

For a pair of bipartite networks expressed as an $m \times n$ matrices \mathbf{A}, \mathbf{B} , we seek to find an $m \times m$ matrix, \mathbf{T} , such that we minimize the loss function

$$L(\mathbf{T}) = \sum_{gene=1}^N \|\mathbf{b}_{gene} - \mathbf{a}_{gene} \mathbf{T}\|^2$$

First calculate covariance matrix, $cov(\mathbf{A}, \mathbf{B})$.

This (and later steps) is made easier by centering and scaling both \mathbf{A} and \mathbf{B} . So define

$$\mathbf{S}_{m \times m} = \mathbf{B}^T \mathbf{A}$$

And calculate the Singular Value Decomposition (SVD), such that

$$\mathbf{S} = \mathbf{U} \mathbf{D} \mathbf{V}^T$$

Define a vector, \mathbf{c} , such that

$$\mathbf{c}_k = \sum_{gene=1}^N [\mathbf{a}_{gene} \mathbf{V}]_k [\mathbf{b}_{gene} \mathbf{U}]_k$$

and a corresponding diagonal matrix, \mathbf{E} with diagonal equal to $sign(\mathbf{c})$.

We then define

$$\mathbf{T} = \mathbf{V} \mathbf{E} \mathbf{U}^T$$

and \mathbf{T} is the solution to the least squares transition matrix problem.