Robotics and Computer Vision

January 5th, 2021

Basic information:

- Points Robotics: 60
- Points Computer Vision: 60
- Total time: 4 hours, 2h Robotics, 2h Vision
- Questions indicate the number of points you can earn. 1point = 2minutes. Coordinate your effort accordingly.
- Use of the Internet is NOT allowed during the exam.
- Use of any files stored on your computer or USB stick (except the document to hand in and the Matlab library rvctools) is NOT allowed during the exam.
- Final, single, pdf document must be handed in on Digital Exam.
- For computation questions (marked with):
    - Please provide the:
        - MATLAB (or similar) expressions used or
        - give your steps (equations) used to arrive at the result.
    - Make sure to include all the derivations so that it is clearly seen why something goes wrong. A result without an argument for arriving at that result has essentially no value.
    - **Highlight your final result.**
- We would prefer you to use the option to scan material during the exam sparingly. Questions, where we feel scanning might be required, are marked with .
- All multiple-choice questions in this exam have one and only one correct answer.
- This assignment has 11 pages in total.

Part PR1 – MCQ (2 points per question)

R1. The following equation represents:

$$\begin{bmatrix} \dfrac{\partial x}{\partial \vartheta_1} & \dfrac{\partial x}{\partial \vartheta_2} \\ \dfrac{\partial y}{\partial \vartheta_1} & \dfrac{\partial y}{\partial \vartheta_2} \end{bmatrix}$$

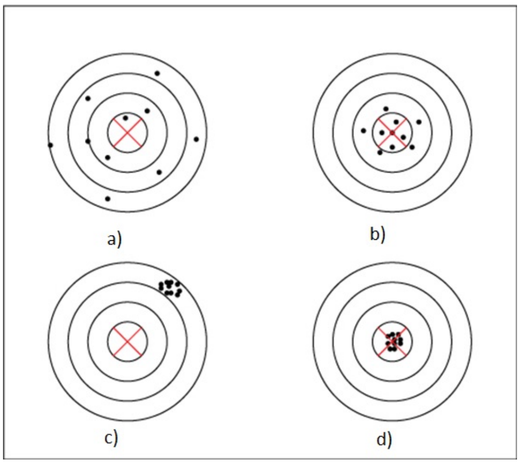
a. The Jacobian of a 4 DOF mechanism,

b. The Jacobian of a 2 DOF mechanism,

c. The transformation function of a 4 DOF mechanism,

d. The transformation function of a 2 DOF mechanism.

R2. Which figure represents <u>good</u> accuracy and <u>bad</u> repeatability?



a.

b.

c.

d.

R3. The equivalent angle axis representation of orientation is defined as:

a. [unit vector, RPY],
b. [unit vector, angle],
c. [rotation matrix, angle],
d. [angle, quaternion].

R4. The definition of quaternion normalization is given as:

a. $q' = \frac{q}{|q|}$,
b. $q' = \sqrt{w^2 + v^2}$,
c. $q' = |q + q|$,
d. $q' = qq^*$.

R5. For planning applications using the RRT algorithm, the parameter ϵ controls the:

a. number of obstacles in configuration space,
b. distance covered by RRT in one planning step,
c. number of RRT executions,
d. manipulators goal configuration.

R6. Let's consider two quaternions $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$, if the distance between them is small, we can simplify the interpolation to:

a. Multiplication of two quaternions
b. Addition of two quaternions
c. Linear interpolation between two points.
d. The logarithm of two quaternions.

Part PR2 – Answer the following questions with a detailed explanation

R7. Explain the difference between linear interpolation, and cubic polynomial interpolation between 3 consecutive frames:

a. Give a written explanation of the two methods **(3 points)**,

b. Explain the difference based on the velocity, acceleration and jerk **(3 points)**,

c. Sketch the position, velocity, acceleration and jerk trajectories for the two methods **(4 points)**.

