DSC 40A

Theoretical Foundations of Data Science I

In This Video

- We’ll define the Law of Total Probability and Bayes Theorem.

Getting to Campus

- You conduct a survey:
  - How did you get to campus today? Walk, bike, or drive?
  - Were you late?

	Late	Not Late
Walk	6%	24%
Bike	3%	7%
Drive	36%	24%

Getting to Campus

	Late	Not Late
Walk	6%	24%
Bike	3%	7%
Drive	36%	24%

What is the probability that a randomly selected person is late?

A. 24%
B. 30%
C. 45%
D. 50%

Getting to Campus

	Late	Not Late
Walk	6%	24%
Bike	3%	7%
Drive	36%	24%

- Since everyone either walks, bikes, or drives,

P(Late) = P(Late AND Walk) + P(Late AND Bike) + P(Late AND Drive)

- This is called the **Law of Total Probability**.

Getting to Campus

	Late	Not Late
Walk	6%	24%
Bike	3%	7%
Drive	36%	24%

Suppose someone tells you that they walked. What is the probability that they were late?

A. 6%
B. 20%
C. 25%
D. 45%

Getting to Campus

	Late	Not Late
Walk	6%	24%
Bike	3%	7%
Drive	36%	24%

- Since everyone either walks, bikes, or drives,

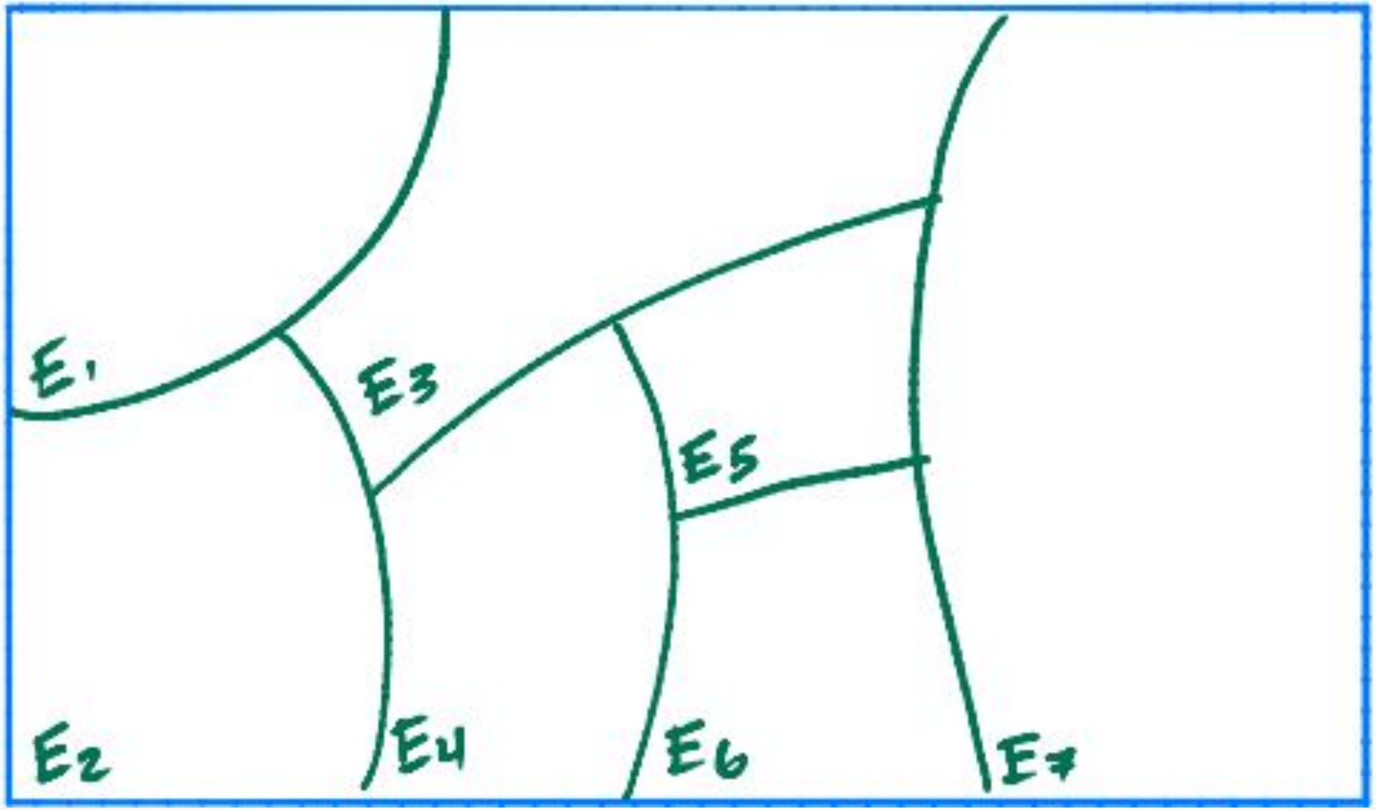
P(Late) = P(Late AND Walk) + P(Late AND Bike) + P(Late AND Drive)

