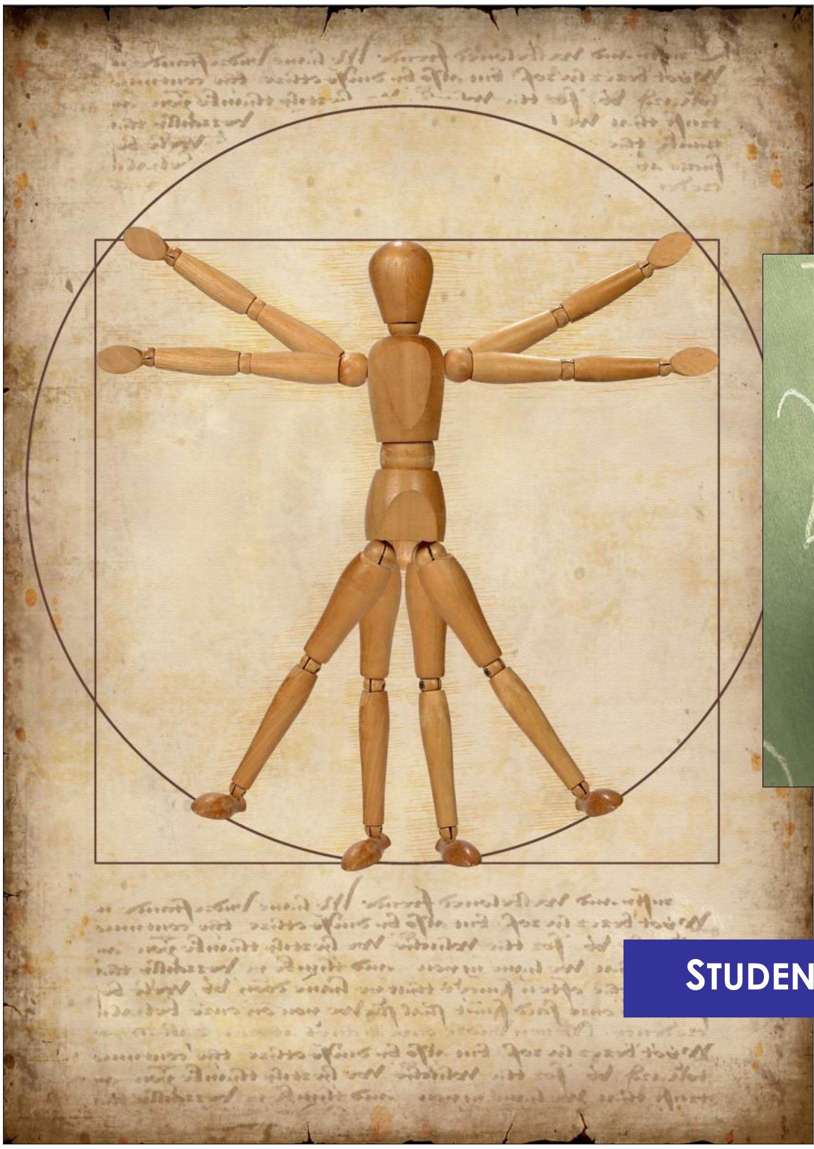
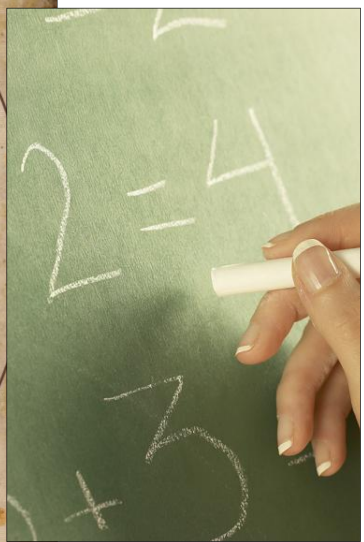
Mathematics:

A Christian Perspective





STUDENT VERSION

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9 Do Statistics Always Tell The Truth? Simpson's Paradox

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The goal of this lesson is to encourage you to become wary of making decisions before seeing all the evidence, or hearing both sides of an argument. As Christians, our jobs are to understand the information presented to us and to interpret that information in a God-pleasing manner. You will work with two-way tables, proportions, and percents. You will also encounter ideas related to data analysis by observing Simpson's paradox when viewing aggregates of data.

