PHYS 381 – Computational Physics I (Winter 2023)

Assignment #4: Fourier Analysis Using Python

Due date: 27 March 2023, 3 pm

Group members (add your name and UCID below):

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Please, only add text in this report template using Times New Roman font, non-italic, non-bold, font size 12. Equations can be typed using the default font of the equation editor of your choice.

Authors' contributions:

Scott took the main lead on the code, however all 4 of us communicated and brainstormed solutions on the code together. Gabe took a lead on the Abstract, while Elham worked on the Introduction, Methods and Conclusion. Grady and Scott both worked on the Results and Analysis section of the report.

We all worked together throughout the entire lab, and communicated constantly throughout the whole process. We feel the work distribution was fair.

Abstract (0.5 points):

Provide a summary of your assignment work and findings. The abstract is a paragraph where you summarize what you studied in this computational lab and what are your main findings. Please, stay on the limit size of up to 300 words max. **Abstract** cannot go over page 2 of this template.

In this laboratory, we explore the Fourier Series and the Fourier Transform. These mathematical techniques are used to break down periodic and non-periodic functions into a sum of sine and cosine functions with corresponding coefficients.

The first part of the lab focuses on computing these coefficients using Simpson's rule, a numerical integration method. We test the accuracy of this algorithm by calculating the Fourier series coefficients numerically for a handful of sinusoidal functions, and then apply this algorithm on the square wave function and on the pulse train function. We are able to observe in this part of the lab that increasing the amount of calculated "k" coefficients increases the accuracy of the reconstructed signal dramatically.

We then explore how to apply Fourier Series to non-periodic functions using a Discrete Fourier Transform (DFT). We first apply a DFT to two functions: $sin(0.45\pi t)$ and $cos(6\pi t)$. We observe how a DFT allows us to find the Power Spectrum of a function which gives the dominant frequency of the given function. We were also able to determine that as you decrease the time spacing constant, "h", the accuracy of the reconstructed signal also increases dramatically.

Finally, we analyzed a given text file denoted as "*pitch.txt*" that depicted measurement of air vibrations over a period of time. A DFT was applied to the list of values, and a filter mechanism was applied to "de-noise" the signal depicted in the pitch.txt file. This shows how useful and important Fourier Transforms are and how vital they are to modern telecommunications.

Overall, the lab provides an in-depth understanding of the Fourier Series and its applications in signal processing.

Introduction (0.5 points):

Provide an overview of the topics studied in the assignment. Concentrate on the 'physics' part, in this case, the Fourier Series, DFT, processing of signals, power spectrum, time/frequency domain, etc. **Introduction** cannot go over page 3 of this template.

Fourier Analysis and Discrete Fourier Transform (DFT) are powerful mathematical tools that have revolutionized the way we study signals and analyze physical phenomena using Fourier series. These techniques allow us to break down complex signals into simpler components and study their behavior in both the time and frequency domain.

In this report, we will explore the use of Fourier Analysis and DFT to analyze both periodic and non-periodic signals using Python. We will start by discussing the concept of signals and their representation in the time and frequency domain. We will then introduce the Fourier series and its use in decomposing periodic signals into their constituent frequencies. We will also discuss the Discrete Fourier Transform and how it can be used to analyze non-periodic signals.

Fourier analysis, put simply, decomposes the periodic signals into its constituent harmonic vibrations. But it is also useful when dealing with non periodic signals with some modification to it i.e. Discrete Fourier Transform.

Periodic signals are those that repeat themselves after a fixed period of time. Examples of periodic signals include sine waves, square waves, and sawtooth waves. The representation of a periodic signal in the time domain is a waveform that repeats itself after a fixed interval. In contrast, the representation of a periodic signal in the frequency domain is a series of spikes at the fundamental frequency and its harmonics.

Using a Fourier series framework, the signal gets split into a combination of sine and cosine signals which are referred to as harmonic functions with discrete frequencies and coefficients, usually denoted as a or b, indicating the 'amount' of sine and cosine terms present in the function representing the signal. The signals thus have their corresponding amplitudes. Fourier series help us identify the fundamental frequency and its harmonics, which carry most of the power in the signal.

On the other hand, non-periodic signals are those that do not repeat themselves after a fixed period of time. Examples of non-periodic signals include speech signals, music signals, and signals from sensors that measure environmental variables such as temperature or pressure. The representation of a non-periodic signal in the time domain is a waveform that does not repeat itself, and its representation in the frequency domain is a continuous spectrum of frequencies.

To be able to analyze such signals, a non periodic function representing the corresponding signal is expanded in terms of cosine and sine terms but in this case, the expansion is a Fourier integral over a continuous range of frequencies. Then DFT is used to convert a finite sequence of samples of the signal into its frequency domain representation, which can be used to recognize the frequencies that carry most of the power of the signal.

The power spectrum of a DFT is a plot of the power, or square of the magnitude, of each frequency component in the discrete signal. It allows for the identification and analysis of frequency components in a signal.

Methods (1 point):

Describe the used numerical methods. Provide your own explanations about the two numerical methods used in this assignment: Fourier Series (page 4) and DFT (page 5). **Methods** cannot go over page 5 of this template.

Fourier series is an expansion of a periodic function into a sum of trigonometric functions. This series was first used by Joseph Fourier to find solutions to the heat equation and quickly became a popular mathematical tool that helps analyze signals in the form of periodic functions which is usually a sum of sine and cosine terms with varying frequencies, amplitudes and phases.

The period of the repetitive function is denoted by T, where T = 2π/ω. And the Fourier series of a periodic function is given by:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$

where ω = 2π/T, and $a_0/2$, referred to as the DC component, is the average value of the function over one period, T.

