Scalable Algorithm Design

The “Map Reduce” Programming Model

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Eurecom

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- Anand Rajaraman, Jeffrey D. Ullman, Jure Leskovec, "Mining of Massive Datasets", Cambridge University Press, 2013
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What is Big Data?

- **Vast repositories of data**
  - The Web
  - Physics
  - Astronomy
  - Finance
- **Volume, Velocity, Variety**
- **It's not the algorithm, it's the data!**
  - More data leads to better accuracy
  - With more data, accuracy of different algorithms converges

What is the “Map Reduce” Programming Model?

- **A distributed programming model**:
  - Inspired by functional programming
  - Inspired by Bulk Synchronous Parallelism (BSP)
- **An instance of an execution framework**:
  - Designed for large-scale data processing
  - Designed to run on clusters of commodity hardware

Key Principles

Scale out, not up!

- **For data-intensive workloads, a large number of commodity servers is preferred over a small number of high-end servers**
  - Cost of super-computers is not linear
  - But datacenter efficiency is a difficult problem to solve
- **Some numbers ($\sim$ 2012):**
  - Data stored/processed by Google every day: $O(EB)$
  - Data stored/processed by Facebook every day: $O(PB)$

Implications of Scaling Out

- **Processing data is quick, I/O is very slow**
  - 1 Mechanical HDD $\sim$ 100 MB/sec
  - 1000 Mechanical HDDs $\sim$ 100 GB/sec

- **Sharing vs. Shared nothing**:
  - Sharing: manage a common/global state
  - Shared nothing: independent entities, no common state

- **Sharing is difficult**:
  - Synchronization, deadlocks
  - Finite bandwidth to access data from SAN
  - Temporal dependencies are complicated (restarts)

Failures are the norm, not the exception

- LALN data [DSN 2006]
  - Data for 5000 machines, for 9 years
  - Hardware: 60%, Software: 20%, Network 5%
- DRAM error analysis [Sigmetrics 2009]
  - Data for 2.5 years
  - 8% of DIMMs affected by errors
- Disk drive failure analysis [FAST 2007]
  - Utilization and temperature major causes of failures
- Amazon Web Service(s) failures [Several!]
  - Cascading effect

Implications of Failures

- **Failures are part of everyday life**
  - Mostly due to the scale and shared environment
- **Sources of Failures**
  - Hardware / Software
  - Electrical, Cooling, ...
  - Unavailability of a resource due to overload
- **Failure Types**
  - Permanent
  - Transient

Move Processing to the Data

- **Drastic departure from high-performance computing model**
  - HPC: distinction between processing nodes and storage nodes
  - HPC: CPU intensive tasks
- **Data intensive workloads**
  - Generally not processor demanding
  - The network becomes the bottleneck
  - Framework generally assumes processing and storage nodes to be collocated

  $\rightarrow$ **Data Locality Principle**
- **Distributed filesystems are necessary**

Process Data Sequentially and Avoid Random Access

- **Data intensive workloads**
  - Relevant datasets are too large to fit in memory
  - Such data resides on disks
- **Disk performance is a bottleneck**
  - **Seek times** for random disk access are **the problem**
    - Example: 1 TB DB with 10^{10} 100-byte records. Updates on 1% requires 1 month, reading and rewriting the whole DB would take 1 day[1]
  - Organize computation for sequential reads

[1] From a post by Ted Dunning on the Hadoop mailing list

Implications of Data Access Patterns

- **Systems designed for:**
  - **Batch processing**
  - involving (mostly) **full scans** of the data
- **Typically, data is collected "elsewhere" and copied to the distributed filesystem**
  - E.g.: Apache Kafka, Hadoop Sqoop, $\cdots$
- **Data-intensive applications**
  - Read and process the whole Web (e.g. PageRank)
  - Read and process the whole Social Graph (e.g. LinkPrediction, a.k.a. "friend suggest")
  - Log analysis (e.g. Network traces, Smart-meter data, $\cdots$)

Hide System-level Details

- **Separate the *what* from the *how***
  - Framework abstracts away the “distributed” part of the system
  - Such details are handled by internal primitives
- **BUT: In-depth knowledge of the framework is key**
  - Custom data reader/writer
  - Custom data partitioning
  - Memory utilization
- **Auxiliary components**
  - Too many to list!

