Spatial correlations in a turbulent MHD laboratory plasma

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Correlation analysis is used to determine the Taylor microscale and magnetic Reynold's number in a turbulent laboratory MHD plasma. We find that radial correlation length is shorter for a colliding MHD wind tunnel plasma than for a single plume. An unambiguous measure of the magnetic Reynolds number is estimated from the Taylor microscale and the correlation scale, then compared to a calculation using the Spitzer resistivity.

I. INTRODUCTION

A useful measure of fully developed turbulence is the spatial correlation function. For a magnetohydrodynamic (MHD) plasma, the radial correlation function of the magnetic field can be written

$$R(r) = \langle \mathbf{b}(\mathbf{x}) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}) \rangle \tag{1}$$

where $\mathbf{b}$ is the fluctuating part of a turbulent magnetic field ($\mathbf{B}(x,t) = \mathbf{B_0} + \mathbf{b}$). For well-behaved turbulence, the magnetic fluctuations at two points should become uncorrelated at large spatial separation and the correlation function should vanish ($R \to 0$ as $r \to \infty$).

Two point velocity correlation functions have been measured in conventional fluids for decades (see, for example[7]) but two point magnetic correlations in plasmas are less common. The first proper two-point single time measurements of the magnetic correlation function in the solar wind plasma were performed by Matthaeus, et al[1]. They used simultaneous magnetic field data from several spacecraft, including the four Cluster spacecraft flying in tetrahedral formation. Simultaneous measurements were performed with separations ranging from 150 km (using pairs of Cluster satellites) to 350 R_E (2.2×10^6 km). From measurements of the outer correlation scale, and the Taylor microscale (discussed below), they report an effective magnetic Reynolds number of the solar wind $R_m^{eff} = 230,000$.

In a set of follow-up papers, Weygand, et al[2–5] have modified and improved the earlier result. In particular, they describe a method using fits of Cluster separations from 100 to 10^6 km^2, and extrapolating the Taylor microscale down to zero separation. We discuss this method below. These more detailed measurements confirm the earlier work[1] and find a solar wind magnetic Reynolds number of $R_m^{eff} = 260,000 \pm 20,000$. In addition, using data in the magnetospheric plasma sheet (tailward of Earth), they find a much smaller Reynolds number $R_m^{eff} = 111 \pm 12$ since the outer correlation scale is much smaller in the plasma sheet. Anisotropies in the correlation function parallel and perpendicular to the local magnetic field were studied in separate papers[3,4], with longer correlation lengths measured parallel to the local field. Variations with solar wind speed were also studied[5].

II. THEORY AND TECHNIQUES

As noted above, the magnetic correlation function can be written

$$R(r) = \langle \mathbf{b}(\mathbf{x}) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}) \rangle \tag{2}$$

where $\mathbf{b}$ is the fluctuating part of a turbulent magnetic field ($\mathbf{B}(x,t) = \mathbf{B_0} + \mathbf{b}$).

The magnetic Taylor microscale can be formally defined as

$$\lambda_T^2 = \frac{\langle \mathbf{b}^2 \rangle}{\langle (\nabla \times \mathbf{b})^2 \rangle}. \tag{3}$$

This definition identifies the Taylor microscale as the scale associated with mean square spatial derivatives of the fluctuating magnetic field $\mathbf{b}$. A similar definition of the Taylor microscale in conventional fluids involves spatial derivatives of the fluctuating velocity field[7]. It is at this scale that one would expect dissipation effects to become important, although actual dissipation likely occurs at smaller, kinetic scales ($k_D \lambda_T = R_m^{1/4}$)[6]. We expect that the Taylor microscale should be on the order of but larger than the Larmor scale ($\rho_i \approx 1$ mm in the SSX wind tunnel) and the ion inertial scale ($c/\omega_{pi} \approx 5$ mm in SSX).

The correlation scale can be evaluated

$$\lambda_C = \frac{1}{\langle b^2 \rangle} \int_0^\infty R(r)dr \tag{4}$$

which is of the order of the system size (eg. the radius of the SSX wind tunnel).

