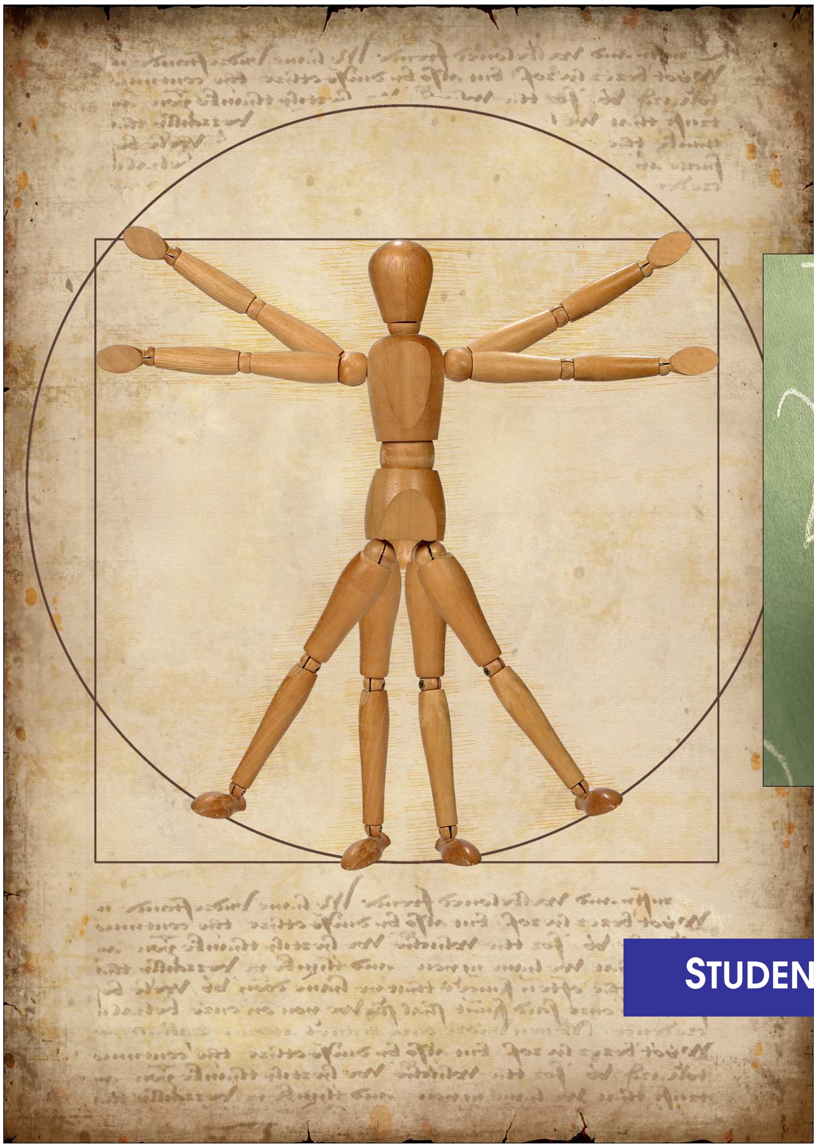
Mathematics:
A Christian Perspective



STUDENT VERSION

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Table of Contents

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6

Paper or Plastic? No, Thanks!

God's earth is sacred and it is our responsibility as good stewards to care for the created world. However, our habits exploit the earth for our own ends, thereby destroying the interdependence of all creation. For example, the population of the United States is only five percent of the planet's human population, but the United States produces one-quarter of the world's carbon emissions, consumes a quarter of its natural riches and generates enormous amounts of waste per person. The United States Environmental Protection Agency (EPA) states that paper, plastic, and food scraps make up approximately 58 percent of the total municipal solid waste. A few cities in this country and cities in other nations are working to reduce the amount of paper and plastic that is consumed.

The first two lessons in this unit will introduce a variety of functions. In the third lesson you will choose the appropriate function to model data provided by the EPA. In each lesson you will also be given the opportunity to discuss our role as good stewards of our created world. As good stewards and consumers, we need to be able to "think outside the bag."

Polynomial functions

In this lesson you will learn about graphs of polynomial functions and how to analyze them. You will see that data can be described by a polynomial model and learn how to find the model for specific data points, with the help of your graphing calculator.

