Analysis of the Effects of non-pointlike Detectors in Hanbury-Brown and Twiss Interferometery

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Abstract

In this paper we analyse the effect of spatially extended, non-pointlike, detectors on the measurements of Hanbury Brown and Twiss (HBT) Interferometers by simulating measurements done with a disk-shaped detector. We find that the results do not fundamentally change compared to measurements done with point-size detectors, but the contrast gets worse.

1 Introduction

The Hanbury Brown and Twiss (HBT) effect describes the fact that light from an incoherent source when measured at two points will produce a coincidence rate which is slightly higher than the expected Poisson coincidence rate. This additional term in the coincidence rate is the HBT effect and it can be used to do intensity interferometry on stars with two detectors as done by Hanbury Brown & Twiss in 1958 [2]. More recently, research has been done on the feasibility of using three detectors to recover the phase information a well. Most of this research has been done assuming that only the source has a spatial extent, with the detectors being point-sized. In this paper we analyze if and how detectors with spatial extent affect the result. First we present an analytical solution in a simplified 1-Dimensional double-slit situation with two detectors. We then expand our model into two dimensions where the two point sources are represented by one disk made up of many point sources and analyze the results for two detectors.

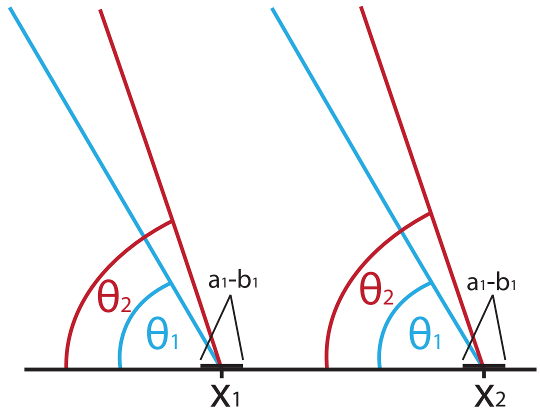

Figure 1: 2 detectors (at x_1, x_2) receiving light from 2 sources with different incident angles θ_1 and θ_2

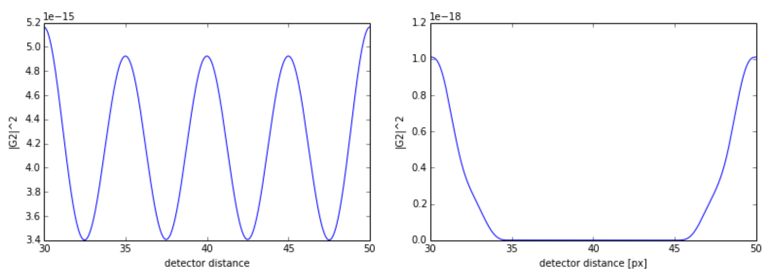

Figure 2: Signal measured at the detector in the case that a) the detector is smaller than the fringe width and b) the detector is about the size of the fringe width and some edge effects

2 Methods: Expanding the detectors

2.1 1-Dimensional Case

In its simplest form, the HBT effect can be thought of as Youngs double slit experiment done with an incoherent light source. Roy Glauber [1] shows that for such a case the correlation at any point on the screen can be expressed by the second order correlation function

$$G^{(2)}(x_1x_2x_2x_1) = G^{(1)}(x_1x_1)G^{(1)}(x_2x_2) + G^{(1)}(x_1x_2)G^{(1)}(x_2x_1)$$

where $G^{(1)}$ are first order correlation functions defined as

$$G^{(1)}(x_nx_m) = \langle E^{(-)}(x_n)E^{(+)}(x_m)\rangle$$

with $x_n = r_nt_n$. x_1 and x_2 are the positions of the two detectors. $E^{(\pm)}(x)$ are the two terms to define the oscillating electric field E and can also be expressed as

$$E^{(\pm)}(x) = E_{max}e^{\pm ikx}$$

Since we are only interested in a qualitative analysis, we will omit constant factors such as the amplitude E_{max} from here on.

