

# States of an Ensemble of Two-Level Atoms with Reduced Quantum Uncertainty

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We generate entangled states of an ensemble of  $5 \times 10^4$   $^{87}\text{Rb}$  atoms by optical quantum nondemolition measurement. The resonator-enhanced measurement leaves the atomic ensemble, prepared in a superposition of hyperfine clock levels, in a squeezed spin state. By comparing the resulting reduction of quantum projection noise [up to 8.8(8) dB] with the concomitant reduction of coherence, we demonstrate a clock input state with spectroscopic sensitivity 3.0(8) dB beyond the standard quantum limit.

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Atomic clocks [1–3] and atom interferometers [4] are reaching the standard quantum limit (SQL) of precision [1,5,6], set by the quantum projection noise inherent in measurements on a collection of uncorrelated particles. In the canonical Ramsey interferometer with  $N_0$  particles, a quantum mechanical phase is converted into occupation probabilities for two states and read out as a population difference  $N$  between them. Entanglement can reduce the projection noise  $\Delta N$  by redistributing it to another variable that does not directly affect the experiment precision. The resulting “squeezed spin state” [5–16] can be used as an input state to an interferometer to overcome the SQL [5,6,8,9].

Formally, the system can be described by an ensemble spin vector  $\mathbf{S} = \sum \mathbf{s}_i$  that is the sum over the (pseudo-) spins  $\mathbf{s}_i$  of the individual (spin-1/2) particles [5–7]. The ensemble spin  $S$  with  $\langle \mathbf{S}^2 \rangle = S(S+1)$  can take on values in the range  $0 \leq S \leq S_0$ , where  $S_0 = N_0/2$ . For a given  $S$ , the minimum variance  $\Delta S_z^2$  of  $S_z = N/2$  for an unentangled state is realized by the coherent spin state (CSS), and is given by  $\Delta S_{z,\text{CSS}}^2 = S/2 = |\langle \mathbf{S} \rangle|/2$ , where it is assumed that the mean ensemble spin vector  $\langle \mathbf{S} \rangle$  lies in the  $xy$  plane. A spin state can be defined as squeezed if it satisfies  $\zeta_e \equiv 2\Delta S_z^2/|\langle \mathbf{S} \rangle| < 1$  (entanglement criterion [7,11]), or  $\zeta_m \equiv 2\Delta S_z^2 S_{\text{in}}/|\langle \mathbf{S} \rangle|^2 < 1$  (criterion for metrological gain [5,6]), where  $S_{\text{in}}$  is the initial spin of the uncorrelated ensemble before the squeezing).  $\zeta_m^{-1}$  represents the increase in the squared signal-to-noise ratio  $|\langle \mathbf{S} \rangle|^2/\Delta S_z^2$  over the value  $2S_{\text{in}}$  for the initial uncorrelated state. Since  $|\langle \mathbf{S} \rangle| \leq S_{\text{in}}$ , we have  $\zeta_e \leq \zeta_m$ ; i.e., metrological gain guarantees entanglement.

The process utilized for spin squeezing can reduce  $|\langle \mathbf{S} \rangle|$  below the initial spin  $S_{\text{in}}$  before the squeezing, thereby reducing the minimum variance  $\Delta S_z^2$  that is consistent with an unentangled state [11]. Therefore, measurements of both spin noise  $\Delta S_z$  and average spin length after squeezing  $|\langle \mathbf{S} \rangle|$  are necessary to verify spin squeezing or quantify metrological gain. While reduction of spin noise alone has sometimes been referred to as “spin squeezing” [17,18] or “number squeezing” [19,20], we take spin squeezing to require at least demonstrated entanglement,

$\zeta_e < 1$ , although we are primarily interested in metrological gain,  $\zeta_m < 1$ .

