PHYS 381 – Computational Physics I (Winter 2023)

Assignment #3: Projectile motion under air resistance

Due date: 03 March 2023, 3 pm

Group members (add your name and UCID below):

Member #1: David (Scott) Salmon - 30093320

Member #2: Gradyn Roberts - 30113797

Member #3: Gabriel Komo - 30164690

Please, only add text in this report template using Times New Roman font, non-italic, non-bold, font size 12. Equations can be typed using the default font of the equation editor of your choice.

Authors' contributions:

Provide a brief description of how the work of this assignment was distributed among group members. Assignment workload distribution must be balanced among group members, i.e., all group members should be involved equally with coding and written materials for the report. **Authors' contribution** description cannot go over page 1 of this template.

We all worked together on all of the parts of the assignment. We worked together in and outside of class on the code, and Scott finished up sections 3 and 4 over reading week. We also all typed up parts of the assignment, Gabe mostly worked on the introduction and abstract, Gradyn worked on the Methods section and the conclusion, and Scott worked on the Results and Analysis section, and the code workflow section.

Abstract (0.5 points):

Provide a summary of your assignment work and findings. The abstract is a paragraph where you summarize what you studied in this computational lab and what are your main findings. Please, stay on the limit size of up to 300 words max. **Abstract** cannot go over page 2 of this template.

This computational lab was about determining the type(s) of air resistance that are non-negligible to a sphere based on the sphere's diameter and velocity. It was discovered that when the product of the two was lower than $7.0 * 10^{-5}$ (m^2/s), the sphere experienced primarily linear air resistance, and when the product of the two was higher than $7.0 * 10^{-3}$ (m^2/s), the sphere experienced primarily quadratic air resistance. This range was then used to determine that a baseball (and most macroscopic objects moving in general) experience primarily quadratic air resistance, really tiny objects the size of a dust particle or a tiny drop of oil moving really slowly experience primarily linear air resistance, and then objects that are in between those such as a raindrop moving fairly slowly experience both types of air resistance.

It was then discovered that as the mass of the sphere increased, the longer it took for the sphere to reach terminal velocity. In addition, we compared the results from Euler's method and the analytical method to solving the differential equation (1) from the lab manual [1], and determined that it is imperative that a very low Δt must be chosen for Euler's method to be accurate.

Finally, we compared how linear and quadratic air resistance stacked up to the motion of a projectile in a vacuum. We saw how in a vacuum the projectile moved in a perfect arc, whereas when air resistance was applied, the peak of the arc was lower and the horizontal displacement was also lower. We also discovered that when a mass is sufficiently low, the optimal angle of launch to maximize horizontal displacement is NOT always 45 degrees when the sphere experiences air resistance, as is the case when there is no air resistance.

Introduction (0.5 points):

Provide an overview of the topics studied in the assignment. Concentrate on the ‘physics’ part, in this case, the physics of projectile motion with air resistance. **Introduction** cannot go over page 3 of this template.

The assignment was split into four main sections. The first main section was about investigating the relationship between the product of the diameter and velocity of a sphere and what type of air resistance is non-negligible and needed to be accounted for. A rough range that determined when the types of air resistance were relevant was discovered and then applied this range to determine what type of air resistance 3 different sphere’s with various diameters and travel speeds experienced.

The second section of the assignment was focused on analyzing the vertical motion of a specified dust particle. The first step to this section was solving what the mass was of the sphere through calculating the volume of a sphere and multiplying it by the mass density. Then the terminal velocity needed to be calculated so the Dv product could be determined. Using this product, it was determined that the dust particle experienced primarily linear air resistance, and therefore “c” was assumed to be equal to 0. Then Euler’s method was utilized to create a velocity vs. time plot. Next, Euler’s method was compared to a more accurate analytical method for determining the velocity of the dust particle at a specified time. Finally, some calculations were put to the test and it was determined how long it took the specified dust particle to fall from a height of 5 meters while experiencing (linear) air resistance. The last part of this section included an investigation into “the often-quoted statement that all objects fall together with the same acceleration regardless of their masses”. [1]

The third section of the assignment was mostly about comparing how projectile motion differed when the object experienced no air resistance (i.e. traveling in a vacuum), and when it experienced linear air resistance. Another topic explored in this section included how the optimal angle to launch an object at for the projectile motion to maximize horizontal displacement differed for the sphere when it experienced air resistance, and how this angle was dependent on the mass of the sphere.

