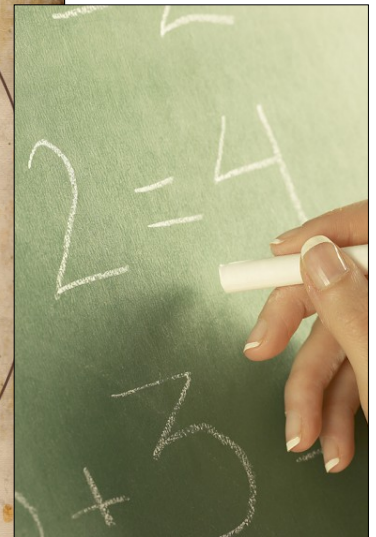# Mathematics:

A Christian Perspective

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STUDENT VERSION

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Hypercubes

We all know what a 3-dimensional cube is. In these activities, we'll explore figures that are analogous to a cube, but in a variety of dimensions.

1) Specific features of 3-dimensional cubes

Let's start by reviewing some features of familiar 3-D cubes. Some of you may find it helpful to look at a physical model of a cube (wooden or plastic block, sugar cube, etc.) as you answer the following questions; others may be able to answer the questions by referring to a mental image of a cube in their "mind's eye."

a. Faces
How many faces does a 3-D cube have?

What geometric term describes each face of a 3-D cube?

What's the "dimensionality" of each face?

b. Edges
How many edges does a 3-D cube have?

What geometric term describes each edge of a 3-D cube?

What's the "dimensionality" of each edge?

How are the edges related to the faces?

c. Vertices
How many vertices does a 3-D cube have?

What geometric term describes each vertex of a 3-D cube?

What's the "dimensionality" of each vertex?

How are the vertices related to the faces and edges?

2) "Cubes" in many dimensions

a. What does a "cube" in a 0-dimensional universe look like? Sketch it.

b. What does a "cube" in a 1-dimensional universe look like? Sketch it.
How is this 1-dimensional cube related to cubes in 0 dimensions?

c. What does a "cube" in a 2-dimensional universe look like? Sketch it.
    How is this 2-dimensional cube related to cubes in 1 dimension?

d. What does a "cube" in a 3-dimensional universe look like? Try to sketch it.
    How is this 3-dimensional cube related to cubes in 2 dimensions?

e. What does a "cube" in a 4-dimensional universe look like? Can you sketch it?
    How is this 4-dimensional cube related to cubes in 3 dimensions?

    A 4-dimensional cube is sometimes called a tesseract. In general, cube-like objects in n dimensions are often called n-cubes: if n > 3, the objects are sometimes called hypercubes.

f. Let's use the term n-cube to stand for an n-dimensional "cube." (So "2-cube" is another name for what we usually call a square, a 3-cube is the cube with which we're all familiar, and "4-cube" is another name for a tesseract.) Complete the following table on the next page.

|	Number of Vertices	Number of Edges from Each Vertex	Total Number of Edges	Number of Components of One Less Dimension
0-cube				
1-cube				
2-cube				
3-cube				
4-cube				
5-cube				
6-cube				
n-cube				

The Bible tells us that humans are created in God's image, and responsible for caring for the world and everything in it (Genesis 1:26–28). One aspect of being created in God's image is that we are able to use our imaginations to go beyond our physical senses. For example, in this activity, we were able to imagine using the process we discovered for building cubes to construct cubes of any dimension, even though the universe as we perceive it is only 3-dimensional. We used our minds to discover patterns in the structure of cubes that we can see, and then extended and generalized those patterns to discover characteristics of higher dimensional cubes that are beyond our perceptions. As we do mathematics, we detect patterns God has built into his creation, we imagine how to generalize those patterns, and we take advantage of analogies to help us overcome the limitations of our physical senses. Christians should celebrate such activity as an illustration of our status as image-bearers of God!
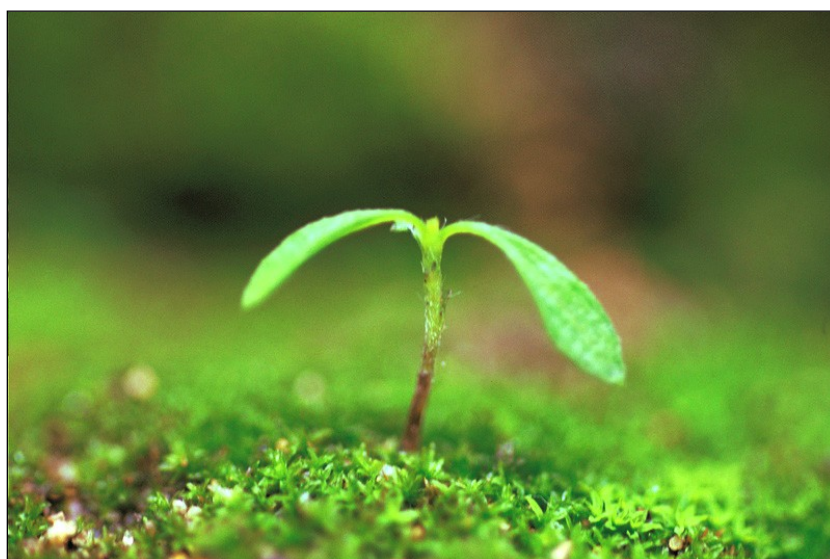
Discussion Questions