Spin noise has been modified by atomic collisions [19–21] and by absorption of squeezed light [15]. In dilute atomic systems, quantum nondemolition (QND) measurements with light [10–13,17,18,22] have reduced the projection noise of rotating [17] and stationary [18] spins. Spin squeezing has been achieved with two ions [8], and spectroscopic sensitivity further improved with a maximally entangled state of three ions [9]. Recently, spin squeezing with a Bose-Einstein condensate (BEC) in a multiple-well potential has been reported [23]. Demonstrated metrological gains over the SQL include  $\zeta_m^{-1} = 3.2(1)$  dB in the three-ion system [9];  $\zeta_m^{-1} \sim 4$  dB by light-induced squeezing within individual atoms of large spin  $s = 3$  [24]; and  $\zeta_m^{-1} = 3.8(4)$  dB for the BEC [23].

In this Letter, we demonstrate the generation of squeezed spin states of  $5 \times 10^4$  trapped  $^{87}\text{Rb}$  atoms on an atomic-clock transition by resonator-aided QND measurement with a far-detuned light field, as proposed by Kuzmich, Bigelow, and Mandel [10]. We verify the entanglement by comparing the observed reduction in projection noise below that of a coherent spin state [up to 8.8(8) dB] with the accompanying reduction in clock signal, and achieve a 3.0(8) dB improvement in precision over the SQL.

The light-induced spin squeezing presented here requires strong ensemble-light coupling [10,12–14] (large collective cooperativity [25]). This is achieved by means of a near-confocal optical resonator with, at the  $2\pi/k = 780$  nm wavelength of the probe light, a finesse  $\mathcal{F} = 5.6(2) \times 10^3$ , a linewidth  $\kappa = 2\pi \times 1.01(3)$  MHz, and a mode waist  $w = 56.9(4)$   $\mu\text{m}$  at the atoms’ position, corresponding to a maximal single-atom cooperativity  $\eta_0 = 24\mathcal{F}/(\pi k^2 w^2) = 0.203(7)$  [25]. Our experiments are performed on an ensemble containing up to  $N_a = 5 \times 10^4$  laser-cooled  $^{87}\text{Rb}$  atoms optically trapped inside the resonator in a standing wave of 851-nm light (Fig. 1).

One resonator mode is tuned 3.57(1) GHz to the blue of the  $|5^2S_{1/2}, F=2\rangle \rightarrow |5^2P_{3/2}, F'=3\rangle$  transition in  $^{87}\text{Rb}$ , such that the atomic index of refraction results in a

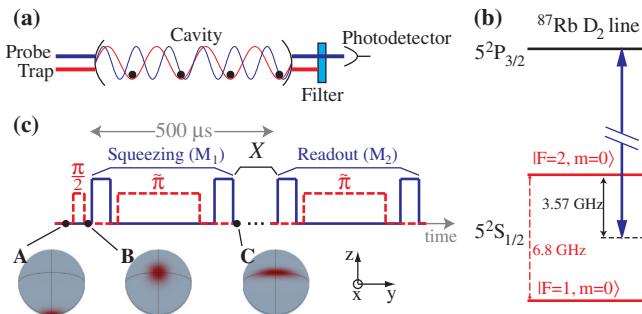


FIG. 1 (color online). (a) Experimental setup. (b) Atomic level structure. (c) Experimental sequence. Timing of probe pulses (solid line) and microwave pulses (dashed line) in preparation and readout of a squeezed state.  $\tilde{\pi}$  designates a composite  $\pi$  pulse [25]. Various procedures are inserted at X, as described in the text, to measure the CSS variance, measure the noise of a spin component other than  $S_z$ , or operate a clock. A–C illustrate semiclassical probability distribution functions for the Gaussian states discussed in the text.

mode frequency shift  $\omega$  that is proportional to the population difference  $N = N_2 - N_1$  between the hyperfine clock states  $|1\rangle = |5^2S_{1/2}, F = 1, m_F = 0\rangle$  and  $|2\rangle = |5^2S_{1/2}, F = 2, m_F = 0\rangle$ . The transmission of a probe laser tuned to the slope of this mode thus directly measures  $S_z = N/2$ , and is insensitive to total atom number (Fig. 1). The atom-resonator coupling also gives rise to a differential light shift between the clock states, which we use to verify experimentally the coupling strength calculated from first principles using spectroscopically determined resonator parameters. We measure a phase shift of 250(20)  $\mu\text{rad}$  per transmitted photon for a maximally coupled atom (on the resonator axis at an antinode of the probe standing wave), in excellent agreement with the calculated value 253(8)  $\mu\text{rad}$  [25].