The fourth section was an extension of the third section, except this time a third plot was introduced that depicted the motion of the sphere experiencing quadratic air resistance. We were then tasked with trying various masses, initial velocities, and launch angles to see how they change the motion in all three cases.

Methods:

Describe the used numerical methods. Provide your own explanations about the two numerical methods used in this assignment, the Euler method. **Methods** cannot go over page 4 of this template.

Euler's method is a method that deals with differential equations through using extremely small Δt values to slowly "step" through an equation. This method is assuming that as Δt approaches 0 (i.e. gets smaller), all other relevant values will stay constant. For example, in the lab manual [1], Equation (1) is a ordinary differential equation of the form:

$$\frac{dv_Y}{dt} = g - \frac{b}{m}v_Y \qquad (1)$$

and using Euler's method, can be simplified to:

$$\Delta v_Y = g\Delta t - \frac{b}{m}v_Y\Delta t \qquad (2)$$

This method is assuming that when $\Delta t \approx dt$, both sides of (1) can be multiplied by Δt to find the change in velocity (over a very small time period). dt is an infinitesimally small number, so $\Delta t \neq dt$ and therefore Euler's method can only (at best) produce a good approximation, and Δt selected needs to be extremely small to do so.

This change in velocity can then be applied to the current velocity of the sphere, and you now have the velocity of the sphere at time ($t + \Delta t$). Euler's method is particularly useful when solving problems computationally, because using extremely small values for Δt and iterating a problem potentially thousands of times is actually feasible when using a computer.

Code workflow (1 point):

Explain how the code you used in exercise 4(a) works. Do not simply copy and paste lines from your code. Use your own words to explain its procedures and algorithm. **Code workflow** cannot go over page 5 of this template.

- First, parameters that will apply for all 3 plots are initialized. These include: gravity, spherical air resistance coefficients, diameter and mass of the sphere, an initial velocity for the sphere, and a designated change in time variable (dt). The user is asked to input a value for θ in degrees that the projectile is launched at, and this angle is then converted to radians. Finally, now that we have an angle, the x and y components of the initial velocity are computed and stored.

- The general workflow of the code is very similar for all 3 plots. First, variables and lists that every plot needs are either initialized or reset to initial values. Next, each plot is split into two loops: one loop accounts for the motion of the projectile while the projectile is moving in the +y direction, and the other accounts for the motion while the projectile is moving in the -y direction but before it hits the ground.
    - In every iteration of both loops, the acceleration value(s) are calculated and then applied to the velocities in the x and y directions over a very small change in time (notated as dt in the code). Then the positions are updated and stored into lists in the x and y directions over the same small change in time.
    - In the vacuum plot, the only acceleration present is gravity (which is constant), so the velocity in the x direction is completely unchanged throughout the whole motion and the velocity in the y direction is fairly simple to compute.
    - In the linear plot, there is linear air resistance present in both the x and y directions, as well as gravity in the y direction. As the projectile is moving upwards, the air resistance and gravity accelerations work together to slow the projectile down. However, once the projectile begins to move downwards, the air resistance vector switches directions and opposes the gravity vector. Equations (4) and (5) we were given in the lab manual [1] describe the accelerations present in both directions throughout the motion.
    - Similarly, in the quadratic plot there is quadratic air resistance present in both the x and y directions, as well as gravity in the y direction. The air resistance itself acts very similarly to the linear air resistance in terms of direction as the projectile moves through the air, however in magnitude it is very different to the linear air resistance. Again, like above, Equations (6) and (7) that were given in the lab manual [1] describe the accelerations present in both directions throughout the motion.

- Finally, there is a series of print statements that print easier to digest values about the maximum height, distance traveled and time of flight of the projectile throughout each of the three motions. Then there are a bunch of matplotlib commands that print the graph, and add extra detail such as red dots at the peak on all three plots or a dotted line along the x-axis.

Results and analysis:

Present all plots and discussions requested in Sections 1, 2, 3, and 4 of the assignment. Discuss and explain the plots and their results. **Results and analysis** cannot go over page 10 of this template. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

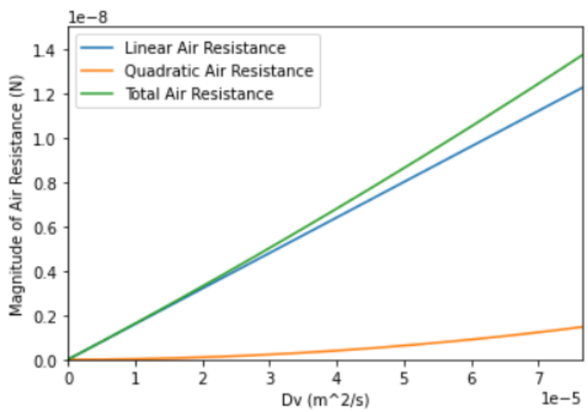

Figure 1: Range where Linear Resistance Dominates

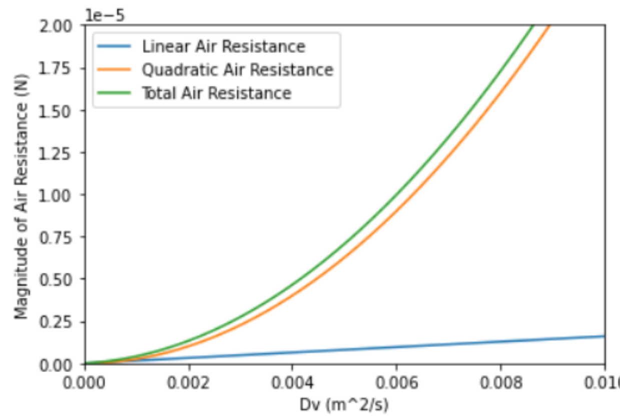

Figure 2: Range where Quadratic Air Resistance Dominates

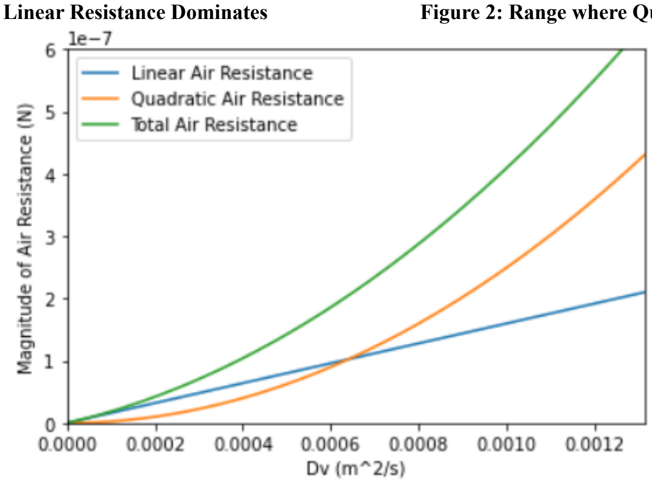

Figure 3: Range where both Linear and Quadratic are relevant

- As seen above in Figures 1, 2, and 3, different Dv's of a sphere indicate the primary type of air resistance affecting a projectile. Based on Figure 1, we know that when the Dv is about $7.0 * 10^{-5}$ (m^2/s) or lower that the vast majority of the air resistance is linear. Similarly, from Figure 2 we know that when Dv is roughly $7.0 * 10^{-3}$ (m^2/s) or higher, the vast majority of the air resistance must be quadratic. Therefore, a general range that both linear and quadratic would be relevant would be: $7.0 * 10^{-5}\frac{m^2}{s} < Dv < 7.0 * 10^{-3}\frac{m^2}{s}$.