An equivalent formulation of the Taylor scale in terms of the spectrum is

$$\frac{1}{\lambda_T^2} = \int_0^\infty dk k^2 E(k) / \int_0^\infty dk E(k) \tag{5}$$

The correlation function can be approximated

$$R(r) \approx \langle b^2 \rangle \left(1 - \frac{r^2}{2\lambda_T^2} \right) \tag{6}$$

We will fit our measured 16-point correlation function $R(r)$ to this form in order to extract the Taylor microscale λ_T.

An effective turbulent magnetic Reynolds number can be written[7]

$$R_m^{eff} = \left(\frac{\lambda_C}{\lambda_T} \right)^2 \qquad (7)$$

which is related to the actual magnetic Reynolds number

$$R_m = \left(R_m^{eff} \right)^2. \qquad (8)$$

Note that the formal definition of the magnetic Reynolds number involves the plasma (Spitzer) conductivity

$$R_m = \mu_0 V \sigma_{SP} L. \qquad (9)$$

Using $T_e = 10 \ eV$ for the SSX wind tunnel and the radius of the tunnel for the outer scale ($L = R = 0.078 \ m$), we calculate $R_m = 300$ for a flow speed of 50 km/s.

The Fourier transform of the correlation function is the spatial energy spectrum, $E(k)$:

$$E(k) = \frac{1}{2\pi} \int_0^L e^{ikr} R(r) dr \qquad (10)$$

We can compare a measurement of the energy spectrum $E(k)$ to the Fourier transform of the measured correlation function $R(r)$.

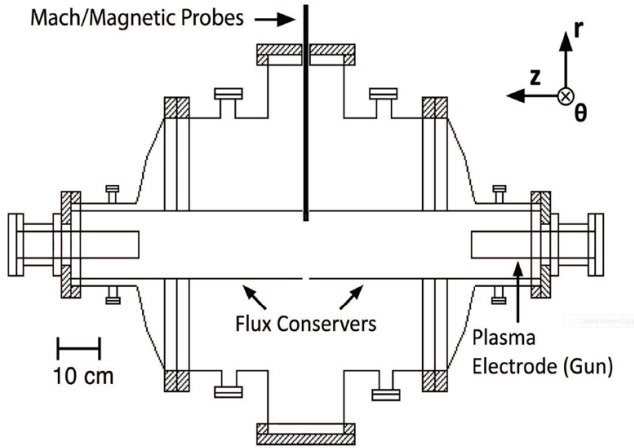## III. EXPERIMENTAL APPARATUS

FIG. 1. Cross-section of SSX chamber in MHD wind-tunnel configuration

TODO: Need some standard experiment setup description. Plasma production, lab info? Definitely need to talk about the magnetic probes: tri-directional $\dot{B}$ probes inserted radially at the mid-plane of the flux conserver. Measures change in magnetic field at 16 points along radius.

The SSX operates in two configurations, relaxation and merging. In the relaxation configuration one plasma gun is fired and the relaxation of the plasma plume (reference the Taylor relaxation paper) is observed. In the merging configuration, two plasmas with opposing helicities are created and their subsequent interaction is studied.

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IV. RESULTS

Fig. 2 shows the averaged autocorrelation function for integrated B-field in each of the three probe directions. 75 runs of the single-plume relaxation configuration were sampled, the $40.0 - 60.0 \ \mu s$ epoch being used in each case. The darkened data points indicate which points were used in plotting the fit equation (6). Each distinct point represents a correlation measured from a given probe tip: thus there are $16 - n$ points for a n tip separation. Apart from providing a visual representation of experimental spread in data, this method also reveals locational variations in the correlation function that persist over multiple shots. For example, in Fig. 2(b), it appears that there are two clusters of data, one with a generally higher correlation strength than the other. The higher correlation strength points correspond to tips further from the wall of the flux conserver; that this pattern is apparent after an averaging of 75 sets of data suggests that B_θ correlation strengths are influenced by wall proximity. Fig. 2(b) shows how the parabolic fit (7) can be applied to a subset of the data — in this case the fit shown was performed only on the averaged correlation strengths from the four outer probe tips.

Fig. 2 provides an enlightening contrast between the three measured directions of magnetic field. The B_r field is highly correlated throughout, while in contrast the B_z correlation strengths are highly scattered and less suggestive of any distinctive pattern. As such, it is prudent to focus on the correlation strengths of the B_θ signal, which are well-clustered and exhibit a sharper decay in correlation strength, thus giving a more precise measurement of the Taylor microscale λ_T. We proceeded to compare the correlation strengths of B_θ from merging and relaxation configurations of SSX with corresponding simulations.

