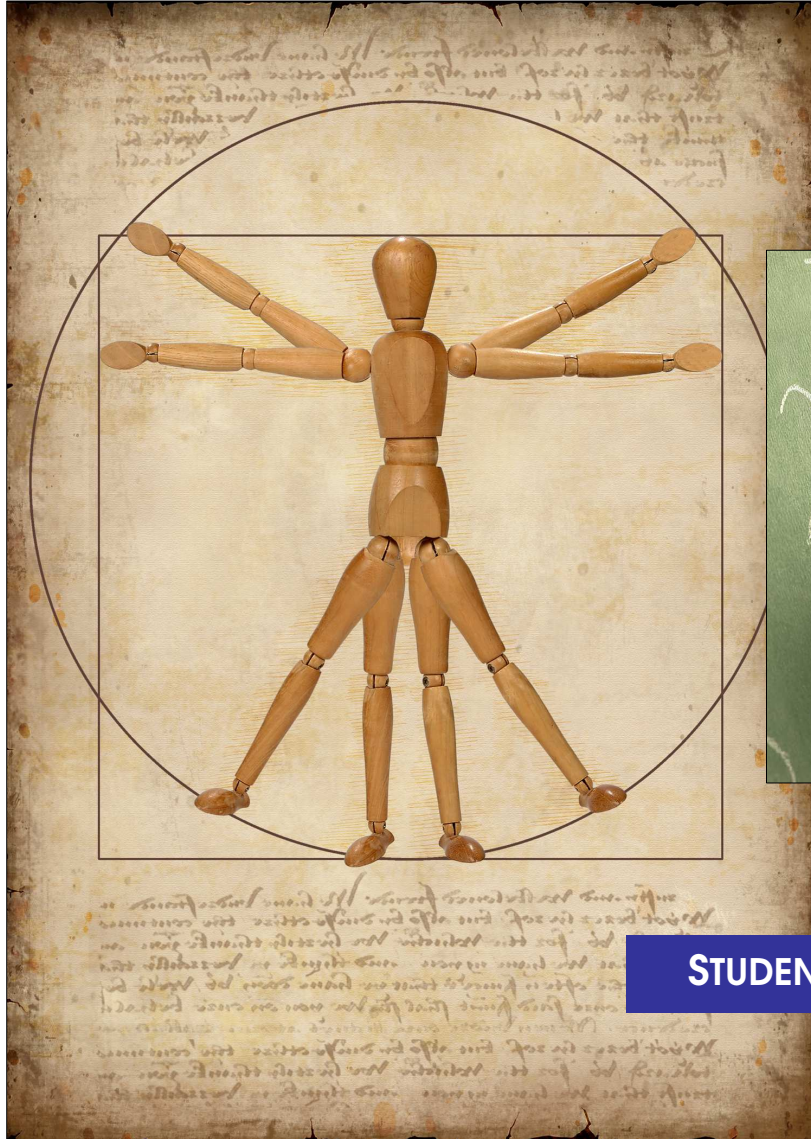


Mathematics:

A Christian Perspective



STUDENT VERSION

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Exponential Functions

Today, we explore several real world applications related to exponential functions.

In 1798, the English economist Thomas Malthus wrote a paper entitled, "An Essay on the Principle of Population as it Affects the Future Improvement of Society." In this paper, he argued that the world population increases according to an exponential function, while the resources available to feed this



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population increase according to a linear function. He concludes that cataclysmic problems, including mass starvation, would result in the coming decades. His predictions did not come true, in part because he failed to account for advancements in technology to increase food production. Furthermore, modern demographers have decided that a logistic model more accurately describes world population. Nonetheless, analysis of world population data can help illustrate the potential use and limitations of exponential growth models.

1) Analysis of World Population Data

As of July 2005, the world population stood at 6,457,000,000. Current mathematical models project a world population of over 9 billion in 2050. Here's a table of world population data for the past millennium, based upon estimates from the United Nations (see <http://census.gov/ipc/www/worldhis.html> for more information):

Year	Total World Population	Year	Total World Population	Year	Total World Population
1000	310,000,000	1900	1,650,000,000	1960	3,024,000,000
1250	400,000,000	1910	1,750,000,000	1970	3,697,000,000
1500	500,000,000	1920	1,860,000,000	1980	4,442,000,000
1750	790,000,000	1930	2,070,000,000	1990	5,280,000,000
1800	980,000,000	1940	2,300,000,000	2000	6,086,000,000
1850	1,260,000,000	1950	2,520,000,000		

- a. Graph this data. Use a graphing calculator if available. For the TI 83+ and similar machines, place the year data in L1 and the Population data in L2. Turn your statplot 1 on and graph. Describe the shape of this graph.
- b. How can you use the data table to justify that a linear model is unreasonable?
- c. Since these data do not fit a linear model, we cannot apply linear regression techniques to find an equation which “best” fits the data. Instead, use the “ExpReg” option on your graphing calculator. Record the result below. Note that the form for a general exponential function is: $f(x) = a(b^x)$. If no graphing calculators or equivalent technological tools are available, then your teacher will supply the exponential model that best fits this set of data.
- d. How well does this model “fit” the data? There are three ways to answer this question.