- After calculating the Dv of the given objects in 1b), we can conclude that:

$$Dv_{BASEBALL} = 0.07m * 5\frac{m}{s} = 0.350\frac{m^2}{s} > 7.0 * 10^{-3}\frac{m^2}{s}$$

$$Dv_{OIL\ DROP} = 1.50 * 10^{-6}m * 5 * 10^{-5}\frac{m}{s} = 7.50 * 10^{-11}\frac{m^2}{s} < 7.0 * 10^{-5}\frac{m^2}{s}$$

$$Dv_{RAIN\ DROP} = 0.001m * 1\frac{m}{s} = 0.001\frac{m^2}{s},\ 7.0 * 10^{-5}\frac{m^2}{s} < 0.001\frac{m^2}{s} < 7.0 * 10^{-3}\frac{m^2}{s}$$

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- Therefore, the primary type of air resistance experienced by the baseball would be quadratic, the primary for the oil drop would be linear, and the raindrop would experience considerable amounts of air resistance of both types.

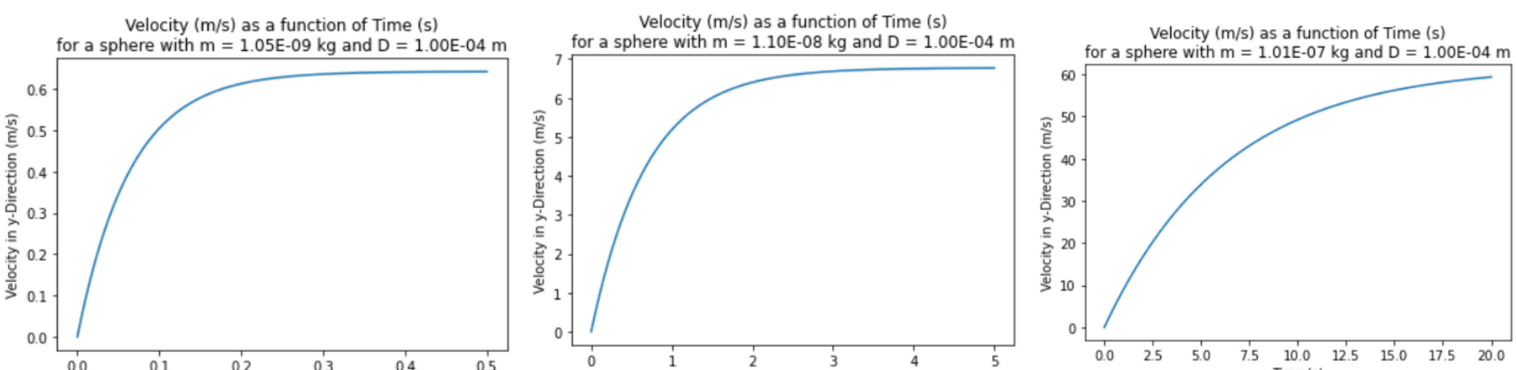
- When the following formula from the lab manual [1] is used: $\frac{dv_y}{dt} = g - \frac{b}{m}v_y$, we know that $\frac{dv_y}{dt} = 0$ when the object reaches terminal velocity (because when v_y reaches terminal velocity, the object stops accelerating downwards, so the net acceleration must be 0). Therefore, a terminal velocity and the Dv for the dust particle outlined in 2 can be calculated as follows:

$$0 = g - \frac{b}{m}v_T \Rightarrow v_T = \frac{mg}{b} = \frac{mg}{BD}$$

$$v_T = \frac{(1.05 * 10^{-9}\, kg) * (9.81 \frac{m}{s^2})}{(1.6 * 10^{-4} \frac{N*s}{m^2}) * (1.0*10^{-4} m)} = 6.42 \ * \ 10^{-1} \frac{m}{s}$$

$$Dv_T = (1.0 \ * \ 10^{-4} m) \ * \ (6.42 \ * \ 10^{-1} \frac{m}{s}) \ = 6.42 \ * \ 10^{-5} \frac{m^2}{s} < 7.00 \ * \ 10^{-5} \frac{m^2}{s}$$

  - Because the Dv when $v = v_T$ is less than 7.00E-5, we can safely assume that quadratic air resistance can be neglected for the dust particle.