R8. For the given equation $J(q)\Delta q = \Delta u$, describe the following:

a. What does $J(q)$ stand for and how do we calculate it **(2.5 points)**?

b. This equation can be related with the Newton – Raphson algorithm. In relation to the algorithm answer the following questions:

  i. Describe the main idea of the algorithm **(2 point)**?

  ii. Write the pseudo- code representation, describing the algorithm **(4 points).**

  iii. Describe the right hand side of the algorithm's main equation **(2.5 points)**?

c. Can we use the equation also with redundant manipulators, what happens with $J(q)$ in this case? **(2 points)**.

R9. Workspace representations:

a. Sketch and give a description of the dexterous workspace of a 2 DOF manipulator **(2.5 points)**.

b. Sketch and describe the difference between a 2 and 6 DOF manipulator's reachable workspace **(2.5 points)**.

Part PR3 – Computation

R10. Compute a linear interpolation between 4 given transformations in Tool space with the following parameters:

```
T1 =[1,0,0,0;0,1,0,0;0,0,1,0;0,0,0,1];
T2 =[0.7071,0,0.7071,0;0.7071,0,-0.7071,2; 0,1,0,1;0,0,0,1];
T3 =[1,0,0,1;0,-1,0,0;0,0,-1,0.5;0,0,0,1];
T4 =[0.5,-0.8660,0,1;0.8660,0.5,0,2;0,0,1,0.5;0,0,0,1];
Samp = 500;
dt= 1/Samp;
Time1=2; %interpolation time [s] from T1 to T2
Time2=2; %interpolation time [s] from T2 to T3
Time3=3; %interpolation time [s] from T3 to T4
```

a. Calculate the linear interpolation between the given poses, use Matlab and the library (rcvtools) provided at the lectures. Plot the entire sequence of interpolations as a continuous trajectory (T1-T4), with subplots. Poses and orientations, should be plotted separately for each axis. Regarding orientation, for calculation purposes use the rotation matrix representation and for plotting convert them to RPY. **Paste the code and plots into the document**. *Every subplot* should contain: the name of the axis plotted and time – use the example code provided on the next page **(10 points).**

b. Calculate and plot the linear and angular (from RPY) velocities, as well as the accelerations for each axis. Use the matlab inbuild functions. **Paste the code and plots into the document.** *Every subplot* should contain: the name of the axis plotted and time – modify the example code **(10 points)**.

```
%%Example Code

figure(1)
%% positions
subplot(2,3,1)
H1=plot(Time,TrjP(:,1),'LineWidth',1.5);
ylabel('$X_{pos}$ [m]','interpreter','latex')
xlabel('$Time$ [s]','interpreter','latex')
legend([H1],'Position trajectory','Location','southwest');
grid on
subplot(2,3,2)
plot(Time,TrjP(:,2),'LineWidth',1.5)
ylabel('$Y_{pos}$ [m]','interpreter','latex')
xlabel('$Time$ [s]','interpreter','latex')
grid on
subplot(2,3,3)
plot(Time,TrjP(:,3),'LineWidth',1.5)
ylabel('$Z_{pos}$ [m]','interpreter','latex')
xlabel('$Time$ [s]','interpreter','latex')
grid on

%% orientations in RPY
subplot(2,3,4)
H1=plot(Time,TrjO(:,1),'LineWidth',1.5);
ylabel('$Roll$ [rad]','interpreter','latex')
legend([H1],'Orientation trajectory','Location','southwest');
xlabel('$Time$ [s]','interpreter','latex')
grid on
subplot(2,3,5)
plot(Time,TrjO(:,2),'LineWidth',1.5)
ylabel('$Pitch$ [rad]','interpreter','latex')
xlabel('$Time$ [s]','interpreter','latex')
grid on
subplot(2,3,6)
plot(Time,TrjO(:,3),'LineWidth',1.5)
ylabel('$Yaw$ [rad]','interpreter','latex')
xlabel('$Time$ [s]','interpreter','latex')
grid on
```