Seamless Scalability

- **We can define scalability along two dimensions**
  - In terms of data: given twice the amount of data, the same algorithm should take no more than twice as long to run
  - In terms of resources: given a cluster twice the size, the same algorithm should take no more than half as long to run
- **Embarrassingly parallel problems**
  - Simple definition: independent (shared nothing) computations on fragments of the dataset
  - How to to decide if a problem is embarrassingly parallel or not?

The Programming Model

Functional Programming Roots

- **Key feature: higher order functions**
  - Functions that accept other functions as arguments
  - **Map** and **Fold**

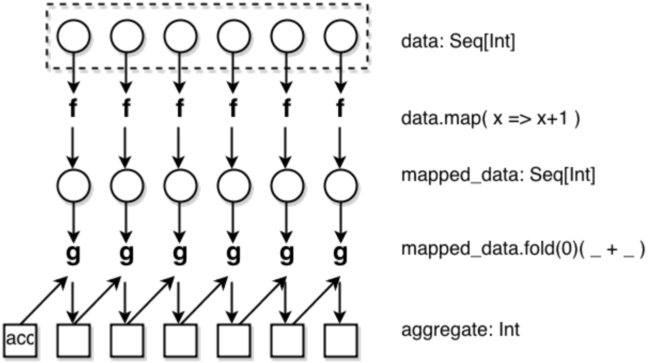


Figure: Illustration of *map* and *fold*.

Functional Programming Roots

- **map phase:**
  - Given a list, *map* takes as an argument a function *f* (that takes a single argument) and applies it to all element in a list
- **fold phase:**
  - Given a list, fold takes as arguments a function *g* (that takes two arguments) and an initial value (an accumulator)
  - *g* is first applied to the initial value and the first item in the list
  - The result is stored in an intermediate variable, which is used as an input together with the next item to a second application of *g*
  - The process is repeated until all items in the list have been consumed

Functional Programming Roots

- **We can view map as a transformation over a dataset**
  - This transformation is specified by the function *f*
  - Each functional application happens in **isolation**
  - The application of *f* to each element of a dataset can be parallelized in a straightforward manner
- **We can view fold as an aggregation operation**
  - The aggregation is defined by the function *g*
  - Data locality: elements in the list must be "brought together"
  - If we can **group** elements of the list, also the fold phase can proceed in parallel
- **Associative and commutative operations**
  - Allow performance gains through local aggregation and reordering

Functional Programming and “Map Reduce”

- **Equivalence of “Map Reduce” and Functional Programming:**
  - The map of Hadoop MapReduce corresponds to the map operation
  - The reduce of Hadoop MapReduce corresponds to the fold operation

- **The framework coordinates the map and reduce phases:**
  - Grouping intermediate results happens in parallel

- **In practice:**
  - User-specified computation is applied (in parallel) to all input records of a dataset
  - Intermediate results are aggregated by another user-specified computation

What can we do with this Programming Model??

- **Introducing the Data Flow abstraction**
  - The "old" Hadoop MapReduce programming model appears quite limited and strict
  - Apache Spark programming model is much more flexible, and operates on a directed acyclic graph representative of the computations
- **Generally, everything can be computed with the "Map Reduce" model**
  - We will focus on illustrative cases
  - We will see in detail "design patterns"
    - How to transform a problem and its input
    - How to save memory and bandwidth in the system

Data Structures

- **Key-value pairs are the basic data structure in "Map Reduce"**
  - Keys and values can be: integers, float, strings, raw bytes
  - They can also be **arbitrary data structures**

- **The design of "Map Reduce" algorithms involves**:
  - Imposing the key-value structure on arbitrary datasets[2]
    - E.g.: for a collection of Web pages, input keys may be URLs and values may be the HTML content
  - In some algorithms, input keys are not used, in others they uniquely identify a record
  - Keys can be combined in complex ways to design various algorithms

[2] There's more about it: here we only look at the input to the map function.