To find $G^{(2)}$ for 2 detectors with a spatial extent we integrate the individual $G^{(1)}$ that make up $G^{(2)}(x_1x_2x_2x_1)$ (See fig. 1) and can then take the absolute value squared to get the signal as measured by detectors with a spatial extent $b_i - a_i$. The first product, $G^{(1)}(x_1x_1)G^{(1)}(x_2x_2)$, will just yield a constant, hence we can omit this and only integrate

$$|G^{(2)}|^2 = |\int_{a_1}^{b_1}\int_{a_2}^{b_2} G^{(1)}(x_1, x_2)\mathrm{d}x_1\mathrm{d}x_2 * \int_{a_1}^{b_1}\int_{a_2}^{b_2} G^{(1)}(x_2, x_1)\mathrm{d}x_1\mathrm{d}x_2|^2$$

Where

$$\begin{aligned}
&\int_{a_1}^{b_1}\int_{a_2}^{b_2} G^{(1)}(x_1, x_2)\mathrm{d}x_1\mathrm{d}x_2 \\
&= \int_{a_1}^{b_1}\int_{a_2}^{b_2} (\mathrm{e}^{ik(x1-x2)\theta_1} + \mathrm{e}^{ik(x1-x2)\theta_2})\mathrm{d}x_1\mathrm{d}x_2 \\
&= (\mathrm{e}^{ik(b_1-b_2)\theta_1} - \mathrm{e}^{ik(a_1-b_2)\theta_1} - \mathrm{e}^{ik(b_1-a_2)\theta_1} + \mathrm{e}^{ik(a_1-a_2)\theta_1}) \\
&\quad +(\mathrm{e}^{ik(b_1-b_2)\theta_2} - \mathrm{e}^{ik(a_1-b_2)\theta_2} - \mathrm{e}^{ik(b_1-a_2)\theta_2} + \mathrm{e}^{ik(a_1-a_2)\theta_2})
\end{aligned}$$

and

$$\begin{aligned}
&\int_{a_1}^{b_1}\int_{a_2}^{b_2} G^{(1)}(x_2, x_1)\mathrm{d}x_1\mathrm{d}x_2 \\
&= \int_{a_1}^{b_1}\int_{a_2}^{b_2} (\mathrm{e}^{ik(x2-x1)\theta_1} + \mathrm{e}^{ik(x2-x1)\theta_2})\mathrm{d}x_1\mathrm{d}x_2 \\
&= (\mathrm{e}^{-ik(b_1-b_2)\theta_1} - \mathrm{e}^{-ik(a_1-b_2)\theta_1} - \mathrm{e}^{-ik(b_1-a_2)\theta_1} + \mathrm{e}^{-ik(a_1-a_2)\theta_1}) \\
&\quad +(\mathrm{e}^{-ik(b_1-b_2)\theta_2} - \mathrm{e}^{-ik(a_1-b_2)\theta_2} - \mathrm{e}^{-ik(b_1-a_2)\theta_2} + \mathrm{e}^{-ik(a_1-a_2)\theta_2})
\end{aligned}$$

respectively. x_1 and x_2 are the positions of the two detectors and k is the wave number. θ_1 and θ_2 are the incidence angles (see fig. 1).

To verify our result, we test the case in which the detector size is approximately the same as the fringe width of the interference pattern created by the double slit. As expected, the measured signal drops to zero (See fig. 2). The integral is basically a convolution of the expected signal at point detectors convolved with detectors with a spatial extent. Hence, the same result can also be found by taking the inverse fourier transform of $G^{(1)}(x_1x_2)$, which then represents the source, and multiplying it with the inverse fourier of the detectors, represented by rect-functions. The multiplication in the position space is equal to the convolution in the fourier space. By taking the fourier transformation and then the absolute value squared of it this yields the same result as our analytical solution, except for a constant factor which has to be added due to the non-steady nature of the (inverse) fast fourier transform functions

