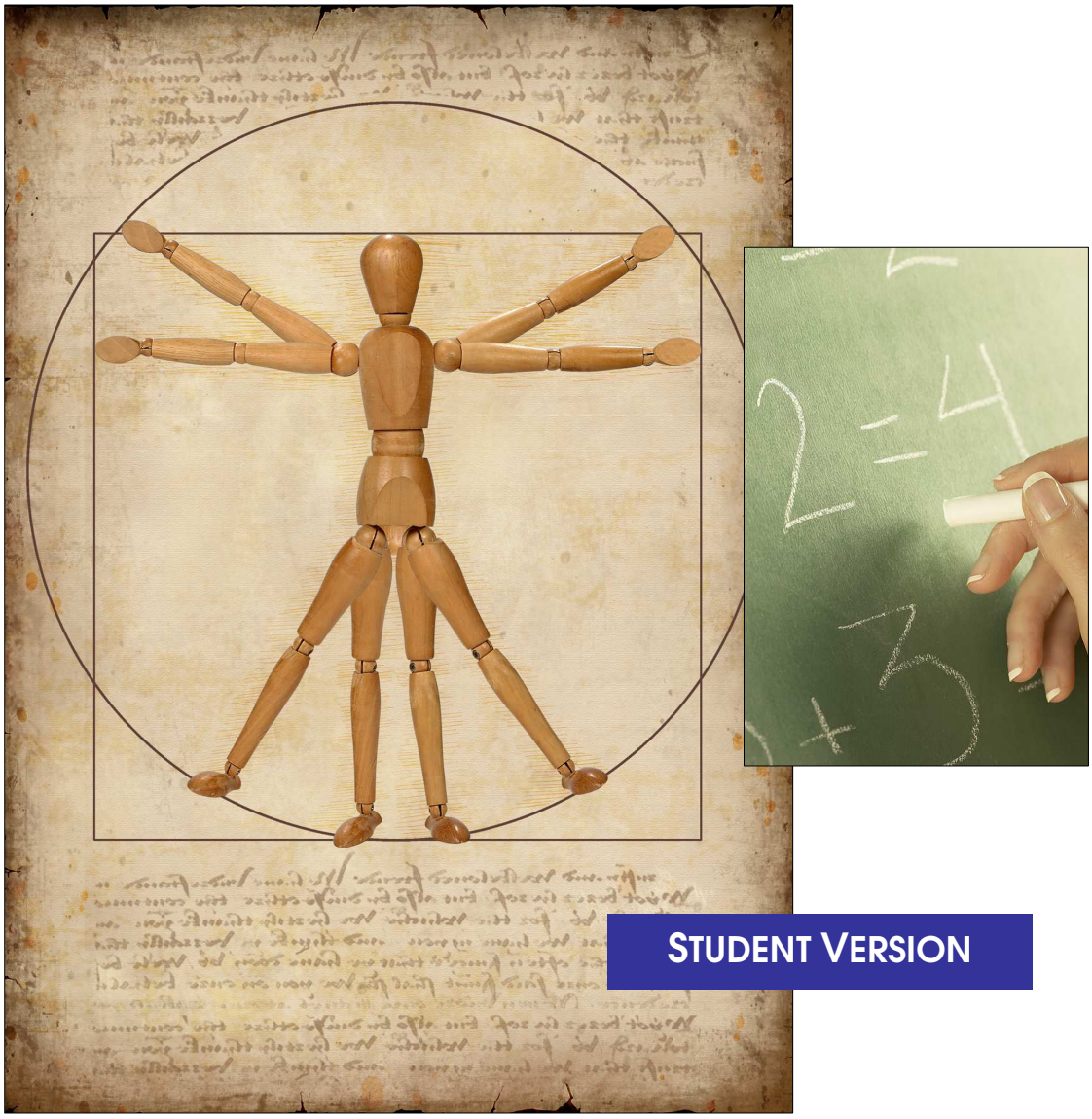
Mathematics:

A Christian Perspective



STUDENT VERSION

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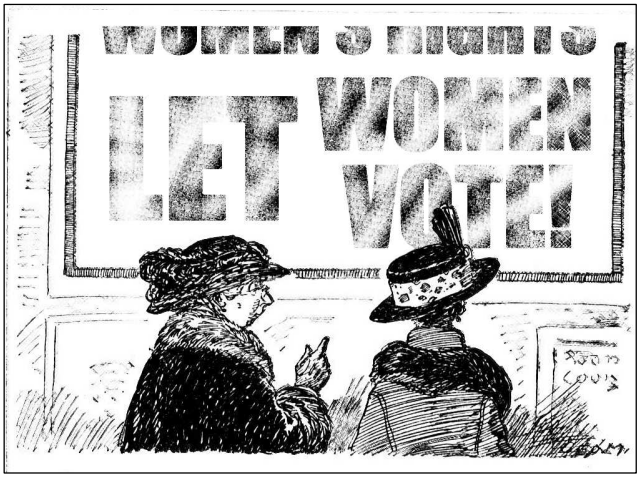
8 The Gender Gap



Equality is something we all hold dear. No one wants to be treated unfairly or taken advantage of. In the last 150 years, the United States has seen a dramatic change in how it values women. Women have gained the right to vote, the right to own property, and access to the workplace. Women were even granted the right to a fair wage under the Equal Pay Act of 1963. But the fight for equality has not been easy; nor does it seem to be over. With the defeated Fair Pay Act of 1999, and the proposed pay equity legislation in 2000, apparently some people still think they are being treated unjustly. Is there really a problem? If there is a problem, how big is it? Together we are going to try to find an answer to those questions.

The table on page 88 includes median incomes in the United States for full-time year-round workers (all races) from 1955 to 2003 by gender.

1) Looking at the shape of the distribution.

   a. Using the information in the table (page 88), we are going to explore possible relationships between median wages for men and women. Due to the large amount of information, it is best if you have access to either a graphing calculator or a computer to use for this lesson. What are some reasons we might be using the *median* incomes for *full-time year-round* workers?

Year	Male	Female	Year	Male	Female	Year	Male	Female
2003	$41,503	$31,653	1986	$25,894	$16,843	1970	$9,184	$5,440
2002	40,507	30,970	1985	24,999	16,252	1969	8,668	5,077
2001	40,136	30,420	1984	24,004	15,422	1968	7,814	4,568
2000	38,891	29,123	1983	22,508	14,479	1967	7,289	4,198
1999	37,374	27,370	1982	21,655	13,663	1966	6,955	4,026
1998	36,252	26,855	1981	20,692	12,457	1965	6,598	3,816
1997	35,248	26,029	1980	19,173	11,591	1964	6,284	3,710
1996	33,538	24,935	1979	17,479	10,531	1963	6,070	3,556
1995	32,199	23,777	1978	16,062	9,641	1962	5,826	3,457
1994	31,612	23,265	1977	15,070	8,814	1961	5,663	3,341
1993	31,077	22,469	1976	13,859	8,312	1960	5,434	3,296
1992	30,832	22,093	1975	12,934	7,719	1959	5,241	3,206
1991	30,331	21,245	1974	12,162	7,174	1958	4,949	3,101
1990	28,979	20,591	1973	11,468	6,488	1957	4,722	3,007
1989	28,419	19,638	1972	10,538	6,053	1956	4,467	2,829
1988	27,342	18,545	1971	9,631	5,701	1955	4,241	2,735
1987	26,681	17,564						

Source: US Census Bureau Current Population Survey 2003. The data can be found at the link http://www.census.gov/hhes/income/histinc/p36.html as of 2005.

b. To get an idea about the types of relationships that might exist, we first need to get an idea for the shape of the distribution. What patterns do you notice in the table? Be as specific as you can.

c. Create a single scatterplot showing the median wages for men (Year, Wage) and the median wages for women (Year, Wage). Use a separate symbol for men and women. When looking at the scatterplot for median incomes from 1955 to 2001, what patterns do you notice? Be as specific as you can.

