DSC 140A - Homework 03

Due: Wednesday, February 1

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Unless otherwise noted by the problem's instructions, show your work or provide some justification for your answer. Homeworks are due via Gradescope at 11:59 p.m.

Problem 1.

Suppose that $f : \mathbb{R}^d \to \mathbb{R}$ is convex and $g : \mathbb{R} \to \mathbb{R}$ is convex and non-decreasing. That is, if $a > b$, then $g(a) \geq g(b)$.

Show that the composition of these functions, $h(\vec{x}) = g(f(\vec{x}))$, is also convex.

Hint: you'll want to go back to the definition to show this is true. A similar problem was solved in discussion.

Solution:

Problem 2.

Recall that the *hinge loss* is

$$L_{\text{hinge}}(\vec{w}, \vec{x}, y) = \max\{0, 1 - y\,\vec{w} \cdot \text{Aug}(\vec{x})\}$$

The Soft-SVM problem aims to minimize the regularized empirical risk:

$$R(\vec{w}) = \frac{C}{n} \sum_{i=1}^{n} L_{\text{hinge}}(\vec{w}, \vec{x}^{(i)}, y_i) + \|\vec{w}\|^2$$

Show that $R(\vec{w})$ is a convex function of $\vec{w}$.

Hint: you will probably *not* want to use the formal definition of convexity here. Instead, you'll want to show that R is composed of simpler functions which themselves are convex.

Solution: