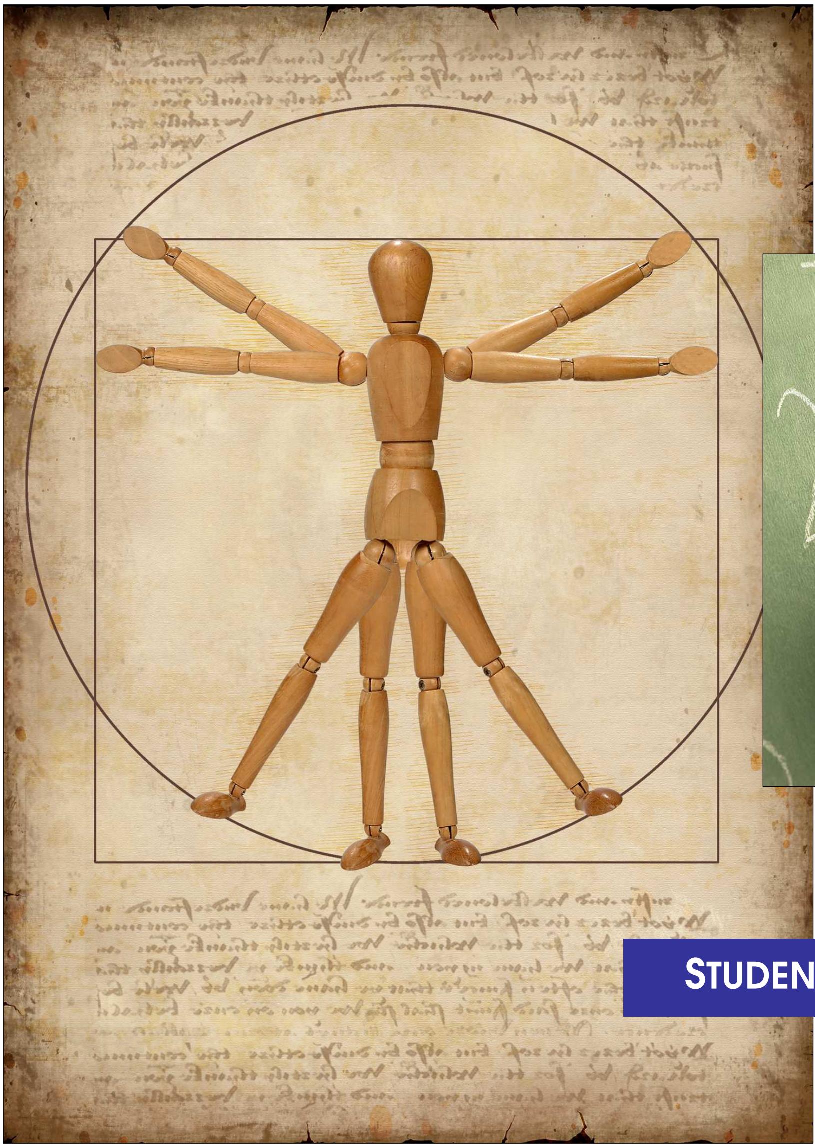
Mathematics:

A Christian Perspective



STUDENT VERSION

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Table of Contents

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7 The Indian Ocean Tsunami: December 26, 2004

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What is a tsunami? A tsunami is an ocean wave that is generated by a sudden displacement of the sea floor. This displacement can occur as a result of earthquakes. Tsunami is a Japanese word for "harbor wave." The December 26, 2004, Indian Ocean tsunami was a natural disaster that caused unimaginable grief and sorrow to millions of people. The mathematics of logarithmic scales will help us understand how earthquakes are measured. By graphing trigonometric functions, we will study the parameters that define waves; and the study of quadratics will enable us to understand how supplies can be dropped to stranded people. Various sections of the unit will encourage you to consider questions as they relate to a Christian's response to natural disasters.

1) Create a tsunami in a cake pan

Materials:
- plastic cake pan
- 16 oz water
- scissors
- sturdy plastic wrap
- tape

Directions: Cut a hole in the bottom of the plastic cake pan approximately 4 inches in diameter. Tape the plastic wrap on the outside of the pan, covering the hole and making sure it is water-tight. Fill the pan with water. Gently tap on the plastic wrap and observe what happens. Why? What happens if you alter the strength of your tap?