TODO get Slava's description here? Relevant details such as the paradigm used (Hall/MHD), etc... The simulation data consisted of magnetic field measured along 8 equally-spaced radii at the midplane of the simulated flux conserver.

The simulation epoch used was $43 - 53\mu s$, chosen to capture fluctuations in the simulated B-field that qualitatively resembled the $40 - 60 \ \mu s$ epoch of the experimental data. TODO: need to try different epochs of the simulation data.

Fig. 3 compares the measured auto-correlation functions from merging and relaxation configurations of the experiment and of simulation. Fig. 3(a) presents the results from the two configurations of SSX, namely single plume (relaxation) and merging. The steeper decay
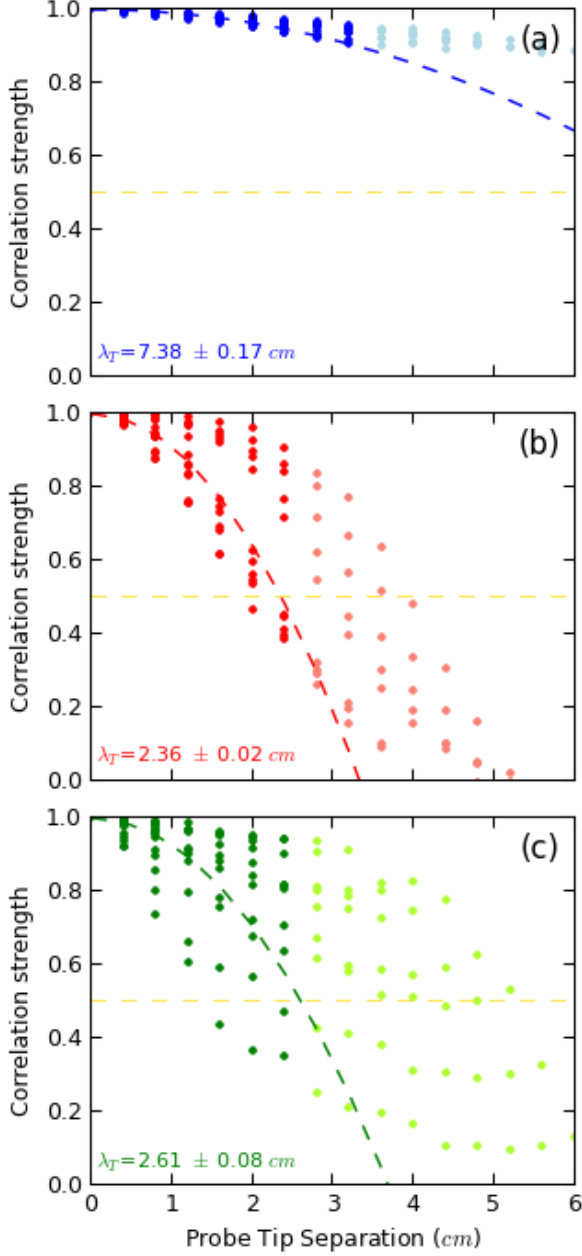
FIG. 2. Single plume correlation functions for magnetic field in the (a) $\hat{r}$, (b) $\hat{\theta}$, (c) $\hat{z}$ directions

of the merging configuration correlation function is indicative of a more turbulent plasma during the sampled epoch, which is intuitively expected.

The curves in Fig. 3(b) correspond to the relaxation configuration for SSX and simulation. While the two correlation curves look qualitatively different, it is enlightening that the radii of curvature (i.e. the curve obtained by fitting (7)) obtained from experiment and from simulation are close. mini-conclusion?

Fig. 3(c) presents results from the merging configuration in SSX and simulation. The contrast between the laboratory and simulated results motivates deeper consideration of details of the simulation that differ from an experimental setup. One potential source of differences between the two is that the simulated plasmas always merge exactly at the midplane, and thus exactly where the measurements of B-field are obtained. Using measurements slightly offset from the midplane of the simulation could imitate the variability in the merging plane of lab plasmas.

