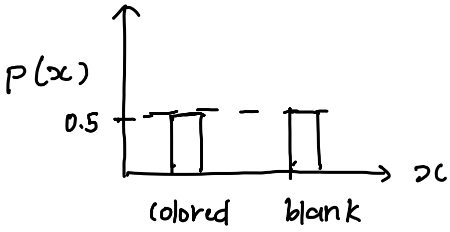
Assuming uniform distribution initially since we are uncertain about the initial state and the states are discrete.

Thus,



Given,

$p(x_{t+1} = \text{colored} \mid x_t = \text{blank}, u_{t+1} = \text{paint}) = 0.9$

$p(z = \text{colored} \mid x = \text{blank}) = 0.2$

$p(z = \text{colored} \mid x = \text{colored}) = 0.7$

It is given that painting operation has been performed & sensor indicates that object is colored.

$$p(x_{t+1} = \text{blank}) = ?$$

Using Bayes filter,

$p(x_{t+1} = \text{blank} \mid z_{t+1} = \text{colored}, u_{t+1} = \text{paint})$

$$= \frac{p(z_{t+1} = \text{colored} \mid x_{t+1} = \text{blank}, u_{t+1} = \text{paint}) \times p(x_{t+1} = \text{blank} \mid u_{t+1} = \text{paint})}{p(z_{t+1} = \text{colored} \mid u_{t+1} = \text{paint})}$$

Here, $p(x_{t+1} = \text{blank} \mid u_{t+1} = \text{paint})$

$$= \sum_{x_t} p(x_{t+1} = \text{blank} \mid x_t, u_{t+1} = \text{paint}) \times p(x_t \mid u_{t+1} = \text{paint})$$

for $x_t \in \{\text{blank}, \text{color}\}$

$$= 0 \times 0.5 + 0.1 \times 0.5 = 0.05$$

$$p(z_{t+1} = \text{colored} \mid u_{t+1} = \text{paint}) = \sum_{x_{t+1}} p(z_{t+1} = \text{colored} \mid x_{t+1}, u_{t+1} = \text{paint}) \times p(x_{t+1} \mid u_{t+1} = \text{paint})$$

$x_{t+1} \in \{\text{color}, \text{blank}\}$

$$= 0.7 \times 0.95 + 0.2 \times 0.05$$

$$= 0.665 + 0.01 = 0.675$$

[Assuming sensor probability is independent of control command)

$\therefore p(x_{t+1} = \text{blank} \mid z_{t+1} = \text{colored}, u_{t+1} = \text{paint})$

$$= \frac{0.2 \times 0.05}{0.675} = 0.0148$$