Figures 4, 5, 6: Velocity as a function of Time graphs for a sphere with the same diameter but with varying masses

- As can be seen in Figures 4-6, as the mass increases, it takes much longer for the velocity to level out and for the spheres to reach terminal velocity. This is because as the mass increases, so does the terminal velocity of the sphere. In Figure 4, the sphere has the dimensions of the dust particle given to us in the lab manual [1], and the sphere achieves terminal velocity at approximately 0.4 seconds. In Figure 5 however, the mass has been

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increased by 1.00E-8 kg, and it takes significantly longer for the sphere to reach terminal velocity, approximately 5 seconds. In Figure 6, the mass has again been increased, this time by 1.00E-7 kg, and it takes more than 20 seconds for the sphere to achieve terminal velocity. It can be seen that as the mass of the sphere increases, it takes longer and longer for linear air resistance to make the sphere reach terminal velocity.

- It is also likely that since the Dv_T values would also be increasing for the spheres depicted in Figures 5 and 6 that and they likely would experience quadratic drag in addition to linear drag, which would also affect the results given here. The results in Figures 4 - 6 only take into account linear air resistance.

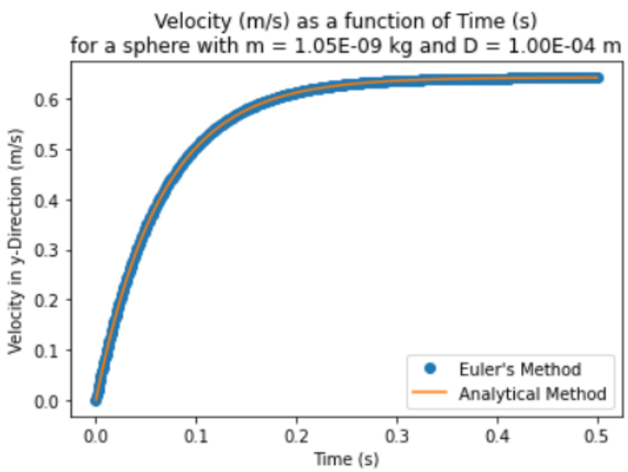

Figure 7: Velocity as a function of Time for the Dust Particle both Eulers and the Analytical Method

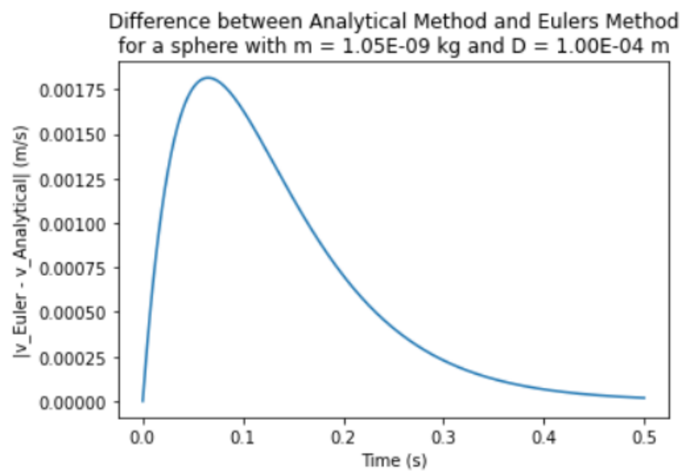

Figure 8: The Difference in the calculated Velocity between the Analytical Method and Euler's Method

- Using Equation (3) from the lab manual [1], a more accurate calculation for the velocity over time can be made. This method (more thoroughly explained in the textbook we used for Classical Mechanics [3]),] is solving the ODE (Equation (1)) using the separable DE method and then integrating both sides. Therefore it is slightly more accurate than Euler's method, because instead of using a really small Δt like Euler's method does, integrating essentially uses an infinitesimally small Δt, which produces even more accurate results.

