Ex: 1

(a) The Bayes filter is used to obtain the current belief of a robot given its initial observations and controls

$$bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$$

It comprises of two steps, prediction step and correction step. It recursively defines the belief of a robot.

$$p(x_t \mid z_{1:t}, u_{1:t}) = p(x_t \mid z_t, z_{1:t-1}, u_{1:t})$$
$$= \frac{p(z_t \mid x_t) \times \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \times bel(x_{t-1})\, dx_{t-1}}{p(z_t \mid u_{1:t})}$$

Prediction step: $\int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \times bel(x_{t-1})\, dx_{t-1}$

Correction step: $\eta \times p(z_t \mid x_t) \times \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \times bel(x_{t-1})\, dx_{t-1}$

(b) $bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$ ⇝ (It is the probability distribution of the current position of the robot conditional to its previous observations and controls)

$p(z_t \mid x_t)$ ⇝ The probability of obtaining a given observation given the robot's current position.

$p(x_t \mid u_t, x_{t-1})$ ⇝ The probability of the current position of the robot given its initial position and control step.

(c) $p(x_t \mid x_{t-1}, u_t) \sim \mathcal{N}(g(u_t, x_{t-1}), R_t)$

$x_t = g(u_t, x_{t-1}) + \epsilon_t$

$h(g(u_t, x_{t-1}))$

$p(z_t \mid x_t) \sim \mathcal{N}(h(x_t), Q_t)$

x_t

$bel(x_t) \sim \mathcal{N}(\mu_t, \Sigma_t)$

μ_t is the final mean obtained after EKF slam & Σ_t is the covariance matrix.

"Considering ONLINE SLAM"

(d) μ_t is the final mean of the $bel(x_t)$ obtained from EKF SLAM. Its dimensions are $(2n+3) \times 1$ if there are n land marks. It is the best estimate of the position of the robot.

Σ_t is the covariance matrix of the $bel(x_t)$. It tells us how the different variables $(x, y, \theta, m_1, \ldots, m_n)$ vary with each other. The final $bel(x_t) \sim \mathcal{N}(\mu_t, \Sigma_t)$

Dimensions: $(2n+3) \times (2n+3)$

$$\begin{array}{cc} (3\times3) & (\times n) \end{array}$$

$$\Sigma_t = \begin{pmatrix} \Sigma_{xx} & \Sigma_{mx} \\ \Sigma_{xm} & \Sigma_{mm} \end{pmatrix} \begin{array}{c} x \\ y \\ \theta \\ 1 \end{array}$$

g: It is the function which represents the motion model. $g(x_{t-1}, u_t)$ & gives the prediction of the bot after control.

$\bar{\mu}_t = g(\mu_{t-1}, u_t)$

G_t^x: It is the Jacobian of the motion model which is used to use Kalman Filter for non-linear models. It makes motion model linear. dimension: (3×3)

$$G^x = \begin{pmatrix} G_t^x & 0 \\ 0 & I \end{pmatrix}$$ dimension $(2n+3) \times (2n+3)$

It is obtained after expanding dimensions of (G_t^x) to ensure that the landmark positions remain same.

$g(x_{t-1}, u_t)$

R_t^x : It is the covariance matrix for the noise in the motion model. dimension : (3×3)

R_t : It is the extended matrix obtained after $F_x^T R_t^x F_x$
dimension: $(2N+3 \times 2N+3)$ It comprises of all the covariances between possible variables.

h : It is the function representing the sensor model. $h(x_t)$ gives the expected observation.

H_t : It is the Jacobian for the function h which linearizes it. For a particular observation z_t^i, we obtain H_t^i which is the Jacobian of $h(x_t^i)$ about the

$h(y_x) = h(\mu_t) + H_t^i(\cancel{y_t} - \mu_t)$

Q_t : It is the covariance matrix corresponding to the sensor noise. Dimension: (3×3)

K_t : It is the Kalman gain. It incorporates the sensor measurement to correct our belief.

