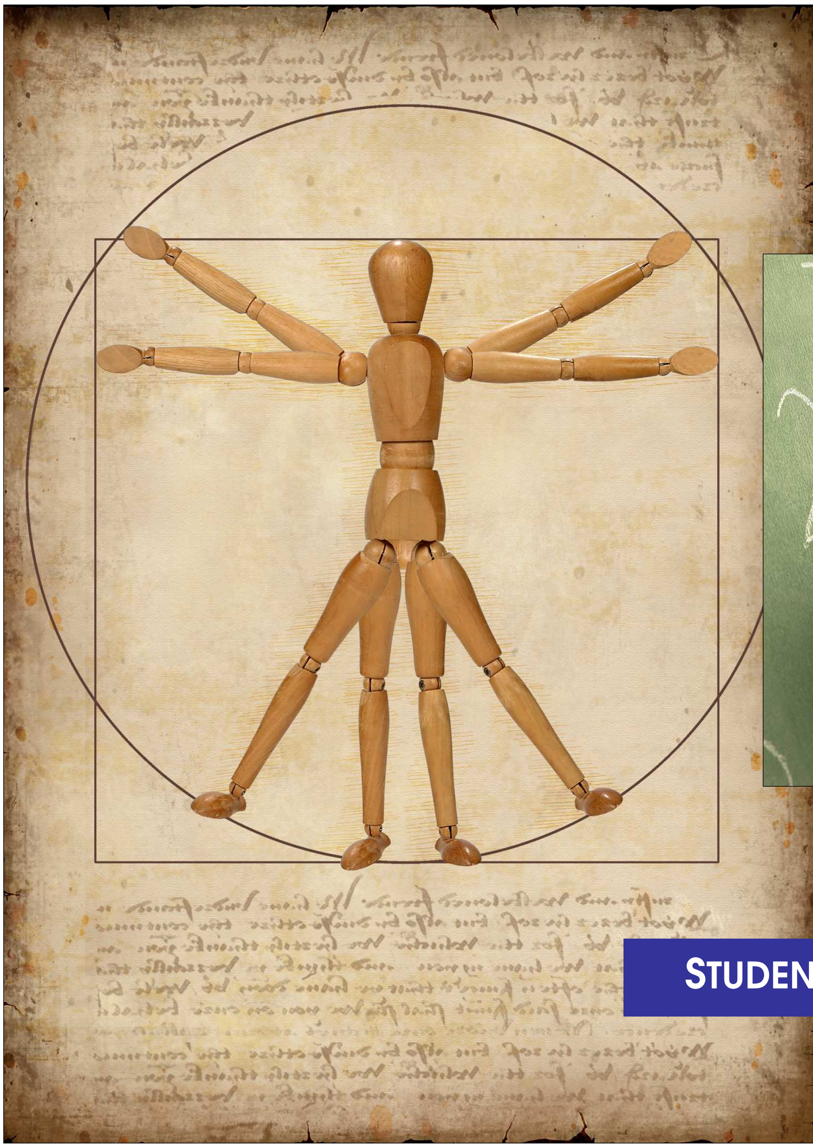
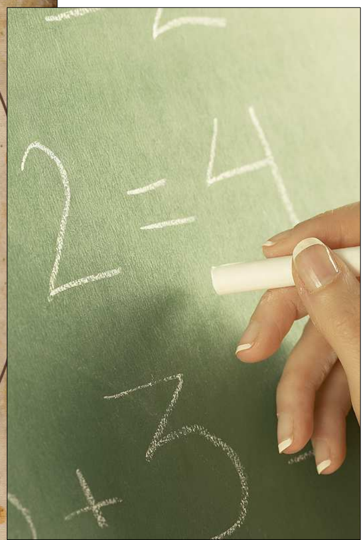
Mathematics:

A Christian Perspective





STUDENT VERSION

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Materials prepared on behalf of the Kuyers Institute of Calvin College, March 2006

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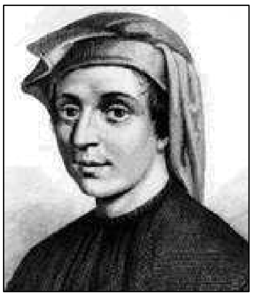
3 Fibonacci Numbers and the Golden Ratio

Today's lesson continues previous work on the analysis, extension, and generalization of numerical patterns. In particular, we look at one special pattern called the *Fibonacci Sequence*.