A Generic "Map Reduce" Algorithm

- **The programmer defines a mapper and a reducer as follows**[3,4]:
  - map: $(k_1, v_1) \rightarrow [(k_2, v_2)]$
  - reduce: $(k_2, [v_2]) \rightarrow [(k_3, v_3)]$

- **In words**:
  - A dataset stored on an underlying **distributed** filesystem, which is split in a number of **blocks** across machines
  - The mapper is applied to every input key-value pair to generate intermediate key-value pairs
  - The reducer is applied to all values associated with the same intermediate key to generate output key-value pairs

[3] We use the convention $[\cdots]$ to denote a list.

[4] Pedices indicate different data types.

Where the magic happens

- **Implicit between the map and reduce phases is a parallel "group by" operation on intermediate keys**
  - Intermediate data arrive at each reducer in order, sorted by the key
  - No ordering is guaranteed across reducers
- **Output keys from reducers are written back to the distributed filesystem**[5]
  - The output may consist of *r* distinct files, where *r* is the number of reducers
  - Such output may be the input to a subsequent phase[6]
- **Intermediate keys are transient**:
  - They are not stored on the distributed filesystem
  - They are "spilled" to the local disk of each machine in the cluster

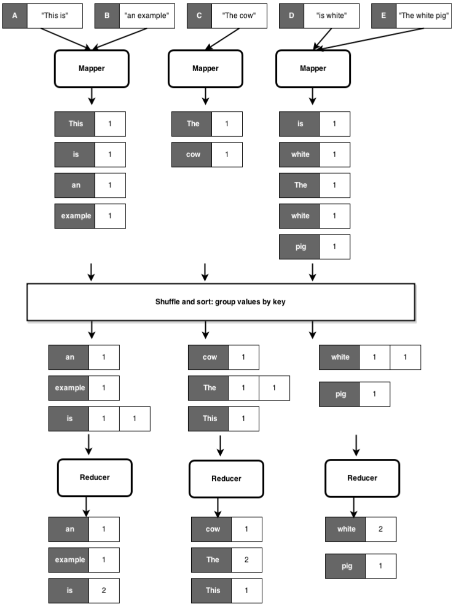
[5] This is true for Hadoop MapReduce. Apache Spark instead keeps in memory intermediate data.

[6] Think of **iterative algorithms**.

"Hello World" in "Map Reduce"

```
class Mapper
    method Map(offset a, line l)
        for all term t ∈ line l do
            Emit(term t, count 1)

class Reducer
    method Reduce(term t, counts [c1, c2, ...])
        sum ← 0
        for all count c ∈ counts [c1, c2, ...] do
            sum ← sum + c
        Emit(term t, count sum)
```


A
"This is"
B
"an example"
C
"The cow"
D
"is white"
E
"The white pig"
Mapper
Mapper
Mapper
This 1
is 1
an 1
example 1
The 1
cow 1
is 1
white 1
The 1
white 1
pig 1
Shuffle and sort: group values by key
an 1
example 1
is 1 1
cow 1
The 1 1
This 1
white 1 1
pig 1
Reducer
Reducer
Reducer
an 1
example 1
is 2
cow 1
The 2
This 1
white 2
pig 1

"Hello World" in "Map Reduce"

