Percent Variance Explained Quiz CSE 255, Spring 2019

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Name: _____

PID: _____

This is the first quiz of CSE255/DSE230
On your desk you should have only the exam paper and writing tools.
No hats or hoods allowed (unless religious items).
There is one question in this quiz.
Write your answer in the lines provided.
You have 5 minutes to complete the exam.
Start by writing your name and PID on this page.
Good Luck!

Suppose $\vec{x}$ is drawn from a distribution D over R^d.
Let C be the covariance martix for the distribution D and suppose the spectral decomposition of C is $C = A^T DA$ where A is an orthonormal matrix and D is the diagonal matrix:

$$D = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ 0 & \dots & \sigma_3^2 \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & \sigma_d^2 \end{bmatrix}$$

where $\sigma_1^2 \geq \sigma_2^2 \geq \cdots \geq \sigma_d^2$ are the variances of the pricipal components. Equivalently σ_i^2 is the eigen-value associated with the ith eigen-vector of C.
Write an expression for the fraction of the total variance that is explained by components 1 to $k \leq d$.