To account for the spatial variation in coupling between standing-wave probe light and atoms, we define an effective atom number  $N_0 = (\langle \eta \rangle_e^2 / \langle \eta^2 \rangle_e) N_a \approx 0.66 N_a$ , where the single-atom cooperativity  $\eta$ , proportional to the local intensity of probe light, is averaged over the ensemble containing  $N_a$  atoms [25]. The definition is chosen so that the projection noise variance of the effective atom number measured via the mode shift  $\omega \propto N_a \langle \eta \rangle_e$  satisfies the usual relation  $\Delta N_0^2 = N_0$ . This avoids carrying near-unity factors through the equations and allows direct comparison to a spatially uniform system of collective cooperativity  $N_0 \eta_{\text{eff}}$ , where  $\eta_{\text{eff}} = (2/3) \langle \eta^2 \rangle_e / \langle \eta \rangle_e = 0.47(1) \eta_0$ , taking into account the oscillator strength 2/3 of the D<sub>2</sub> line and the measured rms transverse cloud radius of 8.1(8)  $\mu\text{m} \ll w$ . The mode frequency shift per effective atom of population difference  $N$  between the clock states is  $d\omega/dN = 4.5(2) \times 10^{-5} \kappa$  [25].

To quantify spin squeezing, we need to measure  $\Delta S_z^2$  and  $|\langle \mathbf{S} \rangle|$ . The latter can be obtained from the observed contrast  $C$  of Rabi oscillations as  $|\langle \mathbf{S} \rangle| = CS_0$ , where the maximum spin  $S_0 = N_0/2$  is measured by optically pumping the

atoms between the two hyperfine states  $F = 1, 2$ . For large  $S_0$ , the cavity shift  $\omega$  exceeds  $\kappa$  ( $\omega \leq 1.8\kappa$ ), which we take into account by correcting for the (accurately measured) Lorentzian line shape of the resonator. To verify the atom numbers  $2S_0$  thus obtained, we have also directly measured the cavity mode frequency shift  $\omega \propto S_z$ , finding agreement to within 2(4)% [25].  $\Delta S_z^2$  is obtained from transmission measurements that always remain in the linear regime, with  $2|\Delta S_z|d\omega/dN \leq 0.01\kappa$ .

The probe laser is frequency-stabilized to a far-detuned, negligibly shifted mode [25]. Each measurement of  $S_z$  employs two probe light pulses of duration  $T = 50 \mu\text{s} \gg \kappa^{-1} = 158 \text{ ns}$  separated by a 280  $\mu\text{s}$  delay, during which we apply a microwave  $\pi$  pulse sequence [25] to suppress inhomogeneous light shifts (spin-echo sequence). Each probe light pulse contains  $10^5$  to  $10^6$  photons which, after traversing the resonator, are detected with an overall quantum efficiency  $Q_e = 0.43(4)$ . From the detected photon numbers in the two pulses, we deduce two cavity shifts  $\omega_{\pm}$  whose difference constitutes a single measurement  $M$  of  $S_z = (\omega_+ - \omega_-)/(4d\omega/dN)$ . In a typical experiment [Fig. 1(c)], after initializing the ensemble spin state by optical pumping into  $|1\rangle$  (A) and applying a  $\pi/2$  microwave pulse to rotate the CSS into an equal superposition of  $|1\rangle$  and  $|2\rangle$  (B), we perform two measurements  $M_1$  and  $M_2$  to induce and verify conditional spin squeezing. We quantify spin noise  $\Delta S_z$  by extracting variances from 100 repetitions of such a sequence.

