Turbulence statistics for double-helix flux rope plasmas

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Abstract. We have previously generated elongated Taylor double-helix flux rope plasmas in the SSX MHD wind tunnel. These plasmas are remarkable in their rapid relaxation (about one Alfvén time) and their description by simple analytical Taylor force-free theory despite their high plasma β and high internal flow speeds. We discuss here the possibility that the turbulence facilitating access to the final state supports coherent structures and intermittency revealed by non-Gaussian signatures in the statistics. Comparisons to a two-fluid simulation show a similarity in several statistical measures.

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1. Overview

Flux ropes observed in the heliosphere have two striking properties. First is their rapid emergence. Whether in the magnetosphere or in the solar corona, these large scale structures emerge rapidly, often in just a few Alfvén crossing times of the system. Second is their long lifetimes. Once formed, these structures persist for long times despite being embedded in turbulent MHD plasma.

Flux ropes have recently been observed *in situ* at the subsolar magnetopause [1]. In this remarkable coordinated observation using three THEMIS spacecraft, a flux rope is caught in the process of forming, revealing properties that are fundamentally 3D. Since magnetospheric flux ropes evolve rapidly, observations of flux ropes in the magnetosphere tend to be detected at later stages of their evolution. The newly formed flux rope reported here appeared to be flanked by two active X-lines as part of the formation process.

Flux ropes are also observed remotely in the solar atmosphere [2]. On July 19, 2012, an eruption occurred on the solar surface producing dynamical magnetic activity resulting in a destabilized flux rope, the acceleration of a fast (1000 km/s) coronal mass ejection, and a long-lived solar flux loop. The long-lived structure is remarkable in its nearly semi-circular topology, and the persistence of a "coronal rain" from the loop top for nearly 24 hours. The observation was made with the Solar Dynamics Observatory's AIA instrument on the sun's lower right hand limb. This represents the first direct evidence of a fast CME driven by the prior formation and destabilization of a coronal magnetic flux rope.

Video at http://www.youtube.com/watch?v=HFT7ATLQQx8

It is interesting to note that in MHD simulations, the peak in the mean square current density $\langle j^2 \rangle$ is also achieved rapidly, within a fraction of an Alfvén time. At this time, the turbulence is fully developed, the peak of small scale activity is achieved, and coherent structures appear.

We have recently reported on the observation of a long-lived helical flux rope called a Taylor double-helix in the SSX MHD wind tunnel [3]. The Taylor double-helix is the natural relaxed state of MHD plasma confined in a long, perfectly conducting cylinder [4]. In the case of an infinite cylinder, the minimum energy state has a helical pitch of $ka = 1.234$, where k is the wave number associated with the z axis.

In the SSX experiments, a magnetized plasma gun launches a magnetized plasma plume into a long flux conserving cylinder. The plasma rapidly relaxes to the double-helix state in about 1 Alfvén crossing time and subsequently decays resistively. In the paper, we postulated that the physics of selective decay was at play as the initially turbulent plasma relaxed to the double-helix state. The selective decay hypothesis posits that the energy selectively decays relative to the magnetic helicity because the energy spectra peaks at higher wave numbers, where dissipation is higher [5]. The wind tunnel's minimum energy state possesses $ka = 1.292$, which is within 5% of the infinite cylinder's $ka = 1.234$.

Servidio et al. [6, 7] detail simulations which observe the rapid and simultaneous magnetohydrodynamic relaxation into localized patches of plasma with near alignment of **J** and **B**. These patches of locally relaxed plasma can then negotiate with adjacent patches to reach a globally relaxed state on a longer time scale. However, many of the characteristics of the relaxed state will be evident locally. This localized relaxation might explain the rapidity of the transition observed in the double-helix plasmas. A fully relaxed Taylor state would be expected to have a flat lambda profile (where $\nabla \times B = \lambda B$ governs the equilibrium). The reported lack of a flat radial lambda profile could also be a consequence of a patchy relaxation.