For this approximation to be complete, we need to find the Fourier coefficients, a_n and b_n. So, we look to the orthogonality properties of the sines and cosines.

$$\int_0^{\pi} \sin(nx)\sin(mx)dx = \begin{cases} 0, & \text{if } n \neq m \\ \frac{\pi}{2}, & \text{if } n = m \end{cases}$$

$$\int_0^{\pi} \cos(nx)\cos(mx)dx = \begin{cases} 0, & \text{if } n \neq m \\ \frac{\pi}{2}, & \text{if } n = m \end{cases}$$

$$\int_0^{\pi} \sin(nx)\cos(mx)dx = 0$$

The a_n and b_n are the Fourier coefficients, given by:

$$a_n = \frac{2}{T}\int_0^{T} f(t)\cos(n\omega t)dt$$

$$b_n = \frac{2}{T}\int_0^{T} f(t)\sin(n\omega t)dt$$

The Fourier coefficients are evaluated using the orthogonality properties of sines and cosines.

The Fourier series representation of a periodic function can be thought of as a weighted sum of harmonically related sine and cosine functions. The terms with n=1 represent the fundamental frequency, or the frequency with the largest amplitude, and its harmonics. The higher-order terms represent higher frequencies with decreasing amplitudes. The accuracy of the Fourier series representation depends on the number of terms used in the summation. As more terms are included, the approximation of the original function becomes more accurate and approaches the original periodic function, as it captures more of its constituent frequencies. However, Fourier series cannot be used to approximate arbitrary functions because most functions have infinitely many terms in their Fourier series, and the series do not always converge.

Methods:

Describe the used numerical methods. Provide your own explanations about the two numerical methods used in this assignment: Fourier Series (page 4) and DFT (page 5). **Methods** cannot go over page 5 of this template.

To analyze non periodic signals, we use a modification of the Fourier series approximation referred to as the Discrete Fourier Transform (DFT). However, it is different when it comes to how it analyzes data when compared to the standard Fourier analysis. DFT can analyze any finite sequence of data points in the time domain.

While breaking down what the DFT does, we first need to understand what a Fourier transform is. The Fourier transform is a mathematical technique used to transform a function of time, f(t), into a function of frequency, F(ω). The Fourier transform, F(ω) of a function f(t) is defined as,

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

where i is the imaginary unit, ω is the angular frequency, and the integral is taken over all time.

In practice, the numerical solution of the equation above will involve replacing the integration with a discrete summation. The exact integrals however are approximated by a DFT as defined below.

DFT is a discrete-time version of the Fourier transform, which is used to transform a signal from the time domain to the frequency domain.

We take a signal represented by f(t) and we sample N times at intervals at h from t = 0 to t = (N-1)h. Then, we discretize the timeline $t_m = mh$ with m = 0, 1, 2,...N.

In simpler terms, the DFT uses the Fourier transform to analyze a discrete set of sampled values of a time-dependent physical signal. The signal is approximated by a set of values at specific intervals, and the period of the approximated function is determined by the longest time of interest, τ and we assume that:

$$f(t) = f(t+\tau) \text{ i.e. } f(t_m) = f(t_{m+N}) \text{ or in shorthand form, } f_m = f_{m+N}$$

If the function is periodic with period T, then the lowest frequency in the DFT will be, v_1 =1/T=1/(Nh). The frequency spectrum is then given by equally spaced values, where each frequency is a multiple of the lowest frequency. This set of frequencies is determined by the number of samples N and the sampling interval h, and is given by the formula v_n = n/(N*h) for n = 1, 2, ..., N.

The DFT calculates the complex amplitudes for each frequency component of the signal, which can then be used to analyze the signal in the frequency domain. The power spectrum, which is the square of the magnitude of the complex amplitudes, provides information about the strength of each frequency component.

To reconstruct the signal accurately from its DFT, one can perform an inverse DFT, which uses the complex amplitudes to calculate the original sampled values of the signal

Code workflow (1 point):

Explain how the code you used in exercise 2.2(b): Radio and TV transmission works. Do not simply copy and paste lines from your code. Use your own words to explain its procedures and algorithm. **Code workflow** cannot go over page 6 of this template.

First, the data file "pitch.txt" is opened and read. A list with all the data files is then created so we can use the pitch file information in further calculations. A graph is then created using this data file list with each vibration value recorded as a function of time, with a 1 second time interval between each data point.

Next, all the lists and parameters used are initialized. N is the amount of data points in the list, h is the time interval between each data point (1 second), and then empty lists for all the real and imaginary components are initialized.

Next, a nested loop creates all of the "real" and "imaginary" components of the fourier series. Eq (24) and Eq (25) from the lab manual are used to create these components.

$$Re\{F_N\} = \sum_{m=0}^{N-1} f_m cos(\frac{2\pi nm}{N}) \quad [24] \qquad Im\{F_N\} = \sum_{m=0}^{N-1} f_m sin(\frac{2\pi nm}{N}) \quad [25]$$

The first loop is used to find F_1 through to F_N, and the nested loop is doing the summation in Eq (24) and (25). Using these components, we can then do a fourier transform to reconstruct the signal. The "reconstruction" function is called, where Eq (26) from the lab manual is used:

$$f_m = \frac{1}{N} \sum_{n=0}^{N-1} [\, Re\{F_N\}\, cos(\frac{2\pi mn}{N}) \; + \; Im\{F_N\}\, sin(\frac{2\pi mn}{N})] \qquad [26]$$

Again, the first loop is used to find F_1 through to F_m, and the nested loop is doing the summation in Eq (24) and (25). When doing the nested for loop in the "reconstruction" function, there is an additional threshold value to "denoise" the signal. If the magnitude of F_N is less than 50, (i.e $\sqrt{Re\{F_N\}^2 + Im\{F_n\}^2} < 50$) then F_N is set to 0. This removes any extra signals that are making "noise" around the desired signal.

After the reconstructed signal is calculated, all of the results are plotted using matplotlib functions.

Results and analysis (1 point):

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. **Results and analysis** section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

The point of Section 1.2 is about proving that the code for finding the coefficients for the fourier series is working properly. The program is fed a sum of known sinusoidal functions with varying coefficients to ensure the correct coefficients are calculated based on Eq (1) from the lab manual [1].

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n cos(n\omega t) + b_n sin(n\omega t)]$$

So say for example, we were trying to find the correct coefficients in this fourier series to create $3sin(\omega t)$. We would expect every "a" coefficient to be 0 because there are no cosine functions present. Next, because the value of "n" inside the sine is equal to 1 and there are no other sine terms present, we know that every "b" coefficient except for b_1 is also equal to 0. Thus we know that $b_1 sin(\omega t) = 3sin(\omega t) \Rightarrow b_1 = 3$.

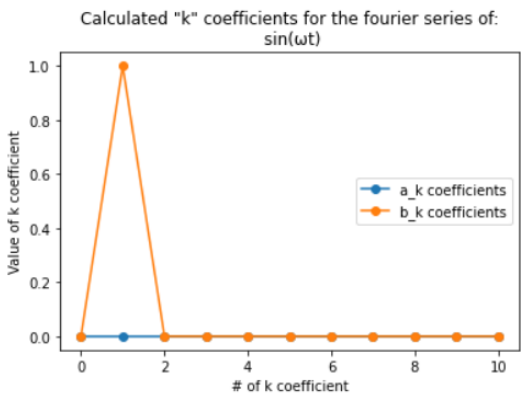


Figure 1: Fourier series for sin(ωt)

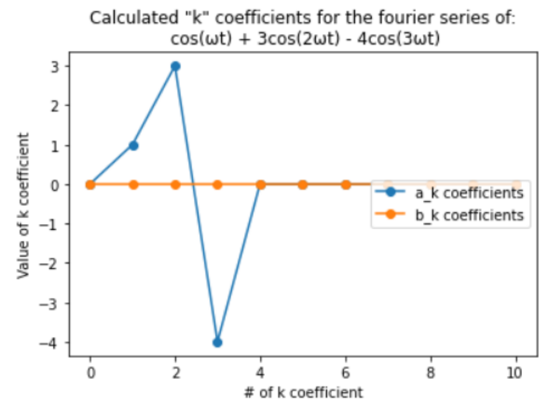


Figure 2: Fourier series for cos(ωt) + 3cos(2ωt) - 4cos(3ωt)

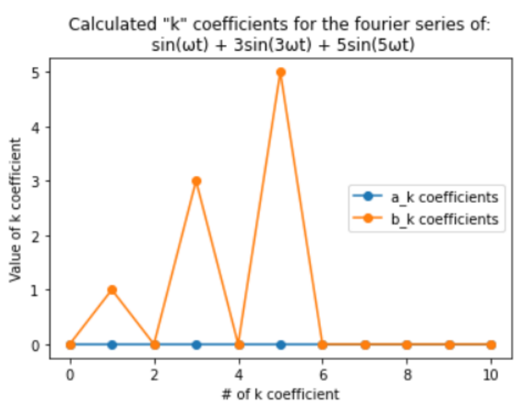


Figure 3: Fourier series for sin(ωt) + 3sin(3ωt) + 5sin(5ωt)

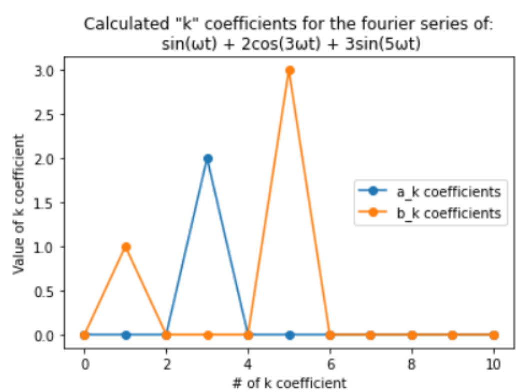


Figure 4: Fourier series for sin(ωt) + 2sin(3ωt) + 3sin(5ωt)

Results and analysis:

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. Results and analysis section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

All four figures in Section 1.2 use the same line of logic, and will be explained using the example of Figure 3. In Figure 3, we are trying to find the coefficients of a and b in the Fourier series to create $sin(\omega t) + 3sin(3\omega t) + 5sin(5\omega t)$. Since there are no cosine functions present, the value of a will be zero for all k in Eq (1). The values of k inside $sin(k\omega t)$ correspond to 1,3, and 5, meaning that we need the values of b_1, b_3, and b_5. These correspond to 1, 3, and 5 respectively. In the case where a cosine is present as in Figure 2, the same process is used to find the coefficients of both a and b.

This line of logic works for all four figures in Section 1.2, meaning that our code is working as expected. We are now ready to apply our algorithm to a more complex function.

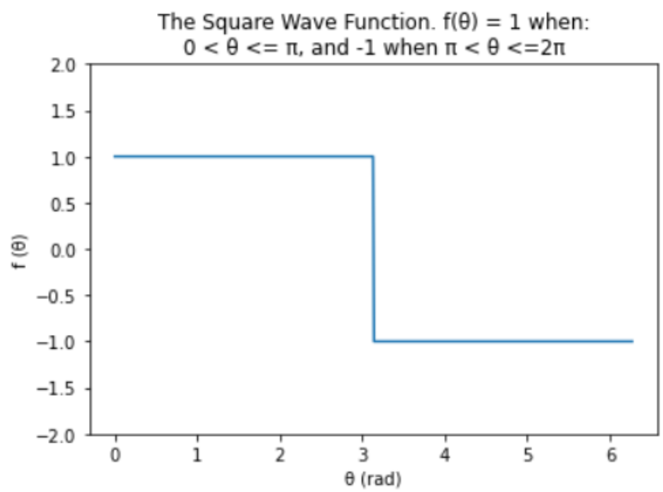


Figure 5: Square Wave Function, as defined in Lab Manual [1]

The function above is the "Square Wave". This wave has a period of T = 2π. When $0 \leq \theta \leq \pi$, $f(\theta)$ returns 1, and when $\pi < \theta \leq 2\pi$, $f(\theta)$ returns -1. The analytical solutions for the coefficient in the fourier series were given in the lab manual [1] and are as follows:

$$a_k = 0 \quad k = 1, 3, 5, \ldots$$

$$b_k = \begin{cases} \dfrac{4}{\pi k} & k = 1, 3, 5, \ldots \\ 0 & k = 2, 4, 6, \ldots \end{cases}$$

Results and analysis:

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. Results and analysis section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

We can use the exact same code that was used above to find coefficients for the sum of sinusoidal functions to computationally calculate the fourier series coefficients for the square wave function, and then compare.

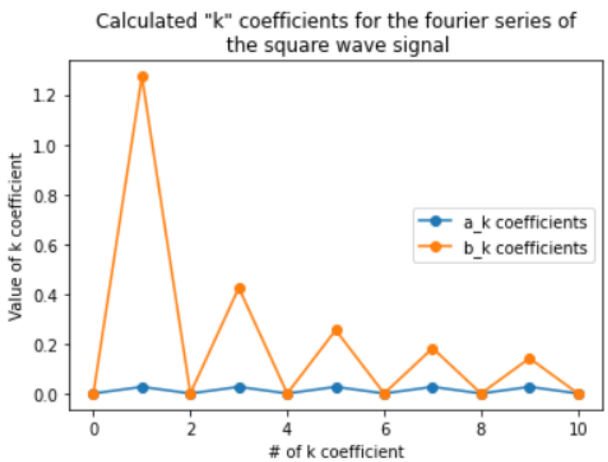


Figure 6: Ten calculated "k" coefficients

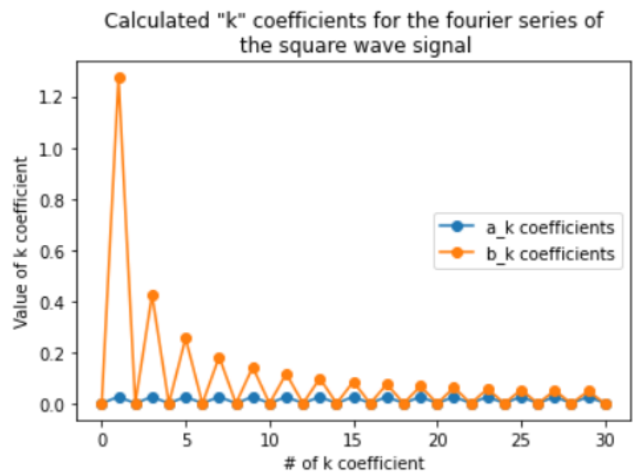


Figure 7: Thirty calculated "k" coefficients

Above are two figures that show 10 and 30 calculated "k" constants of the square wave function using the numerical solution discussed earlier. We can compare these values to the analytically determined coefficients by graphing the absolute value of the Analytical value subtracted by the Numerical for each coefficient, as is done below in Figure 8.

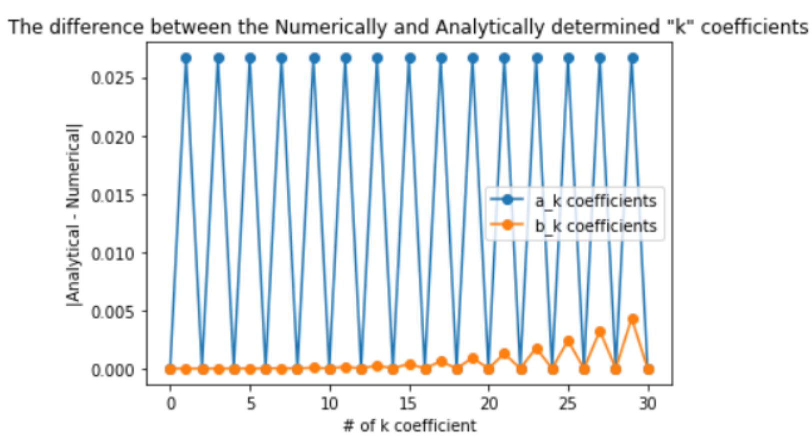


Figure 8: Difference between Analytical and Numerically determined Fourier Coefficients

Results and analysis:

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. Results and analysis section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

As can be seen, there are large amounts of error present in the "a" coefficients. That is because the analytical solution states that the "a" coefficient is equal to 0 for every odd "a" coefficient, however when solving this problem numerically there are approximations made that do not perfectly equal 0. Do notice however that the y-axis is in small units, so this amount of error looks larger than it truly is. In general, this shows that the numerically calculated are pretty accurate. Next, we can use the fourier series coefficients we just calculated in Eq (1) from the lab manual [1] to "reconstruct" the original square wave signal as follows:

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n cos(n\omega t) + b_n sin(n\omega t)$$

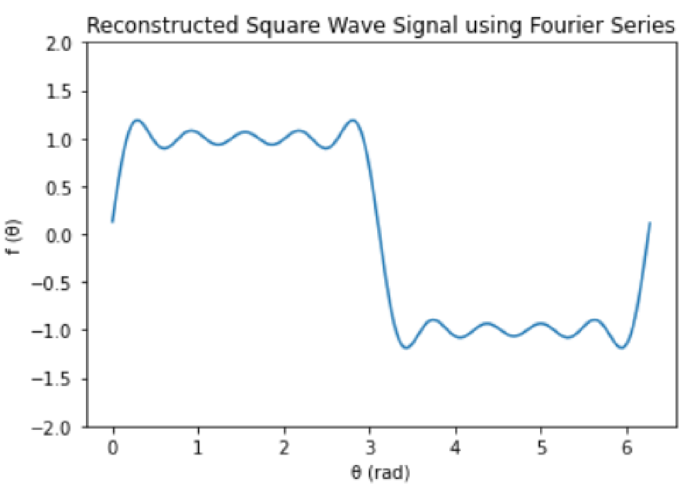


Figure 9: Reconstructed wave (10 "k" coefficients)

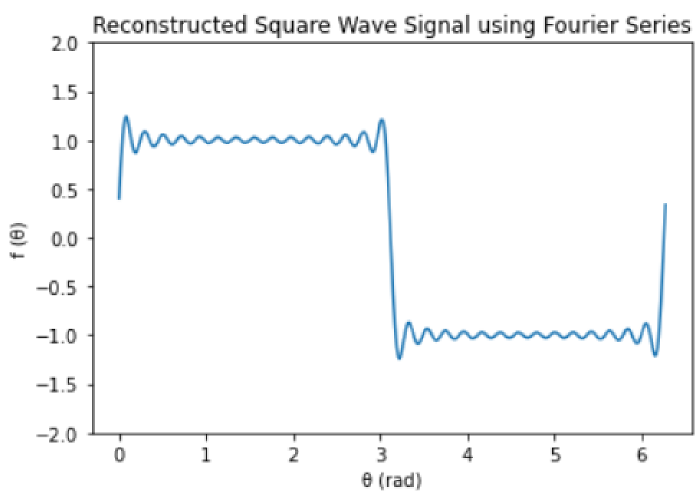


Figure 10: Reconstructed wave (30 "k" coefficients)

The above figures are the "reconstructed" square waves. As you can see, when more "k" coefficients are used, the more the reconstructed wave looks like the original square wave depicted in Figure 5. We can therefore conclude as you increase the amount of "k" coefficients, the reconstruction algorithm improves in accuracy.

Results and analysis:

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. Results and analysis section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

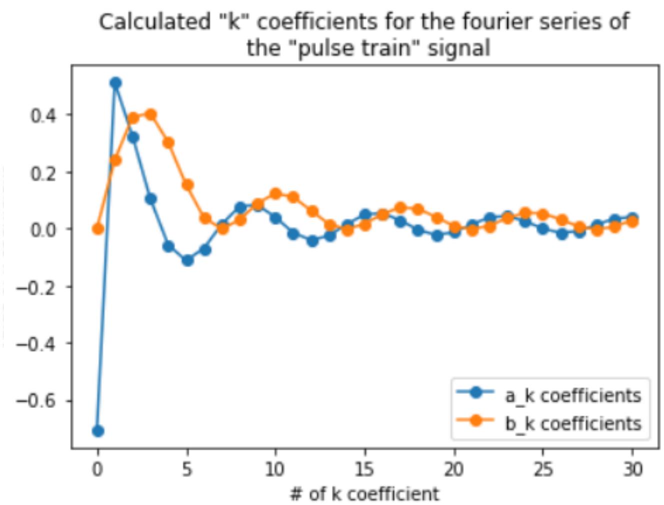


Figure 11: Calculated coefficients for the pulse train signal.

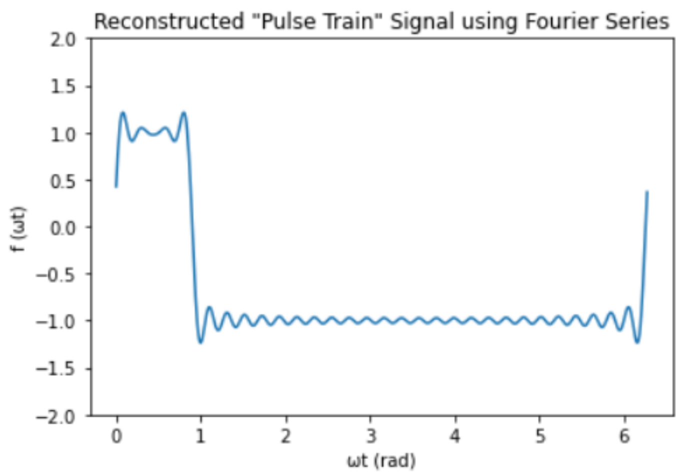


Figure 12: Reconstructed pulse train signal using using Fourier Series

The "pulse train" signal is similar to the square wave signal, except instead of being split at the halfway point of 2π, it is split at a predetermined parameter $\frac{2\pi}{\alpha}$. For these graphs, we chose $\alpha = 7$. As discovered above, the accuracy of this algorithm improves as the amount of "k" coefficients increases, so we chose to do the reconstruction with 30 coefficients. We can compare the calculated results to the analytical results for the "a" and "b" coefficients given in the lab manual [1] as follows:

$$
\begin{aligned}
a_0 &= \frac{2}{\alpha} - 1 \\
a_k &= \frac{2}{k\pi}\sin\left(\frac{2k\pi}{\alpha}\right) \qquad k = 1, 2, 3, \ldots \\
b_k &= \frac{2}{k\pi}\left[1 - \cos\left(\frac{2k\pi}{\alpha}\right)\right] \qquad k = 1, 2, 3, \ldots
\end{aligned}
$$

Again, we can compare these values to the analytically determined coefficients by graphing the absolute value of the Analytical value subtracted by the Numerical value for each coefficient, as is done below in Figure 13.

Results and analysis:

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. Results and analysis section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