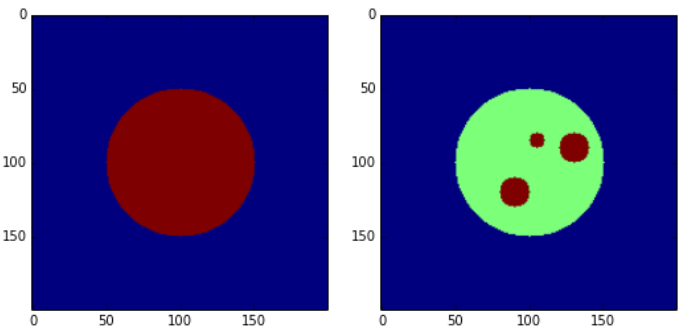

Figure 3: 2-Dimensional disk-shapes used for the analysis: a) homogenous disk used as detectors, b) spotted disk used as source

used to compute the results. In our case this constant factor Δx for the fourier transformation and Δx^{-1} for the inverse fourier transformation has the order of magnitude of arc seconds respectively arc seconds^{-1}. It has to verify the relation $N\Delta x\Delta k = 2\pi$ where N is the size of our position space, which corresponds to 512 units of space (pixels) in our case. Δk is the correction constant in the fourier space.

2.2 2-Dimensional Case

In the 2-Dimensional case the source is not a double slit anymore but a disk made up of individual point sources. This source can either be homogenous or an inhomogenous source with different valued spots (see fig. 3, modelled after [3]). The sources are being created by the function *source2d* and *randsource2d* respectively (See Appendix A).

The detectors are also modelled with a uniform disk. Because they move and detect in k-space they are disks in the fourier space. We then multiply the source with the inverse fourier transformation of the detectors and transform it back in the fourier space to get $G^{(2)}$ for the 2d case.

3 Results

Plots of the source, the absolute squared visibility as well as the real and imaginary parts of the visibility can be seen in figure 5 for detectors with a radius of 50 pixels and as a comparison in figure 4 the same values for approximately point-sized detectors of radius 1 pixel. The grid size is 512*512 pixels.

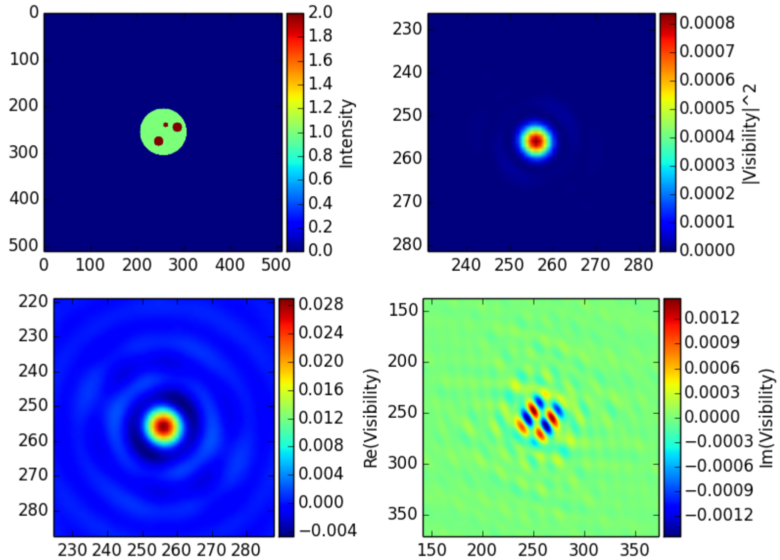

Figure 4: Resulting signal at two detectors with radius 1 pixel (point-size detector approximation), slightly enlarged to see more details

