Resampling worksheet for sensitivity analysis of movement rates and recruitment rates on per-capita growth rate of Big Head Carp in the Illinois River

```
(*Get directory of notebook*)
dir = NotebookDirectory[];
(*Set working directory to 'dir'*)
SetDirectory[dir];
(*Load posterior samples of movement probs,
see README.md for details*)
pPSI = Import["psi_bhcp.csv", "CSV", "HeaderLines" → 1];
```

In[•]:=
```
Dimensions[pPSI]
```

```
(*Create function to turn into a transition matrix*)
getpsi[x_] :=
 ArrayReshape[{0, pPSI[[x, 1 ;; 6]], 0,
    pPSI[[x, 7 ;; 12]], 0, pPSI[[x, 13 ;; 18]], 0,
    pPSI[[x, 19 ;; 24]], 0, pPSI[[x, 25 ;; 30]], 0},
   {6, 6}]ᵀ
```

In[•]:=
```
μpsi = Median[pPSI]
```

In[•]:=
```
meanpsi =
 ArrayReshape[{0, μpsi[[1 ;; 6]], 0, μpsi[[7 ;; 12]],
    0, μpsi[[13 ;; 18]], 0, μpsi[[19 ;; 24]], 0,
    μpsi[[25 ;; 30]], 0}, {6, 6}]ᵀ

(*examine output in matrix-form*)
meanpsi // MatrixForm
```

```
(*Create month based matrix*)

In[•]:= A1 = Table[ϕ[i, j], {j, 6, 1, -1}, {i, 6, 1, -1}]

In[•]:= (*Replace the diagonal elements with 1-
 off diagonal*)
A1[[1, 1]] = 1 - Total[A1[[2 ;; 6, 1]]]
A1[[6, 6]] = 1 - Total[A1[[1 ;; 5, 6]]]
A1[[2, 2]] =
 1 - Total[Flatten[{A1[[1, 2]], A1[[3 ;; 6, 2]]}]]
A1[[3, 3]] =
 1 - Total[Flatten[{A1[[1 ;; 2, 3]], A1[[4 ;; 6, 3]]}]]
A1[[4, 4]] =
 1 - Total[Flatten[{A1[[1 ;; 3, 4]], A1[[5 ;; 6, 4]]}]]
A1[[5, 5]] =
 1 - Total[Flatten[{A1[[1 ;; 4, 5]], A1[[6, 5]]}]]

In[•]:= (*Create seasonal matrix*)
A2 = A1.A1.A1;

In[•]:= (*Create full matrix*)
A3 =
  A2.A2.(A2 + DiagonalMatrix[{b6, b5, b4, b3, b2, b1}]).
   A2;
vars = Drop[Flatten[A1], {1, 36, 7}];

(*Create function to sample posterior and
 calculate sensitive of each ϕ*)
```

```
(*test*)
ind = 34;
phiests = getpsi[ind];
params =
  Flatten[Table[ϕ[i, j] → phiests[[7 - j, 7 - i]],
    {j, 6, 1, -1}, {i, 6, 1, -1}]];
params2 = Drop[params, {1, 36, 7}];
A1 /. params2 // MatrixForm

(*Create a resampling function for transition
 probabilites*)
sensResamp[bz_] := Block[{ind},
  ind = RandomInteger[{1, 30 000}];
  phiests = getpsi[ind];
  params =
   Flatten[Table[ϕ[i, j] → phiests[[7 - j, 7 - i]],
     {j, 6, 1, -1}, {i, 6, 1, -1}]];
  params2 = Drop[params, {1, 36, 7}];
  Do[
   ev[i] = Eigenvalues[
     A3 /. Dispatch[
       Flatten[{{b3 → 0, b2 → 0, b1 → 0, b4 → bz,
          b5 → bz, b6 → bz}, Drop[params2, {i}]}]]],
   {i, 1, 30}];
  pos = Flatten[Table[Ordering[ev[i] /. params2, -1],
     {i, 1, 30}]];
  sens = Table[D[ev[i][[pos[[i]]]], vars[[i]]] /.
     params2, {i, 1, 30}];
  Return[sens]
 ]

(*Run it for 50 samples about 2 min each*)
```

```
iter = 50;
```

