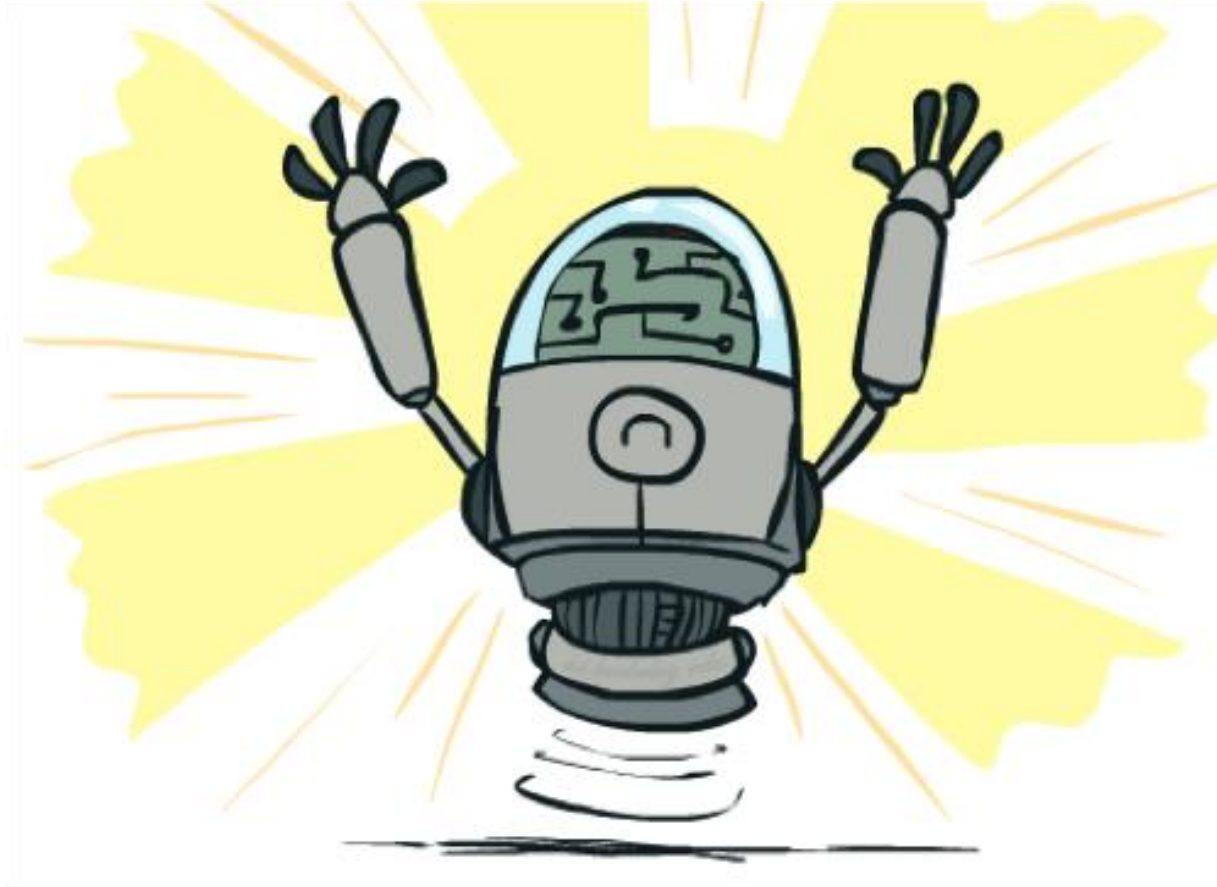
$$0 \leq h(n) \leq h^*(n)$$

  where $h^*(n)$ is the true cost to a nearest goal

- Examples:

- Coming up with admissible heuristics is most of what's involved in using A* in practice.