- **Input:**
  - Key-value pairs: (offset, line) of a file stored on the distributed filesystem
  - a: unique identifier of a line offset
  - l: is the text of the line itself
- **Mapper:**
  - Takes an input key-value pair, tokenize the line
  - Emits intermediate key-value pairs: the word is the key and the integer is the value
- **The framework:**
  - Guarantees all values associated with the same key (the word) are brought to the same reducer
- **The reducer:**
  - Receives all values associated to some keys
  - Sums the values and writes output key-value pairs: the key is the word and the value is the number of occurrences

Combiners

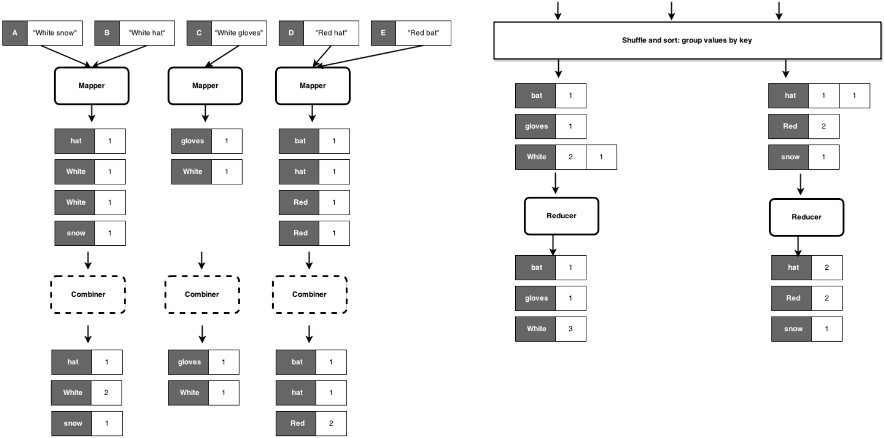
- **Combiners are a general mechanism to reduce the amount of intermediate data**
  - They could be thought of as "mini-reducers"
- **Back to our running example: word count**
  - Combiners aggregate term counts across documents processed by each map task
  - If combiners take advantage of all opportunities for local aggregation we have at most $m \times V$ intermediate key-value pairs
    - m: number of mappers
    - V: number of unique terms in the collection
  - Note: due to Zipfian nature of term distributions, not all mappers will see all terms

A word of caution

- **The use of combiners must be thought carefully**
  - In Hadoop, they are optional: the correctness of the algorithm cannot depend on computation (or even execution) of the combiners
  - In Apache Spark, they're mostly automatic

- **Combiners I/O types**
  - Input: $(k_2, [v_2])$ [Same input as for Reducers]
  - Output: $[(k_2, v_2)]$ [Same output as for Mappers]

- **Commutative and Associative computations**
  - Reducer and Combiner code may be interchangeable (e.g. Word Count)
  - This is not true in the general case


A "White snow"
B "White hat"
C "White gloves"
D "Red hat"
E "Red bat"
Mapper
Mapper
Mapper
hat 1
White 1
White 1
snow 1
gloves 1
White 1
bat 1
hat 1
Red 1
Red 1
Combiner
Combiner
Combiner
hat 1
White 2
snow 1
gloves 1
White 1
bat 1
hat 1
Red 2
Shuffle and sort: group values by key
bat 1
gloves 1
White 2 1
hat 1 1
Red 2
snow 1
Reducer
Reducer
bat 1
gloves 1
White 3
hat 2
Red 2
snow 1

Algorithmic Correctness: an Example

- **Problem statement**
  - We have a large dataset where input keys are strings and input values are integers
  - We wish to compute the mean of all integers associated with the same key
    - ⋆ In practice: the dataset can be a log from a website, where the keys are user IDs and values are some measure of activity

- **Next, a baseline approach**
  - We use an **identity mapper**, which groups and sorts appropriately input key-value pairs
  - Reducers keep track of running sum and the number of integers encountered
  - The mean is emitted as the output of the reducer, with the input string as the key