In[•]:=
```
Do[sphi[i] = sensResamp[2], {i, 1, iter}]
```

```
(*Put results in table form*)
```

In[•]:=
```
sphitab = Table[sphi[i], {i, 1, iter}];
```

In[•]:=
```
λ1 =
 Eigenvalues[
   A3 /.
    Dispatch[
     Flatten[{{b3 → 0, b2 → 0, b1 → 0, b4 → 2, b5 → 2,
        b6 → 2}, params2}]]][[1]]
```

```
(*Calculate mean and standard error of regional
 elasticity*)
elast = Mean[(sphitab^T * μpsi / λ1)^T]
elastse = StandardDeviation[(sphitab^T * μpsi / λ1)^T] /
  Sqrt[iter]
(*Calculate mean and standard error of regional
 sensitivities*)
```

In[•]:=
```
sensitivites = Mean[sphitab]
sensitivitesse =
 StandardDeviation[sphitab] / Sqrt[iter]
(*Output the results*)
Export["psi_sensi.csv",
 {vars, sensitivites, sensitivitesse, elast, elastse},
 "CSV"]
```

```
(*Function for doing sensitivity of recruitment
  rates b on metapopulation growth-rate*)
sensResampl[bz_] := Block[{ind},
  ind = RandomInteger[{1, 30 000}];
  phiests = getpsi[ind];
  params =
   Flatten[Table[ϕ[i, j] → phiests[[7 - j, 7 - i]],
     {j, 6, 1, -1}, {i, 6, 1, -1}]];
  params2 = Drop[params, {1, 36, 7}];
  btab = {b6, b5, b4};
  evb = Eigenvalues[
    A3 /. Dispatch[
      Flatten[{{b3 → 0, b2 → 0, b1 → 0}, params2}]]];
  pos =
   Flatten[
    Table[Ordering[evb /. params2 /.
       {b4 → bz, b5 → bz, b6 → bz}, -1], {i, 1, 3}]];
  sensb =
   Table[D[evb[[pos[[i]]]], btab[[i]]] /. params /.
     {b4 → bz, b5 → bz, b6 → bz}, {i, 1, 3}];
  Return[sensb]
 ]
```

In[•]:=
```
Do[sb[i] = sensResampl[1], {i, 1, iter}]
```

```
(*Put in table form*)
sbtab = Table[sb[i], {i, 1, iter}];
```

In[•]:=
```
Mean[sbtab]
StandardDeviation[sbtab] / Sqrt[iter]
```

In[•]:=
```
Mean[sbtab / λ1]
StandardDeviation[sbtab / λ1] / Sqrt[iter]
```

```
(*Output the results*)
Export["b_sensi.csv",
 {btab, Mean[sbtab],
  StandardDeviation[sbtab] / Sqrt[iter],
  Mean[sbtab / λ1], StandardDeviation[sbtab / λ1] /
   Sqrt[iter]}, "CSV"]


(*Function for calculating local sesitivity of
 movement rates on Pool 1*)
```

In[•]:=
```
spsilocal := Block[{ind},
  ind = RandomInteger[{1, 30 000}];
  phiests = getpsi[ind];
  params =
   Flatten[Table[ϕ[i, j] → phiests[[7 - j, 7 - i]],
     {j, 6, 1, -1}, {i, 6, 1, -1}]];
  params2 = Drop[params, {1, 36, 7}];
 sens1 =
   Table[
    D[A3[[6]] /. Dispatch[
          Flatten[{{b3 → 0, b2 → 0, b1 → 0, b4 → 1,
             b5 → 1, b6 → 1}, Drop[params2, {i}]}],
          {i, 1, 30}], vars[[i]]] /. params2 /.
     {b4 → 1, b5 → 1, b6 → 1}, {i, 1, 30}];
  Return[sens1]]
```