We determine the CSS projection noise level  $\Delta S_{z,\text{CSS}}^2 = N_0/4$  from the measured atom number  $N_0$  and verify it [5,6,15,17] either by evaluating the variance  $\text{Var}(M_1)$  of the set of single measurements  $M_1$ , or by inserting between two measurements  $\tilde{M}_1$  and  $\tilde{M}_2$  a second CSS preparation, consisting of optical pumping into state  $|1\rangle$  and a  $\pi/2$  pulse, and evaluating  $\text{Var}(\tilde{M}_1 - \tilde{M}_2)/2$ . Figure 2 shows the dependence of the corresponding quantities in atom number units,  $y_1 = 4\text{Var}(M_1)$  (open triangles) or  $y_2 = 2\text{Var}(\tilde{M}_1 - \tilde{M}_2)$  (open circles), on  $N_0$ . The contribution of CSS projection noise scales as  $\Delta S_{z,\text{CSS}}^2 \propto N_0$ , while atom-number-dependent technical noise, e.g., due to microwave power fluctuations or any sensitivity to atom number fluctuations, generically scales as  $\Delta S_{z,\text{tech}}^2 \propto N_0^2$ . A quadratic fit  $y_{1,2} = a_0 + a_1 N_0 + a_2 N_0^2$  yields  $a_1 = 1.3(1)$  and  $a_2 = 1(2) \times 10^{-6}$  (not shown in Fig. 2), but the data are also well fit by setting  $a_1 = 1$ , as required by independently measured cavity and atomic properties with no free parameters [25], and allowing a small technical noise contribution  $a_2 N_0^2 < N_0$  with  $a_2 = 9(3) \times 10^{-6}$  (solid curve). Slow drifts in microwave power of 0.4% over the set of measurements could account for the technical noise of  $y_1$ , which vanishes if the data are analyzed by comparing only adjacent cycles of the experiment [25]. Our ability to prepare an unentangled state close to a CSS—with  $S_z$  variance  $\Delta S_{z,\text{prep}}^2 \sim 1.3S_0/2$  for our largest atom number—is not a prerequisite for spin squeezing but does provide independent confirmation of the CSS refer-

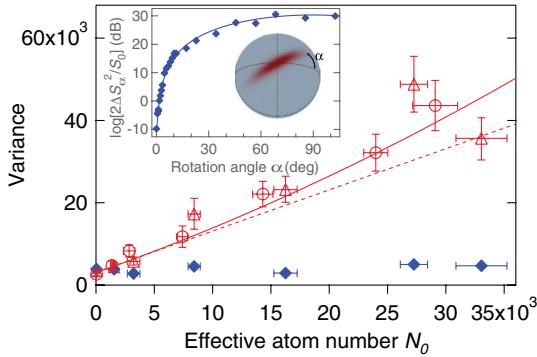


FIG. 2 (color online). Spin noise measurements (see text):  $y_1 = 4\text{Var}(M_1)$  for a single CSS preparation (open triangles),  $y_2 = 2\text{Var}(M_1 - M_2)$  for two independently prepared CSSs (open circles), and  $4\Delta S_z^2_{\text{meas}} = 2\text{Var}(M_1 - M_2)$  for two measurements after a single CSS preparation (solid diamonds), all in units of atom number. Vertical error bars are statistical; horizontal error bars indicate standard deviation of measured atom numbers. The solid (dashed) line corresponds to  $a_1 = 1$ ,  $a_2 = 9 \times 10^{-6}$  ( $a_2 = 0$ ). Inset: Variance  $\Delta S_z^2$  of  $S_z$  after rotating a squeezed state by an angle  $\alpha$  about  $\langle \mathbf{S} \rangle$ , with parameter-free theory curve.

ence level for spin noise measurements. We emphasize that, in quantifying spin squeezing below, we conservatively normalize to the CSS noise  $4\Delta S_z^2_{\text{CSS}} = N_0$  as obtained from our cavity parameters (dashed line), not to the 30% larger slope of the unconstrained quadratic fit to  $y_{1,2}$ .