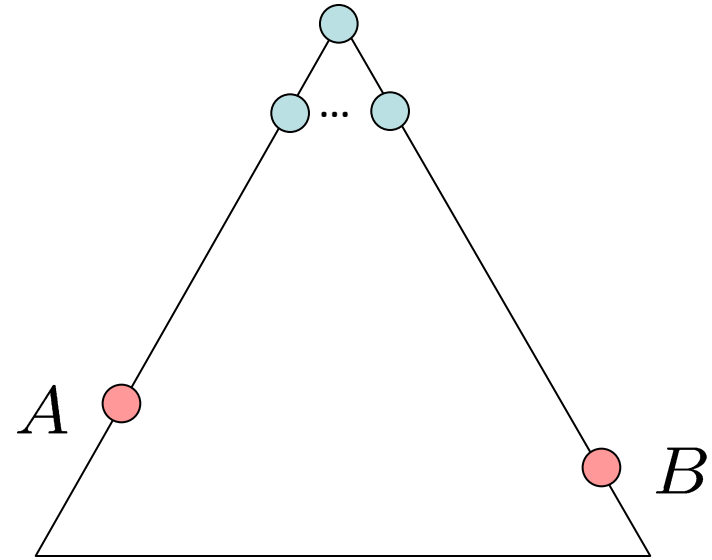# Optimality of A* Tree Search

Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
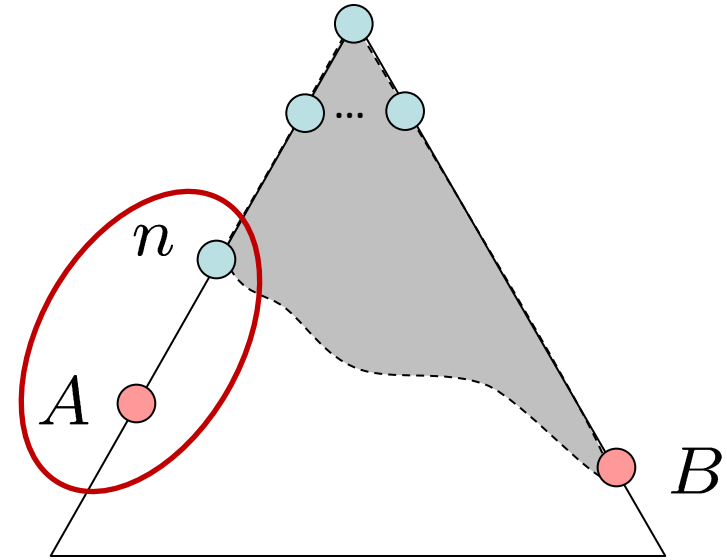- h is admissible

Claim:

- A will exit the fringe before B

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe

- Some ancestor *n, that is* along the optimal path to A, is on the fringe, too (maybe A!)

- Claim: *n* will be expanded before B

  1. f(n) is less or equal to f(A)

$$f(n) = g(n) + h(n) \quad \text{Definition of f-cost}$$

$$f(n) \leq g(n) + h^*(n) \quad \text{Admissibility of h}$$

$$= g(A) \quad \text{h = 0 at a goal}$$

$$= f(A)$$

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe

- Some ancestor *n, that is* along the optimal path to A, is on the fringe, too (maybe A!)

- Claim: *n* will be expanded before B

  1. f(n) is less or equal to f(A)
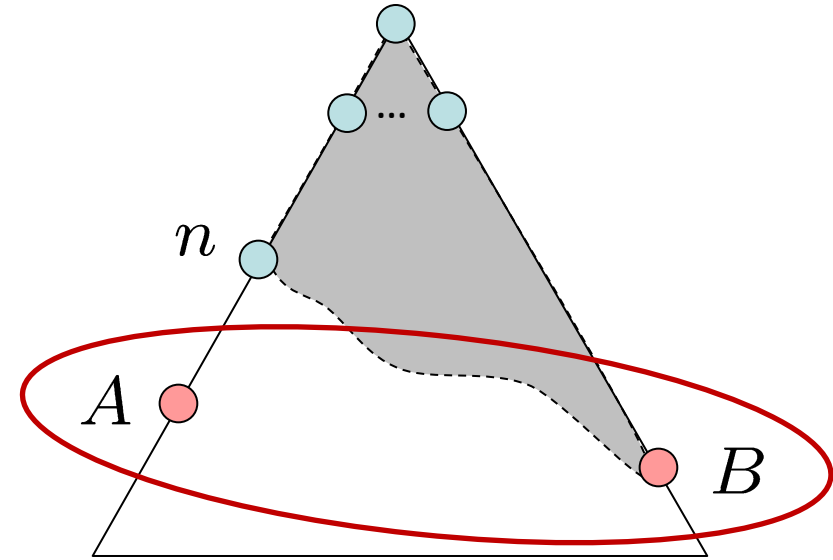 2. f(A) is less than f(B)

$$g(A) < g(B)$$
$$f(A) < f(B)$$

B is suboptimal

h = 0 at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n, that is* along the optimal path to A, is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors along optimal path to A expand before B
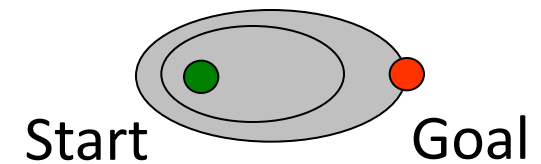- A expands before B
- A* search is optimal
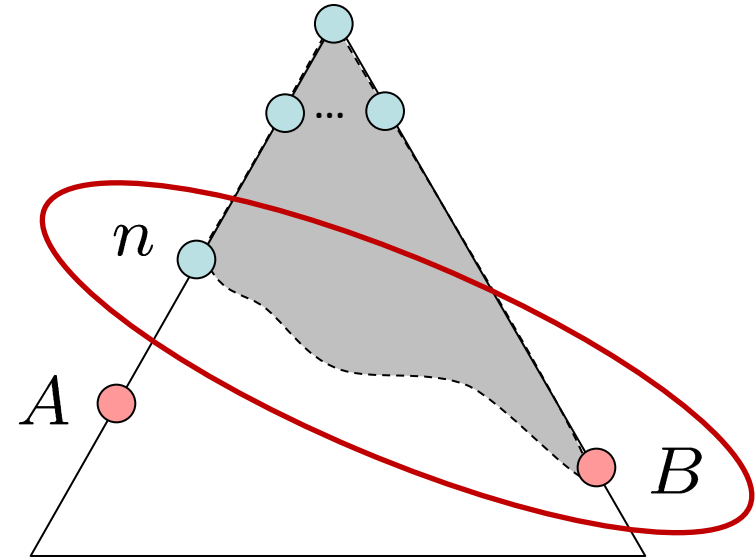
$$f(n) \leq f(A) < f(B)$$