1) Classification of a polynomial by degree

   The **degree of a polynomial** is the same of that of its term with the greatest degree. The **leading coefficient** is the coefficient of the term with the highest degree. The **degree of a monomial** is the sum of the exponents of its variables. See the table below for degrees, corresponding names and examples:

Degree	Name	Example
n=0	constant	3
n=1	linear	$5x + 7$
n=2	quadratic	$x^2 + 5x - 8$
n=3	cubic	$4x^3 + 7x^2 - 15x + 5$
n=4	quartic	$7x^4 - 3x^3 + 7x^2 + 15x - 3$

Example: Classify each polynomial by degree, name, and leading coefficient.

a. $2x^3 - 5x + 7$

b. $-8x^4 + 5x^3 + 11x + 5$

2) Graphs of Polynomial Functions

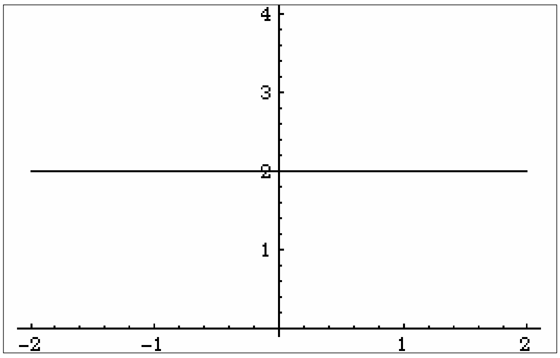
a. The general shapes of the graphs of several polynomial functions are shown below.

These graphs show the maximum number of times the graph of each type of polynomial may intersect the x-axis. The x-coordinate of the point at which the graph intersects the x-axis is called the **zero** of a function. Notice how the degree compares to the maximum number of real zeros.

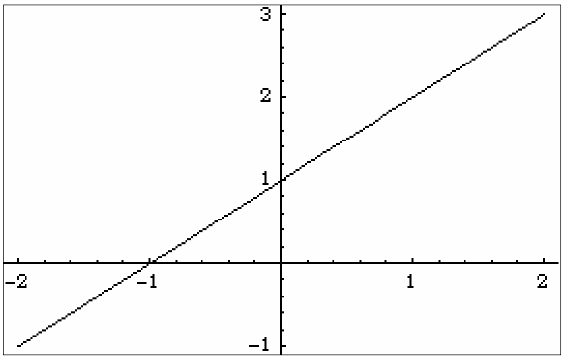
Also notice the shapes of the graphs for even-degree polynomial functions and odd-degree polynomial functions. The degree and leading coefficient of a polynomial function determine the graph's shape.

End-behavior is the terminology used to describe where the graph is going as x is approaching negative and positive infinity.