- **Inefficiency problems in the shuffle phase**

Example: Computing the mean

class MAPPER
    method MAP(string t, integer r)
        EMIT(string t, integer r)

class REDUCER
    method REDUCE(string t, integers $[r_1, r_2, \ldots]$)
        $sum \leftarrow 0$
        $cnt \leftarrow 0$
        for all integer $r \in$ integers $[r_1, r_2, \ldots]$ **do**
            $sum \leftarrow sum + r$
            $cnt \leftarrow cnt + 1$
        $r_{avg} \leftarrow sum/cnt$
        EMIT(string t, integer r_{avg})

Algorithmic Correctness

- **Note: operations are not distributive**
  - `Mean`(1,2,3,4,5) $\neq$ `Mean`(`Mean`(1,2), `Mean`(3,4,5))
  - Hence: a combiner cannot output partial means and hope that the reducer will compute the correct final mean
- **Rule of thumb:**
  - Combiners are optimizations, the algorithm should work even when "removing" them

Example: Computing the mean with combiners

```
class Mapper
    method Map(string t, integer r)
        Emit(string t, pair (r, 1))
class Combiner
    method Combine(string t, pairs [(s1, c1), (s2, c2) ...])
        sum ← 0
        cnt ← 0
        for all pair (s, c) ∈ pairs [(s1, c1), (s2, c2) ...] do
            sum ← sum + s
            cnt ← cnt + c
        Emit(string t, pair (sum, cnt))
class Reducer
    method Reduce(string t, pairs [(s1, c1), (s2, c2) ...])
        sum ← 0
        cnt ← 0
        for all pair (s, c) ∈ pairs [(s1, c1), (s2, c2) ...] do
            sum ← sum + s
            cnt ← cnt + c
        r_avg ← sum/cnt
        Emit(string t, integer r_avg)
```

Basic Design Patterns

Algorithm Design

- **Developing algorithms involve:**
  - Preparing the input data
  - Implement the mapper and the reducer
  - Optionally, design the combiner and the partitioner
- **How to recast existing algorithms in "Map Reduce"?**
  - It is not always obvious how to express algorithms
  - Data structures play an important role
  - Optimization is hard
- **Learn by examples**
  - "Design patterns"
  - "Shuffle" is perhaps the most tricky aspect

Algorithm Design

- **Aspects that are not under the control of the designer**
  - *Where* a mapper or reducer will run
  - *When* a mapper or reducer begins or finishes
  - *Which* input key-value pairs are processed by a specific mapper
  - *Which* intermediate key-value pairs are processed by a specific reducer

- **Aspects that can be controlled**
  - Construct data structures as keys and values
  - Execute user-specified initialization and termination code for mappers and reducers
  - Preserve state across multiple input and intermediate keys in mappers and reducers
  - Control the sort order of intermediate keys, and therefore the order in which a reducer will encounter particular keys
  - Control the partitioning of the key space, and therefore the set of keys that will be encountered by a particular reducer

Algorithm Design

- **"Map Reduce" algorithms can be complex**
  - Hadoop MapReduce requires algorithm decomposition in several jobs
  - Apache Spark is much simpler
  - In general, iterative algorithms require a driver

- **Basic design patterns**[7]
  - Local Aggregation
  - Pairs and Stripes
  - Order inversion

[7] You will see them in action during the laboratory sessions.

Local Aggregation

- **In the context of data-intensive distributed processing, the most important aspect of synchronization is the exchange of intermediate results**
  - This involves copying intermediate results from the processes that produced them to those that consume them
  - In general, this involves **data transfers over the network**
  - In Hadoop, also disk I/O is involved, as intermediate results are written to disk
- **Network and disk latencies are expensive**
  - Reducing the amount of intermediate data translates into algorithmic efficiency
- **Combiners and preserving state across inputs**
  - Reduce the number and size of key-value pairs to be shuffled