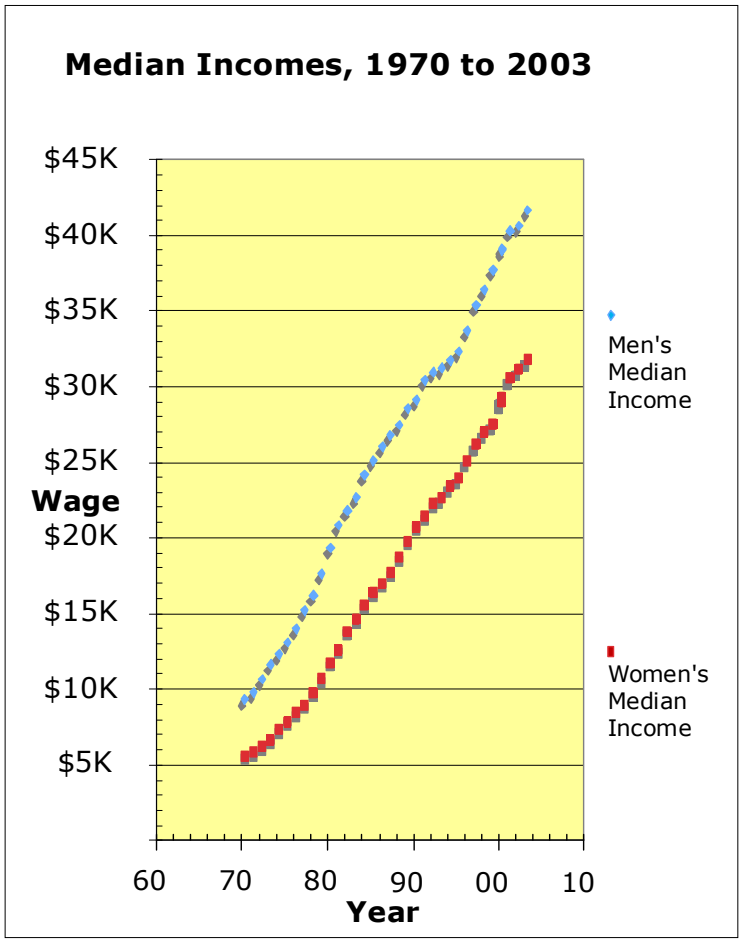
d. Could this data be modeled well by a line? Why or why not?

e. Why do you think the median incomes have this shape?

2) Line of Best-fit (Trend Lines)

a. The curve at the beginning of the graph poses a problem. Is there a point in the graph when the data becomes approximately linear? If so when?

b. There are several possible answers for when the median incomes become somewhat linear. For the remainder of the lesson, let us agree to use the data from 1970 onward. Below is a scatterplot using only the data from the years 1970 to 2003. Your teacher should provide you with a handout of this scatterplot. Using a ruler, draw in best-fit lines for both men and women's median incomes on the handout.



c. Why did you choose the lines you did?

d. Compare your lines with those of your group. Decide as a group which ones work best and use those for the following questions.

e. Find the equation for your line of best-fit for women's median incomes by following these steps.

i) Find two points from the women's income figures that lie on your line of best-fit. Using those two points, find the slope of the line.

ii) Using the slope, find the y-intercept of the line.

iii) Write out your equation in y = ax + b form.

f. Find the equation for your line of best-fit for men's median incomes by following these steps.

i) Find two points from the men's income figures that lie on your line of best-fit. Using those two points, find the slope of the line.

ii) Using the slope, find the y-intercept of the line.

iii) Write out your equation in $y = ax + b$ form.

g. Compare your median income equations for women and men.

i) How are the slopes similar and different?

ii) Using the context of 'Women's and Men's Median Incomes', what does the number for slope stand for in real life? Using this idea, how do the trends for the median incomes for women and men compare?

iii) How are the y-intercepts similar? How are they different?

iv) What does the "y-intercept" stand for in this situation?

h. Lines of best-fit can be very useful in helping us estimate what might happen to the data over time. With your equation, use tables, graphs, or symbols to estimate the median income for women in the years 1963, 1983, 2003, 2023, and 2053.

Do your answers make sense? Explain.

i. With your best-fit equation for men's median incomes, use tables, graphs, or symbols to estimate the median income for men in the years 1963, 1983, 2003, 2023, and 2053.

Do your answers make sense? Explain.

j. As a class, compare your estimates for the median incomes of women and men. Are they the same? If not, what might account for the differences?