As shown in Figures 2 and 3, (7) can be fitted to functions of correlation strength to obtain a value for the Taylor microscale λ_T. Following the example of Matthaeus et al.[1], we vary the degrees of freedom given in fitting (7) to the data and extrapolate to a zero-point fit. This theoretically corresponds to the infinitesimal measurement of the correlation function's radius of curvature, and thus to the most physically accurate value of the Taylor microscale λ_T. (TODO check the corresponding section in Bill's paper and use their language)

Fig. 4 corresponds to Fig. BLAH in[1], and shows the results of extrapolating the measured λ_T values to a zero-point fit. The simulation data show a curious inflection at points fitted ~ 5, which could be caused by BLAH. The laboratory results, however, fit well to a linear function and yield well-defined values of the Taylor microscale: $\lambda_T = 2.06cm$ for the relaxation configuration, and $\lambda_T = 1.56cm$ for the merging setup.

Once this extrapolated λ_T value has been obtained, calculating R_m requires a measurement of the correlation length λ_C. A variety of ways exist for calculating λ_C, which is the fundamental length scale of the experiment: the radius of the flux conserver is a natural maximum length scale for radial correlation measurements; fitting the fitted correlation function to a Gaussian yields an integration scale or an e-folding time.

Sample calculation of R_m values: "w/ radius" means using $\lambda_C = 7.8\ cm$ (?), "w/ Int. Scale" means performing the fit to find e-folding time (Talk about measuring correlation length: Gaussian/other fits), computed means using the formula for R_m with Spitzer resistivity. Hopefully they all end up in the same ballpark.

Table I compares the magnetic Reynolds numbers obtained from four different data sets: experimental single-plume and merging configurations, and their corresponding simulations. It compares the values of R_m obtained when the various λ_C calculations are performed: the "R_m w/ Radius" column uses the radius of the flux chamber
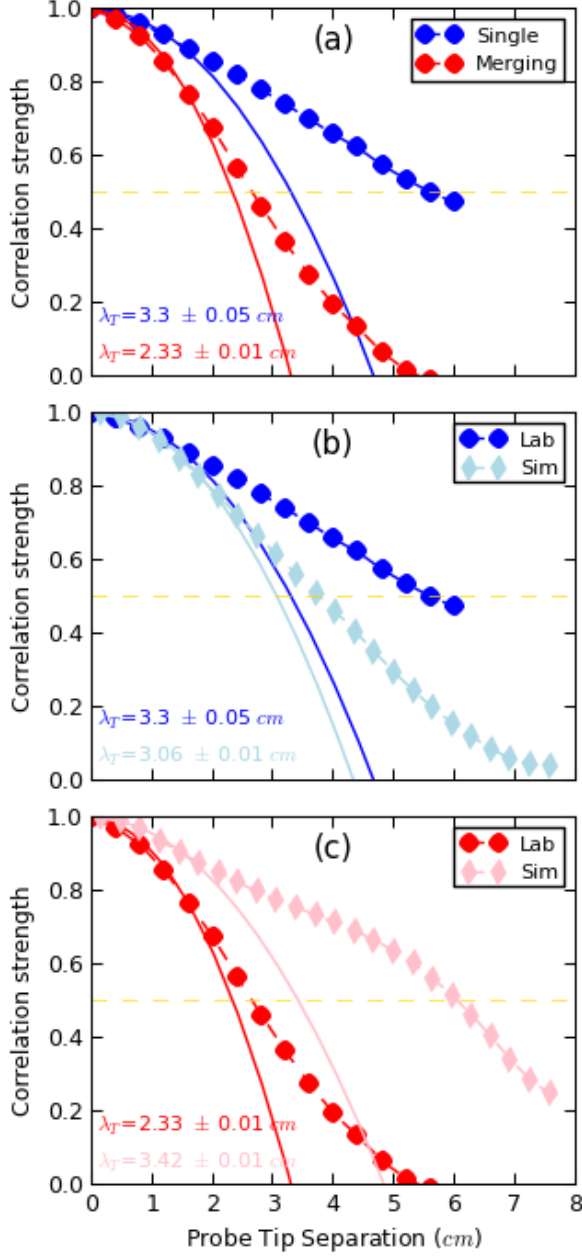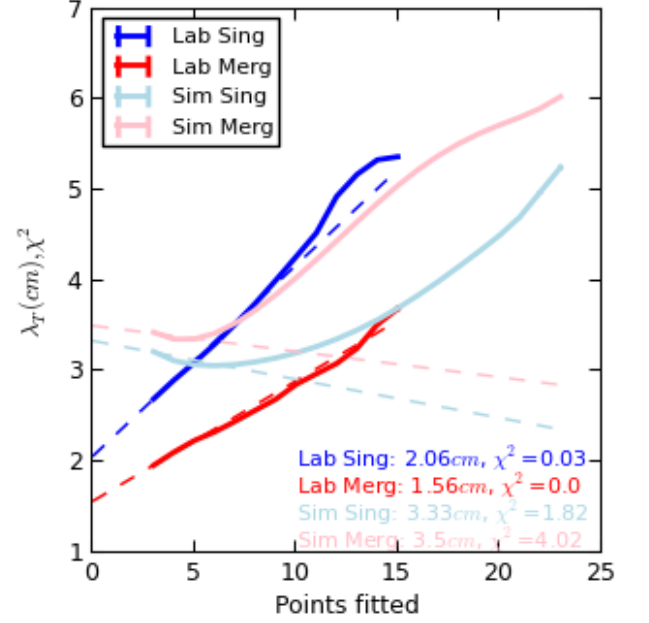
FIG. 4. λ_T vs. ndf. of parabola fit with error