What are some other human characteristics that you think show that we're created in God's image? You may consider physical, intellectual, emotional, or spiritual aspects.
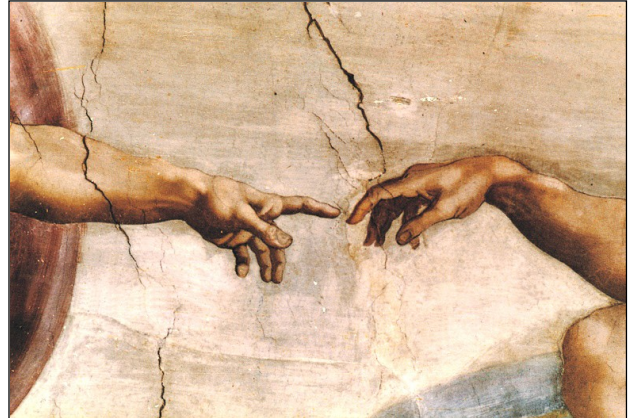
Have you ever seen a <u>perfect</u> square or 3-cube? Do 2-cubes and 3-cubes really exist?

Do you think that 4-cubes, 5-cubes, 6-cubes, etc., actually exist, even though we can't see them? If so, where and in what sense do they exist? If not, why not?

Choose a 4-digit number. Add a row to the preceding table for a hypercube of dimension equal to your selected number. It's quite likely that no other person in human history has ever considered your specific hypercube! You may currently know more about that specific hypercube than anyone ever has! Did you create that hypercube, or did you discover it?

Are any animals creative? Are all humans creative? Some Christians say that humans can be creative because God, the great Creator, made us in his image. Others say that anything people create has already been conceived of by God, and in that sense, the human creator is really "just" a discoverer. Do you think human creativity exists? Why or why not?

How does mathematics help us obey the "Cultural Mandate" that we take care of the world God has created? We know more mathematics now than the Israelites of the Old Testament knew; does that make us better stewards of God's creation than they were? Why or why not?

What can this hypercube activity tell us about what God is like?

3) 2-dimensional representations of 3-dimensional cubes

    a. Make a perspective sketch of a cube, trying to show its "3-dimensionality" on the flat page.

    b. Make a projective sketch of a cube. Imagine the cube being made of black wires for its edges, with transparent faces; also think of the cube being suspended between a flat page and a light source directly above the cube. What would the shadow of the cube look like on the page?

    c. Make a net for a cube. A net for a 3-D cube is a 2-dimensional pattern that can be folded up to form a cube. Indicate (by using colors or numbers) which sides of the squares in your pattern will be joined during folding to form edges of the cube.

d. For each of the representations in parts a-c, answer these questions:

What are the characteristics of a 3-cube that are accurately shown in the 2-D representation?

What are the characteristics of a 3-cube that are "distorted" in the 2-D representation, or how is the 2-D representation unclear, inaccurate, or misleading?

e. Here's a nice website:
http://www.math.union.edu/~dpvc/math/4D/welcome.html illustrates how these representations for a 3-D cube are related to one another. Click on the section titled "Folding Cubes and Hypercubes" and watch the first movie there.

Summary: In this activity, we have learned that each 2-D representation of a 3-D cube provides us with accurate information about <u>some</u> of the properties of the cube, but none of the representations provides accurate information about <u>all</u> properties. Each representation distorts some features of the cube. This isn't surprising, since two dimensions are in a sense not "big enough" to hold a complete representation of a 3-D cube. There are inherent limitations when we try to "squish" a higher dimensional object into lower dimensions.

Discussion Questions

Some Christians suggest that the mathematical situation described in the preceding paragraph has similarities to our ability as created beings to obtain complete knowledge of a transcendent Creator, his works, and his plans. Explain the parallels you think they might see. What do Isaiah 40:28 and Isaiah 55:8–9 have to do with this issue? Will we ever have complete knowledge of God? Why or why not?