Properties of A*

UCS vs A* Contours
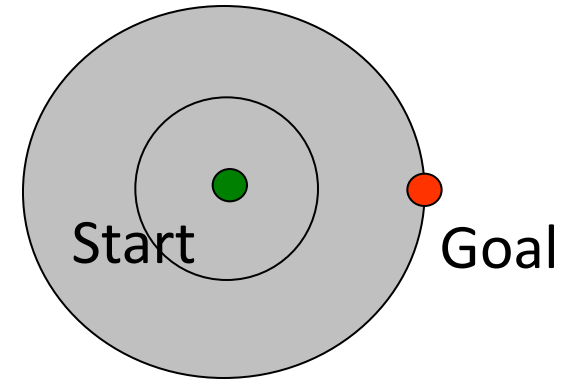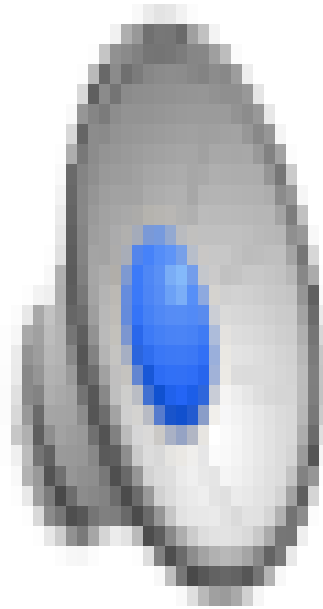
- Uniform-cost expands equally in all "directions"

- A* expands mainly toward the goal, but does hedge its bets to ensure optimality

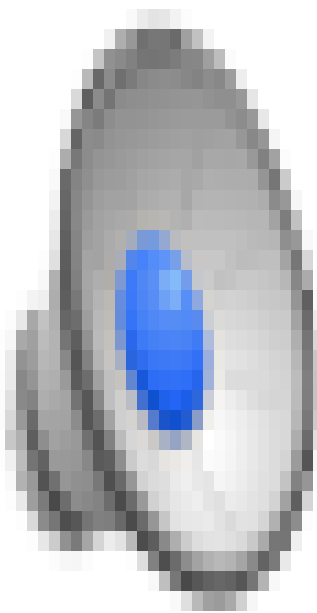# Video of Demo Contours (Empty) -- UCS
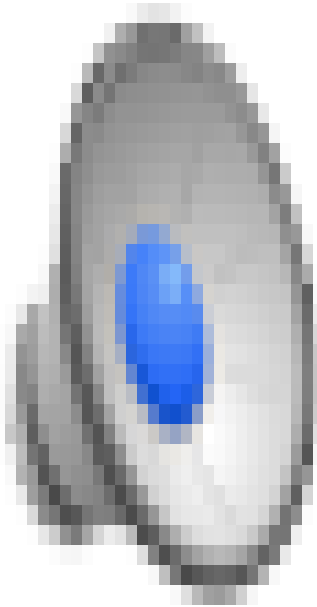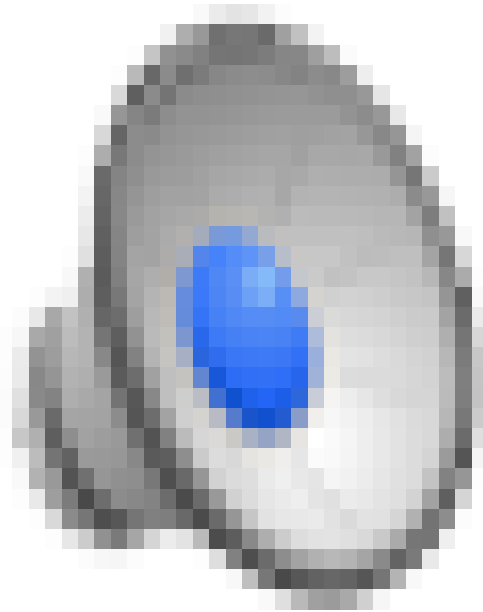
Video of Demo Contours (Empty) -- Greedy

Video of Demo Contours (Empty) – A*

Video of Demo Contours (Pacman Small Maze) – A*

Comparison

Greedy　　　　　　Uniform Cost　　　　　　A*

A* Applications

A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- ...

Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available
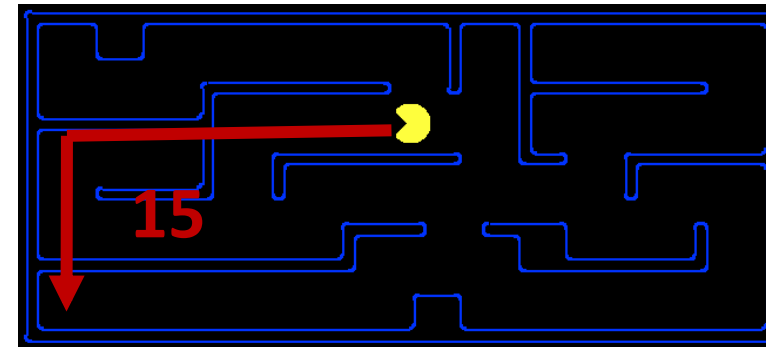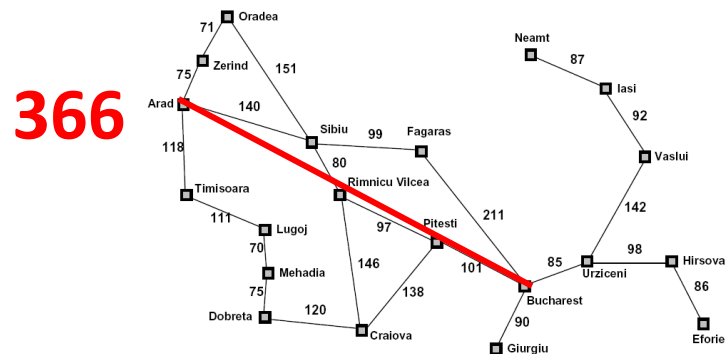
- Inadmissible heuristics are often useful too

Example: 8 Puzzle
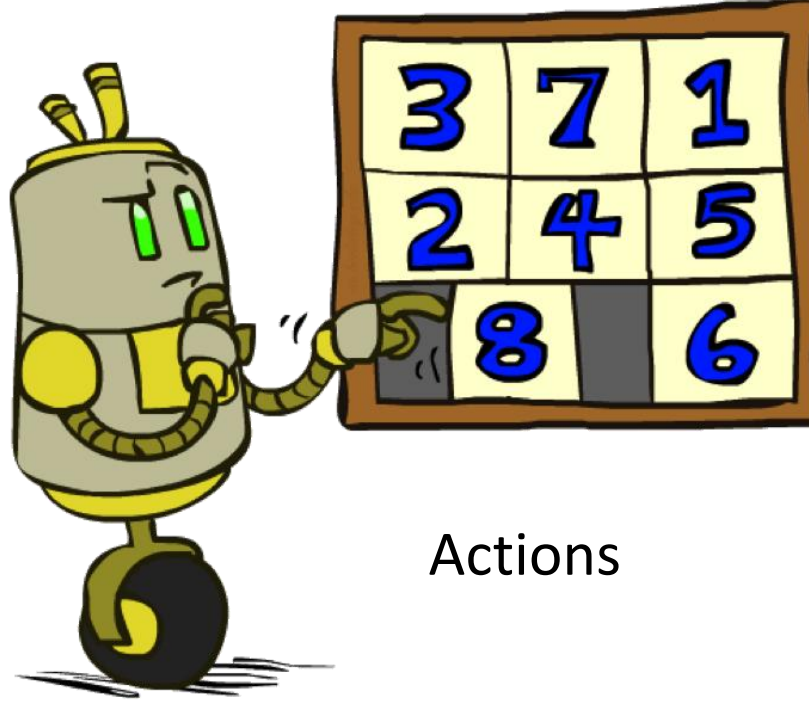
Start State

Actions

Goal State

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

8 Puzzle I

- Heuristic: Number of tiles misplaced

- Why is it admissible?

- h(start) = 8

- This is a *relaxed-problem* heuristic
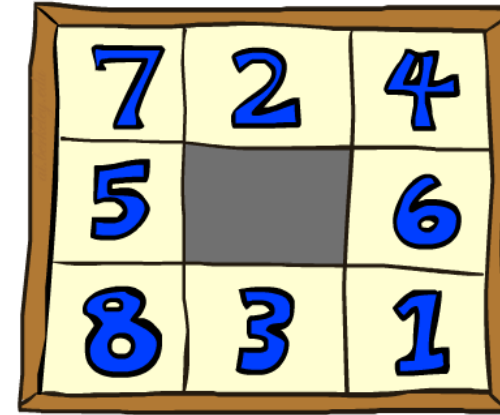
Start State   Goal State
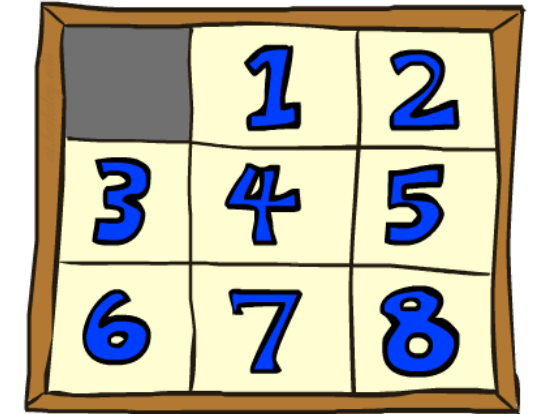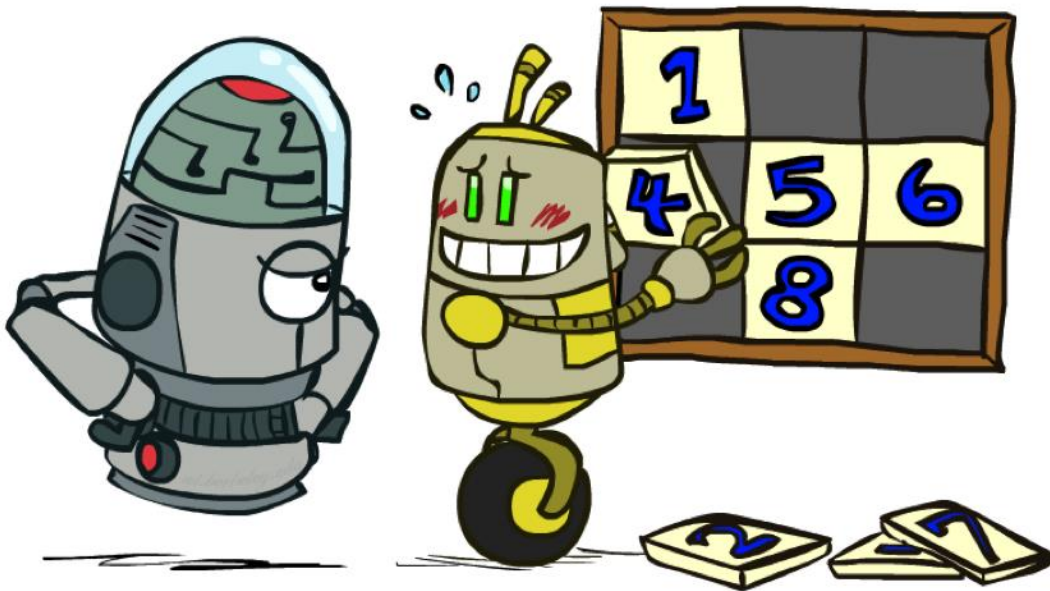
	Average nodes expanded when the optimal path has...		
	...4 steps	...8 steps	...12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