In-Mapper Combiners

- **In-Mapper Combiners, a possible improvement over vanilla Combiners**
  - Hadoop does not[8] guarantee combiners to be executed
  - Combiners can be costly in terms of CPU and I/O
- **Use a hash map to cumulate intermediate results**
  - The data structure is also know as "associative array" or "dictionary"
  - The array is used to tally up term counts within a single "document"
  - The `Emit` method is called only after all `InputRecords` have been processed
- **Example (see next slide)**
  - The code emits a key-value pair for each unique term in the document

[8] Actually, combiners are not called if the number of map output records is less than a small threshold, *i.e.*, 4

In-Memory Combiners

```
class Mapper
    method Map(offset a, line l)
        H ← new HashMap
        for all term t ∈ line l do
            H{t} ← H{t} + 1
        for all term t ∈ H do
            Emit(term t, count H{t})
```

In-Memory Combiners

- **Taking the idea one step further**
  - Exploit implementation details in Hadoop
  - A Java mapper object is created for each map task
  - JVM reuse must be enabled

- **Preserve state within and across calls to the `Map` method**
  - `Initialize` method, used to create an across-map, persistent data structure
  - `Close` method, used to emit intermediate key-value pairs only when all map task scheduled on one machine are done

In-Memory Combiners

```
class Mapper
    method Initialize
        H ← new HashMap
    method Map(offset a, line l)
        for all term t ∈ line l do
            H{t} ← H{t} + 1
    method Close
        for all term t ∈ H do
            Emit(term t, count H{t})
```

In-Memory Combiners

- **Summing up: a first "design pattern", *in-memory combining***
  - Provides control over when local aggregation occurs
  - Designer can determine how exactly aggregation is done

- **Efficiency vs. Combiners**
  - There is no additional overhead due to the materialization of key-value pairs
    - Un-necessary object creation and destruction (garbage collection)
    - Serialization, deserialization when memory bounded
  - With combiners, mappers still need to emit all key-value pairs; combiners "only" reduce network traffic

In-Memory Combiners

- **Precautions**
  - In-memory combining breaks the functional programming paradigm due to **state preservation**
  - Preserving state across multiple instances implies that algorithm behavior might depend on execution order
    - Works well with commutative / associative operations
    - Otherwise, order-dependent bugs are difficult to find
- **Memory capacity is limited**
  - In-memory combining strictly depends on having sufficient memory to store intermediate results
  - A possible solution: "block" and "flush"

Further Remarks

- **The extent to which efficiency can be increased with local aggregation depends on the size of the intermediate key space**
  - Opportunities for aggregation arise when multiple values are associated to the same keys

- **Local aggregation also effective to deal with reduce stragglers**
  - Reduce the number of values associated with frequently occurring keys

Computing the average, with in-mapper combiners

- Partial sums and counts are held in memory (across inputs)
- Intermediate values are emitted only after the entire input split is processed
- The output value is a pair

```
class Mapper
    method Initialize
        S ← new HashMap
        C ← new HashMap
    method Map(term t, integer r)
        S{t} ← S{t} + r
        C{t} ← C{t} + 1
    method Close
        for all term t ∈ S do
            Emit(term t, pair (S{t}, C{t}))
```

Pairs and Stripes

- **A common approach in MapReduce: build complex keys**
  - Use the framework to group data together
- **Two basic techniques:**
  - *Pairs*: similar to the example on the average
  - *Stripes*: uses in-mapper memory data structures
- **Next, we focus on a particular problem that benefits from these two methods**