P(Late) = P(Late|Walk)*P(Walk) + P(Late|Bike)*P(Bike)+P(Late|Drive)*P(Drive)

Partitions

- A set of events $E_1, E_2, \ldots, E_k$ is a **partition** of S if
  - $P(E_i \cap E_j) = 0$ for all i,j
  - $P(E_1) + P(E_2) + \ldots + P(E_k) = 1$

Partitions



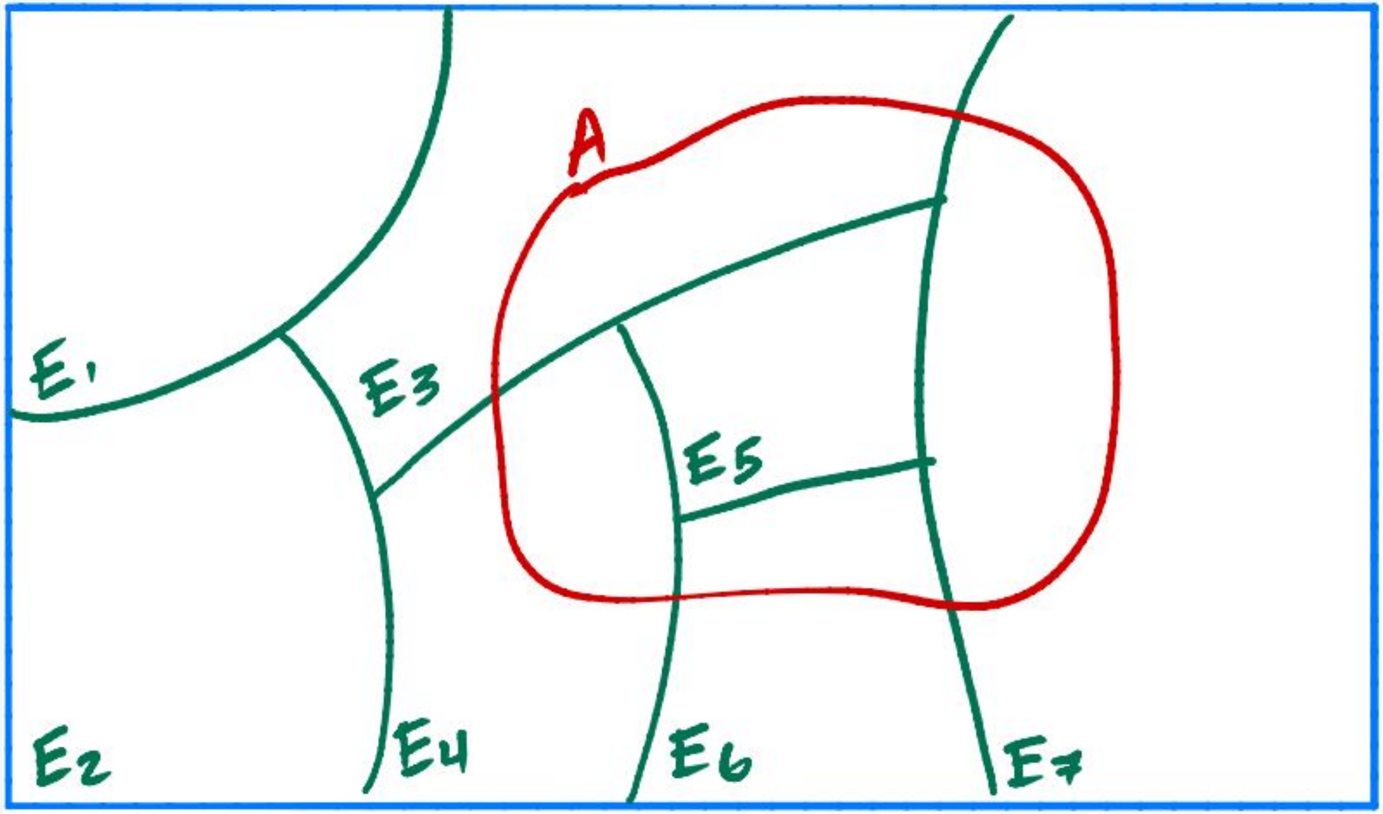
Law of Total Probability

- If A is an event and $E_1, E_2, \ldots, E_k$ is a **partition** of S, then

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \ldots + P(A \cap E_k)$$

$$= \sum_{i=1}^{k} P(A \cap E_i)$$

Partitions



Law of Total Probability

- If A is an event and E_1, E_2, …, E_k is a **partition** of S, then

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \dots + P(A \cap E_k)$$
$$= \sum_{i=1}^{k} P(A \cap E_i)$$