- Total *Manhattan* distance

- Why is it admissible?

- h(start) = 3 + 1 + 2 + ... = 18
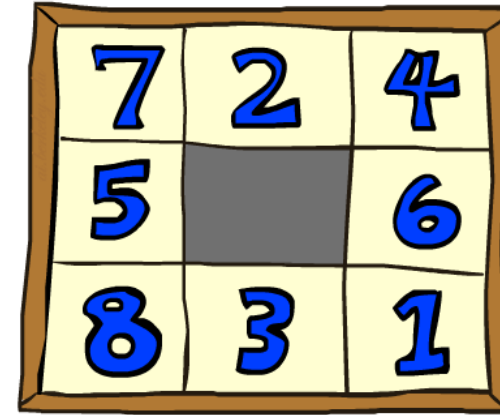
Start State   Goal State
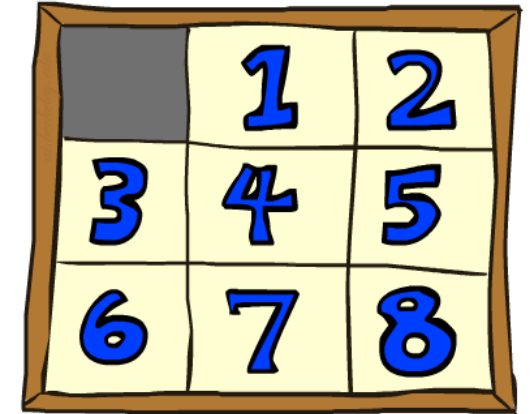
	Average nodes expanded when the optimal path has...		
	...4 steps	...8 steps	...12 steps
TILES	13	39	227
MANHATTAN	12	25	73

Heuristics

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?
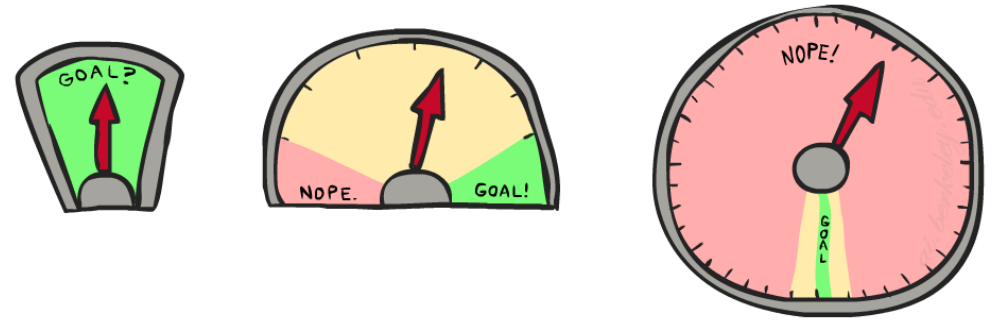
- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                if STATE[child-node] is not in closed then   fringe ← INSERT(child-node, fringe)