- The biggest discrepancy between Figures 7 and 8 comes when the time elapsed is extremely small, i.e. t<0.4s. This makes sense because that is when the speed of the sphere is changing the most rapidly, and the infinitesimally small Δt is going to catch more of the small changes then the Δt we set Euler's method to. A very small Δt was already used to produce these results (1.00E-3), however the results could be improved while using Euler's method if an even smaller Δt is to be used. Euler's method will never be quite as accurate as integration is, but it is a good approximation when Δt is small enough.

Results and analysis:

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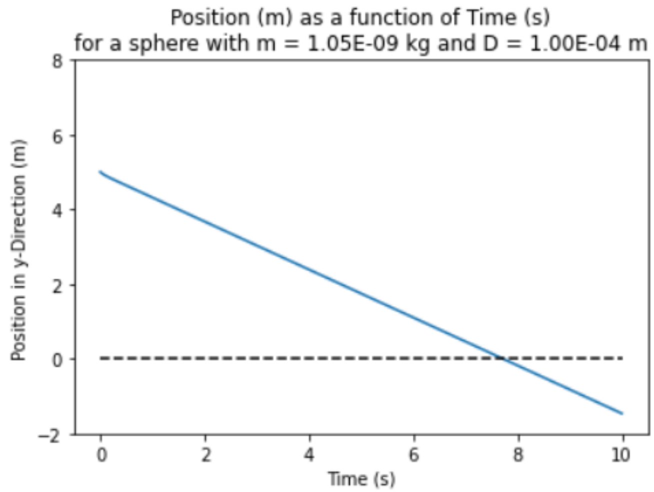


Figure 9: Position as a Function of Time of a Dust Particle falling from an initial height of 5 meters

- Above in Figure 9, the position over time of the specified dust particle being dropped from a height of 5 meters is shown. If you look really closely, there is a slight curve at the beginning of the motion. This curve is from the dust particle accelerating until it reaches its terminal velocity. If we go back to Figures 4 and 7, we know the dust particle reaches terminal velocity at approximately 0.5 seconds. After this, the position changes linearly because the particle will be moving at a constant speed downwards.
- The total fall time for the particle is approximately 7.72 seconds.

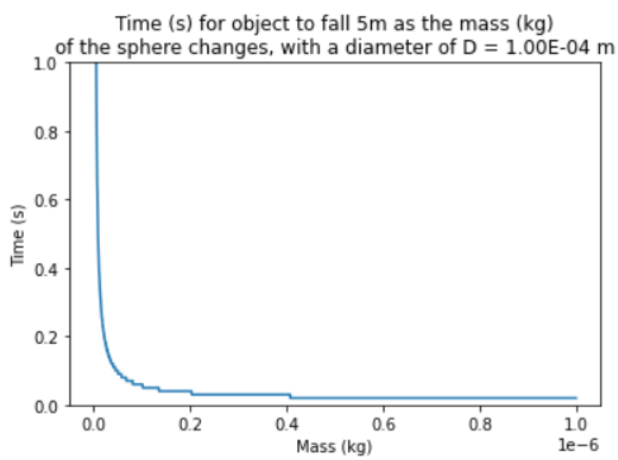


Figure 10: The total time it takes for a sphere (with a constant diameter) to fall 5m as the mass of the sphere changes

- The common saying that "all objects fall together with the same acceleration regardless of their masses" is accurate most of the time, but falls apart for extremely small masses. This is because when the mass decreases, the magnitude of the gravitational force drops much faster than the air resistance force drops and therefore the net force in the downwards direction is much less as the particle gets smaller. For the dust particle, based on Figure 10 the statement is true once the mass is approximately 5E-5 kg or larger.