First, you can compare the graph of the exponential function obtained in #3 to the graph of the actual data. This is easily done with the graphing calculator, and can otherwise be done carefully by hand. Describe your findings below.

Second, you can compare the numerical values of the actual data and the corresponding values “predicted” by the exponential model. Complete the table:

Third, you can inspect the value of “r” that was produced with the exponential function by your calculator. This value is analogous to the linear correlation coefficient that you may have encountered in prior work. Its value ranges from -1.00 to 1.00, where values of

Year	Total World Population	Predicted World Population
1000	310,000,000	
1250	400,000,000	
1500	500,000,000	
1750	790,000,000	
1800	980,000,000	
1850	1,260,000,000	
1900	1,650,000,000	
1910	1,750,000,000	
1920	1,860,000,000	
1930	2,070,000,000	
1940	2,300,000,000	
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1980	4,442,000,000	
1990	5,280,000,000	
2000	6,086,000,000	

± 1.00 represent a perfect fit. Based upon all three of the above analyses, describe how well the exponential model fits the data.

- e. Discuss some of the potential effects of ever larger populations on the use of the earth's resources, on pollution, and on global warming. How do these potential effects relate to our responsibility to be stewards of the creation?

2) "Think Globally—Act Locally"

Regardless of the trends in population growth, there are decisions that each individual person makes that will affect the impact on the environment. We first consider the amount of garbage that we discard each week. Suppose that you made a commitment to reduce the amount of garbage that you produce this year by 5%. This may mean reducing the amount of packing (e.g., declining a bag at the grocery store), reusing a product (e.g., taking your own empty bags to the grocery store), or recycling a product (e.g., beverage can). Furthermore, suppose that you commit to a 5% reduction of this lower amount of garbage in the following year, and so forth.



- a. What is the general formula that would model this situation?

- b. If you currently produce 500 pounds of garbage per year, then how many pounds will you produce 10 years from now?

- c. How many years must you continue this commitment to cut your amount of garbage in half? (Note: It is not necessary to know the initial amount of garbage. Why not?)



- d. Next, suppose that higher gasoline costs encourage motorists to purchase more fuel efficient (and less polluting) vehicles. At the end of 2005, there were approximately 2,000,000 hybrid vehicles in use worldwide. This total is expected to grow by 20% per year for the foreseeable future. What exponential function would model this situation?

- e. If this trend continues, when will the number of hybrid cars in use first exceed 10 million? What is the time required for the number of hybrid cars to double?



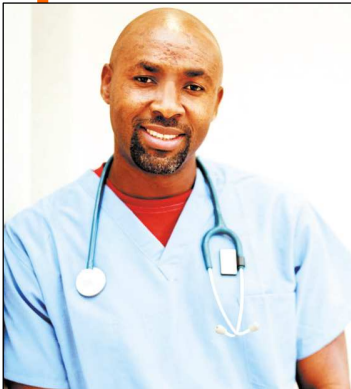
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3) Harnessing the Power of Exponential Growth: Saving for Retirement!

Many students in high school have recently found their first part-time jobs. There is a temptation to spend the money earned from this job as fast as (or even faster than) it comes in! However, two important points should be made. First, as Christians we view all of our financial resources as a part of God's provision. As such, we respond with gratitude and give a portion of these resources to a local church or other charitable organization.



Second, it is important to think about saving for future needs. Although most students will not retire from the workforce for 50 years or more, it would still be a wise decision to start investing now for those eventual retirement needs. To help students understand the impact of starting early, let's investigate the retirement planning strategies of two twins, John and Jane.



- a. John decides that he will wait to save for retirement until he turns 40 years old. At this point, he plans to put aside \$5,000 and let it continue to grow for 27 years. Assuming that this investment earns an average annual rate of 6%, compute the amount of money that John will have saved when he retires at age 67.

- b. John does not think that this will be enough money. So, he decides to invest an additional \$5,000 each year after age 40 until he retires at age 67. Let's assume that his last investment is at age 66, so a total of 27 separate investments of \$5,000 are made. Compute the total amount of money that John will have saved by retirement, assuming an average annual rate of return of 6%.

- c. Now it's Jane's turn. She decides to begin saving for retirement as soon as she gets her first full-time job at age 22. She invests \$5,000 this year and repeats this investment until age 40. Note that this means a total of 19 such investments. At this point, she stops investing money for retirement. Assuming a 6% annual rate of return, compute the total that Jane will have saved for retirement at age 67.

- d. Explain why Jane's amount is so much more than John's amount, even though Jane invested a total of only \$95,000 (19 years @ \$5,000 per year) while John invested a total of \$135,000 (27 years @ \$5,000 per year).



- e. Explain how early retirement planning can connect to charitable giving.

- f. Now it's your turn! Decide on the age that you wish to start investing for retirement. Decide on the amount of money that you will commit to investing each year until retirement. Use either a recent newspaper or an online resource to determine a reasonable annual rate of return for your investments. Compute the estimated value of your investments when you reach retirement age.