We suspect that the MHD turbulent flow in the SSX wind tunnel contains patches of locally relaxed plasma with reconnection sites at the boundaries. A fully relaxed flow might be expected to exhibit Gaussian statistics in its fluctuations and power law behavior for the power spectra. A flow containing coherent structures and reconnection sites should exhibit non-Gaussian statistics. Simulations show that coherent structures appear rapidly, in less than one dynamical time. Large numbers of reconnection sites can be identified statistically in MHD turbulence studies [8, 9]. A statistical way to find these coherent structures is to identify rapid changes in the magnetic field vector. A useful technique is to generate a probability distribution function PDF of vector increments [12, 13].

Non-Gaussian statistics and characteristic coherent structures are initiated almost identically in dissipative and ideal systems [14]. Therefore we postulate that the origins of coherence and intermittency are essentially ideal, with dissipation acting only to limit growth of the smallest scale structures. The fact that our Lundquist number is modest (S = 1000) shouldn't impact the emergence of coherent structures.

We are interested here in the decay phase of the double-helix, in particular, processes that rapidly evolve the state such as patchiness and the evolution of coherent structures. We present MHD turbulence statistics that suggest the emergence of non-Gaussian structures.

I think the theme of the paper should be that we see this state [3], and in the PRL we hypothesized that there might be rapidly formed patches of relaxation. These can be exposed by statistical studies, esp departures from Gaussian statistics. Connection to HiFi could be some direct comparisons of turbulent statistics.

2. Experiment

The flux ropes under investigation are formed in a "'wind-tunnel"' configuration of the Swarthmore Spheromak Experiment. A copper cylindrical flux conserver serves as the tunnel capped by two plasma gun electrodes whose extents limits the length of the tunnel to 86cm as can be seen in Figure 1. The radius of the cylinder is 7.75cm making the aspect ratio of the this configuration, L/a = 11. Though slightly shorter than the tunnel reported in previous work [3], the aspect ratio is considered still large enough for comparison to an infinite cylinder in Taylor relaxtion theory. The plasma itself is formed by the discharge of a 1mF, 4.0keV capacitor across a few centimeter gap between the tunsten-coated gun inner electrode and outer wall into a puffed volume of hydrogen gas. After ionization, currents of over 100kA across the gap push plasma into the main section of the flux conserver through $v \times B$ forces. Magnetic coils coaxial to the gun electrode and flux conserver contributes the stuffing flux which allows for the formation of a spheromak at the gun edge. Given the high aspect ratio, the spheromak tilts, eventually forming a twisted double-helix Taylor state; this sequence has been shown to occur in a very short time span [3]. Magnetic fluctuations are measured using an arrayed $\dot{B}$ probe; a single loop, approximately 0.5cm in diameter measures $\dot{B}$ in three orthoganal directions (r, θ and z) at 16 locations separated by 0.4cm and beginning 1cm from the cylindrical axis. Signals are aquired using a DTaq digitizer at 14-bit resolution and 65MHz sampling rate. Magnetic field vectors are computed through numerical integration of the $\dot{B}$ signal. Mach number fluctuations, as a proxy for velocity fluctuations are measured using a Mach probe oriented along the axial/z-axis [17] and located 1.5cm from the inner edge of the flux conserver. Density measurements are made using a HeNe interferometer at a diameter 21.5cm from the midplane toward the plasma source. In the data presented here, it is assumed that the plasma has already achieved a relaxed Taylor state by the time it reaches the midplane where most of the diagnostics are located. The chamber is pumped down to $\sim 7 \times 10^{-8}$ Torr and undergoes a helium glow discharge between runs so it is assumed that any influence on the plasma by impurities is low.