Warm-up

a. What evidence would you want to see before accusing someone of wrongdoing?

b. For instance, let us imagine it is the end of the marking period. When your teacher posts grades, one of you notices that everyone with brown eyes has higher grades than the rest of the class. Would you be suspicious?

c. What if we changed the scenario, and at the end of the marking period every boy in the classroom had a higher grade than the girls? Would you want to accuse your teacher of something?

d. How would you go about numerically creating an argument that your teacher was biased in his or her grading?

e. Would this be enough information to convince you that your teacher did something wrong? If so, why? If not, what would convince you?

1) Simpson's paradox—heart disease patients

Today we are going to look at common numerical arguments often used in newspapers and in television. Often when newspapers and other media provide statistics, we do not know how data was gathered, or the different ways data was manipulated before being presented to us. This lesson will look at how, on the surface, the numbers may tell us one thing; but as we dig deeper, the numbers may tell us something entirely different.

The following example is a generalized case of what happened several years ago in the United States. A large drug manufacturer presented information from their study suggesting that their drug reduced the number of heart attacks in patients with heart disease. Patients were randomly selected from several hospitals and administered the new drug. The rest of the patients continued to receive normal heart care. The table shows the results.

	Normal Treatment	New Drug Treatment
Heart Attack	920	70
No Heart Attack	8880	765
Total	9800	835

From the table, we can calculate the proportion of patients that had heart attacks from both the normal and the new drug treatments.

a. Calculate the proportions of heart attacks for both the normal treatment and the new drug treatment.

b. Is there a difference between the two treatments? Which treatment is better?

The news media started talking about the new wonder drug and berating the medical community for not supplying this drug to all of its heart patients, since the new treatment was about the same price as the normal treatment. The medical community held a press conference to respond. They produced the following tables:

Low Blood Pressure		
	Normal	New Drug
Heart Attack	220	30
No Heart Attack	3980	500
Total	4200	530

High Blood Pressure		
	Normal	New Drug
Heart Attack	700	40
No Heart Attack	4900	265
Total	5600	305

Pooled Results		
	Normal	New Drug
Heart Attack	920	70
No Heart Attack	8880	765
Total	9800	835

c. Calculate the proportions of heart attacks for both the low and high blood pressures for the treatments.

d. Do you still agree with your choice when we only had the pooled results? Make sure to support your answer.

The media was shocked. Not only is the new drug worse for low blood pressure patients but it is also worse for high blood pressure patients. The media had just learned a hard lesson in statistics; not everything is as straightforward as it seems.

Simpson's Paradox

An association or comparison that holds for all of several groups can reverse direction when combining data to form a single group. This reversal is called *Simpson's paradox* (Moore, 2003). In other words, two variables may seem to be related, but when we take into account a third *lurking variable*, the apparent association can vanish or even reverse. This highlights the importance of exploring the data (and listening to both sides of an argument) before coming to a conclusion.

2) Hospital Comparison

There are two hospitals in Sometown, USA: hospital A and hospital B. Your sick grandmother happens to live in Sometown and needs surgery. Being an astute math student and concerned grandchild, you decide to compare the hospitals to determine which is better. Figuring that living through the surgery is by far the most

important outcome, you decide to look at the survival rates of patients after surgery. (To survive the surgery, we are assuming they live at least 3 months after the surgery.) The table summarizes the results from your data gathering.

	Hospital A	Hospital B
Died	490	100
Survived	2000	1095
Total	2490	1195
Proportions A = 0.196787		B = 0.083682

From the results, clearly Hospital B is the safer choice. 20% of Hospital A's patients die after surgery while only 8% of Hospital B's patients die. In fact, you are surprised the city health officials have not shut down Hospital A. However, after remembering the problem with the New Drug for Heart Disease, you decide to dig a little deeper. The following tables break down the surgery patients into two categories: patients who arrive at the hospital in good condition, and patients who arrive at the hospital in bad condition (i.e., arrive by air transport or ambulance). [Problem adapted from Moore, 2003.]

GOOD CONDITION		
	Hospital	
	A	B
Died	40	50
Survived	1000	1000
Total	1040	1050

BAD CONDITION		
	Hospital	
	A	B
Died	450	50
Survived	1000	95
Total	1450	145

a. Calculate the proportions of deaths to total patients for both conditions of the hospitals.

b. Do the data support one hospital as being safer than the other?

c. What other variable do we need to take into account that the data does not show?

3) UC Berkeley Admissions

The table is actual data from the University of California Berkeley's admissions office in 1973 (Bickel, 1975). As always, the University published its admissions numbers from the graduate departments. However, in 1973, the information caught the eye of the public.

All Majors		
	Male	Female
Applied	2691	1835
Admitted	1198	557
% Admitted	0.44519	0.3035

a. Might the data indicate gender-based discrimination in the admission procedures of UC Berkeley? What would convince you?

