Lecture 22 – Conditional Independence, Classification

DSC 40A, Fall 2022 @ UC San Diego

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Agenda

- Conditional independence
- Classification.

Conditional independence

Conditional independence

- Sometimes, events that are dependent *become* independent, upon learning some new information.
- Or, events that are independent can become dependent, given additional information.

Example: cards

♥: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A
♦: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A
♣: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, A
♠: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

- Your dog ate the King of Clubs. Suppose you draw one card from a deck of 51.
  - A is the event that the card is a heart.
  - B is the event that the card is a face card (J, Q, K).

- Are A and B independent?

Example: cards

♥: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♦: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♣: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, A

♠: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

- Your dog ate the King of Clubs. Suppose you draw one card from a deck of 51.
  - A is the event that the card is a heart.
  - B is the event that the card is a face card (J, Q, K).

- Suppose you learn that the card is red. Are A and B independent given this new information?

Conditional independence

- Recall that A and B are independent if

$$P(A \cap B) = P(A) \cdot P(B)$$

- A and B are **conditionally independent** given C if

$$P((A \cap B)|C) = P(A|C) \cdot P(B|C)$$

- Given that C occurs, this says that A and B are independent of one another.

Assuming conditional independence

- Sometimes we assume that events are conditionally independent to make calculations easier.
- Real-world events are almost never exactly conditionally independent, but they may be close.

Example: Harry Potter and TikTok

Suppose that 50% of UCSD students like Harry Potter and 80% of UCSD students use TikTok. What is the probability that a random UCSD student likes Harry Potter and uses TikTok, assuming that these events are conditionally independent given that a person is a UCSD student?

Independence vs. conditional independence

- Is it reasonable to assume conditional independence of
  - liking Harry Potter
  - using TikTok

  given that a person is a UCSD student?
- Is it reasonable to assume independence of these events in general, among all people?

Discussion Question

Which assumptions do you think are reasonable?

A) Both
B) Conditional independence only
C) Independence (in general) only
D) Neither

To answer, go to `menti.com` **and enter 5686 2173.**

Independence vs. conditional independence

In general, there is **no relationship** between independence and conditional independence. All of these are possibilities, given three events A, B, and C.

- A and B are independent, and are conditionally independent given C.
- A and B are independent, and are conditionally dependent given C.
- A and B are dependent, and are conditionally independent given C.
- A and B are dependent, and are conditionally dependent given C.

Example: constructing events

- Consider a sample space $S = \{1, 2, 3, 4, 5, 6\}$ where all outcomes are equally likely.
- For each scenario, specify events A, B, and C that satisfy the given conditions. (e.g. $A = \{2, 5, 6\}$)
- Choose events that are neither impossible nor certain, i.e. $0 < P(A), P(B), P(C) < 1$.

Scenario 1: A and B **are not** independent. A and B **are** conditionally independent given C.

Example: constructing events

- Consider a sample space $S = \{1, 2, 3, 4, 5, 6\}$ where all outcomes are equally likely.
- For each scenario, specify events A, B, and C that satisfy the given conditions. (e.g. $A = \{2, 5, 6\}$)
- Choose events that are neither impossible nor certain, i.e. $0 < P(A), P(B), P(C) < 1$.

Scenario 2: A and B **are not** independent. A and B **are not** conditionally independent given C.

Example: constructing events

- Consider a sample space $S = \{1, 2, 3, 4, 5, 6\}$ where all outcomes are equally likely.
- For each scenario, specify events A, B, and C that satisfy the given conditions. (e.g. $A = \{2, 5, 6\}$)
- Choose events that are neither impossible nor certain, i.e. $0 < P(A), P(B), P(C) < 1$.

Scenario 3: A and B **are** independent. A and B **are** conditionally independent given C.

Example: constructing events

- Consider a sample space $S = \{1, 2, 3, 4, 5, 6\}$ where all outcomes are equally likely.
- For each scenario, specify events A, B, and C that satisfy the given conditions. (e.g. $A = \{2, 5, 6\}$)
- Choose events that are neither impossible nor certain, i.e. $0 < P(A), P(B), P(C) < 1$.