Part PV1 – Multiple choice questions

V1. What are the inhomogeneous versions of the following vectors $\begin{bmatrix} 8 \\ 6 \\ -1 \end{bmatrix}, \begin{bmatrix} 8 \\ 6 \\ 0.5 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -1 \\ 10 \end{bmatrix}, \begin{bmatrix} -9 \\ 6 \\ 9 \\ 3 \end{bmatrix}$? **(2 points)**

a. $\begin{bmatrix} -8 \\ -6 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 20 \\ 30 \\ -10 \\ 1 \end{bmatrix}, \begin{bmatrix} -27 \\ 18 \\ 27 \\ 1 \end{bmatrix}$

b. $\begin{bmatrix} 8 \\ 6 \end{bmatrix}, \begin{bmatrix} 8 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} -9 \\ 6 \\ 9 \end{bmatrix}$

c. $\begin{bmatrix} -8 \\ -6 \\ 1 \end{bmatrix}, \begin{bmatrix} 16 \\ 12 \\ 1 \end{bmatrix}, \begin{bmatrix} 0.2 \\ 0.3 \\ -0.1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 3 \\ 1 \end{bmatrix}$

d. $\begin{bmatrix} -8 \\ -6 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 20 \\ 30 \\ -10 \end{bmatrix}, \begin{bmatrix} -27 \\ 18 \\ 27 \end{bmatrix}$

e. $\begin{bmatrix} -8 \\ -6 \end{bmatrix}, \begin{bmatrix} 16 \\ 12 \end{bmatrix}, \begin{bmatrix} 0.2 \\ 0.3 \\ -0.1 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 3 \end{bmatrix}$

V2. Stereo matching: Stereo matching relies heavily on the assumption that the surfaces in the scene are Lambertian. Some objects/materials do not adhere to that assumption and create problems for stereo thereby. Which of the following materials/objects **do adhere to it**? **(2 points)**

a. A transparent glass object

b. A highly reflective, slightly textured metal surface

c. A uniform white coloured piece of paper

V3. Which of the following is a valid projection matrix (P) for a camera? (There is no typo in this question/answers!) **(2 points)**

a. $\begin{pmatrix} 0 & 0 & 1.0 & -0.5 \\ 0 & 1.0 & 0 & 0 \\ -1.0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & 1.0 \end{pmatrix}$

b. $\begin{pmatrix} 300.0 & 0 & 900.0 & 1050.0 \\ 300.0 & -900.0 & 0 & -300.0 \\ 1.0 & 0 & 0 & 2.0 \end{pmatrix}$

c. $\begin{pmatrix} 0 & 0 & 1.0 \\ 0 & 1.0 & 0 \\ -1.0 & 0 & 0 \end{pmatrix}$

d. $\begin{pmatrix} 1234.0 & 0 & 700.0 & 0 \\ 0 & 1234.0 & 700.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

V4. Which parameters does the P3P approach estimate? **(2 points)**

a. Only the object position
b. Only the object orientation
c. The object position + object orientation
d. The camera intrinsics
e. All of the above

V5. What does the RANSAC algorithm do? **(2 points)**

a. Given an image area in the left and one in the right image, it computes the similarity of both.

b. It resamples point densities in the Particle filter by equalising the distribution of particles in space

c. It computes the binary patterns used for a structured light scanner

d. It computes the rigid transformation between two matched point sets which minimizes the least squared error

e. Probabilistically finds the parameters for the best-fitting model to noisy data

V6. How many corresponding 2D point pairs are minimum needed to estimate a homography (represented as a 3x3 matrix)? **(2 points)**

a. 2

b. 3

c. 4

d. 5

e. 9

f. 10

V7. What is the difference between an ordered and an unordered point cloud? **(2 points)**

a. In an ordered point cloud, the points are sorted according to distance to the sensor

b. In an ordered point cloud, the points are sorted according to the grey-scale-intensity / colour information

c. In an ordered point cloud, the neighbourhood relationship between points is known