        end
    end
```
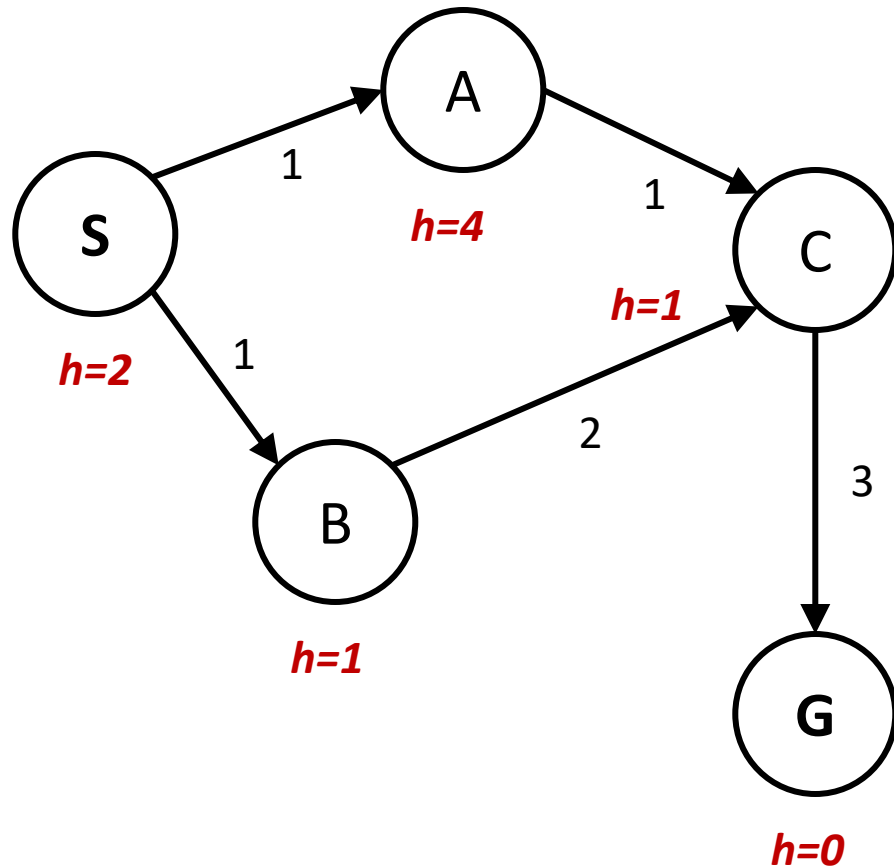
Use this version for the homeworks, projects, and exams!

A* Graph Search Gone Wrong?
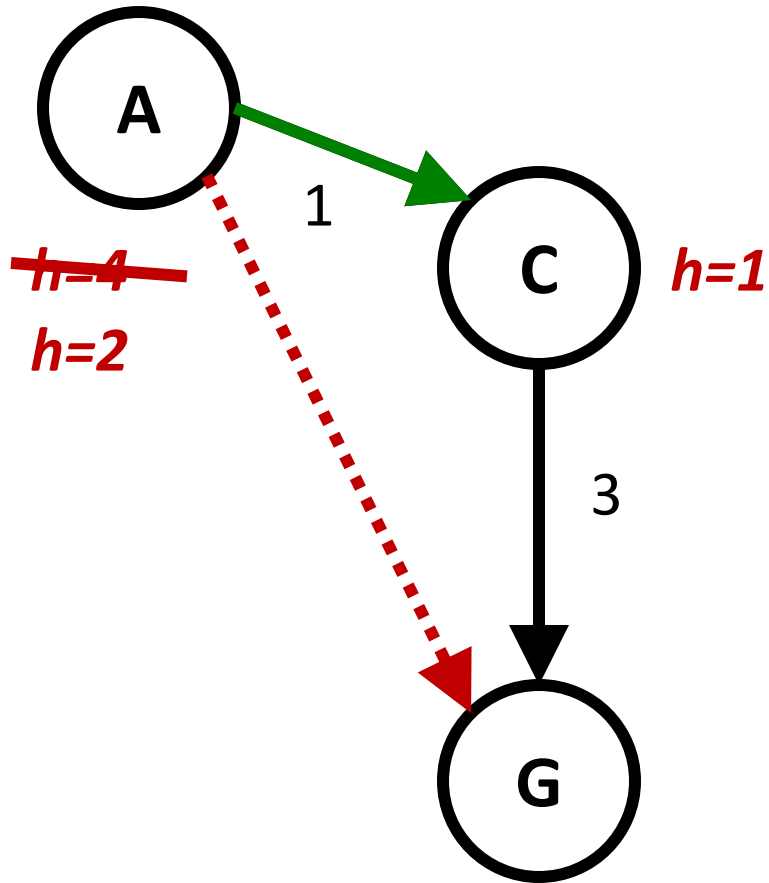
State space graph

Search tree

S (0+2)

A (1+4)   B (1+1)

C (2+1)   C (3+1)

G (5+0)   G (6+0)
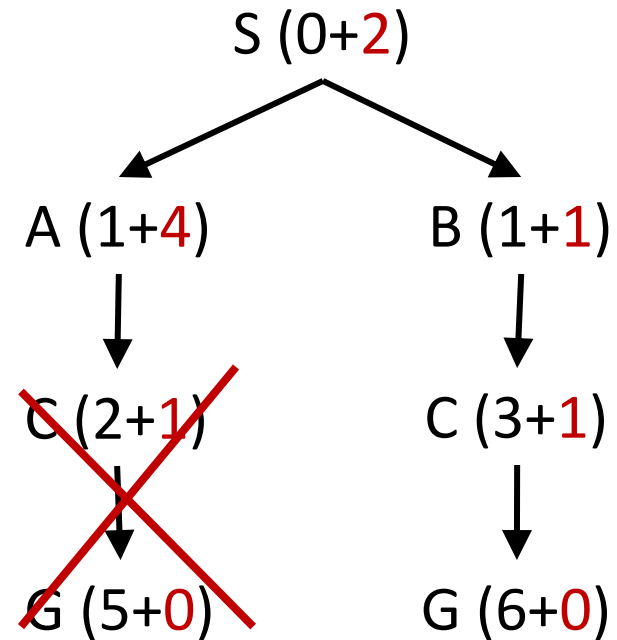
C is already in closed set
so not expanded again

Consistency of Heuristics

- Main idea: estimated heuristic costs ≤ actual costs

    - Admissibility: heuristic cost ≤ actual cost to goal

        h(A) ≤ actual cost from A to G

    - Consistency: heuristic "arc" cost ≤ actual cost for each arc