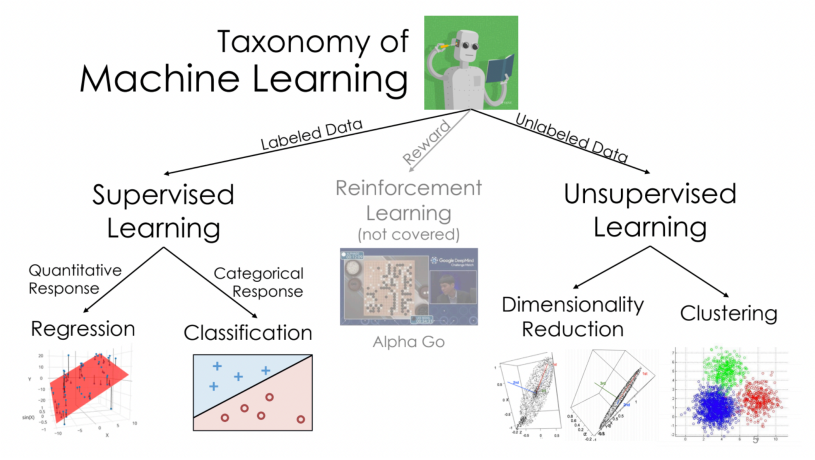
Scenario 4: A and B **are** independent. A and B **are not** conditionally independent given C.

Recap: Bayes' theorem, independence, and conditional independence

- Bayes' theorem: $P(A|B) = \frac{P(A)P(B|A)}{P(B)}$.
- A and B are **independent** if $P(A \cap B) = P(A) \cdot P(B)$.
- A and B are **conditionally independent** given C if $P((A \cap B)|C) = P(A|C) \cdot P(B|C)$.
  - In general, there is no relationship between independence and conditional independence.
  - **See the Campuswire post on conditional independence if you're still shaky on the concept.**

Classification

Taxonomy of machine learning



Classification problems

- Like with regression, we're interested in mkaing predictions based on data we've already collected (called **training data**).
- The difference is that the response variable is **categorical**.
- Categories are called **classes**.
- Example classification problems:
  - Deciding whether a patient has kidney disease.
  - Identifying handwritten digits.
  - Determining whether an avocado is ripe.
  - Predicting whether credit card activity is fraudulent.

Example: avocados

You have a green-black avocado, and want to know if it is ripe.

color	ripeness
bright green	unripe
green-black	ripe
purple-black	ripe
green-black	unripe
purple-black	ripe
bright green	unripe
green-black	ripe
purple-black	ripe
green-black	ripe
green-black	unripe
purple-black	ripe

Question: Based on this data, would you predict that your avocado is ripe or unripe?

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green-black	unripe
purple-black	ripe

Strategy: Calculate two probabilities:

$$P(\text{ripe} | \text{green-black})$$

$$P(\text{unripe} | \text{green-black})$$

Then, predict the class with a **larger** probability.

Estimating probabilities

- We would like to determine $P(\text{ripe}|\text{green-black})$ and $P(\text{unripe}|\text{green-black})$ for all avocados in the universe.

- All we have is a single dataset, which is a **sample** of all avocados in the universe.

- We can estimate these probabilities by using sample proportions.

$$P(\text{ripe}|\text{green-black}) \approx \frac{\text{\# ripe green-black avocados in sample}}{\text{\# green-black avocados in sample}}$$

- Per the **law of large numbers** in DSC 10, larger samples lead to more reliable estimates of population parameters.

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green-black	ripe
green-black	unripe
purple-black	ripe

$P(\text{ripe}|\text{green-black}) =$

$P(\text{unripe}|\text{green-black}) =$

Bayes' theorem for classification

- Suppose that A is the event that an avocado has certain features, and B is the event that an avocado belongs to a certain class. Then, by Bayes' theorem:

$$P(B|A) = \frac{P(B) \cdot P(A|B)}{P(A)}$$

- More generally:

$$P(\text{class}|\text{features}) = \frac{P(\text{class}) \cdot P(\text{features}|\text{class})}{P(\text{features})}$$

- What's the point?
  - Usually, it's not possible to estimate $P(\text{class}|\text{features})$ directly from the data we have.
  - Instead, we have to estimate $P(\text{class})$, $P(\text{features}|\text{class})$, and $P(\text{features})$ separately.

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$$P(\text{class}|\text{features}) = \frac{P(\text{class}) \cdot P(\text{features}|\text{class})}{P(\text{features})}$$

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$$P(\text{class}|\text{features}) = \frac{P(\text{class})\cdot P(\text{features}|\text{class})}{P(\text{features})}$$

Shortcut: Both probabilities have the same denominator. The larger one is the one with the larger numerator.

$$P(\text{ripe}|\text{green-black})$$

$$P(\text{unripe}|\text{green-black})$$

Summary

Summary

- Two events A and B are conditionally independent if they are independent given knowledge of a third event, C.
  - Condition: $P((A \cap B)|C) = P(A|C) \cdot P(B|C)$.

- In general, there is no relationship between independence and conditional independence.

- In classification, our goal is to predict a discrete category, called a **class**, given some features.

- The Naive Bayes classifier works by estimating the numerator of $P(\text{class}|\text{features})$ for all possible classes.