Exercise 2:

(a)

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta_{trans} \cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans} \sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix} = \begin{pmatrix} x_{t-1} + \delta_{trans} \cos(\theta_{t-1} + \delta_{rot1}) \\ y_{t-1} + \delta_{trans} \sin(\theta_{t-1} + \delta_{rot1}) \\ \theta_{t-1} + \delta_{rot1} + \delta_{rot2} \end{pmatrix}$$

$$= g(u_t, \bar{x}_{t-1}) = \begin{pmatrix} g_x \\ g_y \\ g_\theta \end{pmatrix}$$

$$G_t^x = \begin{pmatrix} \frac{\partial g_x}{\partial x_{t-1}} & \frac{\partial g_x}{\partial y_{t-1}} & \frac{\partial g_x}{\partial \theta_{t-1}} \\ \frac{\partial g_y}{\partial x_{t-1}} & \frac{\partial g_y}{\partial y_{t-1}} & \frac{\partial g_y}{\partial \theta_{t-1}} \\ \frac{\partial g_z}{\partial x_{t-1}} & \frac{\partial g_z}{\partial y_{t-1}} & \frac{\partial g_z}{\partial \theta_{t-1}} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & -\delta_{trans}\sin(\theta_{t-1}+\delta_{rot1}) \\ 0 & 1 & +\delta_{trans}\cos(\theta_{t-1}+\delta_{rot1}) \\ 0 & 0 & 1 \end{pmatrix}$$

(b) low H_t^i

$$h(\bar{\mu}_t, j) = z_t^i = \begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(\bar{\mu}_{j,x}-\bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y}-\bar{\mu}_{t,y})^2} \\ \operatorname{atan2}(\bar{\mu}_{j,y}-\bar{\mu}_{t,y}, \bar{\mu}_{j,x}-\bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix}$$

$\uparrow$

expected measurement

$$h(x_t, j) = h(\bar{\mu}_t, j) + \underbrace{\text{low } H_t^i}_{(2\times 3)}(\bar{x}_t - \bar{\mu}_t) \qquad h(x_t, j) = \begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix}$$

$$\text{low } H_t^i = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} & \frac{\partial r}{\partial \theta} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial \theta} \end{pmatrix}$$

$$\frac{\partial r_t^i}{\partial \bar{\mu}_{t,x}} = \frac{1 \times 2(\bar{\mu}_{j,x}-\bar{\mu}_{t,x})}{2\sqrt{(\bar{\mu}_{j,x}-\bar{\mu}_{t,x})^2+(\bar{\mu}_{j,y}-\bar{\mu}_{t,y})^2}} \qquad \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,\theta}} = -1$$

$$\frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,x}} = \frac{+(\bar{\mu}_{j,y}-\bar{\mu}_{t,y})}{(\bar{\mu}_{j,y}-\bar{\mu}_{t,y})^2+(\bar{\mu}_{j,x}-\bar{\mu}_{t,x})^2} \qquad \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,y}} = \frac{(\bar{\mu}_{j,x}-\bar{\mu}_{t,x})\times -1}{(\bar{\mu}_{j,y}-\bar{\mu}_{t,y})^2+(\bar{\mu}_{j,x}-\bar{\mu}_{t,x})^2}$$

let $(\bar{\mu}_{j,x}-\bar{\mu}_{t,x}) = \Delta x$, $(\bar{\mu}_{j,y}-\bar{\mu}_{t,y}) = \Delta y$, $(\bar{\mu}_{j,y}-\bar{\mu}_{t,y})^2 + (\bar{\mu}_{j,x}-\bar{\mu}_{t,x})^2 = a$

$$\therefore \text{low } H_t^i = \begin{pmatrix} \frac{\Delta x}{\sqrt{a}} & \frac{\Delta y}{\sqrt{a}} & 0 \\ \frac{\Delta y}{a} & -\frac{\Delta x}{a} & -1 \end{pmatrix}$$