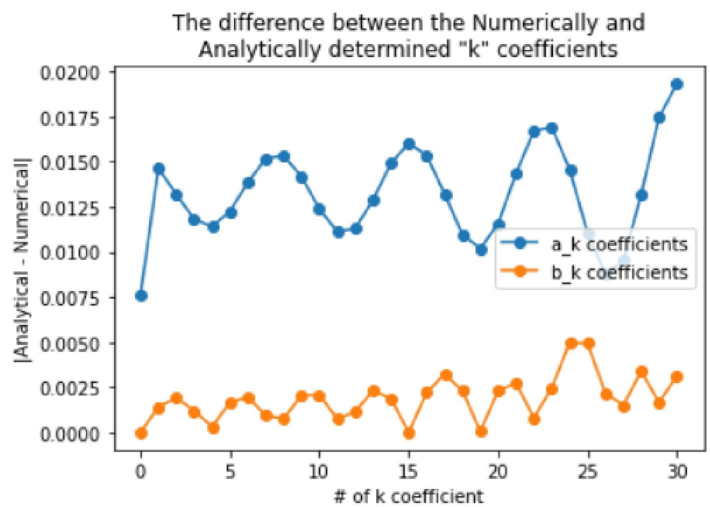
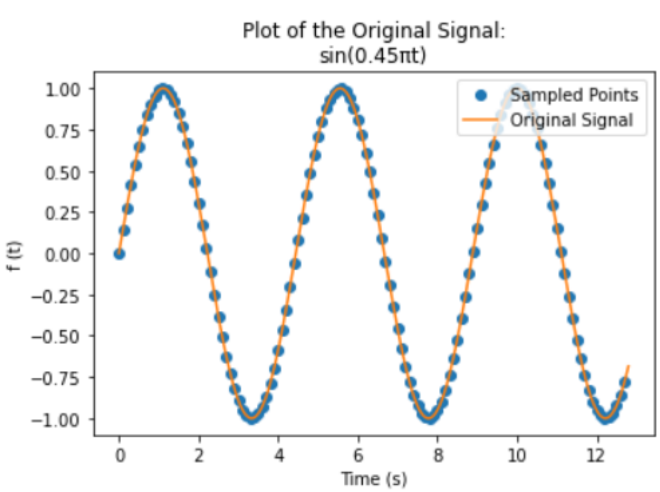
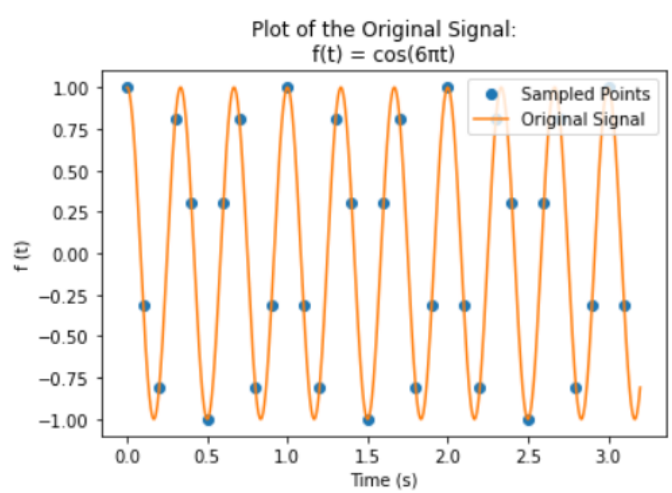


Figure 13: Difference between Analytical and Numerically determined Fourier Coefficients

As can be seen, the difference between the numerical and analytically determined “k” coefficients exists, but because the units on the y-axis are so small, we know that this difference is minor. Therefore, we know the results in Figure 12 are pretty accurate.

Next, we are going to investigate doing a DFT for two given sinusoidal functions: $f(t) = sin(0.45\pi t)$ and $f(t) = cos(6\pi t)$.



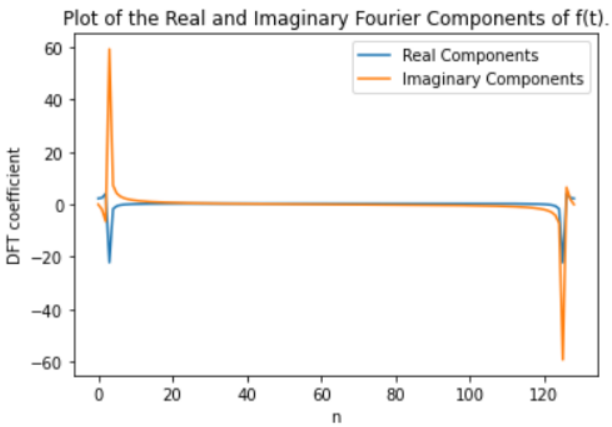


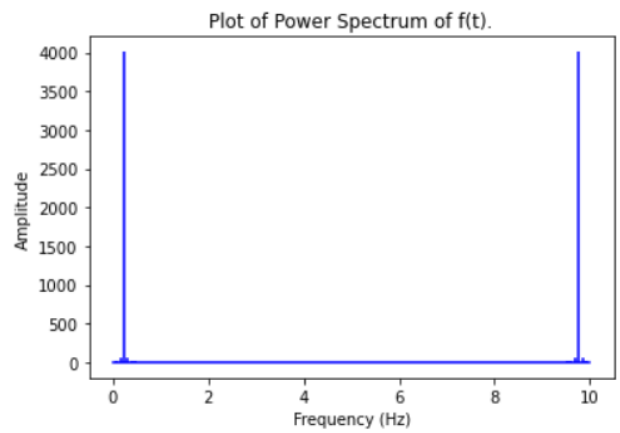
Figures 14 and 15: Plot of Original sin(0.45πt) and cos(6πt), along with their sampled points.

(N = 128, h = 0.1 for sin function. N = 32, h = 0.1 for cos function)

Results and analysis:

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. Results and analysis section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.





Figures 16 and 17: Plot of the Real and Imaginary Fourier Components of sin(0.45πt), and then its Power Spectrum

The frequency of the $sin(0.45\pi t)$ function is determined as follows:

$$T = \frac{2\pi}{0.45\pi} \quad \Rightarrow \quad f = \frac{0.45\pi}{2\pi} = \frac{0.45}{2} = 0.225\ Hz$$

This aligns with the power spectrum. The amplitude at the frequency approximately equal to 0.225Hz spikes dramatically. Based on the plot of the imaginary and real fourier components, we know that there is a dominant fourier component around n=2. When we plug this into the given formula from the lab formula, the expected frequency is:

$$v_2 = \frac{2}{128*0.1} = 0.15625\text{Hz}$$

Which while slightly less than the analytically determined frequency, is pretty close.

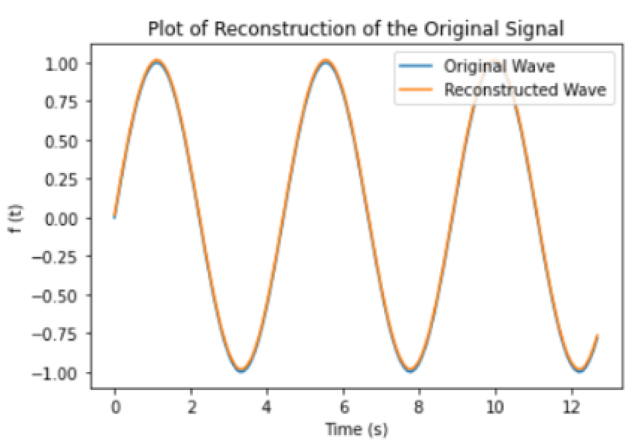


Figure 18: Plot of Reconstructed signal

Results and analysis:

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. Results and analysis section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

As can be seen in Figure 18, the reconstructed signal is very similar to the original signal with only very small error at crests and troughs of the wave.

Similar investigations were done with the $cos(6\pi t)$ function. This time though, the primary exploration was observing what occurs to the final reconstructed signal when the spacing constant "h" is changed.

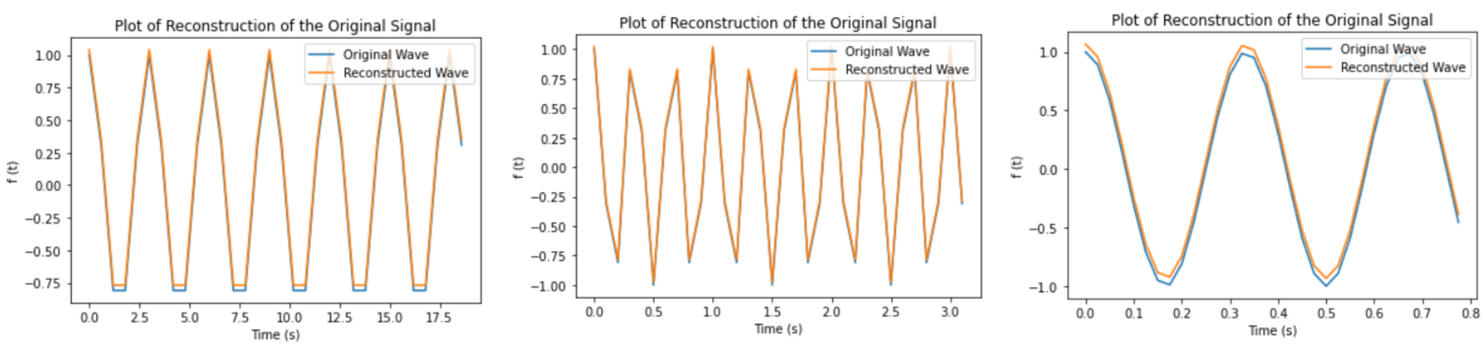


Figure 19, 20 and 21: Reconstructed signal of cos(6πt), with h=0.5, 0.1 and 0.25 respectively

Two observations can be made from these figures. The first observation is that clearly as h becomes lower, the reconstructed signal becomes more and more like it does in Figure 15. The second observation is that when N is held constant and h is decreased, the amount of "time" elapsed is much lower. Notice that when h = 0.5, the x-axis in Figure 19 goes for over 17.5s. This is contrasted by Figure 21 only going for about 0.75s. The main conclusion we can come to is as h is decreased, the reconstruction improves dramatically. This is mostly due to the sampling of the original signal also improving dramatically - as h decreases, the blue dots on Figure 15 become closer and closer together.

Results and analysis:

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. Results and analysis section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

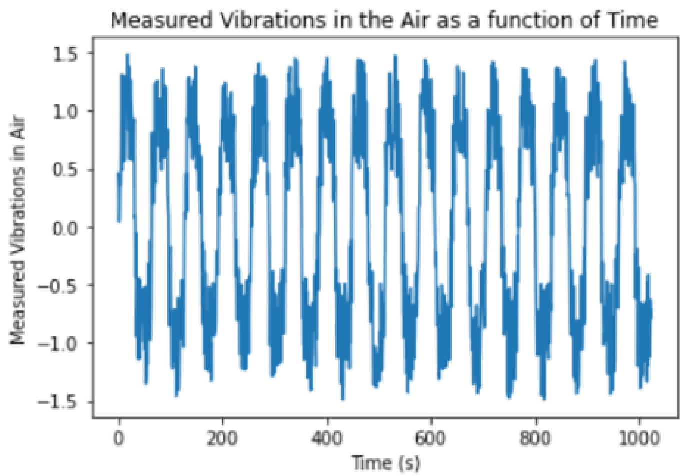
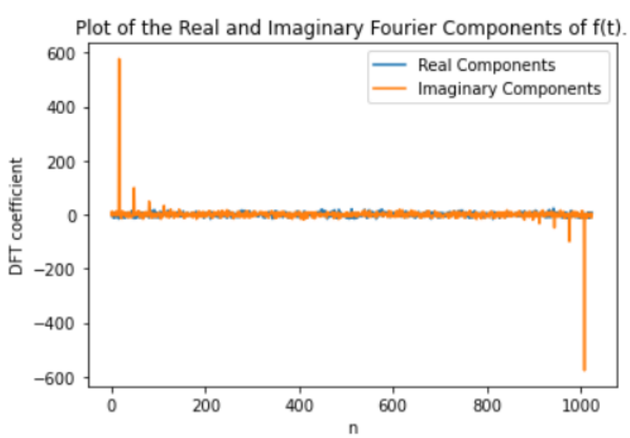


Figure 22: Plot of Measured Vibrations over time from pitch.txt file

Above is the measured vibrations from the given pitch.txt file on the D2L. The data file was a column list of values that represents the vibrations in the air with a 1 second time interval between the recording of each value. There were approximately 1025 values in this data file. We can determine the real fourier coefficients as we did before for the sin(0.45πt) and cos(6πt) functions above. Down below is a plot of the calculated Real and Imaginary Fourier components of the pitch.txt signal, and the power spectrum of the pitch.txt signal.