Part PV2 – Short answer questions

V8. Camera intrinsics: What is the meaning/use of the individual intrinsic parameters $(f, \alpha_u, \alpha_v, s, u_0, v_0)$? And what units are each of them measured in? **(4 points)**

$$K*A = \begin{pmatrix} f*\alpha_u & f*\alpha_u*s & u_0 & 0 \\ 0 & f*\alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

V9. Euclidian segmentation**:** What are the steps in Euclidian clustering? **(3 points)**

V10. Particle filter: Which problem can the particle filter deal with that none of the discussed Kalman filters can deal with? **(2 points)**

V11. What do we know about the location of a 3D point M if we have the corresponding 2D point m on the image plane? (You can assume that you know the projection matrix P.) **(2 points)**

V12. Homography based 2D -> 3D pose estimation: What are the steps involved in estimating the pose of a planar object relative to the camera if we have the homography H (mapping from planar object to camera) and $K * \bar{A}$ (K and A are from the projection matrix formulation, the bar over $\bar{A}$ indicates that the last column has been removed, compared to A).? **(5 points)**

Part PV3 – Computation

V13. Compute the location in the image m where the 3D point M will appear, given the projection Matrix P: **(4 points)**

$$P = \begin{pmatrix} -700.0 & 0 & 1234.0 & 83.0 \\ -700.0 & 1234.0 & 0 & 700.0 \\ -1.0 & 0 & 0 & 1.0 \end{pmatrix}$$
$$M = \begin{pmatrix} -14.0 \\ 6.0 \\ 2.0 \end{pmatrix}$$

```
P = [ -700 0 1234 83; -700 1234 0 700; -1 0 0 1]
M = [ -14; 6; 2]
```

V14. Pose estimation: Estimate the transformation T that aligns the five 3D points in P with the corresponding points in Q: **(10 points)**

$$P = \begin{pmatrix} 31.0 & 33.0 & 21.0 & 45.0 & 25.0 \\ 42.0 & 32.0 & 21.0 & 45.0 & 25.0 \\ 53.0 & 31.0 & 21.0 & 45.0 & 22.0 \end{pmatrix}$$
$$Q = \begin{pmatrix} 58.0 & 36.0 & 26.0 & 50.0 & 27.0 \\ -25.0 & -27.0 & -15.0 & -39.0 & -19.0 \\ -35.0 & -25.0 & -14.0 & -38.0 & -18.0 \end{pmatrix}$$

```
P = [31 33 21 45 25; 42 32 21 45 25; 53 31 21 45 22]
Q = [58.0 36.0 26.0 50.0 27.0; -25.0 -27.0 -15.0 -39.0 -19.0;
-35.0 -25.0 -14.0 -38.0 -18.0]
```

V15. Uncertainty propagation: Propagate uncertainties from the scalar a and the vector $b = \begin{pmatrix} b_x \\ b_y \end{pmatrix}$ to $f = \begin{pmatrix} a^2 * \mathrm{b_y} \\ a^3 * b_x^2 \end{pmatrix}$. $a = 5, b = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $var_a = 5, cov_b = \begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix}$. (Compute the covariance of f). a and b are independent. **(9 points)**

(HINT: $\frac{d}{dx} x^a = a x^{a-1}$.)

```
a = 5
b = [3 2]
var_a = 5
cov_b = [4 1; 2 5]
```

Part PV4 – Drawing

V16. Make a drawing of a stereo setup in case of a **rectified system**. Indicate: **(7 points)**

a. The camera centres (C_L, C_R)

b. A 3D point(P)

c. The image planes

d. The projections onto the image planes (x_L, x_R)

e. To compute the depth of the 3D point three components are required. One is disparity, indicate the other two on the image.

f. The depth Z of the point P.