To prepare a state with (conditionally) reduced  $\Delta S_z^2$  [Fig. 1(c) C], we simply measure  $S_z$  for a CSS on the  $x$  axis with a photon number  $p \approx 5 \times 10^5$  sufficiently large to resolve  $S_z$  beyond the CSS variance. Each such measurement  $M_1$  yields a value of  $S_z$  that is random but known, as verified by a readout measurement  $M_2$ . We plot  $2\text{Var}(M_1 - M_2)$  vs atom number  $N_0$  in Fig. 2 (solid diamonds), finding it a factor of 2 above the photocurrent noise level, with very weak dependence on atom number, and well below the CSS level.

In principle, it is possible for the value of  $S_z$  at the end of the measurement to differ from the average value of  $S_z$  during the measurement. Besides the far-detuned locking light whose effect on  $S_z$  is negligible, only spin-echo microwave composite  $\pi$  pulses, whose fidelity was separately measured to be 98(1)%, and probe light are applied during  $M_1$ . The probe light can only change  $S_z$  through free-space scattering, which adds at most 3.1 (3)% of CSS projection noise at  $p = 5 \times 10^5$  [13,25]. Thus, while the added noise is negligible compared to the CSS level, it can explain part of the small remaining variance of  $M_1 - M_2$ .

Provided  $M_1$  does not alter the state appreciably, and the measurements  $M_1, M_2$  are identical and uncorrelated [25],  $\Delta S_z^2_{\text{meas}} \equiv \text{Var}(M_1 - M_2)/2$  represents the uncertainty of any single such measurement. The conditional variance of the state after measurement  $M_1$  can then be shown to be

$[\Delta S_z^2]_{M_1} = \Delta S_{z\text{prep}}^2 \Delta S_{z\text{meas}}^2 / (\Delta S_{z\text{prep}}^2 + \Delta S_{z\text{meas}}^2)$  [25]. When no new information is gained in measurement  $M_1$  ( $\Delta S_{z\text{meas}}^2 \gg \Delta S_{z\text{prep}}^2$ ), the variance is that of the state-preparation process,  $\Delta S_{z\text{prep}}^2 \equiv \text{Var}(M_1) - \Delta S_{z\text{meas}}^2$  (close to, but above, the CSS value), while information gained reduces the variance, ultimately to the measurement variance  $\Delta S_{z\text{meas}}^2$  of  $M_1$ . At  $N_0 = 3.3(2) \times 10^4$  and  $p = 6 \times 10^5$ , we observe a normalized spin noise  $\sigma^2 \equiv [\Delta S_z^2]_{M_1} / \Delta S_{z\text{CSS}}^2 = -9.1(8)$  dB (see Fig. 3); a slight correction for the effect of photon scattering [25] yields  $\sigma^2 = -8.8(8)$  dB.

The reduction of  $[\Delta S_z^2]_{M_1}$  below the CSS value  $\Delta S_{z\text{CSS}}^2$  is accompanied by a substantial increase in  $\Delta S_y^2$  because the differential light shift of the atomic levels, corresponding to a rotation of the Bloch vector about the  $z$  axis, depends on the intracavity intensity, which in turn depends on  $S_z$ . To observe the antisqueezing, we apply a microwave pulse after the squeezing measurement [at  $X$  in Fig. 1(c)] to rotate the spin state by a variable angle  $\alpha$  about  $\langle \mathbf{S} \rangle$  before reading out  $S_z$ . The variance  $\Delta S_z^2$  of  $S_z$  in the rotated state, displayed in the inset to Fig. 2, is a sinusoid that is well described with no free parameters by our model of the ensemble-cavity interaction [25].