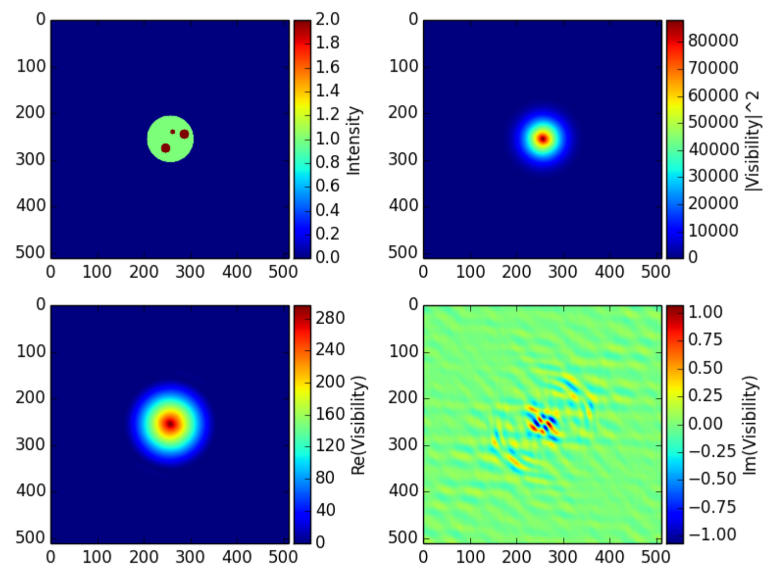

Figure 5: Resulting signal at two detectors, each with radius 50 pixels

4 Interpretation and Conclusion

In comparing the spatially-extended detector case to the point-size detector case, we find three big differences. Firstly, the measured absolute values get much bigger due to the bigger detector area. Secondly, the real part and the squared absolute visibility in particular get much bigger. Thirdly, in all three cases, the contrast gets worse. The first two effects are mostly due to the way the results were computed and would most likely be less important in a practical application due to the distances involved from source to detector and the relative size differences. The third effect however is interesting considering the substructure signal is still visible, albeit with a weaker contrast. This implies that it could indeed be possible to obtain useful results in practical applications.

A Python code used to obtain the results

```
"""
Created on Tue Jul 01 15:56:58 2014

@author: Annina, David
"""

from pylab import *

import numpy as np
import matplotlib.pyplot as plt


#create disk (radius r) filled with 1s, surrounded by 0s (over area x,
def source2d(x,y,r):
    res = np.zeros(dtype=complex,shape=(x,y))
    y,x = np.ogrid[-x/2: x/2, -y/2: y/2]
    mask = (x)**2+(y)**2 <= r**2
    res[mask]=1
    return res


#create disk (radius r) filled with 1s, and spots with 2s, surrounded
def randsource2d(x,y,r):
    res = np.zeros(dtype=complex,shape=(x,y))
    y,x = np.ogrid[-x/2: x/2, -y/2: y/2]
    mask1 = (x)**2+(y)**2 <= r**2
    mask2 = (x-5)**2+(y+15)**2 <= (r-45)**2
    mask3 = (x+10)**2+(y-20)**2 <= (r-40)**2
    mask4 = (x-30)**2+(y+10)**2 <= (r-40)**2
    #res[mask]=np.round(np.random.rand(sum(mask))*2)
    res[mask1]=1
    res[mask2]=2
    res[mask3]=2
    res[mask4]=2
    return res
```

```python
x,y = 512, 512

delx=10**-2
N=x
delk=2*pi/(N*delx)

#wrapper for fftshift(fft2(fftshift(var))) to make editing easier
def fwrp2(var):
    return fftshift(fft2(fftshift(var)))*(delx)**2

def ifwrp2(var):
    return ifftshift(ifft2(ifftshift(var)))/(delx)**2

#disks (in fourierspace)
det1=source2d(x,y,50)
det2 = det1


#disk made up of 1s with spots of 2s, surrounded by 0s in position spa
sources=randsource2d(x,y,50)

#source * inverse fourier of the detectors
sdets=sources*ifwrp2(det1)*ifwrp2(det2)

# fourier of the product to get the result
results=fwrp2(sdets)

G2s=abs(results)**2


# new figure
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2,2)
#plot source
im1 = ax1.imshow(sources.real)
#plot |visibility|^2
im2 = ax2.imshow(G2s)
#plot real-part of result
im3 = ax3.imshow(results.real)
#plot imaginary part of result
im4 = ax4.imshow(results.imag)

plt.tight_layout()

plt.show()
```

References

[1] Roy J. Glauber. Nobel lecture: One hundred years of light quanta. *Rev. Mod. Phys.*, 78:1267–1278, Nov 2006.

[2] R Hanbury Brown and R. Q. Twiss. Interferometry of the intensity fluctuations in light iv. a test of an intensity interferometer on sirius a. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 248:222–237, Nov 1958.

[3] Tina Wentz and Prasenjit Saha. Feasibility of observing hanbury brown and twiss phase. *Mon. Not. R. Astron. Soc.*, 2014.