Results and analysis:

Present all plots and discussions requested in Sections 1, 2, 3, and 4 of the assignment. Discuss and explain the plots and their results. **Results and analysis** cannot go over page 10 of this template. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

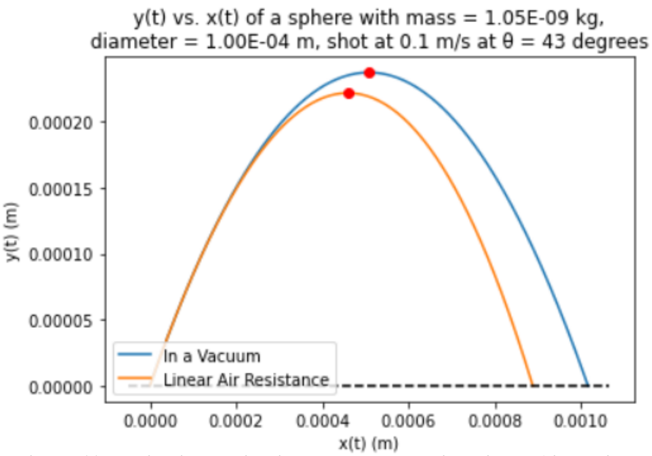


Figure 11: Projectile Motion in a Vacuum vs. with Linear Air Resistance

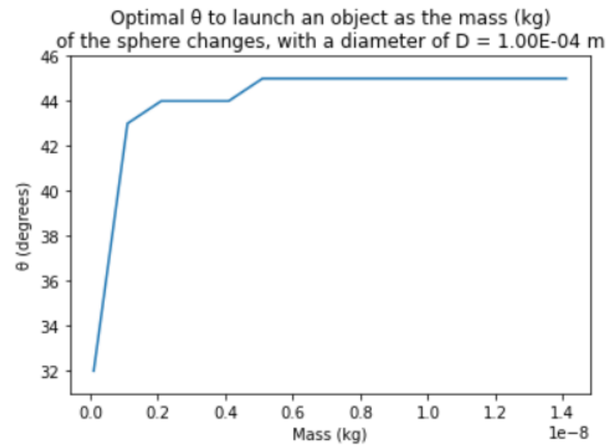
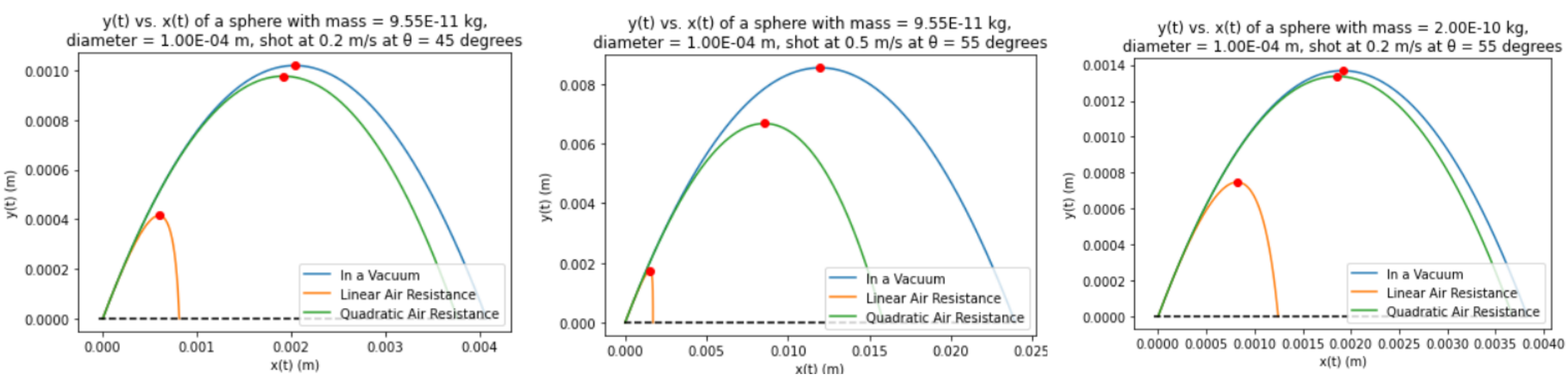