as the correlation scale, $\lambda_C = 7.8\ cm$; the "w/ Int. Scale" performs the integration in (4) to calculate the correlation scale.

These results are comparable to the magnetic Reynolds number calculated using (9), $R_M = 300$.

Any uncertainty in fitting (6) to the data is magnified when the ratio λ_C/λ_T is raised to the fourth power. An estimate of the error in the Lab Single value of R_m calculated using the radius is

$$\Delta R_m = 4\left(\frac{\Delta\lambda_T}{\lambda_T}+\frac{\Delta\lambda_C}{\lambda_C}\right) = 4\left(\frac{0.054}{3.30}+\frac{0.1}{7.8}\right) = 11.7\% \quad (11)$$

TODO is this the right way to estimate error?

TABLE I. Comparison of measured/computed Taylor Microscale and magnetic Reynolds numbers for various cases.

Condition	λ_T	R_m w/Radius	R_m w/Int. Scale
Lab Single	2.06	206	-1
Sim Single	3.33	30.1	-1
Lab Merging	1.56	625	-1
Sim Merging	3.50	24.7	-1

The values obtained for the two configurations in the laboratory straddle the value of R_m calculated from (9). This agreement (check error?) is BLAH — Something about how this method is cool because we were able to obtain a value for two physical parameters (λ_T and R_m) just through a correlation analysis, which basically looks at turbulence/noise? Plus this method is useful for plasmas that we can't directly measure temperature for — good alternative to the calculation with Spitzer resistivity.

FIG. 3. (a) Single vs. Merging. (b) Single, Lab vs. Sim. (c) Merging, Lab vs. Sim.

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REFERENCES

[1] Matthaeus, W. H. and Dasso, S. and Weygand, J. M. and Milano, L. J. and Smith, C. W. and Kivelson, M. G., Phys. Rev. Lett. 95, 231101 (2005) Spatial Correlation of Solar-Wind Turbulence from Two-Point Measurements

[2] Weygand, J. M., Matthaeus, W. H., Dasso, S., Kivelson, M. G., and Walker, R. J. (2007), J. Geophys. Res., 112, A10201.

[3] Weygand, J. M., Matthaeus, W. H., Dasso, S., Kivelson, M. G., Kristler, L. M., and Mouikis, C. (2009), J. Geophys. Res., 114, A07213.

[4] Weygand, J. M., Matthaeus, W. H., El-Alaoui, M., Dasso, S., and Kivelson, M. G. (2010), J. Geophys. Res., textit115, A12250.

[5] Weygand, J. M., Matthaeus, W. H., Dasso, S., and Kivelson, M. G. (2011), J. Geophys. Res., 116, A08120.

[6] Matthaeus W. H., Weygand, J. M., Chuychai, P., Dasso, S., Smith, C. W., and Kivelson, M. (2008), Astrophys. J., 678, L141.

[7] Frisch, U. 1995, *Turbulence* (Cambridge: Cambridge Univ. Press)