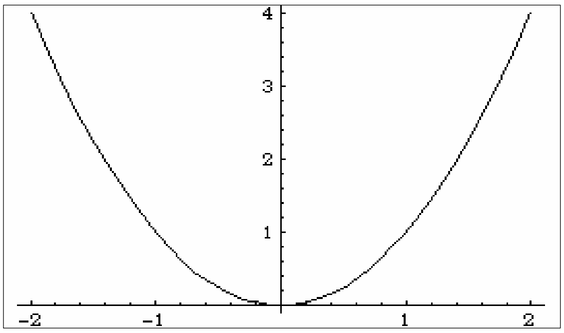
Constant Function
Degree 0



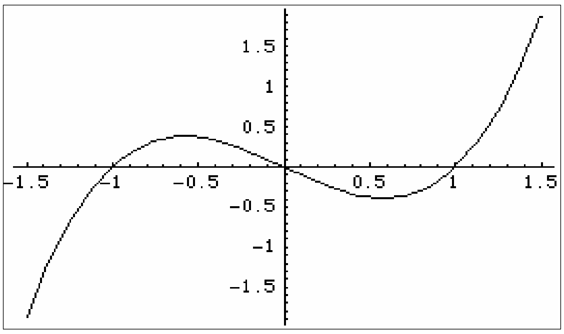
Linear Function
Degree 1



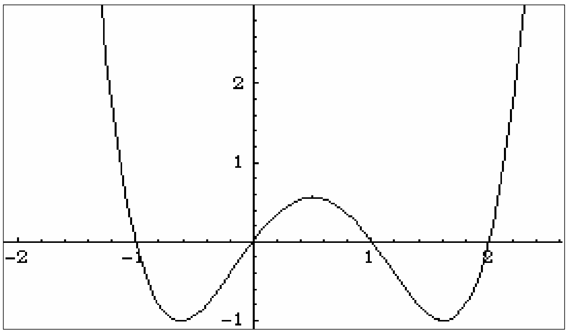
Quadratic Function
Degree 2



Cubic Function
Degree 3

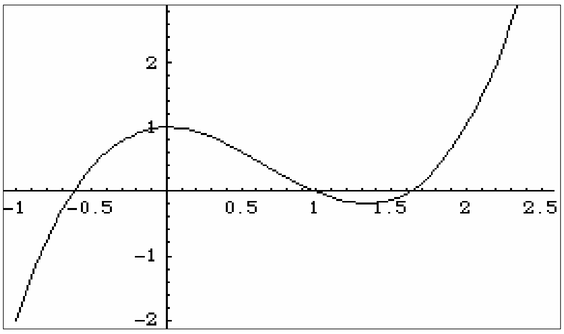


Quartic Function
Degree 4

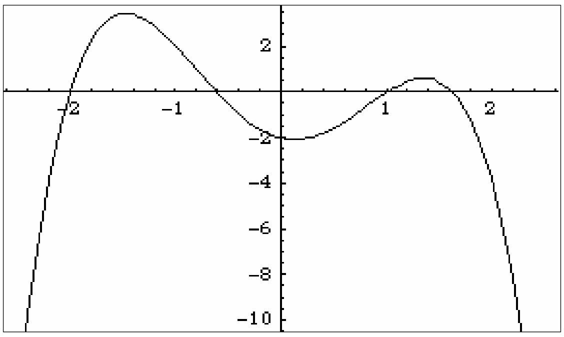


b. Example: For each graph, describe the end behavior, decide whether it represents an odd-degree or an even-degree polynomial function, and state the number of real zeros.

i)



ii)



3) Exploring Graphs of Polynomial Functions

You will need: a graphing calculator or at least one per group.

Directions: Work in groups of three or four. Each student should write a polynomial function. Within your group, state whether each function is an odd-degree or even-degree polynomial function before predicting the end behavior and the number of zeros of the function. Check your predictions by graphing each function.

a. Applications of Polynomials

Example: One of the many areas where we can be better stewards of natural resources is in the area of energy consumption. This example relates to our use of fuel. The average fuel (in gallons) consumed by individual vehicles in the United States from 1960 to 2000 is modeled by the cubic equation:

$$f(t) = 0.025t^3 - 1.5t^2 + 18.25t + 654$$

Where *t* is the number of years since 1960?

i. Graph the equation.

ii. Describe the end behavior of the graph.

iii. Describe the turning points of the graph.

iv. What trends in fuel consumption does the graph suggest?

Discussion Questions

Some Christians respond to the call to be good stewards of their resources by choosing to live lives of simplicity. Frugality demands of us a good recycling program, the elimination of waste, and to use materials efficiently. The EPA states that each person in the U.S. produces approximately 4.5 pounds of waste each day. What steps can you take to change this? How can you reduce waste in your home?

In developing countries millions of plastic bags and plastic mineral water bottles clog the underground drainage system, thus causing the monsoon rains to flood the cities, as happened in Mumbai, India, in 1998 and again in 2005. The local Municipal Corporation immediately responded by placing a ban on plastics. But, the very next day, this ban was lifted because a delegation from the plastics industry claimed the ban would adversely affect a large number of workers in the plastics industry. In a country where employment is a necessity for life, what solutions can be found in this situation? Are there valid reasons to put people out of work for the "greater good"? Could there be solutions where both the plastics industry and the Corporation could take steps forward toward a cleaner, healthier environment?

