Homework 7 problem 3

3a) A = "you are dealt two K♡'s"
B = "some other player has a pair of K♡'s"

ways to show independence between A & B

① $P(A \cap B) = P(A) * P(B)$

② $P(A|B) = P(A)$

③ $P(B|A) = P(B)$

$P(B|A) = 0$

$P(B) \neq 0$

$P(B|A) \neq P(B)$

so A + B are <u>not</u> independent

3 b) C = "some other player has a pair of Aces of Hearts"

are A + C independent?

wts:

$P(A \cap C) = P(A) * P(C)$

$$P(A) = \underbrace{\left(\frac{13}{52}\right)}_{\text{deck 1}} * \underbrace{\left(\frac{13}{52}\right)}_{\text{deck 2}} = \frac{1}{16}$$

independence

$$P(C) = \underbrace{\frac{1}{16}}_{\text{prob of 1 person getting pair of Ace ♡'s}} * \underbrace{3}_{\text{3 other player}} = \frac{3}{16}$$

same as:

$$= \underbrace{\frac{39}{52}}_{\text{someone other than you gets}} * \underbrace{\frac{13}{52}}_{\text{2nd deck: same person}} = \frac{3}{16}$$

1st Ace♡'s   gets Ace♡'s

$P(A \cap C) =$

P(you get K♡'s and someone else gets Ace♡'s)   1st deck

*

P(you get the K♡'s and the <u>same person</u> gets the Ace♡'s)   2nd deck

$= P(A^1 \cap C^1) *$

$P(A^2 \cap C^2)$

$= P(A^1) * P(C^1 \mid A^1)$

$$\overset{*}{P}(A^2) * \underbrace{P(C^2|A^2)}_{\text{same person getting Ace ♡'s}}$$

$$= \underbrace{\frac{13}{52}}_{\text{1st K♡'s}} * \underbrace{\frac{39}{51}}_{\text{1st Ace♡'s}} * \underbrace{\frac{13}{52}}_{\text{2nd, K♡'s}} * \underbrace{\frac{13}{51}}_{\text{same person gets 2nd Ace♡'s}}$$

reason its $\frac{39}{51}$:

- you already have K♡'s
- no one else can get K♡'s in the first deck
- so only 51 options for other players

case where you have all 4 kings

$\frac{39}{48} = \frac{39}{52-4}$

you have all 4 kings

3 c) you know you don't have any aces of hearts

are A + C independent

D = you don't have any Ace's of hearts

$P(A|D) = \frac{13}{51} * \frac{13}{51}$

in first deck prob that someone else gets the Ace of ♡'s is 1 bc we know you don't have it.

P(C|D) = 1 * P(the same person gets Ace♡'s given you don't have any ... in the 2nd deck)

$$= 1 * \frac{1}{3} = \frac{1}{3}$$

3 players that can get Ace♡'s

$$P(A \cap C \mid D) = P(A \mid D) * P(C \mid D)$$

conditional independence.

$$P(A) * P(C) = P(A \cap C)$$

independence

2 events don't affect each other's outcomes

$$P(B \mid A) = P(B)$$

B is not affected

by the presence of A

conditional independence

A & B are conditionally independent given C.

$$P(A \mid C \cap B) = P(A \mid C)$$

in the presence of C, B + A do not affect one another.

HW 7 2c)

$A = \{1, 2, 3, 5\}$, $B = \{1, 4\}$

$C = \{1, 2, 4, 7\}$

Homework 6 1f)

$\left.\begin{array}{l} 1/2 - \text{fast ball} \\ 1/3 - \text{breaking} \\ 1/4 - \text{changeup} \end{array}\right\}$ prob of hitting the ball

prob of hitting exactly 7 times

we miss 2 pitches

7 Y's + 2 N's

types of pitches

for 2N's

1. both fastballs
2. both breaking
3. both changeups
4. 1 fast, 1 breaking
5. 1 fast, 1 changeup
6. 1 breaking, 1 changeup

1. both N's fastballs

$\overbrace{F\,F\,F}$ B B B C C C

$$\underbrace{\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) * \left(\frac{1}{3}\right)^3 * \left(\frac{1}{4}\right)^3}_{\text{prob for 1 sequence}}$$

the number of sequences is

$$\binom{3}{2} = \binom{3}{1} = 3$$

1 F F F

2 F F F

3 F F F

$$3 * \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) * \left(\frac{1}{3}\right)^3 * \left(\frac{1}{4}\right)^3$$

② | breaking | changeup

FFF BBB CCC

$$\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 \binom{2}{\ } \left(\frac{1}{\ }\right)^2 \binom{3}{\ }$$

$$\underbrace{\left(\frac{\,}{2}\right)}_{\text{fast}}\ \underbrace{\left(\frac{\,}{3}\right)\left(\frac{\,}{3}\right)}_{\text{breaking}}\ \underbrace{\left(\frac{\,}{4}\right)\left(\frac{\,}{4}\right)}_{\text{changeup}}$$

number of sequences:

$$\binom{3}{1} * \binom{3}{1} = 9$$

$$9 * \left(\frac{1}{2}\right)^3 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right) \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)$$

Groupwork 6 -
problem 1e)

you see that this bitstring has more 0's than 1's.

conditional

Prob there are more 0's than 1's in total.

E = event that there are more 0's than 1s in both strings in total

F = the first bitstring has more zeros than 1s

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$ def of conditional prob.

$P(F) = 5/16$ found in part c)

$P(E \cap F)$

break this up into cases:

first bitstring (for 1st bitstring) | second bitstring

① 4 zeros (0000) — needs <u>at least</u> <u>one</u> zero.

"at least one" → use the complement

case 1 has 15 outcomes

$16 - 1 = 15$

(2nd bitstring is 1111)

② 3 zeros in first bitstring (0100)

2nd bitstring has to have at least 2 zeros

16 − # of outcomes for less than 2 zeros

1111, 1000, 0100, 0010, 0001

$16 - 5 = 11$ outcomes

$\underbrace{\underline{0}\ \underline{1}\ \underline{0}\ \underline{0}}_{3\text{ zeros}}$ $\underbrace{\_\ \_\ \_\ \_}_{11\text{ outcomes}}$

$\binom{4}{1} = 4$

$4 * 11 = 44$

adding up both cases

$1 * 15 + 4 * 11 = 59$

$$P(E|F) = \frac{59/256}{5/16}$$

$$= \frac{59}{80}$$

Probability roadmap

6

any other letter

A __ __ __ __ __ __

we know this

any number

$1 * 26 * 26 * 10 * 10 * 10$

possible outcomes