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## DSC 40B - Discussion 03

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### Problem 1.

Recall that the regularized least squares risk is

$$\tilde{R}(\vec{w}) = \frac{1}{n} \sum_{i=1}^n (\vec{w} \cdot \vec{\phi}(\vec{x}^{(i)}) - y_i)^2 + \lambda \|\vec{w}\|^2$$

Show that

$$\tilde{R}(\vec{w}) = \frac{1}{n} (\vec{w}^T \Phi^T \Phi \vec{w} - 2 \vec{w}^T \Phi^T \vec{y} + \vec{y}^T \vec{y}) + \lambda \vec{w}^T \vec{w},$$

where  $\Phi$  is the matrix whose  $i$ th row is  $\vec{\phi}(\vec{x}^{(i)})$ , and where  $\vec{y} = (y_1, \dots, y_n)^T$ .

### Problem 2.

In lecture, we defined a kernel function to be a function  $k$  which computes the dot product of vectors after they are mapped to some high-dimensional space. The useful thing about kernel functions is that they allow us to compute these dot products without actually mapping vectors them to the high-dimensional space, which can be costly. In this problem, we will consider the the 2nd-order *polynomial kernel*, defined to be

$$k(\vec{x}, \vec{x}') = (1 + \vec{x} \cdot \vec{x}')^2.$$

Let  $\vec{\phi}(\vec{x}) : \mathbb{R}^3 \rightarrow \mathbb{R}^{10}$  be the mapping:

$$\vec{\phi}(\vec{x}) = (1, x_1^2, x_2^2, x_3^2, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_3, \sqrt{2}x_1x_2, \sqrt{2}x_1x_3, \sqrt{2}x_2x_3)^T,$$

where  $x_1, x_2, x_3$  are the components of the input vector,  $\vec{x}$ . That is,  $\vec{\phi}$  is a feature map which maps a vector into a higher-dimensional space.

Show that  $k(\vec{x}, \vec{y}) = \vec{\phi}(\vec{x}) \cdot \vec{\phi}(\vec{y})$ . That is, that  $k$  indeed computes the inner product of vectors in the higher-dimensional space.