To verify spin squeezing, we also need to measure  $|\langle \mathbf{S} \rangle|$ , observable as the interference contrast  $\mathcal{C} = |\langle \mathbf{S} \rangle|/S_0$  of Rabi oscillations induced between measurements  $M_1$  and  $M_2$ . Figure 3 shows  $\mathcal{C}$  as a function of photon number  $p$  used in the state-preparation measurement at  $N_0 = 4.0(1) \times 10^3$ , and we have verified that the contrast  $\mathcal{C}$  is independent of atom number [25]. Both normalized spin noise  $\sigma^2$  and  $\mathcal{C}$  can be fit by simple models (dashed and dotted curves) [25]. From these two measurements, we deduce the metrological squeezing parameter  $\zeta_m$  (solid triangles and solid curve). For  $p = 3 \times 10^5$ , we achieve  $\zeta_m^{-1} = \mathcal{C}^2 / (\sigma^2 \mathcal{C}_{\text{in}}) = 3.0(8)$  dB of metrological gain [and an inverse entanglement parameter  $\zeta_e^{-1} = \mathcal{C}/\sigma^2 = 4.2(8)$  dB, not shown]. The finite initial contrast  $\mathcal{C}_{\text{in}} = S_{\text{in}}/S_0 = 0.7$  in the ensemble without squeezing is due to the resonator locking light, and can be improved by detuning this light further from atomic resonance. The probe-induced contrast reduction probably arises from differential light shifts between the clock states that are imperfectly canceled by the spin-echo technique because of atomic motion. In the absence of any technical noise, a fundamental limit to the spin squeezing, associated with photon scattering into free space, would be  $\zeta_m^{-1} \leq \sqrt{(3/2)N_0 \eta_{\text{eff}}} \sim 18$  dB in our system with cooperativity  $N_0 \eta_{\text{eff}} \sim 3100$  [13,14,25].

For the data presented above, the readout quantifying the entanglement was completed 500  $\mu\text{s}$  after preparation of the squeezed state. We have further verified that the squeezing remains after a Ramsey clock sequence, in which two  $\pi/2$  pulses about the  $x$  axis, separated by a short (70  $\mu\text{s}$ ) precession time, are inserted at  $X$  in Fig. 1(c). Such a clock can achieve precision below the SQL because the first of these  $\pi/2$  rotations initiates it with a phase that

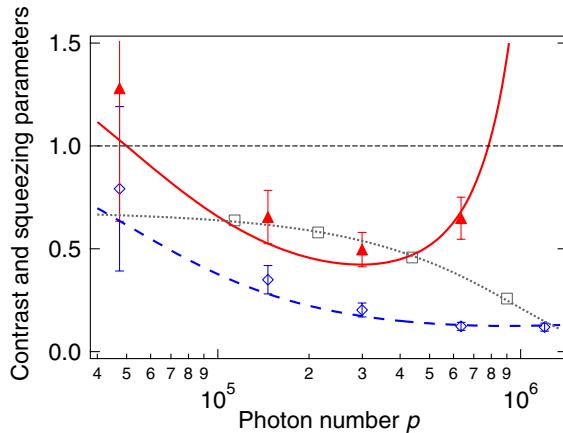


FIG. 3 (color online). Measured data with fits of simple models [25] for normalized spin noise  $\sigma^2 = [\Delta S_z^2]_{M_1}/\Delta S_z^2_{\text{CSS}}$  (open diamonds), contrast  $C$  (open squares), and metrological squeezing parameter  $\zeta_m$  (solid triangles).

is known, from the squeezing measurement, to better than the CSS uncertainty.

The phase coherence time of the unsqueezed CSS in our current trap is 10(2) ms. Both microwave and optical clocks with  $\sim 1$  s coherence times have already been demonstrated with trapped atoms [2,3,26–28]. Whether and to what degree the squeezing technique demonstrated here could benefit such clocks and other precision experiments [4] will depend on the clock characteristics, noise sources [16], and lifetime of the squeezed state. These questions, as well as possible systematic effects, need to be investigated in the future.

The group of E. Polzik independently and simultaneously achieved results similar to ours in a Mach-Zehnder interferometer [29]. We have recently demonstrated a new squeezing method using cavity feedback [30].

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