The Logistic Function

In this lesson you will learn the general equation for a logistic function and use your graphing calculator to find a logistic model to fit given data. We will learn about the logistic function using an example of population growth. This lesson will work best in a small group of three or four.

a. Here is a table of the populations of a small community, in thousands, of people. Plot the data on your calculator and graph the scattergram.

x (years)	y (thousands of people)
1	2
2	3
3	5
4	9
5	13
6	19
7	27
8	32
9	36
10	39

b. At first the population seems to be increasing exponentially with time. On the above figure of your scattergram, and with a color pencil, sketch the graph of an exponential function that would fit the first six data points reasonably well. With another color use the remaining points to sketch the graph of a logarithmic function.

c. Notice that toward the end of the 10-year period, the function seems to be leveling off. A function that models such population growth is called a logistic function. Its general equation is where *x* and *y* are the variables, *e* is the base of natural logarithm and *a, b* and *c* stand for constants. The community has room for about 43 thousand people, meaning c = 43. Calculate *a* and *b* using the first and tenth points. **You will need a separate sheet of paper for this step.** Substitute each point separately into the equation and solve the resulting two equations for *a* and *b.*

$$y = \frac{c}{1+ae^{-bx}}$$

d. Write your equation below.

e. Put this equation into your calculator and graph it on the same screen as the data. Explain whether your equation was a good fit.

f. What does the logistic function indicate the population was at time x = 0 years?

g. What graphical evidence do you have that the maximum population in the community is 43,000?

h. Erase your equation from the calculator. Use the following keystrokes for your calculator to come up with a logistic function to match the data present: STAT→ CALC → LOGISTIC. Write the calculator logistic equation below. Compare this equation with your calculated equation. How close were you?

i. Look up the word *logistic* in the dictionary. Find its meaning and origin.

j. Journal a few sentences below about what you learned as a result of doing this worksheet. Were you a good team member?

Discussion Question

The logistic function is often used to study and model population growth in communities and countries. Countries that have large populations, such as China and India, also have a large percentage of poor and peasant peoples. The plastics industry believes they are "improving the value of life" for all people with their products. However, urban and rural poor people oppose plastic production. One of the reasons given is the displacement of traditional handicrafts; the other is environmental pollution. Both of these problems serve to impoverish communities rather than improve the quality of life.

a. Do you hold stock shares in companies that promote the production of plastics in your own country or in developing nations?

b. Are you a consumer of traditional handicrafts? Do you inquire how products are manufactured and under what conditions?

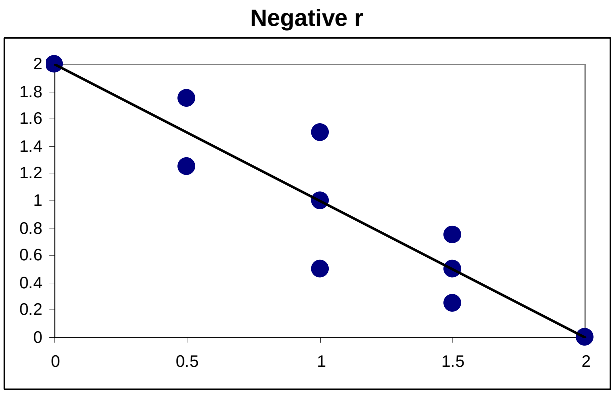
c. Can you become involved in handicrafts in your community? What natural materials can you use to create something beautiful?

Curve Fitting

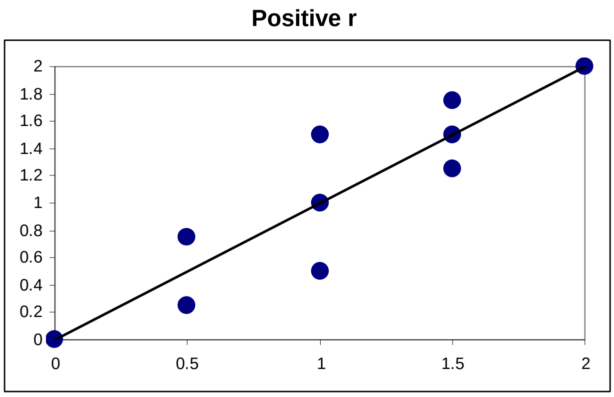
We began this unit by thinking about our role as good stewards of resources available to us. In this lesson we will use data from the United States Environmental Protection Agency (EPA) as our data for the study of curve fitting. As we find various regression functions that best fit the data, we will analyze the results both from a mathematical perspective as well as from the dimension of how to be good stewards.

a. Correlation Coefficient

The number r is called the correlation coefficient. The closer the value of r is to 1 or –1, the closer the data points are to the function of best fit. A negative value of *r* shows that as *x* increases, *y* decreases. A positive value of *r* shows that as *x* increases, *y* increases too.



The number r^2 is called the coefficient of determination. This is the percent of variation that can be accounted for by the least squares regression line. Since this is the value of *r* squared, this will always be a positive number.



