Spatial correlations in a turbulent MHD laboratory plasma

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Correlation analysis is used to determine the Taylor microscale and magnetic Reynold's number in a turbulent laboratory MHD plasma. We find that radial correlation length is shorter for a colliding MHD wind tunnel plasma than for a single plume. An unambiguous measure of the magnetic Reynolds number is estimated from the Taylor microscale and the correlation scale, then compared to a calculation using the Spitzer resistivity.

I. INTRODUCTION

A useful measure of fully developed turbulence is the spatial correlation function. For a magnetohydrodynamic (MHD) plasma, the radial correlation function of the magnetic field can be written

$$R(r) = \langle \mathbf{b}(\mathbf{x}) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}) \rangle \tag{1}$$

where $\mathbf{b}$ is the fluctuating part of a turbulent magnetic field ($\mathbf{B}(x,t) = \mathbf{B_0} + \mathbf{b}$). For well-behaved turbulence, the magnetic fluctuations at two points should become uncorrelated at large spatial separation and the correlation function should vanish ($R \to 0$ as $r \to \infty$).

Two point velocity correlation functions have been measured in conventional fluids for decades (see, for example[8]) but two point magnetic correlations in plasmas are less common. The first proper two-point single time measurements of the magnetic correlation function in the solar wind plasma were performed by Matthaeus, et al[2]. They used simultaneous magnetic field data from several spacecraft, including the four Cluster spacecraft flying in tetrahedral formation. Simultaneous measurements were performed with separations ranging from 150 km (using pairs of Cluster satellites) to 350 R_E (2.2×10^6 km). From measurements of the outer correlation scale, and the Taylor microscale (discussed below), they report an effective magnetic Reynolds number of the solar wind $R_m^{eff} = 230,000$.

In a set of follow-up papers, Weygand, et al[3–6] have modified and improved the earlier result. In particular, they describe a method using fits of Cluster separations from 100 to 10^6 km[3], and extrapolating the Taylor microscale down to zero separation. We discuss this method below. These more detailed measurements confirm the earlier work[2] and find a solar wind magnetic Reynolds number of $R_m^{eff} = 260,000 \pm 20,000$. In addition, using data in the magnetospheric plasma sheet (tailward of Earth), they find a much smaller Reynolds number $R_m^{eff} = 111 \pm 12$ since the outer correlation scale is much smaller in the plasma sheet. Anisotropies in the correlation function parallel and perpendicular to the local magnetic field were studied in separate papers[4,5], with longer correlation lengths measured parallel to the local field. Variations with solar wind speed were also studied[6].

II. THEORY AND TECHNIQUES

As noted above, the magnetic correlation function can be written

$$R(r) = \langle \mathbf{b}(\mathbf{x}) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}) \rangle \tag{2}$$

where $\mathbf{b}$ is the fluctuating part of a turbulent magnetic field ($\mathbf{B}(x,t) = \mathbf{B_0} + \mathbf{b}$).

The magnetic Taylor microscale can be formally defined as

$$\lambda_T^2 = \frac{\langle \mathbf{b}^2 \rangle}{\langle (\nabla \times \mathbf{b})^2 \rangle}. \tag{3}$$

This definition identifies the Taylor microscale as the scale associated with mean square spatial derivatives of the fluctuating magnetic field $\mathbf{b}$. A similar definition of the Taylor microscale in conventional fluids involves spatial derivatives of the fluctuating velocity field[8]. It is at this scale that one would expect dissipation effects to become important, although actual dissipation likely occurs at smaller, kinetic scales ($k_D \lambda_T = R_m^{1/4}$)[7]. We expect that the Taylor microscale should be on the order of but larger than the Larmor scale ($\rho_i \approx 1$ mm in the SSX wind tunnel) and/or(?) the ion inertial scale ($c/\omega_{pi} \approx 5$ mm in SSX).

The correlation scale can be evaluated

$$\lambda_{CS} = \frac{1}{\langle b^2 \rangle} \int_0^\infty R(r) dr \tag{4}$$

which is of the order of the system size (eg. the radius of the SSX wind tunnel).

An equivalent formulation of the Taylor scale in terms of the spectrum is

$$\frac{1}{\lambda_T^2} = \int_0^\infty dk k^2 E(k) / \int_0^\infty dk E(k) \tag{5}$$

The correlation function can be approximated

$$R(r) \approx \langle b^2 \rangle \left(1 - \frac{r^2}{2\lambda_T^2} \right) \tag{6}$$

We will fit our measured 16-point correlation function $R(r)$ to this form in order to extract the Taylor microscale λ_T.

An effective turbulent magnetic Reynolds number can be written?

$$R_m^{eff} = \left(\frac{\lambda_{CS}}{\lambda_T}\right)^2 \quad (7)$$

Note that the formal definition of the magnetic Reynolds number involves the plasma (Spitzer) conductivity

$$R_m = \mu_0 V \sigma_{SP} L. \quad (8)$$

Using $T_e = 10\ eV$ for the SSX wind tunnel and the radius of the tunnel for the outer scale ($L = R = 0.078\ m$), we calculate $R_m = 300$ for a flow speed of 50 km/s.