2) Graphs of cosine functions

The graphs of the cosine and sine functions are important for understanding applications to physical situations. A simple wave closely resembles that of a cosine function that moves regularly in time. These graphs are beautiful and interesting in their own right.

a. *Example 1:* Sketch the graph of each function. Notice how they are similar and different to each other. Write a sentence about your observations.

$f(x) = \cos x$
$f(x) = 2 \cos x$

In general, for $y = a \cos x$, the number *a* is the amplitude and is the largest value these functions attain.

b. Another important parameter is the period of the graph. In respect to the waves, the period is the distance between crests.

For $y = a \cos kx$; $k > 0$

Amplitude = $|a|$

Period = $\frac{2\pi}{k}$

Example 2: Find the amplitude and period of the function and sketch its graph:

$y = 4 \cos 3x$

c. A cosine graph may be shifted horizontally in the plane by an amount *b*. It is shifted to the right if b > 0 or to the left if b < 0. The amount of that shift is known as the "phase shift."

For y = acos k(x - b),

Amplitude = IaI

Period = $\frac{2\pi}{k}$

Phase shift = b

Example 3: Find the amplitude, period, and phase shift of the function; then sketch its graph:

$$y = \frac{3}{4}\cos(2x + \frac{2\pi}{3})$$

3) Practice Problems

a. Find the amplitude and period of the function and sketch its graph.

$y = \cos 4x$

$y = 3 \cos (3x)$

$y = - \cos (1/3\ x)$

$y = 10 \cos (1/2\ x)$

b. Find the amplitude, period, and phase shift of the function; then graph one complete period.

$y = \cos \left(x - \frac{\pi}{2}\right)$

$y = 5 \cos \left(3x - \frac{\pi}{4}\right)$

4) Application Problems

a. As a wave passes by an offshore piling, the height of the water is modeled by the function

$$h(t) = 3 \cos \left(\frac{\pi}{10} t \right)$$

where *h(t)* is height in feet above mean sea level at time *t* seconds.

i. Find the amount of time between crests.

ii. Find the wave height: that is, the vertical distance between the trough and the crest of the wave.

b. A leaf floats on the water bobbing up and down. The distance between its highest and lowest point is 4 centimeters. It moves from its highest point down to its lowest point and back to its highest point every 10 seconds. Write a cosine function that models the movement of the leaf in relationship to the equilibrium point.

Discussion Questions

God is not only Creator and Sustainer but, through the Incarnation, Sufferer and Savior. Do you think God suffers when natural disasters bring tragedy to the lives of humans? Would God suffer differently if God did not become human?

In the midst of terrible natural tragedies, people are tempted to doubt God's mercy. What tools, skills, or resources do we have to find God in the midst of trouble and sorrow?

Logarithmic scales

An earthquake is the sudden release of energy in the form of vibrations caused by rock suddenly moving along fault lines. The portion of the Earth's crust known as the India plate slid under the section known as the Burma plate. According to the United States Geological Survey (USGS), the earthquake that generated the great Indian Ocean tsunami of 2004 is estimated to have released the energy of 23,000 Hiroshima-type atomic bombs.

This energy is too large to represent on a normal number line, so a scale based on exponents is used to measure the magnitude of earthquakes. The idea to use a logarithmic earthquake magnitude scale was first developed by Charles Richter in the 1930s. The Richter scale is an example of an "exponential scale," or a "logarithmic scale".

The word logarithm literally means "ratio of numbers." An increase of 1 unit on the Richter scale roughly corresponds to a multiplication of the energy released by a factor of 10. Algebraically, this may be described as a value of x on the Richter scale corresponds to an energy release of $k \bullet 10^x$, where constant k depends on the units being used.

Richter Magnitude	Description
1	cannot be felt except by instruments
2	cannot be felt except by instruments
3	cannot be felt except by instruments
4	like vibrations from a passing train
5	strong enough to awake sleepers
6	very strong; walls crack, people injured if present
7	ruinous; ground cracks, houses collapse
8	very disastrous; few buildings survive, landslides

(Note: The Richter scale has no upper limit)

1. Intensity of earthquakes

In order to find the difference in intensity of earthquakes, divide the larger energy release by the smaller.

Example 1: How many times more intense is an earthquake with a Richter magnitude of 6.3 than one with magnitude 4.7?