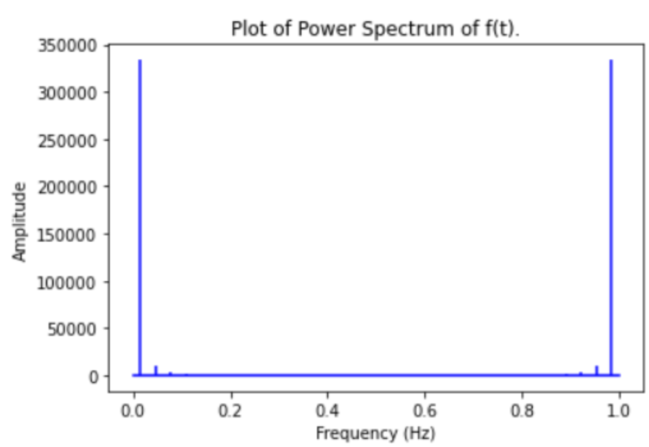


Figure 23 and 24: Plot of the Real and Imaginary Fourier Components of the pitch.txt signal, and the Power Spectrum of the pitch.txt signal

Results and analysis:

Present the plots requested in exercises 1 and 2 of the assignment. Discuss and explain the plots and their results. Results and analysis section does not have a page limit but exercise good judgment in selecting figures that are required and relevant to the assignment. Do not insert too large figure panels just to cover space. Choose figure sizes typically seen in textbooks and scientific manuscripts.

From Figure 24, we know that there are two peaks, one at each end. The power spectrum depicts how a signal's power is allocated among its various frequencies, and it can therefore be used to determine the "dominant" frequency of the signal. The amplitude in the figure represents the "power" detected at each frequency, and the peak(s) on the spectrum represent the dominant frequency present in the signal. If you have a power spectrum of an audio signal, for instance, and you see a peak at 1000 Hz, you can infer that the stream contains a sizable amount of power at that frequency.

Based on this and the plot of the power spectrum-frequency from above, the dominant frequency of the signal from the pitch.txt file is about 0.0125 Hz. A screen ruler was used on Figure 24 to approximate this result.

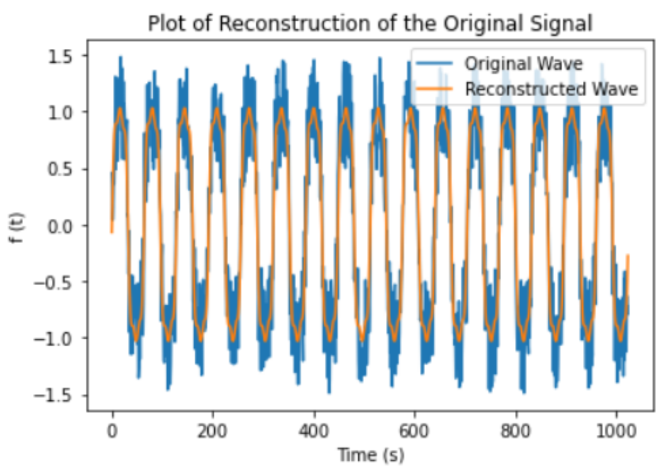


Figure 25: De-noised "pitch.txt" Signal

The Figure above is the de-noised pitch.txt signal. The exact same fourier transform process that we did earlier for the sin(0.45πt) and cos(6πt) functions was done, however we added a "threshold" filter in the reconstruction function that made the recorded signal equal to 0 when the magnitude of the Fourier components (both real and imaginary) was less than 50. This was discussed in greater detail in the code workflow section. The result is the orange plot on the graph, which is the reconstructed signal.

This method is important, and is the same general process that occurs when devices are interpreting radio waves. A specific "threshold" value is chosen when the Fourier Transform is being applied so a sharp and clear signal can be made out from an otherwise very noisy radio wave spectrum. This general process is entirely necessary to make complex communication via electromagnetic waves without a significant amount of interference possible, and is therefore one of the most crucial discoveries in the development of modern telecommunications.

Conclusions (0.5 points):

Wrap up the assignment by highlighting your main findings and results. Please, stay on the limit size of up to 300 words max. **Conclusions** section has a 1-page limit.

Over this lab, we showed that the Fourier transform could be used to break down signals into easier-to-understand components in the time and frequency domains.

By using Simpson's rule and python to calculate Fourier coefficients we explored the use of Fourier Analysis and DFT to analyze both periodic and non-periodic signals using Python. We started by discussing the concept of signals and their representation in the time and frequency domain. We then introduced the Fourier series and its use in decomposing periodic signals into their constituent frequencies. We also discussed the Discrete Fourier Transform and how it can be used to analyze non-periodic signals.

We also cleaned the pitch.txt file provided to us using the Fourier Transform to reconstruct a given signal. We first calculated the Fourier coefficients using the Real and Imaginary components and then reconstructed the original signal. We then de-noised the signal by setting the value of the signal to 0 when the magnitude of the Fourier components was below a certain threshold. The reconstructed signal matched the original signal, demonstrating the effectiveness of the Fourier Transform in signal processing.

Using python, we calculated the coefficients for different sums of sinusoidal functions, namely a square wave function using both analytical and computational methods. The computed coefficients were found to be very close to the analytical solutions, thus validating the computational approach. We also used the Fourier series to reconstruct a square wave signal and found that increasing the number of coefficients improved the accuracy of the reconstruction.

Then, we explored the properties of the pulse train signal and demonstrated its similarity to the square wave signal. We used the Fourier Transform to calculate the Fourier series coefficients for the pulse train signal and used these coefficients to reconstruct the original signal. We found that increasing the number of coefficients also improved the accuracy of the reconstruction.

References:

Include any citations or references used during the preparation of this report and codes. Consulted webpages should also be cited as well as our assignment manuals. Choose a citation style of your preference and adopt it consistently throughout. There is no page limit for the **References** section.

1. Assignment #4 Manual: Projectile Motion under Air Resistance - PHYS 381 Winter term 2023 - Retrieved from https://d2l.ucalgary.ca/d2l/le/content/497542/viewContent/5812333/View
2. Gomes da Rocha, Claudia *In Class Demo: ex_12b_template-wsimpson* - Retreived from https://d2l.ucalgary.ca/d2l/le/content/497542/viewContent/5670807/View
3. Gomes da Rocha, Claudia *In Class Demo: demo_DFT_aliasing* - Retreived from https://d2l.ucalgary.ca/d2l/le/content/497542/viewContent/5670717/View

Other (0.5 points):

This page is to be filled by the instructor or TAs ONLY.

Remaining 0.5 points are granted for following the template and overall quality of the report.

Was the assignment submitted before the due date (mark one option)? YES   NO