Problem statement

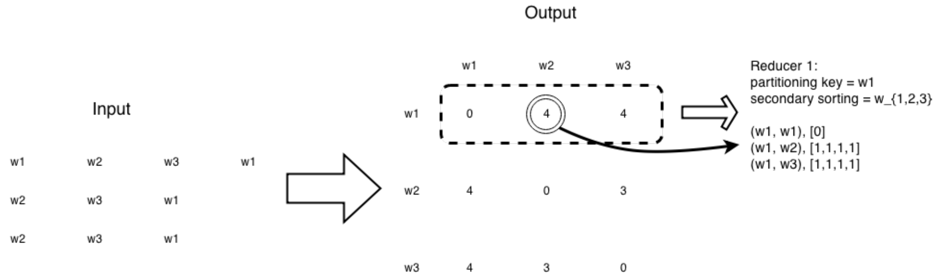
- **The problem: building word co-occurrence matrices for large corpora**
  - The co-occurrence matrix of a corpus is a square $n \times n$ matrix, M
  - n is the number of unique words (*i.e.*, the vocabulary size)
  - A cell m_{ij} contains the number of times the word w_i co-occurs with word w_j *within a specific context*
  - Context: a sentence, a paragraph a document or a window of m words
  - NOTE: the matrix may be symmetric in some cases

- **Motivation**
  - This problem is a basic building block for more complex operations
  - Estimating the distribution of discrete joint events from a large number of observations
  - Similar problem in other domains:
    - ⋆ Customers who buy *this* tend to also buy *that*

Observations

- **Space requirements**
  - Clearly, the space requirement is $O(n^2)$, where n is the size of the vocabulary
  - For real-world (English) corpora n can be hundreds of thousands of words, or even billions of worlds in some specific cases
- **So what's the problem?**
  - If the matrix can fit in the memory of a single machine, then just use whatever naive implementation
  - Instead, if the matrix is bigger than the available memory, then paging would kick in, and any naive implementation would break
- **Compression**
  - Such techniques can help in solving the problem on a single machine
  - However, there are scalability problems

Word co-occurrence: the Pairs approach



Word co-occurrence: the Pairs approach

- **Input to the problem**
  - Key-value pairs in the form of a `offset` and a `line`

- **The mapper:**
  - Processes each input document
  - Emits key-value pairs with:
    - Each co-occurring word pair as the key
    - The integer one (the count) as the value
  - This is done with two nested loops:
    - The outer loop iterates over all words
    - The inner loop iterates over all neighbors

- **The reducer:**
  - Receives pairs related to co-occurring words
    - This requires **modifying the partitioner**
  - Computes an absolute count of the joint event
  - Emits the pair and the count as the final key-value output
    - Basically reducers emit the cells of the output matrix

Word co-occurrence: the Pairs approach

```
class Mapper
    method Map(offset a, line l)
        for all term w ∈ line l do
            for all term u ∈ Neighbors(w) do
                Emit (pair (w, u), count 1)
class Reducer
    method Reduce(pair p, counts [c1, c2, ···])
        s ← 0
        for all count c ∈ counts [c1, c2, ···] do
            s ← s + c
        Emit (pair p, count s)
```

Word co-occurrence: the Stripes approach

Word co-occurrence: the Stripes approach

- **Input to the problem**
  - Key-value pairs in the form of a `offset` and a `line`

- **The mapper:**
  - Same two nested loops structure as before
  - Co-occurrence information is first stored in a hash map
  - Emit key-value pairs with words as keys and the corresponding hash maps as values

- **The reducer:**
  - Receives all hash maps related to the same word
  - Performs an element-wise sum of all hash maps with the same key
  - Emits key-value output in the form of (word, hash map)
    - Basically, reducers emit **rows** of the co-occurrence matrix

Word co-occurrence: the Stripes approach

```
class Mapper
    method Map(offset a, line l)
        for all term w ∈ line l do
            H ← new HashMap
            for all term u ∈ Neighbors(w) do
                H{u} ← H{u} + 1
            Emit (term w, Stripe H)
class Reducer
    method Reduce(term w, Stripes [H1, H2, H3 ···])
        Hf ← new HashMap
        for all Stripe H ∈ Stripes [H1, H2, H3 ···] do
            Sum(Hf, H)
        Emit (term w, Stripe Hf)
```