In[•]:=
```
Do[spil[i] = spsilocal, {i, 1, iter}]
```

```
r2 = A3[[6]] /. μparams2 /.
  {b3 → 0, b2 → 0, b1 → 0, b4 → 1, b5 → 1, b6 → 1}
μparams =
  Flatten[Table[ϕ[i, j] → meanpsi[[7 - j, 7 - i]],
    {j, 6, 1, -1}, {i, 6, 1, -1}]];
μparams2 = Drop[μparams, {1, 36, 7}]

(*Calculate Mean and standard error local
 sensitivity and elasticity*)

In[•]:= spilmean = Table[Mean[Table[spil[i][[j]], {i, 1, 50}]],
    {j, 1, 30}];
spilse =
  Table[
   StandardDeviation[Table[spil[i][[j]], {i, 1, 50}]] /
    Sqrt[iter], {j, 1, 30}];

In[•]:= espilmean =
  Table[
   Mean[Table[spil[i][[j]] / r2 * vars[[j]] /. μparams2,
     {i, 1, 50}]], {j, 1, 30}];
espilse =
  Table[
   StandardDeviation[
     Table[spil[i][[j]] / r2 * vars[[j]] /. μparams2,
      {i, 1, 50}]] / Sqrt[iter], {j, 1, 30}];


(*Export the results*)
```

```
Export["phi_sensi_local.csv", spilmean, "CSV"]
Export["phi_sensi_local_se.csv", spilse, "CSV"]
Export["phi_elast_local.csv", espilmean, "CSV"]
Export["phi_eslast_local_se.csv", espilse, "CSV"]
```

Make tables of results for inspection

In[•]:=
```
rnames =
 Drop[Flatten[Table[ϕi,j, {j, 6, 1, -1}, {i, 6, 1, -1}]],
  {1, 36, 7}]
```

In[•]:=
```
TableForm[Round[spilmean, 0.001],
 TableHeadings →
  {rnames, {"r61", "r51", "r41", "r31", "r21", "r11"}}]
```

In[•]:=
```
TableForm[Round[espilmean, .001],
 TableHeadings →
  {rnames, {"r61", "r51", "r41", "r31", "r21", "r11"}}]
```

```
(*Create function to calculate sensitivity of
 recritment on local rates*)
sblocal := Block[{ind},
  ind = RandomInteger[{1, 30 000}];
  phiests = getpsi[ind];
  params =
   Flatten[Table[ϕ[i, j] → phiests[[7 - j, 7 - i]],
     {j, 6, 1, -1}, {i, 6, 1, -1}]];
  params2 = Drop[params, {1, 36, 7}];
  btab = {b6, b5, b4};
  sensbl =
   Table[
    D[A3[[6]] /. Dispatch[
          Flatten[{{b3 → 0, b2 → 0, b1 → 0}, params2}]],
       btab[[i]]] /. params2 /. {b4 → 1, b5 → 1, b6 → 1},
    {i, 1, 3}];
  Return[sensbl]]
```

In[•]:=
```
Do[sbl[i] = sblocal, {i, 1, iter}]
```

In[•]:=
```
Table[Mean[Table[sbl[i][[j]], {i, 1, 50}]],
 {j, 1, 3}]
```

In[•]:=
```
Table[Mean[Table[sbl[i][[j]] / r2, {i, 1, 50}]],
 {j, 1, 3}]
```

```
(*Calculate Mean and SE of sensitivites and
 elasticities on local rates*)
```

In[•]:=
```
sblmean = Table[Mean[Table[sbl[i][[j]], {i, 1, 50}]],
    {j, 1, 3}];
sblse =
  Table[
   StandardDeviation[Table[sbl[i][[j]], {i, 1, 50}]] /
    Sqrt[iter], {j, 1, 3}];
```

In[•]:=
```
eblmean =
  Table[Mean[Table[sbl[i][[j]] / r2, {i, 1, 50}]],
   {j, 1, 3}];
eblse =
  Table[
   StandardDeviation[Table[sbl[i][[j]] / r2,
      {i, 1, 50}]] / Sqrt[iter], {j, 1, 3}];
```

```
(*Export the results*)
Export["b_sensi_local.csv", sblmean, "CSV"]
Export["b_sensi_local_se.csv", sblse, "CSV"]
Export["b_elast_local.csv", eblmean, "CSV"]
Export["b_eslast_local_se.csv", eblse, "CSV"]
```