The numbers caused quite a stir (as they should), and the administration of the university launched an investigation into its admissions practices.

b. Can you think of possible causes for the discrepancy in admission rates other than gender bias?

At the end of task 1, we discussed the definition of Simpson's paradox and briefly mentioned "lurking variables." A *lurking variable (*or *hidden variable)* is a variable, that when studied, explains the association between two unrelated variables. For example, we can look at several cities in the world and compare their crime rates to their ice cream consumption. The resulting correlation is somewhat surprising: r = 0.637. On the surface, it looks like crime is related to how much ice cream we eat. We know these two variables should be unrelated, and this makes us search for an explanatory variable: a variable that will explain the apparent relationship. For example, here it may or may not be temperature (Fey et al., 2005).

When looking below the surface, a lurking variable was found in the admissions data for Berkeley. It came out that each department was in charge of its own admissions. The admissions data for the six largest graduate programs are as follows:

Major A		
	Male	Female
Applied	825	108
Admitted	512	89

Major B		
	Male	Female
Applied	560	25
Admitted	353	17

Major C		
	Male	Female
Applied	325	593
Admitted	120	202

Major D		
	Male	Female
Applied	417	375
Admitted	138	131

Major E		
	Male	Female
Applied	191	393
Admitted	53	94

Major F		
	Male	Female
Applied	373	341
Admitted	22	24

c. In the table below, calculate the percentage of students admitted to each major by gender. In the last row, include the percentages from the combined data.

Majors	% Male Admitted	% Female Admitted	Your Notes
A			
B			
C			
D			
E			
F			
Total			

d. Which majors were the hardest to get into? Which were the easiest?

e. Overall, is there an association between gender and the difficulty of admission to a major? To put it another way, did one gender tend to apply to easy or difficult majors?

f. Do the individual department data imply gender discrimination in the admissions process? Create an argument, based on the data, either 'for' or 'against' gender discrimination.

4) Hiring Practices

In this task, we will look at the hiring practices of one of the major employers of Sometown, USA: Simpson's Incorporated (Savant, 1996). Simpson's has recently undergone a major expansion and has hired hundreds of new workers this past year. As the head of the human resources department, it is your responsibility to make sure that hiring protocols are followed. The local newspaper has just accused your company of gender discrimination in hiring and used the following numbers to make the case.

White Collar Jobs		
	Male	**Female**
Applied	200	200
Hired	30	20

Blue Collar Jobs		
	Male	**Female**
Applied	400	100
Hired	300	85

Total Jobs		
	Male	**Female**
Applied	600	300
Hired	330	125

a. The board of directors has set up a meeting with you tomorrow to discuss the accusations. Formulate your response.

b. Experiment with the values for the number of women hired for Blue Collar jobs. Of the 100 women who applied for the positions, how many of the women would need to be hired to change the outcome in the Total Jobs table?

5) Discussion

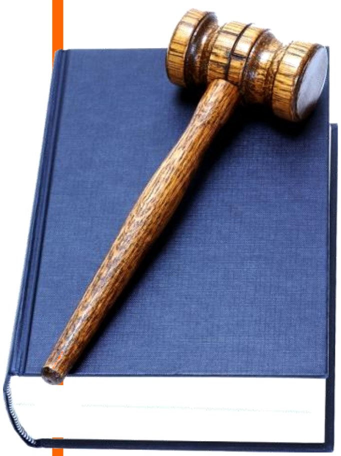
These are just a few of the cases where Simpson's paradox has showed up in our lives. It has been involved in all sorts of strange claims, from asserting that smoking makes you live longer, to "proving" that wearing a seat belt increases your chances of being seriously injured in a car accident. However, much more common is the slight data manipulation that happens when people pick and choose data that supports their opinion, or is convenient for them.

a. Here are some contexts in which this might happen. Think of at least one example of how or why someone might try to manipulate data in each context.

Politics:

Advertising:

Labor Negotiations:



Law (Legal Trials):

Below are two Bible verses. How can we apply these Scripture passages to today's math lesson? List your thoughts below each verse. Be prepared to share your answer with the class.

Ephesians 4:14 "Then we will no longer be infants, tossed back and forth by the waves, and blown here and there by every wind of teaching and by the cunning and craftiness of men in their deceitful scheming."

a. Thoughts:

b. Why might people want to deceive us? Can you think of instances where our believing a lie hurts us? Can you think of instances where our believing a lie hurts someone around us?

Exodus 20:16: "You shall not give false testimony against your neighbor."

a. Thoughts:

b. What damage can false testimony inflict? Does the damage it causes change if the person is found to be innocent?

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