The Fourier transform of the correlation function is the spatial energy spectrum, $E(k)$:

$$E(k) = \frac{1}{2\pi}\int_0^L e^{ikr} R(r) dr \quad (9)$$

We can compare a measurement of the energy spectrum $E(k)$ to the Fourier transform of the measured correlation function $R(r)$.

III. EXPERIMENTAL APPARATUS

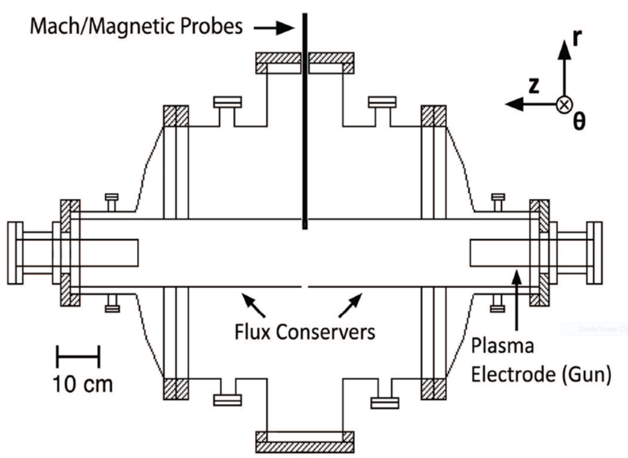

FIG. 1. Cross-section of SSX chamber in MHD wind-tunnel configuration

TODO: Need some standard experiment setup description. Plasma production, lab info? Definitely need to talk about the magnetic probes: tri-directional $\dot{B}$ probes inserted radially at the mid-plane of the flux conserver. Measures change in magnetic field at 16 points along radius.

IV. RESULTS

Fig. 2 shows the averaged autocorrelation function for integrated B-field in each of the three probe directions. 75 runs of the single-plume relaxation configuration were used. The darkened data points indicate which points were used in plotting the fit equation (6). Each distinct point represents a correlation measured from a given probe tip: thus there are $16 - n$ points for a n tip separation. Explain why this is good, e.g. is a visual representation of spread in data, shows how correlation strengths differ based on location in the flux conserver.

Talk about why we want to focus on B_θ now.

Comparison of merging and relaxation runs: correlation clearly decreases much more rapidly for merging case. Indicative of more turbulent plasma, which is intuitive.

Comparsison of lab single and simulated single

Comparison of lab merging and simulated merging

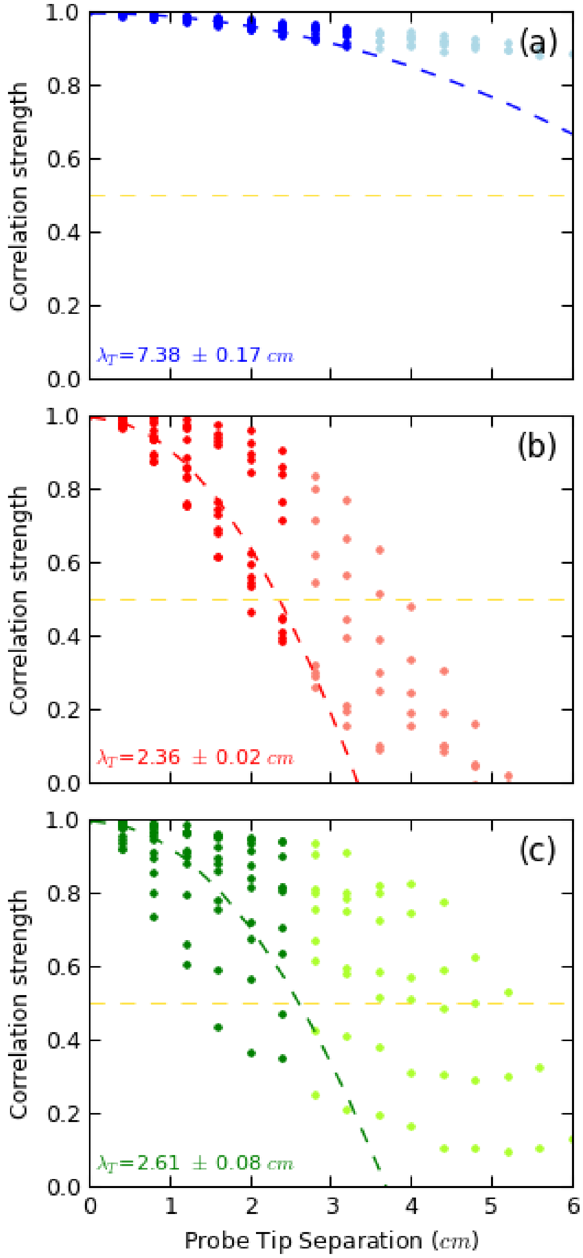

FIG. 2. Single plume correlation functions for magnetic field in the (a) $\hat{r}$, (b) $\hat{\theta}$, (c) $\hat{z}$ directions

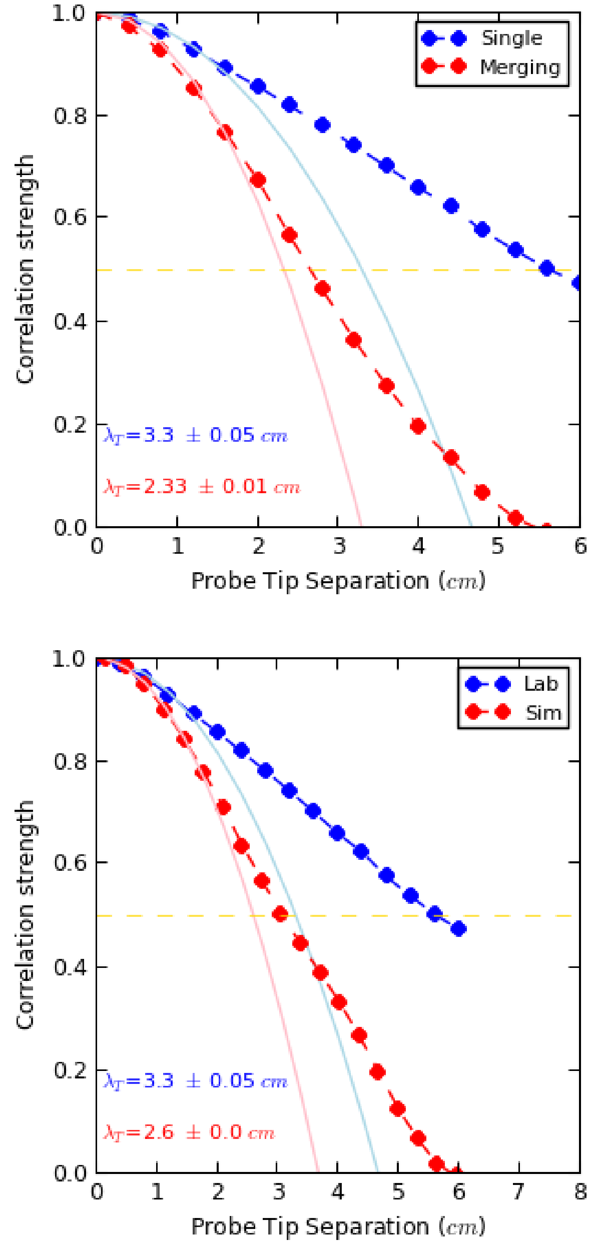

FIG. 3. (a) Single vs Merging. (b) Expt vs Sim.

TABLE I. Comparison of measured/computed Taylor Microscale and magnetic Reynolds numbers for various cases.

Condition	λ_{CS}	λ_T	R_m w/Radius	R_m w/Int. Scale	R_m Computed
Single-Outer	1	1	1	1	1
Single-Inner	1	1	1	1	1
Merging	1	1	1	1	1
Simulation	1	1	1	1	1

TODO: turn these two into single plot, change color for simulation single, fix axes. Do the epoch switches

Sample calculation of R_m values: "w/ radius" means using $\lambda_C = 7.8\,cm$ (?), "w/ Int. Scale" means performing the fit to find e-folding time (Talk about measuring correlation length: Gaussian/other fits), computed means using the formula for R_m with Spitzer resistivity. Hopefully they all end up in the same ballpark.

Can talk about how the error in fitting to the parabola is magnified quickly by taking the fourth power. Do error calculation, i.e. $\%err_{Rm} = 4 * err_{\lambda T}/\lambda_T$. Should get maybe 40%?

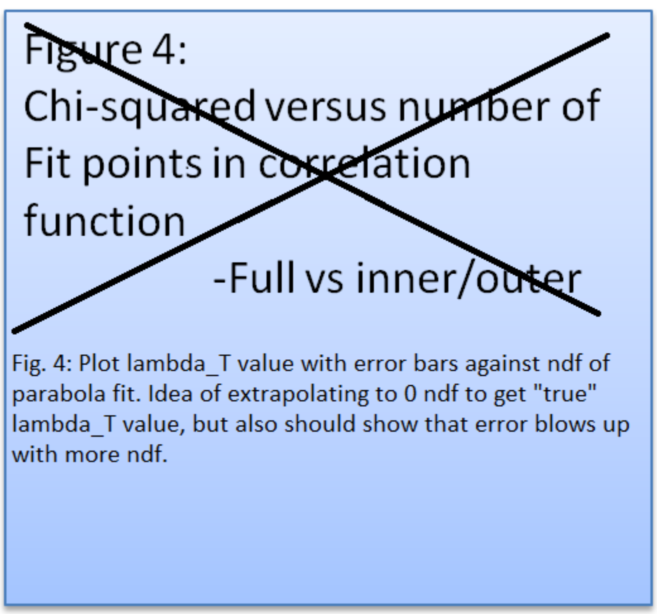

FIG. 4. λ_T vs. ndf. of parabola fit with error

Discuss Fig. 4. We want to extrapolate to a zero-point fit to find the most accurate value (TODO include Bill's plot that does this) for λ_T. Also want to justify our ndf in fitting — we should see error bars blow up after about 5.

ACKNOWLEDGEMENTS

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