        h(A) − h(C) ≤ cost(A to C)

- Consequences of consistency:

    - The f value along a path never decreases

        h(A) ≤ cost(A to C) + h(C)

    - A* graph search is optimal

Semi-Lattice of Heuristics
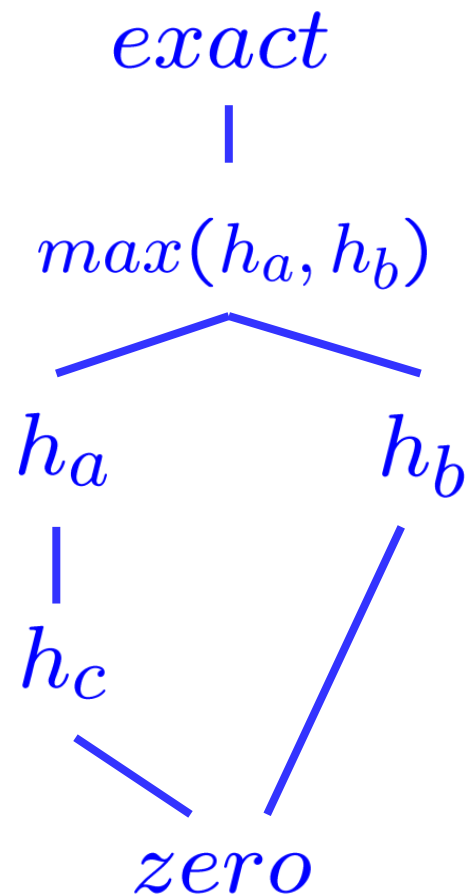
Trivial Heuristics, Dominance

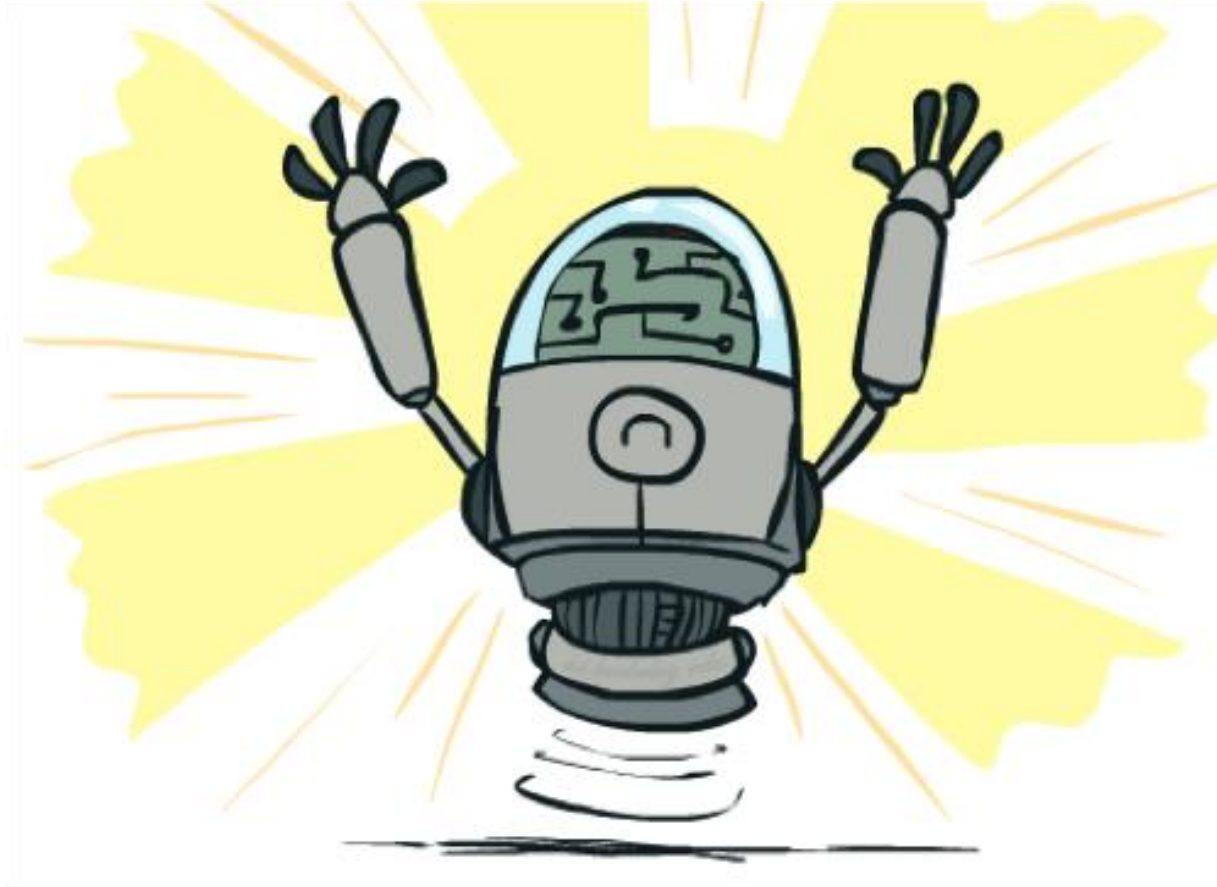- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
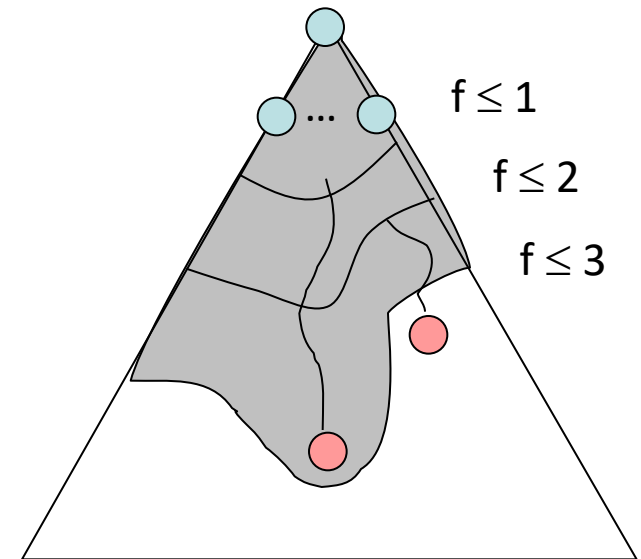 - Top of lattice is the exact heuristic

$exact$

$|$

$max(h_a, h_b)$

$h_a \qquad h_b$

h_c

$zero$

Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:

  - Fact 1: A* expands nodes in increasing total f value (f-contours)