3. Results

The SSX produces a flux-rope plasma that persists on the order of 120μs. Figure 2 shows a timeseries of $|B|$, density, n, Mach number, M, and discharge current, I_{gun} for a single shot and for an average of 75 shots. For time analysis, the shots are divided into epochs to account for the dynamical nature of the plasma discharges as indicated by the shading in Figure 2. The formation/selective decay epoch spans from 30 to 40 μs. As shown previously in [3], the selective decay of the plasma into a Taylor state likely occurs in this time range. The equilibrium epoch ranges from 40 to 60 μs and is the period of turbulent fluctuations must closely analyzed here. This era sees the fully developed turbulence before resistive effects begin to dissipate the flux rope. The epoch from 60 μs on is considered the dissipation epoch and represents the flux-rope struture's resistive decay. Clearly, as in Figure 2(a), the magnetic field diffuses away by 120μs. Remaining unmagnetizied plasma, however, has been shown to persist for many hundreds of microseconds.

The power spectrum as computed using the wavelet technique is shown in Figure 3.

The autocorrelation function is shown in Figure 4

A series of probability distribution functions (PDF's) of $\Delta\dot{B}$ values is shown in Figure 5. These PDF's are constructing by binning a list of differences between $\dot{B}$

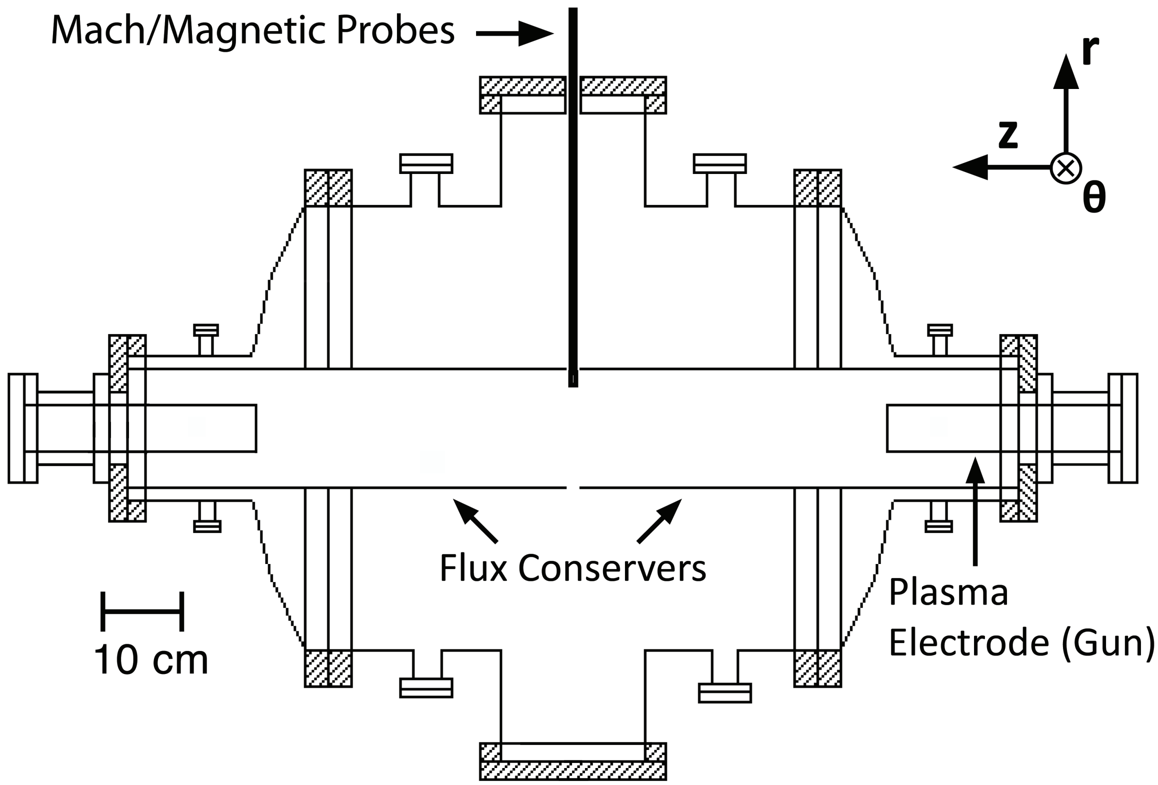

Figure 1. SSX Diagram.

values at different time increments, τ:

$$\Delta \dot{B}_{\tau}(t) = \dot{B}(t+\tau) - \dot{B}(t) \tag{1}$$

The PDFs of these deltas tend to a Gaussian shape as τ increases. This trend is clearly seen in Figure 5 as τ is increased from 0.075μs in Figure 5(a) to 15μs in Figure 5(d).