2. Intensity of Sounds

Another logarithmic scale describes the intensity of sound. Witnesses of the Indian Ocean tsunami said the approaching tsunami sounded like three freight trains or the roar of a jet. The decibel is the unit used to measure sound intensity. Physicists agree that the least intense sound that a human ear can detect is about 10^{-12} watts/square meter (w/m^2). The loudness L(x), measured in decibels (dB), of a sound of intensity x, is defined as

$$L(x) = 10 \log \frac{x}{I_0}$$

Where $I_0 = 10^{-12}$ w/m^2 is the least intense sound that a human ear can detect.

The chart gives the decibel and the corresponding w/m^2 for some common sounds.

Watts/Square meter	Description	Decibels
10^{2}	jet plane 30 m away	140
10^{1}	pain level	130
10^{0}	amplified rock music	120
10^{-1}		110
10^{-2}	noisy kitchen	100
10^{-3}	heavy traffic	90
10^{-4}	ordinary traffic	80
10^{-5}		70
10^{-6}	normal conversation	60
10^{-7}	average home	50
10^{-8}		40
10^{-9}	soft whisper	30
10^{-10}	rustling leaves	20
10^{-11}		10
10^{-12}	barely audible	0

As the decibel values increase by adding 10, the intensities multiply by 10. For example, if you increase the sound intensity by 40dB, you multiply the watts/square meter by $10^4 = 10,000$.

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Example 2: The sound of the tsunami was likened to the roar of a jet. The roar of a jet engine 30 m away is 140 dB. How many times more intense than heavy traffic was the sound of the tsunami?

3. Practice Problems

For a-b, use the Richter scale values given in this lesson.

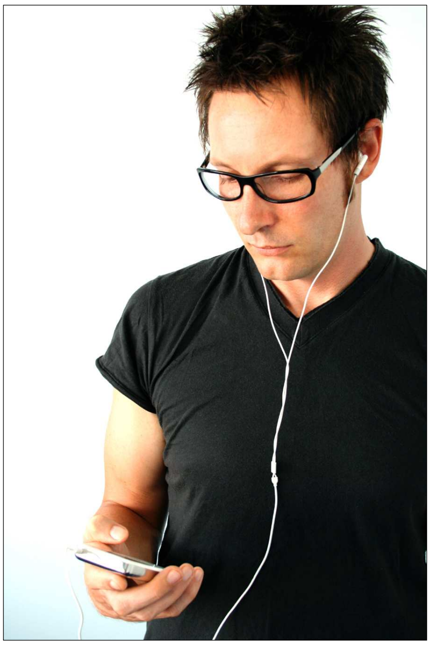
a. Great earthquakes such as the Good Friday Earthquake in Alaska in 1964 had a Richter magnitude of 8.3. How many times more intense was the Indian Ocean tsunami earthquake that was recorded as a Richter magnitude of 9.0?

b. How many times more intense was the motion caused by the 1906 San Francisco earthquake that measured 8.3 on the Richter scale as that caused by the 2001 Bhuj, India earthquake that measured 6.9?

For c-d, refer to the chart of sound intensity levels given in this lesson.

c. A city will make it illegal to create sound in a residential area that exceeds 72 decibels during the day and 55 decibels during the night. How many times more intense is the noise level allowed during the day than at night?

d. For humans, the threshold of pain due to sound averages 130 dB. What is the intensity of such a sound in watts/meter^2?



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Discussion Questions

Is it accidental that developing countries do not have a warning system in place? What role could industrialized nations play in ensuring equity and justice? Can the suffering caused by these inequalities and injustices be prevented in the first place?

The whole creation has fallen, but hope remains for its renewal. How can a Christian respond to recreate a broken world?

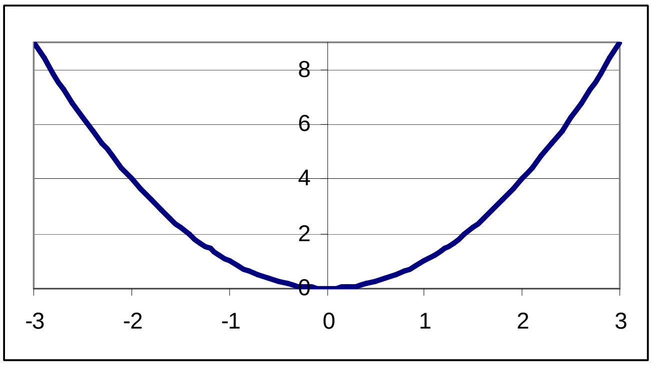
What could be the role of mathematicians who have a heart for service to people? How might they use their knowledge of mathematics in the service of humankind? List your ideas and discuss these in your class.