(Note: If the value of r is not displayed on your calculator, use Diagnostic On from the CATALOG menu).

b. Fitting various functions to data

For our use through this lesson, we will use the table below.

Generation, Materials Recovery, Combustion, and Discards of Municipal Solid Waste (MSW), 1960–2003									
Millions of Tons									
	1960	1970	1980	1990	1995	2000	2001	2002	2003
Total generation	88.1	121.1	151.6	205.2	213.7	234.0	231.2	235.5	236.2
Total materials recovery	5.6	8.0	14.5	33.2	55.8	68.9	69.3	70.5	72.3
Combustion	27.0	25.1	13.7	31.9	35.5	33.7	33.6	33.4	33.1
Discards to landfill	55.5	87.9	123.4	140.1	122.4	131.4	128.3	131.7	130.8
Total discards	82.5	113.0	137.1	172.0	158.0	165.1	161.9	165.0	163.9

Environmental Protection Agency: Facts and Figures for 2003

c. Using your graphing calculator, input the decade years, i.e., 1960 as 1, 1970 as 2 etc. in List 1 and the row of "Total generation" in List 2

i) Graph a scattergram on your calculator.

ii) Use your calculator STAT $\rightarrow$ CALC to choose each of the following functions: Exponential, Cubic, Quadratic and Power. For each function record the equation and the correlation coefficient, r. Depending on the correlation coefficient, choose the function of best fit.

Exponential function: ______________________________

Correlation coefficient: ______________________________

Coefficient of determination: ______________________________

Cubic function: ______________________________

Correlation coefficient: ______________________________

Coefficient of determination: ______________________________

Quadratic function: ______________________________

Correlation coefficient: ______________________________

Coefficient of determination: ______________________________

Power function: ______________________________

Correlation coefficient: ______________________________

Coefficient of determination: ______________________________

iii) What do you notice about the total waste for each decade?

iv) Using your equation of best fit, what would you predict the total generation of solid waste would be in the year 2010?

v) It has been shown that in the years of slow economic conditions, solid waste is less than in the years of a flourishing economy. Discuss the relationship between the economy and amount of waste material.

d. Enter in your calculator the decade years and the “Discards to landfill” row in Lists 1 and 2 respectively.

i) Graph a scattergram on your calculator.

ii) Use your calculator STAT $\rightarrow$ CALC to choose each of the following functions: Exponential, Cubic, Quadratic and Power. For each function record the equation and the correlation coefficient, r. Depending on the correlation coefficient, choose the function of best fit.

<u>Exponential function:</u> ______________________________

Correlation coefficient: ______________________________

Coefficient of determination: __________________________

<u>Cubic function:</u> __________________________________

Correlation coefficient: ______________________________

Coefficient of determination: ______________________________

Quadratic function: ______________________________________

Correlation coefficient: __________________________________

Coefficient of determination: ______________________________

Power function: ____

Correlation coefficient: __________________________________

Coefficient of determination: ______________________________

e. In addition to the equations in c, find a logistic function for this data.

Logistic function: ____

f. What does the constant “c” tell you about the quantity of solid waste that is sent to landfills? How does this value fit with the data for the years 2001–2003?

g. Landfills are engineered areas where waste is placed into the land. The number of landfills in the United States is steadily decreasing, but the capacity has remained relatively constant. New landfills are much larger than in the past. What does this mean for your city, county, or state? Discuss.

h. Landfills can accept household hazardous waste. These are items such as paint, cleaners, oils, batteries, and pesticides. These items contain materials that can be dangerous to your health and the environment. What can you do to reduce the danger that these items cause to the environment?

Discussion Questions

The solid waste hierarchy is given below, ranked from most preferred to least preferred:

i) Source reduction and reuse

ii) Recycling

iii) Composting

iv) Disposal through combustion

v) Disposal through landfills

In your home, school, and community, what could you do to engage in the first three methods of waste management? What changes would you have to make in the way you live now? Record your ideas and discuss these in your class.

Did you know that high tech trash, such as heavy plastics and computers, are sent from industrialized countries to countries in Asia for breakdown and burning? It is the poor in these countries that have to break down and burn this waste. India is a leading importer of industrialized nations waste. As a citizen of an industrialized country, what action would you take? As a citizen of a developing country, what action would you take?