The radial correlation across the flux-rope is shown in Figure 6.

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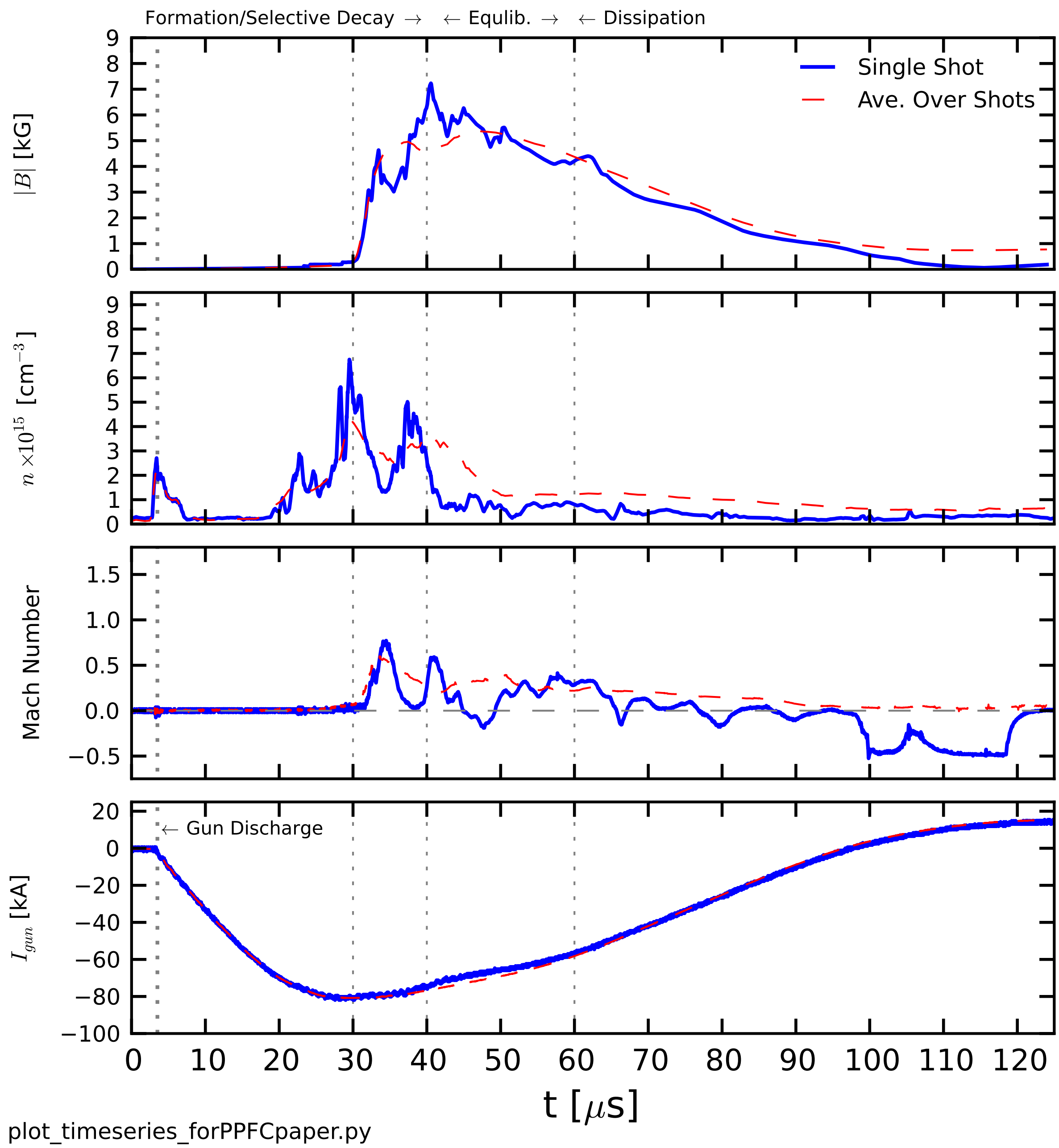

Figure 2. Timeseries of (a) Magnetic Field magnitude, (b) Density, (c) Mach Number, and (d) Discharge current. An example single shot is shown (blue line) as well as the average trace for 75 shots (red dashed).