Leonardo Pisano lived in Pisa, Italy, from the late 12th century until the middle of the 13th century. He was brought to the court of Frederick II, the Holy Roman Emperor. There, Leonardo pursued his mathematical interests under the emperor's sponsorship. The name "Fibonacci" was not used during his lifetime but was suggested by Guillaume Libre, a 19th-century historian.

1. Encountering a new pattern

Consider the following numerical pattern:

1, 1, 2, 3, 5, 8, 13, 21, 34, …

a. What relationship do you see that connects the consecutive terms in the sequence? Describe it in words and compute the next three terms.

b. Next, consider the following definition of a numerical sequence:

Term #1 = a

Term #2 = b

For Term #n, n > 2, Term #n = Term #(n - 1) + Term #(n - 2)
Using a = b = 1, find the first 10 terms of this sequence.
What do you observe?

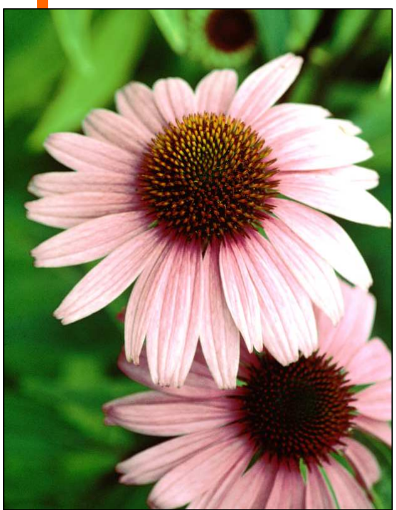
c. What happens if a = 2 and b = 3? How do your results compare to the previous case?

d. What if a = 2 and b = 5?

2) Linking the Fibonacci Sequence to the Natural World

The numerical pattern that you encountered in Task #1 is called the *Fibonacci Sequence*. Several aspects of the natural world are connected to this special sequence.

a. Select one of the objects provided by your teacher. Working with a partner, and following your teacher's example, carefully count the following:



The number of distinct clockwise spirals that wind outward from the base of the object.

The number of distinct counter-clockwise spirals that wind outward from the base of the object.

b. Appoint a representative from your group to record your findings on the board. What patterns do you observe concerning the collected data?

3) Mathematical Extensions

a. Let's reconsider the original Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, … Now, focus your attention on the ratio of consecutive terms in the sequence. Begin by completing the table below:

Terms	Value of Terms	Value of NextTerms	Ratio of Terms
1	1	1	1.00
2	1	2	2.00
3	2	3	1.50
4	3	5	1.66666
5	5		
6	8		
7			
8			
9			
10			

Describe what is happening in the "Ratio of Terms" column as you move further in the sequence.

b. There are numerous examples in art and architecture where the ratio of the length to the width of a rectangular object (or even the frame itself!) follows this common ratio quite closely. From three of the examples provided by the teacher, compute this ratio.

c. A more sophisticated analysis of these ratios would lead to the conclusion that over the long run, these ratios will approach a common value. One way to motivate this conclusion is to analyze the drawing below:

The line segment above was constructed so that the distance from point B to point C is exactly 1 unit (inch). The distance from point A to point B is not currently known and will be denoted by x. Point B was chosen in such as way as to divide the entire segment in “extreme and mean” ratio. That is: AB/BC = AC/AB. Using our given values,

$$\frac{x}{1} = \frac{x+1}{x}$$

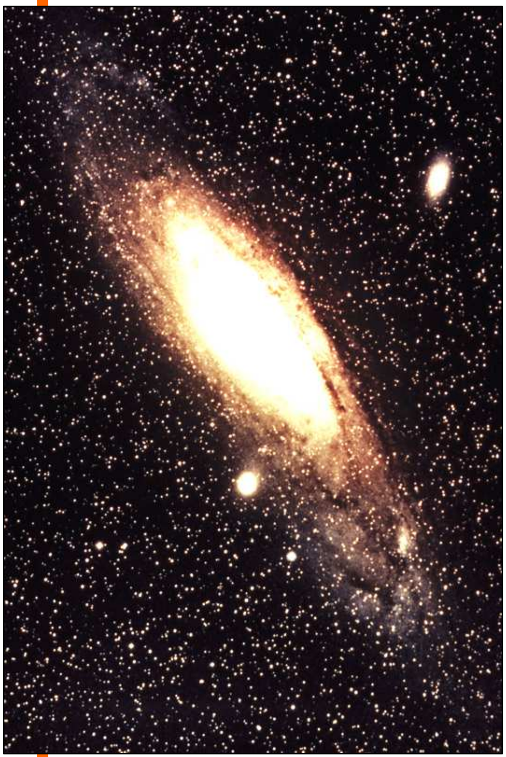
solve this proportion for x using the following steps:

First, cross-multiply and reexpress the resulting equation in the form

$$ax^2 + bx + c = 0$$

Next, solve this quadratic equation using the quadratic formula. Note that you will obtain two answers, but one can be rejected (why?). Give a numerical approximation of the answer to 6 significant digits. This value is commonly referred to as the "golden ratio."