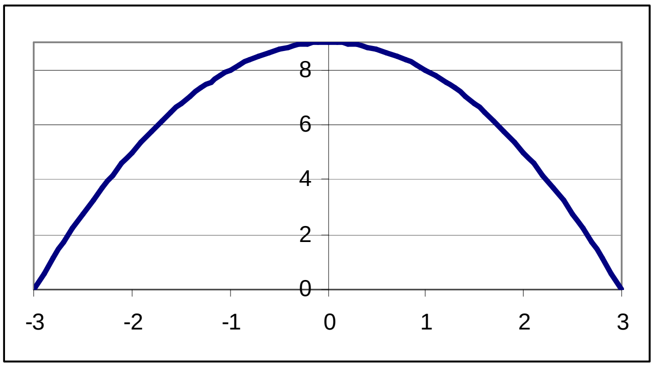
Quadratics

This lesson deals with the mathematics involved in dropping relief supplies. These activities are applications of the quadratic function.

1) Parabolas

In general, a quadratic function is any function that can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is called a parabola. Two types of parabolas are given here.





Notice that each parabola has an axis of symmetry—a mirror line. The vertex of the parabola is either the lowest or highest point on the graph, depending on whether the graph opens "up" or "down." When $a > 0$, the parabola opens up and the function has a minimum value at the vertex. If $a < 0$, the parabola opens down and has a minimum value, again at the vertex.

Practice Problems

State whether the parabola opens up or down, and whether it has a minimum or maximum point at its vertex. If you have a graphing calculator, check by graphing.

$f(x) = x^2 + x - 6$

$f(x) = 5 + 7x - x^2$

2) Newton's formula for falling objects

In the 17th century, Isaac Newton discovered that an object thrown into the air or moving in space can be modeled by a quadratic function. The quadratic equation that involves the force of gravity on a falling object is given as

$$h(t) = -\tfrac{1}{2}gt^2 + v_0t + h_0$$

where h(t) is the height at time t, g is the gravitational constant (32 ft/sec^2 or 9.8 m/sec^2), v_0 is the initial velocity in units per sec and h_0 is the initial height in units of the object.

Practice Problem

A package of blankets is dropped from the top of a building 20 meters tall.

a. Write an equation describing the relation between the height, h, of the ball above the ground and time, t.

b. Graph the equation.

c. Estimate how much time it takes the package to fall to the ground.

3) Applications of quadratics

The Indian Ocean tsunami destroyed thousands of miles of coastlines. It submerged islands, and destroyed roads and airports. Many thousands of people were stranded in areas that were inaccessible. The following problems deal with the dropping of relief supplies.

Practice Problems

a. A rescue helicopter hovering 68 feet above a fishing boat in trouble drops a life raft.

   i) Write an equation to model this situation where h(t) is the height of the raft at time, t, in seconds.

   ii) How many seconds after the raft is dropped will it hit the water?

b. A crate of blankets and food is dropped without a parachute from a helicopter hovering at an altitude of 110 feet.

   i) Write a function where h(t) models the crate's height above the ground and where t is the time in seconds after it is dropped.

   ii) How long will it take the crate to reach the ground?

Discussion Questions

Investigate:

What is a debt moratorium?

What will a debt moratorium enable a country to accomplish?

In 2005, and every year until 2009, the Indonesian Central Bank reports that US $7 billion will be spent in servicing external debt. This is more than the government is able to spend on health and education in the country. Many developing countries find themselves in this situation. Wealthy nations play a role in causing poor nations to become poorer. Discuss how injustices such as these might be remedied.

Why did India and Thailand refuse a debt moratorium?

How do your knowledge of mathematics and your ability to analyze enable you to make decisions regarding matters of money? Give an example.

You may feel called to bring compassion to a hurting world, with the help of God's Holy Spirit. What should a Christian's response be when others are in trouble?

In the past few years, the world has seen several natural disasters that have caused immense damage and loss of lives. Charitable giving in many countries has reached an all time high. What is your response when local authorities note that charitable giving is being depleted in your community as a result of charity being dispersed to other countries?