- Written another way,

$$P(A) = P(A \mid E_1) \cdot P(E_1) + \dots + P(A \mid E_k) \cdot P(E_k)$$
$$= \sum_{i=1}^{k} P(A \mid E_i) \cdot P(E_i)$$

Getting to Campus

	Late	Not Late
Walk	6%	24%
Bike	3%	7%
Drive	36%	24%

Suppose someone is late. What is the probability that they walked? Choose the best answer.

A. Close to 5%
B. Close to 15%
C. Close to 30%
D. Close to 40%

Getting to Campus

- Suppose all you know is
  - P(Late) = 45%
  - P(Walk) = 30%
  - P(Late|Walk) = 20%
- Can you still find P(Walk|Late)?

Bayes' Theorem

Bayes’ Theorem follows from the multiplication rule, or conditional probability.

$$P(A) * P(B|A) = P(A \text{ and } B) = P(B) * P(A|B)$$

Bayes’ Theorem:

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)}$$

Bayes' Theorem

Bayes’ Theorem follows from the multiplication rule, or conditional probability.

$$P(A) * P(B|A) = P(A \text{ and } B) = P(B) * P(A|B)$$

Bayes’ Theorem:

$$\begin{aligned} P(B|A) &= \frac{P(A|B) * P(B)}{P(A)} \\ &= \frac{P(A|B) * P(B)}{P(B) * P(A|B) + P(\overline{B}) * P(A|\overline{B})} \end{aligned}$$

not B

Bayes' Theorem: Example

$$P(B|A) = \frac{P(A|B) * P(B)}{P(B) * P(A|B) + P(\overline{B}) * P(A|\overline{B})}$$

A manufacturer claims that its drug test will **detect steroid use 95% of the time**. What the company does not tell you is that 15% of all steroid-free individuals also test positive (the false positive rate). 10% of the Tour de France bike racers use steroids. Your favorite cyclist just tested positive. What's the probability that he used steroids?

What is your first guess?

A. Close to 95%
B. Close to 85%
C. Close to 40%
D. Close to 15%

Bayes' Theorem: Example

$$P(B|A) = \frac{P(A|B) * P(B)}{P(B) * P(A|B) + P(\overline{B}) * P(A|\overline{B})}$$

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Now, calculate it and choose the best answer.

A. Close to 95%
B. Close to 85%
C. Close to 40%
D. Close to 15%

Bayes' Theorem: Example

$$P(B|A) = \frac{P(A|B) * P(B)}{P(B) * P(A|B) + P(\overline{B}) * P(A|\overline{B})}$$

A manufacturer claims that its drug test will **detect steroid use 95% of the time.** What the company does not tell you is that 15% of all steroid-free individuals also test positive (the false positive rate). 10% of the Tour de France bike racers use steroids. Your favorite cyclist just tested positive. What's the probability that he used steroids?

Solution:

B: used steroids

A: tested positive

Bayes' Theorem: Example

$$P(B|A) = \frac{P(A|B) * P(B)}{P(B) * P(A|B) + P(\overline{B}) * P(A|\overline{B})} = \frac{0.95 * 0.1}{0.1 * 0.95 + 0.9 * 0.15} \approx 0.41$$

A manufacturer claims that its drug test will **detect steroid use 95% of the time.** What the company does not tell you is that 15% of all steroid-free individuals also test positive (the false positive rate). 10% of the Tour de France bike racers use steroids. Your favorite cyclist just tested positive. What’s the probability that he used steroids?

Solution:

B: used steroids

A: tested positive

Despite manufacturer’s claims, only **41% chance** that cyclist used steroids.

Preview: Bayes' Theorem for Classification

Bayes' Theorem is very useful for classification problems, where we want to predict a class based on some features.

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)}$$

B = belonging to a certain class
A = having certain features

$$P(\text{class}|\text{features}) = \frac{P(\text{features}|\text{class}) * P(\text{class})}{P(\text{features})}$$

Summary

- When a set of events partitions the sample space, the law of total probability applies.

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \dots + P(A \cap E_k)$$
$$= \sum_{i=1}^{k} P(A \cap E_i)$$

- Bayes Theorem says how to express P(B|A) in terms of P(A|B).
- **Next time:** independence and conditional independence