  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally

  - Result: A* graph search is optimal

$f \leq 1$

$f \leq 2$

$f \leq 3$

Optimality

- **Tree search:**
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)

- **Graph search:**
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)

- **Consistency implies admissibility**

- **In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems**
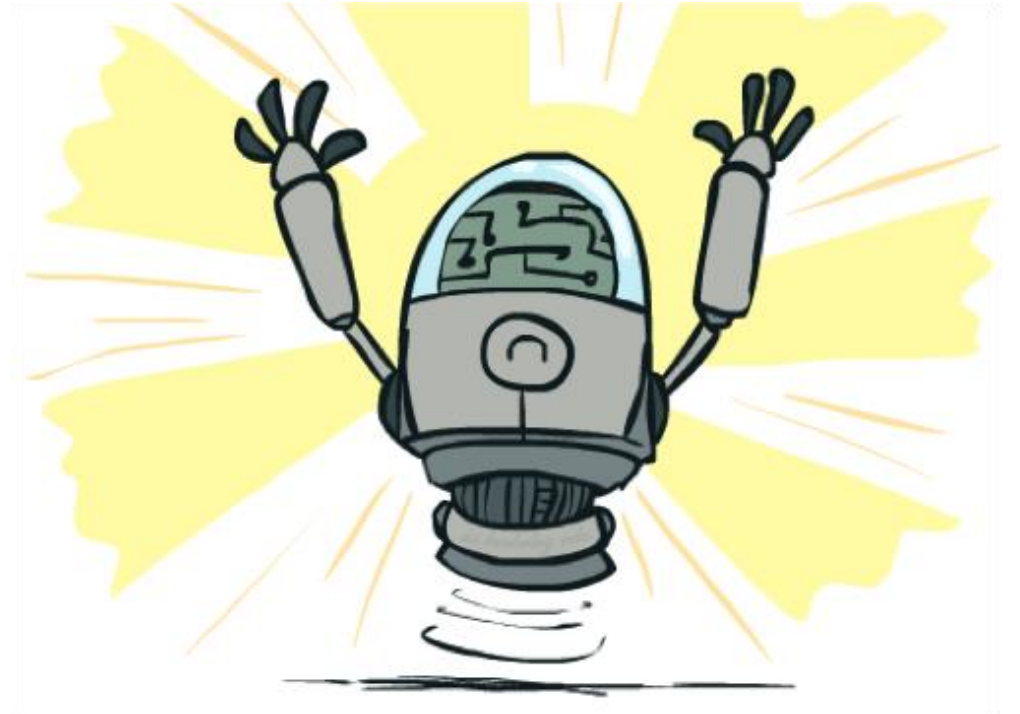
A*: Summary

A*: Summary

- A* uses both backward costs and (estimates of) forward costs

- A* is optimal with admissible / consistent heuristics

- Heuristic design is key: often use relaxed problems