3) Linear Regression

In the previous task, we each guessed at a line we thought came close to all the data. We then used those lines to estimate the median incomes for men and women several years into the future. However, we have a problem. If we are going to estimate incomes, we all want to come up with estimates that are close to each other. To do this, we have to agree on how we are going to find our trend lines. The mathematical community ran into this problem years ago, and adopted a process called linear regression. The idea behind linear regression is to minimize all of the individual errors between our made-up trend line and the real data values. This can be a very time-consuming process to do by hand; thankfully our calculators and computers can do all of the calculations for us. For TI-83 graphing calculators, once we have our data in the lists, we follow these steps: STAT → CALC → LinReg(ax+b) L1, L2.

a. Using the median incomes for men and women starting in 1970, create two linear regression lines: one for women and one for men. Do not graph the lines yet. Make sure everyone in your group has the same equation before moving on.

b. Compare the linear regression equations for men and women's median incomes.
   i) Compare the y-intercepts and slopes of the lines.

   ii) What do the slopes of these lines stand for in their contexts?

   iii) What do the y-intercepts stand for?

   iv) Without graphing the lines, which gender's wage will increase at a faster rate? How do you know?

c. Graph your regression lines for median incomes of men and women (in a different color) on the same scatterplot you used for your trend lines. How do your linear regression equations compare to your equations for best-fit lines?

d. Using your regression equations:
   i) Estimate the median incomes for women and men in the years 1963, 1983, 2003, 2023, and 2053.

   ii) Compare these estimates with those from your best-fit lines. Which do you have more confidence in? Why?

e. What do your equations suggest about the relationship between incomes for women and men in the United States?

f. According to our regression line, will the median incomes for women ever catch the median incomes for men?

[Note: Do not delete the values in your lists, we will use them on the next task.]

4) Cents For Every Dollar (Ratios)

a. Often in news reports about the gender pay gap, the term “cents for every dollar” is used. For example, in the year 2001, women earned fewer than 76 cents for every dollar men earned.

i) How is this number calculated?

ii) How many cents did women earn for every dollar men earned in the year 1960?

b. This idea of “cents for every dollar” offers some new insights into our problem. To help us see any possible patterns we will use a table to organize our information. Use the original data to fill in the following table.

Year	Cents Earned by Women for Every Dollar Earned by Men
1970	
1975	
1980	
1985	
1990	
1995	
2000	

c. What patterns do you see in the table?

d. Are the patterns you see good or bad for women?

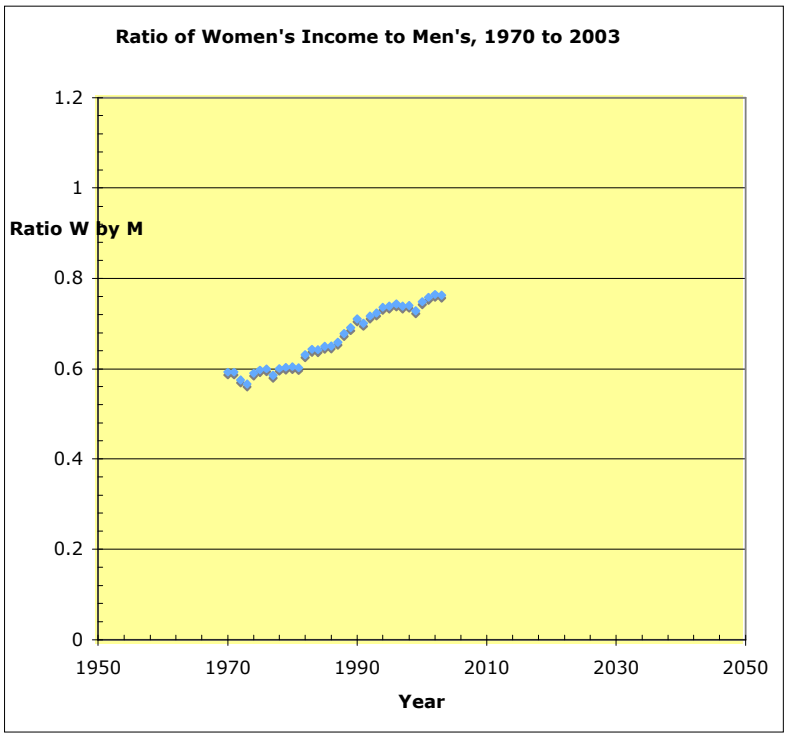
5. Extension

For the following questions, we will use linear regression. Your teacher may decide to have you use best-fit lines instead. Check with your teacher before going on.

a. Using the values from 1970 to 2003, create a list in your calculator that shows the median income ratios of women to men (i.e., cents earned by women for every dollar earned by men). Using this list, create a scatterplot on your calculator of the ratio of women's to men's median incomes from the years 1970 to 2003.