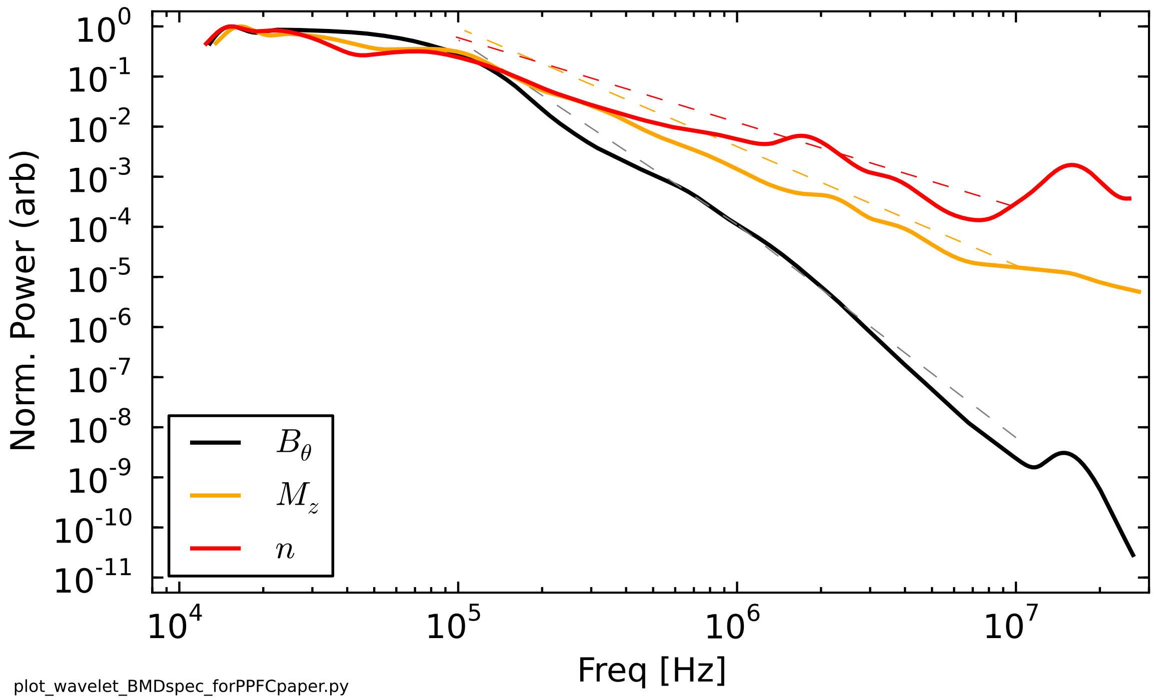

Figure 3. Wavelet Spectrum of B-field, density, and Mach number fluctuations.