Which of the three representations of 3-D cubes discussed in this activity do you prefer? Why do you think it's better than the other representations? Which representation gives the best information about the shape of the faces of a 3-cube? Which representation gives the best information about the number of edges of a 3-cube? Which representation gives the best information about the parallelism of opposite edges of a 3-cube?

Christians believe that God reveals himself to us in three ways: "general revelation" through nature (Psalm 19:1–4), "special revelation" through the Bible (Psalm 19:7–11), and through Jesus (John 14:9). What is an aspect of God that you think is best revealed to us through nature? Through the Bible? Through Jesus?

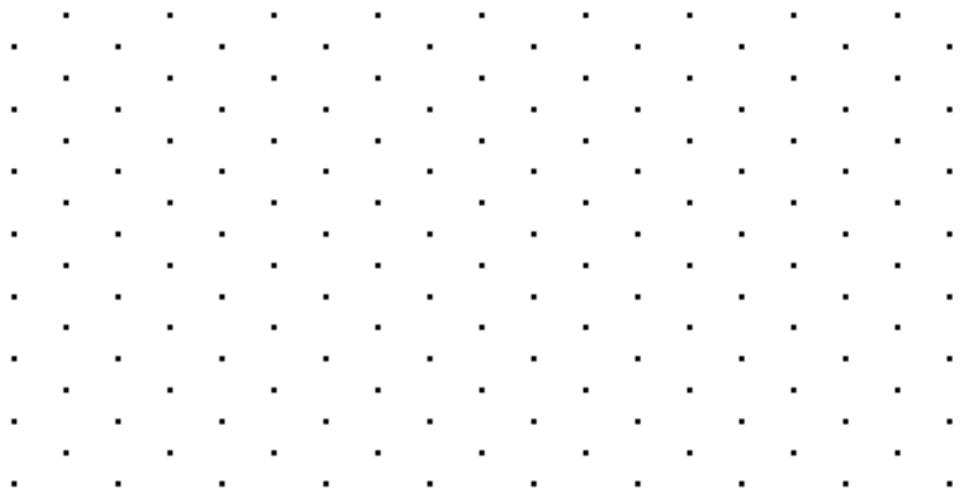4) Representations for a 4-cube

    a. Below are four copies of a "perspective" sketch of a 4-cube (2 distinct 3-cubes with corresponding vertices joined). Use colors to show the eight 3 cubes that make up the 4-cube. One effective way to do this is to show two opposite 3-cubes (ones that share no edges) on each of the four sketches.
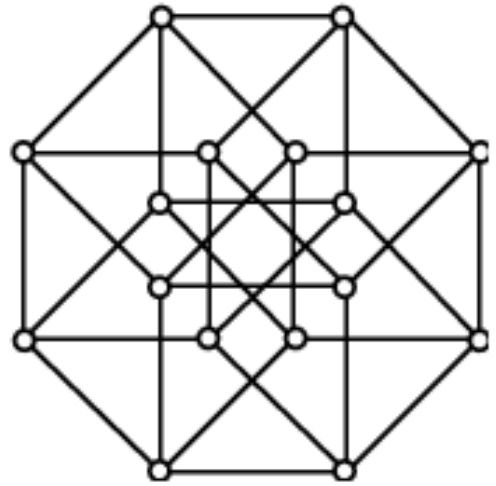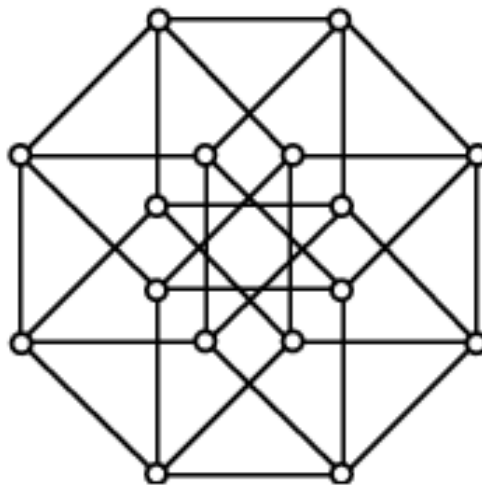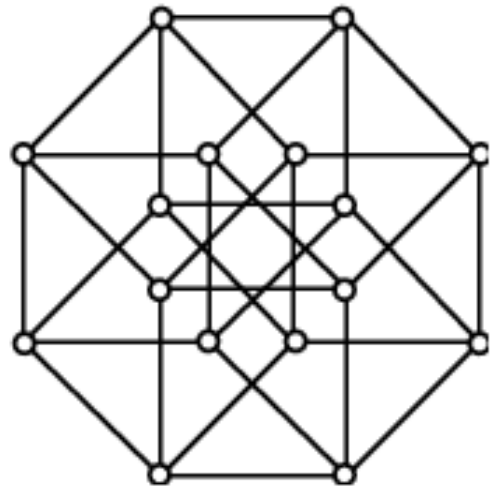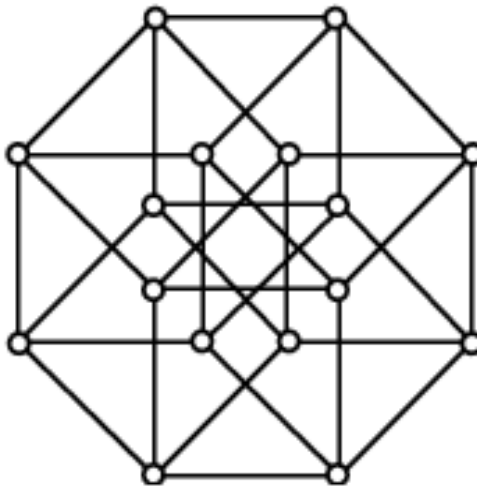
b. Remember from 3b that our projective sketch of a 3-cube looks like a "square-within-a-square." What do you think a similar representation of a 4-cube would look like? Sketch it. Can you see the eight 3-cubes that make up the 4-cube?