Pairs and Stripes, a comparison

- **The pairs approach**
  - Generates a large number of key-value pairs
    - In particular, intermediate ones, that fly over the network
  - The benefit from combiners is limited, as it is less likely for a mapper to process multiple occurrences of a word
  - Does not suffer from memory paging problems

- **The stripes approach**
  - More compact
  - Generates fewer and shorted intermediate keys
    - The framework has less sorting to do
  - The values are more complex and have serialization / deserialization overhead
  - Greatly benefits from combiners, as the key space is the vocabulary
  - Suffers from memory paging problems, if not properly engineered

Computing relative frequencies

- **"Relative" Co-occurrence matrix construction**
  - Similar problem as before, same matrix
  - Instead of absolute counts, we take into consideration the fact that some words appear more frequently than others
    - Word w_i may co-occur frequently with word w_j simply because one of the two is very common
  - We need to convert absolute counts to relative frequencies $f(w_j|w_i)$
    - What proportion of the time does w_j appear in the context of w_i?

- **Formally, we compute:**

$$f(w_j|w_i) = \frac{N(w_i, w_j)}{\sum_{w'} N(w_i, w')}$$

  - $N(\cdot, \cdot)$ is the number of times a co-occurring word pair is observed
  - The denominator is called the marginal

Computing relative frequencies

- **The stripes approach**
  - In the reducer, the counts of all words that co-occur with the conditioning variable (w_i) are available in the hash map
  - Hence, the sum of all those counts gives the marginal
  - Then we divide the joint counts by the marginal and we're done

- **The pairs approach**
  - The reducer receives the pair (w_i, w_j) and the count
  - From this information alone **it is not possible** to compute $f(w_j|w_i)$
  - Fortunately, as for the mapper, also the reducer can preserve state across multiple keys
    - We can buffer in memory all the words that co-occur with w_i and their counts
    - This is basically building the hash map in the stripes method

Computing relative frequencies: a basic approach

- **We must define the sort order of the pair**
  - In this way, the keys are first sorted by the left word, and then by the right word (in the pair)
  - Hence, we can detect if all pairs associated with the word we are conditioning on (w_i) have been seen
  - At this point, we can use the in-memory buffer, compute the relative frequencies and emit

- **We must define an appropriate partitioner**
  - The default partitioner is based on the hash value of the intermediate key, modulo the number of reducers
  - For a complex key, the **raw byte representation** is used to compute the hash value
    - ★ Hence, there is no guarantee that the pair (dog, aardvark) and (dog,zebra) are sent to the same reducer
  - What we want is that all pairs with the same left word are sent to the same reducer

Computing relative frequencies: order inversion

- **The key is to properly sequence data presented to reducers**
  - If it were possible to compute the marginal in the reducer before processing the joint counts, the reducer could simply divide the joint counts received from mappers by the marginal
  - The notion of "before" and "after" can be captured in the ordering of key-value pairs
  - The programmer can define the sort order of keys so that data needed earlier is presented to the reducer before data that is needed later

Computing relative frequencies: order inversion

- **Recall that mappers emit pairs of co-occurring words as keys**

- **The mapper:**
  - additionally emits a "special" key of the form $(w_i, *)$
  - The value associated to the special key is one, that represents the contribution of the word pair to the marginal
  - Using combiners, these partial marginal counts will be aggregated before being sent to the reducers

- **The reducer:**
  - We must make sure that the special key-value pairs are processed before any other key-value pairs where the left word is w_i
  - We also need to modify the partitioner as before, *i.e.*, it would take into account only the first word

Computing relative frequencies: order inversion

- **Memory requirements:**
  - Minimal, because only the marginal (an integer) needs to be stored
  - No buffering of individual co-occurring word
  - No scalability bottleneck
- **Key ingredients for order inversion**
  - Emit a special key-value pair to capture the marginal
  - Control the sort order of the intermediate key, so that the special key-value pair is processed first
  - Define a custom partitioner for routing intermediate key-value pairs
  - Preserve state across multiple keys in the reducer