Does your scatterplot have the same shape as the one below? If not, check the values in your list.

b. What is the shape of the distribution of women's to men's income ratios?



c. Create a linear regression equation (or an equation for line of best-fit) for the ratios of women's to men's median incomes.

d. Graph your equation on the scatterplot and extend the line so it reaches from 1950 to 2050.

e. Using your regression equation (or equation for best-fit line), what are the estimated ratios for women's to men's incomes in the years 1963, 1983, 2003, 2023, and 2053?

i) Do the values seem correct to you? Explain.

ii) What would a value above 1 mean in this situation?

iii) Taking into account what we have previously found out about the relationship between median incomes for women and men, what problem have we just encountered?

iv) Our estimate for the women's to men's income ratio in 2053 is causing a problem. Let's check it by using our regression equations for women and men's incomes. In task 3 you found equations for median incomes for both women and men and calculated the estimates for median incomes in 2053. Use those estimates to find the estimated income ratio for women to men in 2053.

v) How does this ratio for 2053 compare to the previous ratio for 2053?

vi) Why is this happening?

f. Fill in the following table to help correct our estimates of women's to men's income ratios.

Year	Estimated Median Income for Women	Estimated Median Income for Men	Estimated Ratio of Women's to Men's Income
1963			
1983			
2003			
2013			
2023			
2033			
2043			
2053			

i) Looking back at the scatterplot for the ratio of women's to men's incomes and using the table, draw in what the estimated distribution should look like.

5) Practice Solving Equations

For each of the following questions, write equations in the form of $y = ax + b$, then use graphs, tables, or symbols to solve them.

a. In what year are women's median incomes expected to reach $35,000?

b. When will estimated men's incomes be $60,000?

c. When is it expected that women's incomes will reach $0.80 for every dollar men earn?

d. If current trends continue, when should we expect women to make at least 83% of what men make?

If you want to estimate how much income a woman might lose over a lifetime in the United States, here is an interesting site to check out. EQUAL PAY CALCULATOR (2005) http://www.aflcio.org/yourjobeconomy/women/equalpay/calculate.cfm

The following table is taken from the Statistics Division of the United Nations (2005): http://unstats.un.org/unsd/demographic/products/indwm/ww2005/tab5g.htm

Country	Year	Women's wages as a percentage of men's wages
Africa		
Botswana	2003	52
Egypt	2002	68
Eritrea	1996	66
Kenya	1997	123
Swaziland	1997	63
America, North		
Costa Rica	2001	83
El Salvador	2003	69
Mexico	2001	70
Panama	1999	93
Saint Lucia	2002	85
America, South		
Brazil	2002	61
Colombia	2003	65
Paraguay	2003	53
Peru	1995	55
Asia		
Bahrain	2002	44
China		
Hong Kong SAR	2002	64
Macao SAR	2003	67
Cyprus	2002	61
Georgia	2003	62
Iran (Islamic Republic of)	2001	80
Japan	2003	60
Jordan	2001	65
Kazakhstan	2003	70
Malaysia	1997	63
Mongolia	2003	87
Myanmar	1999	112
Occupied Palestinian Territory	2002	49
Philippines	1998	80
Qatar	2001	194
Republic of Korea	2002	56
Singapore	2003	61
Sri Lanka	2003	81
Thailand	2001	72
Turkey	1997	97
Europe		
Austria	2001	60
Belgium	1999	81
Bulgaria	2001	68
Denmark	2002	87
Finland	2002	83
France	2002	78
Germany	2003	74
Greece	1998	82
Hungary	2002	74
Iceland	2003	78
Ireland	2003	69
Latvia	2003	82
Lithuania	2003	77
Luxembourg	1996	63
Malta	2003	92
Netherlands	2000	78
Norway	2003	88
Portugal	1999	64
Sweden	2003	91
Switzerland	2002	133
Ukraine	2003	69
United Kingdom	2003	79
Oceania		
Australia	2002	89
French Polynesia	2003	87
New Zealand	2003	80