

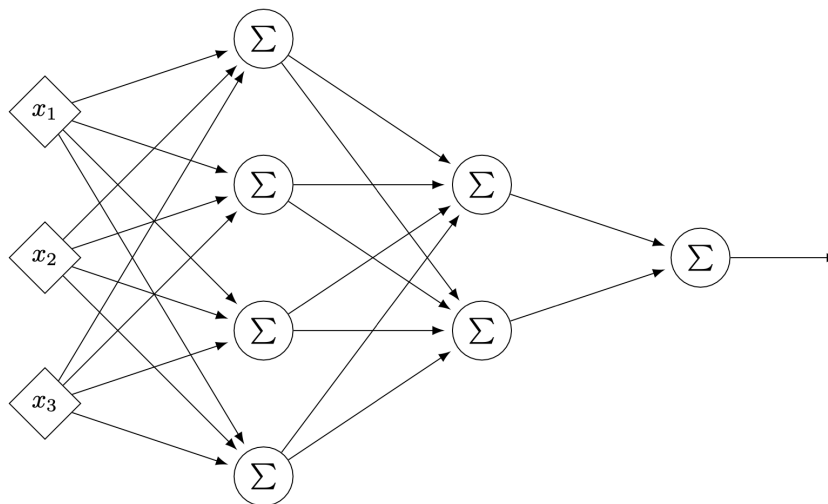
DSC 140A - Homework 05

Due: Wednesday, February 15

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Unless otherwise noted by the problem's instructions, show your work or provide some justification for your answer. Homeworks are due via Gradescope at 11:59 p.m.

Problem 1.

Consider the neural network shown below.



The weights and biases of the network are as follows:

$$W^{(1)} = \begin{pmatrix} -4 & 9 & 4 & 0 \\ -3 & -4 & 8 & 0 \\ 0 & -7 & -3 & -8 \end{pmatrix} \quad W^{(2)} = \begin{pmatrix} -9 & 1 \\ -5 & -9 \\ -10 & 1 \\ 1 & 6 \end{pmatrix} \quad W^{(3)} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

$$\vec{b}^{(1)} = (4 \quad 4 \quad 8 \quad 1)^T \quad \vec{b}^{(2)} = (9 \quad -8)^T \quad \vec{b}^{(3)} = (-6)$$

All neurons use linear activation functions.

You're encouraged to write code to do any and all parts of this problem. If you do, please show your code.

a) What is the output of the network if it is given $\vec{x} = (1, 2, 3)^T$ as input?

b) Recall from lecture that a neural network using linear activation functions is a linear predictor.

More formally, this means that the neural network above should be equivalent to a function $\tilde{H}(\vec{x}) = \vec{w} \cdot \text{Aug}(\vec{x})$, for some \vec{w} .

Find this \vec{w} . That is, find a parameter vector \vec{w} so that the output of the network above is equal to $\vec{w} \cdot \text{Aug}(\vec{x})$ for all $\vec{x} \in \mathbb{R}^2$.

Hint: if $H(\vec{x})$ is the neural network above, write $H(\vec{x})$ using matrix multiplications (you might have already done this in the last part), then simplify as much as possible to get into the form $\vec{a} \cdot \vec{x} + c$. You can check your work by making sure that $\vec{w} \cdot (1, 1, 2, 3)^T$ is the same number you got in the last part.