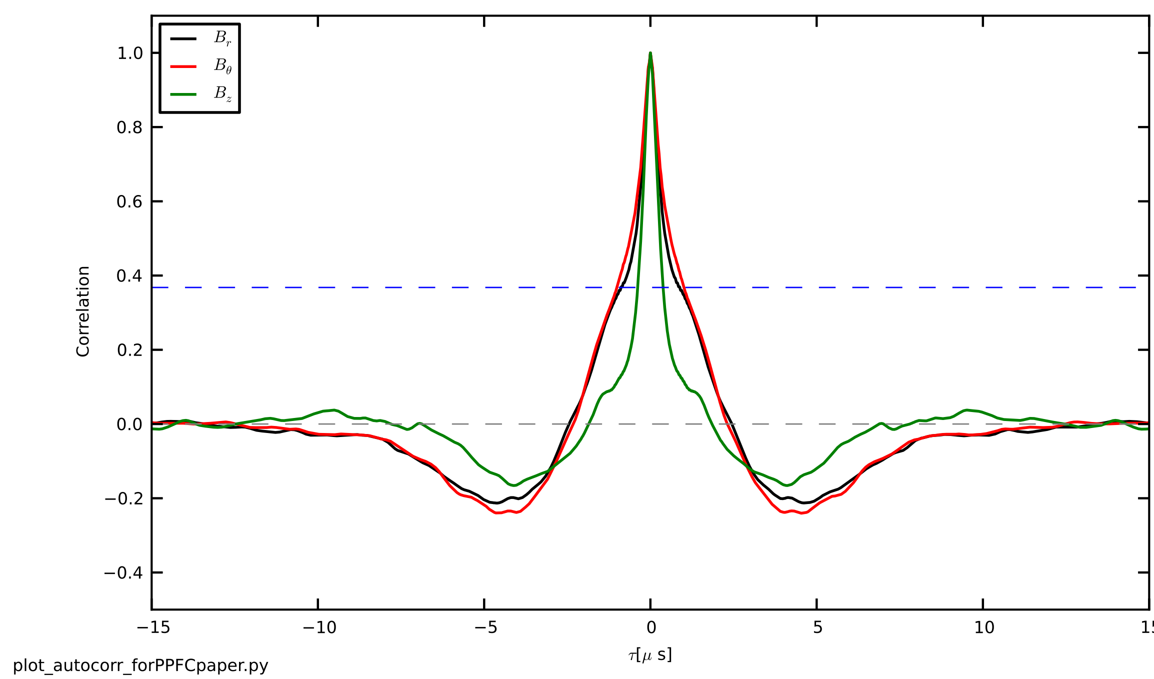

Figure 4. Autocorrelation function for $\dot{B}$ fluctuations in the time range 30 to 60 μs.