Figure 12: Optimal θ to launch the sphere to maximize the distance traveled as the mass increases

- Figure 11 portrays the difference in what the projectile motion of the dust particle would look like in a vacuum to what it looks like when it experiences linear air resistance.
- Figure 12 shows that when the mass is really small, the optimal angle to launch the object to maximize horizontal displacement gets lower as well. As the mass increases, the optimal angle eventually plateaus at 45 degrees, which makes sense because as the mass increases the less relevant linear air resistance would be, and objects act closer and closer to how they act in a vacuum. (without accounting for quadratic air resistance)



Figures 13, 14, 15: Projectile Motion in a Vacuum, with only Linear Air Resistance, and with only Quadratic Air Resistance. In each figure, the sphere is launched at a different speed, angle or mass to show how the motion differs for all 3 scenarios.

- As the initial speed increases, air resistance (both linear and quadratic) becomes more impactful. As the mass increases, the net acceleration due to gravity experienced by the object increases and therefore the max height achieved is lower for all curves. The angle most significantly changes the horizontal displacement that the object travels, with increasing it over 45 degrees (typically) decreasing displacement, and lowering it under 45 degrees (again, typically 45 degrees - see Figure 12 to see how the optimal angle changes as the mass changes) also decreasing displacement.

Conclusions (0.5 points):

Wrap up the assignment by highlighting your main findings and results. Please, stay on the limit size of up to 300 words max. **Conclusions** cannot go over page 11 of this template.

There were multiple goals and findings achieved throughout the lab. Firstly, a range of Dv products for a sphere were established, when this product was lower than $7.0 * 10^{-5}$ (m^2/s), the sphere experiences primarily linear air resistance, and when the product of the two was higher than $7.0 * 10^{-3}$ (m^2/s), the sphere experienced primarily quadratic air resistance. We then applied this range to determine what type of air resistance primarily affected multiple objects of varying sizes and moving at various velocities.

Another investigation done in the lab was depicting the difference between Euler's method to solving ordinary differential equations, and the more accurate analytical method. It was discovered that the analytical method is always slightly more accurate, however if the Δt chosen for Euler's method is sufficiently low enough, Euler's method is a pretty good approximation. Afterwards, we discovered how mass impacts how long it takes for an object to reach terminal velocity, and therefore once an object hits a certain mass it can be assumed that they all "fall together with the same acceleration, regardless of mass" [1], but this is only true at higher masses.

Finally, the last major investigation done throughout the lab was seeing how the arc from projectile motions of a sphere changed when the sphere experienced no air resistance, only linear air resistance, or only quadratic air resistance. We saw how in a vacuum, the projectile moved in a symmetrical arc, whereas when air resistance was applied (both linear and quadratic), the peak of the arc was lower and the horizontal displacement was less. Other factors such as mass of the sphere, launch angle and initial speed also changed how much both linear and quadratic air resistance changed the motion of the projectile.

References:

Include any citations or references used during the preparation of this report and codes. Consulted webpages should also be cited as well as our assignment manuals. Choose a citation style of your preference and adopt it consistently throughout. There is no page limit for the **References** section.

1. Assignment #3 Manual: Projectile Motion under Air Resistance - PHYS 381 Winter term 2023 - Retrieved from https://d2l.ucalgary.ca/d2l/le/content/497542/viewContent/5765259/View
2. Gomes da Rocha, Claudia *In Class Demo: exercise 2 linear motion* - Retreived from https://d2l.ucalgary.ca/d2l/le/content/497542/viewContent/5774224/View
3. Taylor, John R. *Classical Mechanics* Sausalito, Calif. University Science Books, 2005

Other (0.5 points):

This page is to be filled by the instructor or TAs ONLY.

Remaining 0.5 points are granted for following the template and overall quality of the report.

Was the assignment submitted before the due date (mark one option)? YES NO