c. Look at the net for a 3-cube we made in 3c. What do you think an "unfolded" 4-cube would look like? Sketch it on the dot paper. What faces are joined when you "fold up" this 3-D "net" to make a 4-cube?

d. The website http://www.math.union.edu/~dpvc/math/4D/welcome.html illustrates how some of these representations for a 4-D cube are related to one another. Click on the section titled "Folding Cubes and Hypercubes" and watch the second movie there.

5) How many cubes of lower dimension are contained in each n-cube?

We've used reasoning by analogy to get some insights into characteristics of cubes in various dimensions. The basic idea is to look for patterns in the characteristics of lower dimensional cubes, and then extend those patterns to higher dimensions. The table below summarizes some of our observations about cubes in different dimensions, and gives you a chance to extend and generalize these ideas. Answer the questions that follow the table to help you complete the table. (The table from 2-f will be helpful in answering some of the questions and completing this table).

Number of Components of Various Dimensions							
Dimension of Cube	0-cubes (vertices)	1-cubes (edges)	2-cubes (faces)	3-cubes ("solids")	4-cubes	5-cubes	6-cubes
0-cube							
1-cube							
2-cube							
3-cube							
4-cube							
5-cube							
6-cube							
n-cube							

a. What happens to the number of vertices each time a new dimension is added? How many vertices are in an n-cube? Where should you record this information in the table?

b. What happens to the number of edges <u>at each vertex</u> each time a new dimension is added? How many edges from each vertex are in an n-cube?

c. What happens to the number of faces <u>at each edge</u> each time a new dimension is added? How many faces are at each edge of an n-cube?

d. What happens to the number of "outer boundaries" (points in 1D, segments in 2D, squares in 3D, cubes in 4D, etc.) each time a new dimension is added? How many (n–1)-cubes are contained in an n-cube? Where should you record this information in the table?

e. Are there other patterns you notice in the table? For example, what pattern do you see in the sums of the rows of the table? What if you "attach" alternating positive and negative signs to the entries in a row, and then compute the row sums?

f. How many k-cubes are there at each vertex of an n-cube (k ≤ n)?

    For example, how many 2-cubes are there at each vertex of a 3-cube? The answer for this simple example is obviously three, but a very useful way to obtain the answer is to notice that:

    i) There are three edges at each vertex of a 3-cube,

    ii) Two edges at a vertex completely determine a 2-cube, and thus

    iii) There are C(3, 2) = 3 2-cubes at each vertex of a 3-cube.

g. How many k-cubes are contained in each n-cube (k ≤ n)?

For example, how many 2-cubes are contained in each 3-cube? The answer is obviously 6, but a very useful way to obtain the answer is to notice that:

    i)   The example in the previous question shows that there are three 2-cubes at each vertex of a 3-cube,

    ii) A 3-cube has $2^3 = 8$ vertices, so that gives us 3 x 8 = 24 2-cubes, but

    iii) This counts each 2-cube multiple times, once from each of the $2^2 = 4$ vertices of the 2-cube, so

    iv) Our final count, adjusted for the multiple counting of each 2-cube, is 24 ÷ 4 = 6.

Use similar counting techniques to help you complete the table.