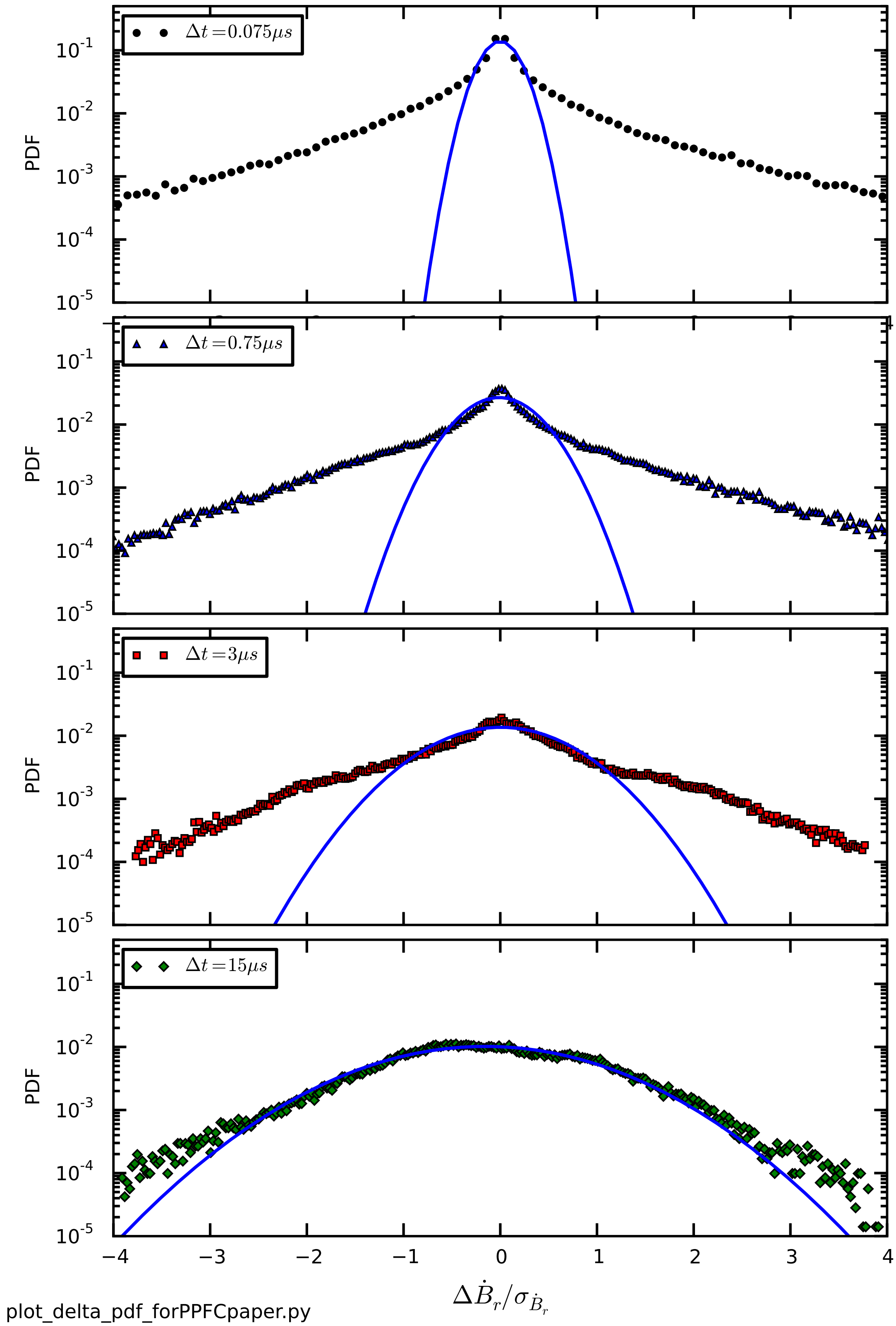

Figure 5. PDFs of $\Delta \dot{B}$ in the time range 30 to 60 μs normalized to the standard deviation for each list.

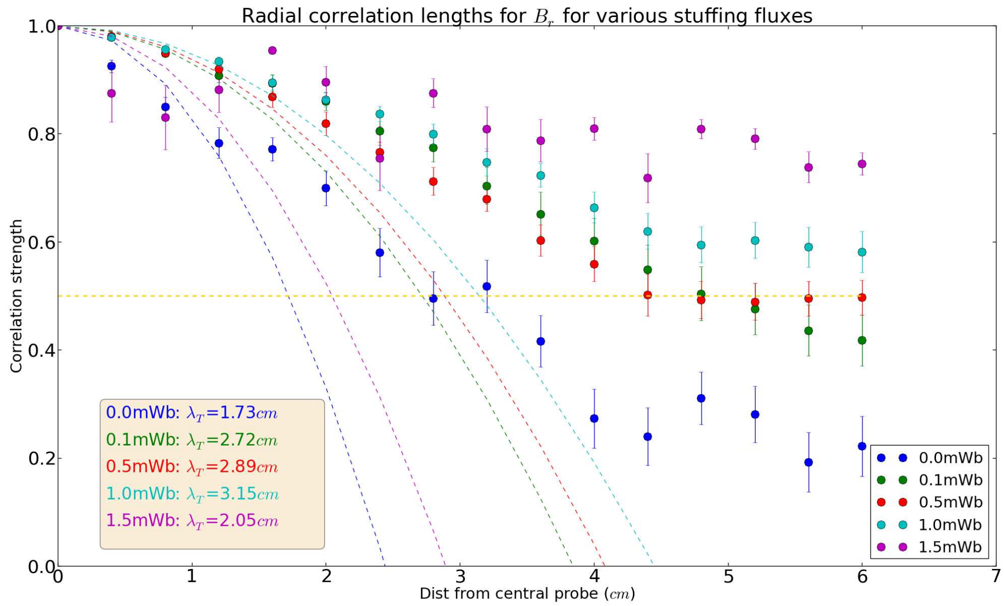

Figure 6. This version of the plot is a place holder for the newer version Adrian is working on.

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