Compare this value, the "limit" of the ratios of terms in the Fibonacci sequence, to the ratios of length:width that you found earlier. In some ancient cultures, including the ancient Greeks and ancient Egyptians, rectangular features in architecture that followed this special ratio (perhaps approximately in some cases) were viewed as aesthetically pleasing.



d. Christians believe that God created the heavens and the earth (see, for example, Genesis 1, Psalm 19). As such, Christians look at the beauty of the world and respond with praise to the Creator. Should Christians expect these "surprising" connections between mathematical ideas and the natural world? Why or why not?

4) Extension: The Golden Spiral (Optional)

There are a number of potential extensions of the golden ratio in a geometric context. One of these is called the Golden Spiral. If you have either the Geometer Sketchpad computer program or the Cabri Geometry package contained in the TI-92 graphing calculator, you can follow the list of instructions below to construct the golden spiral.

Instructions for Creating the Golden Spiral: This is an iterative process. Once you understand the first step, it can simply be repeated several times to produce the final image.

a. Construct a square. Any side length is fine. In Sketchpad, you can begin with any four points, construct the quadrilateral, print the lengths of two adjacent sides, and use your computer mouse to stretch or shrink side lengths until a square is obtained.

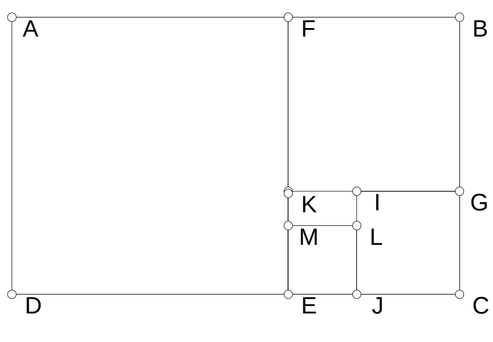
b. Construct the midpoint of the bottom side of the square. Connect this midpoint with the upper right-hand vertex of your square.

c. Using this midpoint as the center and the segment from the midpoint to the upper right-hand vertex as the radius, construct a circle.

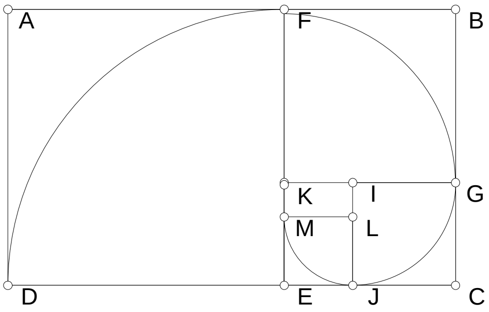
d. Extend the bottom side of your square using the line feature on the toolbar. Construct the point where this line intersects the circle. Note that the segment from the lower right-hand vertex of the square to this newly constructed intersection point forms a ratio with the side of the square that exactly equals the golden ratio. (Why? Hint: Use the Pythagorean Theorem!)

e. To complete the initial "golden" rectangle, you must create a line perpendicular to the bottom side of the square that passes through the newly created intersection point. Finally, find where this line intersects the extension of the top side of the square and construct a point at this intersection. This completes the rectangle. You can hide all unnecessary lines and circles, and you should obtain rectangle ABCD below.

To make additional smaller golden rectangles within this initial construction, you need to create points on segments BC and FE, labeled G and K in the diagram below, such that BG = GK = FK = FB, making FBGK a square. To continue the process, use the leftover rectangle, create a smaller square within it, and repeat the process. Eventually, you will obtain a diagram similar to the one shown:



To obtain the “Golden Spiral,” you need to connect opposite vertices of each square using circular arcs. In Sketchpad, you can select E (as the center) followed by F and D (points on the circle) to construct an arc through points F and D. Repeat this process to obtain the diagram.



Question for reflection: How do these patterns and relationships, and their appearances in the natural world around us, affect your appreciation for the order and regularity in the world around you? What impact do these discoveries have on your personal faith in a God who has created this world?